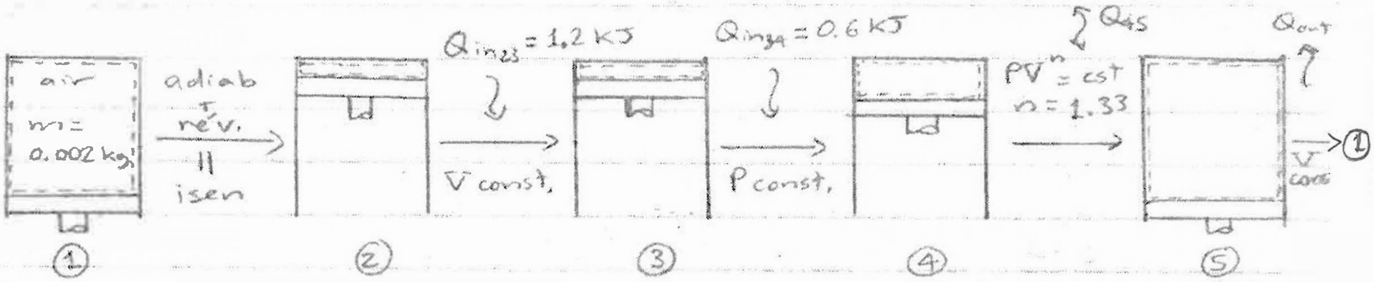
MEC1210 Automne 2024, TD5, Groupe 1: Problème à faire en classe (solutionnaire)



① $P_1 = 100$ kPa, $T_1 = 300$ K

② $V_2 = \dfrac{V_1}{20}$ $(s_2 = s_1)$

③ $(V_3 = V_2)$

④ $(P_4 = P_3)$

⑤ $(V_5 = V_1)$

air → gaz parfait avec C_p, C_v constant : $C_p = 1.005 \ \dfrac{kJ}{kg \cdot K}$, $R = 0.287 \ \dfrac{kJ}{kg \cdot K}$

Suppositions additionnelle
- Δe_c, Δe_p de l'air $\simeq 0$

a) diagramme P-V

b) P, T & V aux états ②, ③, ④, ⑤

c) $W_{12, 23, 34, 45, 51} = ?$

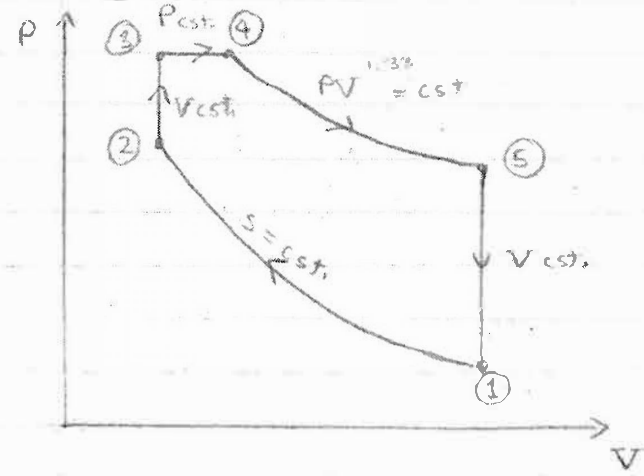
d) $Q_{12, 23, 34, 45, 51} = ?$

e) $\eta_{cycle} = ?$

f) diagramme T-s

g) refaire (b) avec C_p, C_v variables

a)



b) ① $P_1 = 100$ kPa ; $T_1 = 300$ K

$$V_1 = \frac{mRT_1}{P_1} = \frac{(0.002)(0.287)(300)}{100}$$

$$V_1 = 1.722 \times 10^{-3} \ m^3$$

② $V_2 = \frac{V_1}{20} = \frac{1.722 \times 10^{-3}}{20} = \boxed{8.610 \times 10^{-5}\ m^3}$

① → ② scst : $Pv^k = \text{const} \xrightarrow{m\ \text{const.}} P\bar{V}^k = \text{const.}$

$P_2 \bar{V}_2^{\,k} = P_1 \bar{V}_1^{\,k}$ ⇒ $k = \frac{C_p}{C_v}$ → $C_v = C_p - R$

$P_2 = P_1 \left(\frac{\bar{V}_1}{\bar{V}_2}\right)^k$ $= 1.005 - 0.287$

$= 0.718\ \frac{kJ}{kg \cdot k}$

$k = \frac{1.005}{0.718} = 1.4$

$P_2 = (100)(20)^{1.4} = \boxed{6628.9\ kPa}$

$T_2 = \frac{P_2 \bar{V}_2}{mR} = \frac{(6628.9)(8.61 \times 10^{-5})}{(0.002)(0.287)} = \boxed{994.3\ k}$

③ $V_3 = V_2 = \boxed{8.61 \times 10^{-5}\ m^3}$

$T_3 = ?$: bilan d'énergie (1ère loi) sur air entre ② & ③

$\Delta E_{sys_{23}} = E_{in_{23}} - E_{out_{23}}$

$m(u_3 - u_2) + \cancel{\Delta e_{c23}}^{\,0} + \cancel{\Delta e_{p23}}^{\,0} = Q_{in_{23}}$

$mC_v(T_3 - T_2) = Q_{in23}$

$T_3 = T_2 + \frac{Q_{in23}}{mC_v} =$

$T_3 = 994.3 + \frac{(1.2)}{(0.002)(0.718)} = \boxed{1830.0\ K}$

$P_3 = \frac{mRT_3}{V_3} = \frac{(0.002)(0.287)(1830.0)}{(8.61 \times 10^{-5}\ m^3)} = \boxed{12200.0\ kPa}$

④ $P_4 = P_3 = \boxed{12200.0\ kPa}$

$T_4 = ?$: bilan d'énergie (1ère loi) sur air entre ③ & ④

$\Delta E_{sys_{34}} = E_{in_{34}} - E_{out_{34}}$

$m(u_4 - u_3) + \cancel{\Delta e_{c34}}^{\,0} + \cancel{\Delta e_{p34}}^{\,0} = Q_{in_{34}} - \int_{③}^{④} P\,dV$

$m(u_4 - u_3) = Q_{in_{34}} - P_3(\bar{V}_4 - \bar{V}_3)$ → $P_3 = P_4$ (P const.)

$= Q_{in_{34}} - (P_4 \bar{V}_4 - P_3 \bar{V}_3)$

$$m\left[(u_4 + P_4 V_4) - (u_3 + P_3 V_3)\right] = Q_{in_{34}}$$

$$m(h_4 - h_3) = Q_{in_{34}}$$

$$m C_p (T_4 - T_3) = Q_{in_{34}}$$

$$T_4 = T_3 + \frac{Q_{in34}}{m C_p}$$

$$T_4 = 1830.0 + \frac{(0.6)}{(0.002)(1.005)} = \boxed{2128.5\ K}$$

$$V_4 = \frac{m R T_4}{P_4} = \frac{(0.002)(0.287)(2128.5)}{(12200.0)} = \boxed{1.001 \times 10^{-4}\ m^3}$$

⑤ $V_5 = V_1 = \boxed{1.722 \times 10^{-3}\ m^2}$

$P_5 = ?$: $P_5 V_5^{\,n} = P_4 V_4^{\,n}$ $\quad (n = 1.33)$

$$P_5 = P_4 \left(\frac{V_4}{V_5}\right)^n = (12200.0)\left(\frac{1.001 \times 10^{-4}}{1.722 \times 10^{-3}}\right)^{1.33}$$

$$\boxed{P_5 = 277.34\ KPa}$$

$$T_5 = \frac{P_5 V_5}{m R} = \frac{(277.34)(1.722 \times 10^{-3})}{(0.002)(0.287)} = \boxed{832.0\ K}$$

c) i) $W_{12_{\text{par air}}} = ?$: bilan d'énergie (1ère loi) sur air entre ① & ②

$$\Delta E_{sys_{12}} = E_{in_{12}} - E_{out_{12}}$$

$$m(u_2 - u_1) + \cancel{\Delta e_{c12}}^{\,0} + \cancel{\Delta e_{p12}}^{\,0} = -W_{12_{\text{par air}}}$$

$$W_{12_{\text{par air}}} = m(u_1 - u_2)$$

$$= m c_v (T_1 - T_2)$$

$$= (0.002)(0.718)(300 - 994.3)$$

$$\boxed{W_{12_{\text{par air}}} = -0.997\ kJ}$$ (travail fait sur air)

<u>Alternatif:</u> $W_{12_{\text{par sys}}} = \int_{①}^{②} P\,dV \overset{PV^k = cst}{=} \frac{P_2 V_2 - P_1 V_1}{1 - k} = 0.996\ kJ$

ii) $W_{23_{\text{par air}}} = ?$: $W_{23_{\text{par air}}} = \int_{②}^{③} P\,\cancel{dV}^{\,0\ (V\ \text{const})} = \boxed{0}$

iii) $W_{34\,\text{par air}} = ?$: $W_{34\,\text{par air}} = \int_{③}^{④} P\,dV \overset{P_{const}}{=} P_3(V_4 - V_3)$

$= (12200.0)(1.001 \times 10^{-4} - 8.61 \times 10^{-5})$

$\boxed{W_{34\,\text{par air}} = 0.171 \text{ kJ}}$ (travail fait par air)

Alternatif: bilan d'énergie (1ère loi) sur air entre ③ & ④

$\Delta E_{sys_{34}} = E_{in_{34}} - E_{out_{34}}$

$m(u_4 - u_3) + \cancel{\Delta e_{c_{34}}}^{0} + \cancel{\Delta e_{p_{34}}}^{0} = Q_{in_{34}} - W_{34\,\text{par air}}$

$W_{34\,\text{par air}} = m(u_3 - u_4) + Q_{in_{34}}$

$= m C_v (T_3 - T_4) + Q_{in_{34}}$

$= (0.002)(0.718)(1830.0 - 2128.5) + 0.6$

$W_{34\,\text{par air}} = 0.171 \text{ kJ}$

iv) $W_{45\,\text{par air}} = ?$: $W_{45\,\text{par air}} = \int_{④}^{⑤} P\,dV \overset{PV^n = cst}{=} \dfrac{P_5 V_5 - P_4 V_4}{1-n}$

$= \dfrac{(277.34)(1.722 \times 10^{-3}) - (12200)(1.001 \times 10^{-4})}{1 - 1.33}$

$\boxed{W_{45\,\text{par air}} = 2.253 \text{ kJ}}$ (travail fait par air)

Note: On ne peut pas obtenir W_{45} par bilan d'énergie car on ne connaît pas Q_{45}

v) $W_{51\,\text{par air}} = ?$: $W_{51\,\text{par air}} = \int_{⑤}^{①} P\,\cancel{dV}^{\,0\ (V_{const.})} = \boxed{0}$

d) i) $Q_{in_{12}} = ?$: $\boxed{Q_{in_{12}} = 0}$ (adiabatique)

ii) $Q_{in_{23}} = 1.2 \text{ kJ}$ (donné)

iii) $Q_{in_{34}} = 0.6 \text{ kJ}$ (donné)

iv) $Q_{in_{45}} = ?$: bilan d'énergie (1ère loi) sur air pour ④ à ⑤

$$\Delta E_{sys_{45}} = E_{in_{45}} - E_{out_{45}}$$

$$m(u_5 - u_4) + \cancel{\Delta e_{c45}}^{0} + \cancel{\Delta e_{p45}}^{0} = Q_{in_{45}} - W_{45\,\text{par air}}$$

$$Q_{in_{45}} = m(u_5 - u_4) + W_{45\,\text{par air}}$$

$$= m c_v (T_5 - T_4) + W_{45\,\text{par air}}$$

$$= (0.002)(0.718)(832.0 - 2128.5) + 2.253$$

$$\boxed{Q_{in_{45}} = 0.391 \text{ kJ}} \quad (Q \text{ à l'air})$$

v) $Q_{in_{51}} = ?$: bilan d'énergie ($1^{ère}$ loi) sur air pour ⑤ à ①

$$\Delta E_{sys_{51}} = E_{in_{51}} - E_{out_{51}}$$

$$m(u_1 - u_5) + \cancel{\Delta e_{c51}}^{0} + \cancel{\Delta e_{p51}}^{0} = Q_{in_{51}} - \cancel{W_{51\,\text{par air}}}^{0}$$

$$Q_{in_{51}} = m(u_1 - u_5)$$

$$= m c_v (T_1 - T_5)$$

$$= (0.002)(0.718)(300 - 832.0)$$

$$\boxed{Q_{in_{51}} = -0.764 \text{ kJ}} \quad (Q \text{ par air})$$

e) $$\eta_{cycle} = \frac{W_{net}}{Q_{in}} = \frac{(W_{12} + W_{23} + W_{34} + W_{45} + W_{51})_{\text{par air}}}{Q_{in_{23}} + Q_{in_{34}} + Q_{in_{45}}}$$

$$= \frac{[(-0.997) + (0) + (0.171) + (2.253) + (0)]}{(1.2) + (0.6) + (0.391)}$$

$$\boxed{\eta_{cycle} = 0.651}$$

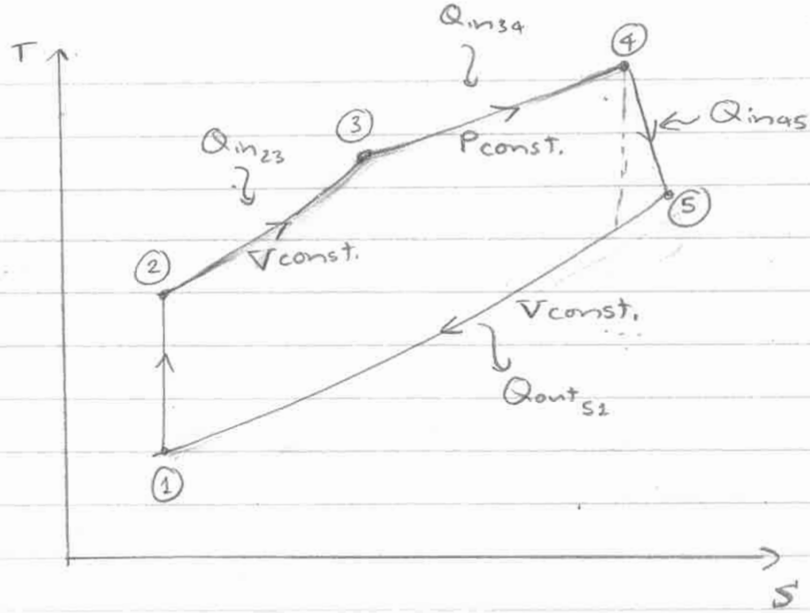
Alternatif : $$\eta_{cycle} = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{Q_{out_{51}}}{Q_{in_{23}} + Q_{in_{34}} + Q_{in_{45}}} \quad \rightarrow Q_{out_{51}} = -Q_{in_{51}} = 0.764 \text{ kJ}$$

$$= 1 - \frac{(0.764)}{(1.2) + (0.6) + (0.391)}$$

$$\eta_{cycle} = 0.651$$

f)



Notes: i) Comment dessiner une évolution avec V constant avec une avec P constante :

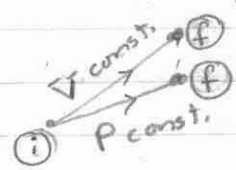
$$\underline{P\ const}: \quad s_f - s_i = C_p \ln \frac{T_f}{T_i} - R \ln \underbrace{\cancel{\frac{P_f}{P_i}}}_{=0}$$

$$T_f = T_i\, e^{\Delta s / C_p}$$

$$\underline{V\ const}: \quad s_f - s_i = C_v \ln \frac{T_f}{T_i} + R \ln \underbrace{\cancel{\frac{V_f}{V_i}}}_{=0}$$

$$T_f = T_i\, e^{\Delta s / C_v}$$

Vu que $C_p > C_v \rightarrow \frac{\Delta s}{C_p} < \frac{\Delta s}{C_v}$ pente de T Vs s plus grande pour V const



ii) évolution 4 → 5 : $S_5 - S_4 = C_p \ln \frac{T_5}{T_4} - R \ln \frac{P_5}{P_4}$

$$S_5 - S_4 = (1.005) \ln \left(\frac{832.0}{2128.5}\right) - (0.287) \ln \left(\frac{277.34}{12200}\right)$$

$$S_5 - S_4 = 0.142 \text{ kJ/kg·K} > 0 \rightarrow S_5 > S_4$$

$$S_5 > S_4$$

g) c_p, c_v variables

① même P_1, T_1, V_1 qu'en (b) ; $T_1 = 300K \xrightarrow{A\text{-}17} V_{r1} = 621.2$

② $V_2 = \frac{V_1}{20}$ même qu'en (b)

$T_2 = ?$: ① → ② isen : $\frac{V_{r2}}{V_{r1}} = \frac{v_2}{v_1} = \frac{V_2}{V_1}$

$$V_{r2} = V_{r1}\left(\frac{V_2}{V_1}\right) = 621.2\left(\frac{1}{20}\right) = 31.06$$

$$\left(u_2 = 700.88\ \frac{kJ}{kg}\right) \quad \boxed{T_2 = 931.4\ K} \leftarrow A\text{-}17$$

$$P_2 = \frac{mRT_2}{V_2} = \frac{(0.002)(0.287)(931.4)}{8.610\times10^{-5}} = \boxed{6209.3\ kPa}$$

③ $V_3 = V_2$ même qu'en (b)

$T_3 = ?$: 1ère loi sur air, comme en (b), donne :

$$m(u_3 - u_2) = Q_{in_{23}}$$

$$\therefore u_3 = u_2 + \frac{Q_{in_{23}}}{m} = 1300.88\ \frac{kJ}{kg}$$

$$\boxed{T_3 = 1602.8\ K} \leftarrow A\text{-}17$$

$$(h_3 = 1760.95\ kJ/kg)$$

$$P_3 = \frac{mRT_3}{V_3} = \frac{(0.002)(0.287)(1602.8)}{(8.61\times10^{-5})} = \boxed{10\,685.3\ kPa}$$

④ $P_4 = P_3 = 10685.3\ kPa$

$T_4 = ?$: 1ère loi sur air, comme en (b), donne :

$$m(h_4 - h_3) = Q_{in_{34}}$$

$$h_4 = h_3 + \frac{Q_{in_{34}}}{m} = 1760.95 + \frac{0.6}{0.002} = 2060.95\ \frac{kJ}{kg}$$

$$\boxed{T_4 = 1846.5\ K} \leftarrow A\text{-}17$$

$$V_4 = \frac{mRT_4}{P_4} = \frac{(0.002)(0.287)(1846.5)}{10685.3} = \boxed{9.919 \times 10^{-5}\ m^3}$$

⑤ $V_5 = V_1$ même qu'en (b)

$P_5 = ?$: $P_5 V_5^{\,n} = P_4 V_4^{\,n}$ ...

$$P_5 = P_4 \left(\frac{V_4}{V_5}\right)^n = (10685.3)\left(\frac{9.919 \times 10^{-5}}{1,722 \times 10^{-3}}\right)^{1,33}$$

$$\boxed{P_5 = 239.97\ kPa}$$

$$T_5 = \frac{P_5 V_5}{mR} = \frac{(239.97)(1.722 \times 10^{-3})}{(0.002)(0.287)} = \boxed{719.91\ k}$$

Note: pour l'application de la 1ère loi dans les partres (c) & (d), il faut obtenir et utiliser u ou h dans la table A-17 correspondant aux états concernés (sans utiliser c_v, c_p)