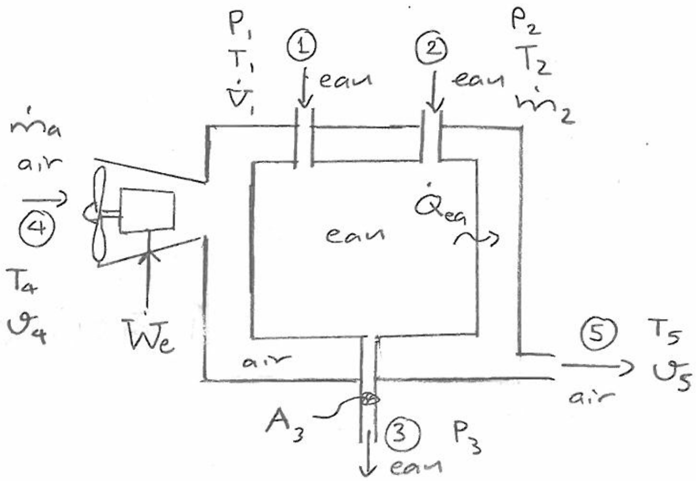
MEC1210 Automne 2024, TD3: Problème à faire en classe (solutionnaire)



eau: $P_1 = 1\ MPa$ $\quad P_2 = 1\ MPa$

$T_1 = 300°C$ $\quad T_2 = 100°C$

$\dot{V}_1 = 1\ \frac{m^3}{s}$ $\quad \dot{m}_2 = 1\ \frac{kg}{s}$

$P_3 = 1\ MPa,\ A_3 = 0.1\ m^2$

air: $\dot{m}_a = 30\ kg/s$

$T_4 = 20°C$ $\quad T_5 = 60°C$

$\vartheta_4 = 3\ m/s$ $\quad \vartheta_5 = 7\ m/s$

- régime permanent
- air → gaz parfait à C_p, C_v const.
  $R = 0.287\ \frac{kPa \cdot m^3}{kg \cdot K}$, $c_p = 1.005\ \frac{kJ}{kg \cdot K}$
- $\dot{Q}_{ext} \simeq 0$
- $\Delta e_{p,air}, \Delta e_{p,eau}, \Delta e_{c,eau} \simeq 0$

a) $T_3, x_3 = ?$

b) $\dot{Q}_{ea} = ?$

c) $\vartheta_3 = ?$

Suppositions additionnelles : aucune

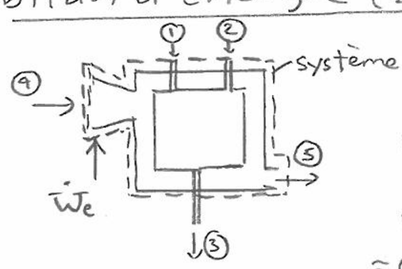
a) T_3, x_3 : on a P_3, il faut trouver h_3 pour définir l'état ③

i) Conservation de la masse : $\sum \dot{m}_{in} = \sum \dot{m}_{out}$ (régime perm.)

eau: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

air: $\dot{m}_4 = \dot{m}_5 = \dot{m}_a$

ii) bilan d'énergie (1ère loi) : $\dot{E}_{in} = \dot{E}_{out}$ (régime perm.)



$$\dot{m}_1\theta_1 + \dot{m}_2\theta_2 + \dot{m}_4\theta_4 + \dot{W}_e = \dot{m}_3\theta_3 + \dot{m}_5\theta_5$$

$$\dot{m}_1\theta_1 + \dot{m}_2\theta_2 + \dot{m}_a\theta_4 + \dot{W}_e = (\dot{m}_1 + \dot{m}_2)\theta_3 + \dot{m}_a\theta_5$$

$$\dot{m}_1(\theta_3 - \theta_1) + \dot{m}_2(\theta_3 - \theta_2) = \dot{m}_a(\theta_4 - \theta_5) + \dot{W}_e$$

$$\dot{m}_1(h_3 - h_1 + \cancel{\Delta e_{c13}}^{\simeq 0} + \cancel{\Delta e_{p13}}^{\simeq 0}) + \dot{m}_2(h_3 - h_2 + \cancel{\Delta e_{c23}}^{\simeq 0} + \cancel{\Delta e_{p23}}^{\simeq 0})$$

$$= \dot{m}_a\left(h_4 - h_5 + \frac{\vartheta_4^2 - \vartheta_5^2}{2} + \cancel{\Delta e_{p54}}^{\simeq 0}\right) + \dot{W}_e$$

$$(\dot{m}_1 + \dot{m}_2)h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_a\left(h_5 - h_4 + \frac{\vartheta_5^2 - \vartheta_4^2}{2}\right) + \dot{W}_e$$

$$\Rightarrow h_5 - h_4 = c_p(T_5 - T_4) \quad \text{(gaz parfait)}$$

Note: $\Delta h = c_p \Delta T$ mais $h \neq c_p T$ excepté pour un gaz caloriquement parfait

$$h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_a\left[c_p(T_5 - T_4) + \frac{v_5^2 - v_4^2}{2}\right] + \dot{W}_e}{\dot{m}_1 + \dot{m}_2}$$

$\Rightarrow h_1 = ?$: $P_1 = 1000$ kPa, $T_1 = 300°C$ } $T_1 > T_{sat@1000kPa} \overset{A-5}{=} 179.88°C \rightarrow$ vapeur surchauffée

Table A-6 : $h_1 = 3051.6 \frac{kJ}{kg}$

$(v_1 = 0.25799\ m^3/kg)$

$\Rightarrow h_2 = ?$: $P_2 = 1000$ kPa, $T_2 = 100°C$ } $T_1 < T_{sat@1000kPa} \overset{A-5}{=} 179.88 \rightarrow$ liquide comprimé

$h_2 \simeq h_{f@100°C} \overset{A-4}{=} 419.17$ kJ/kg

$$\Rightarrow \dot{m}_1 = \frac{\dot{V}_1}{v_1} = \frac{1\ m^3/s}{0.25799\ m^3/kg} = 3.876\ kg/s$$

$$h_3 = \frac{(3.876 \frac{kg}{s})(3051.6 \frac{kJ}{kg}) + (1 \frac{kg}{s})(419.17 \frac{kJ}{kg}) - (30 \frac{kg}{s})\left[(1.005 \frac{kJ}{kg \cdot K})(60-20)°C \cdot \frac{K}{°C} + \frac{(7^2 - 3^2)}{2} \frac{m^2}{s^2} \cdot \frac{1 kJ/kg}{10^3 \frac{m^2}{s^2}}\right] + 10.6 \frac{kJ}{s}}{(3.876 + 1)\ kg/s}$$

$h_3 = 2266.44$ kJ/kg

L'état ③ est donc défini par $P_3 = 1000$ kPa & $h_3 = 2266.44 \frac{kJ}{kg}$

Table A-5 : Pour $P_{sat} = 1000$ kPa, $T_{sat@1000kPa} = 179.88°C$

$h_f = 762.51 \frac{kJ}{kg}$

$h_g = 2777.1 \frac{kJ}{kg}$

$h_{fg} = 2014.6 \frac{kJ}{kg}$

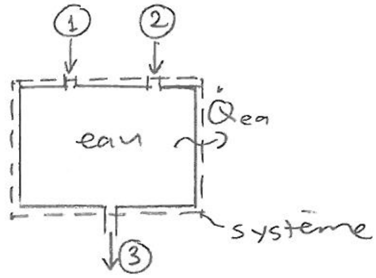
$h_f < h_3 < h_g \longrightarrow$ mélange liquide-vapeur saturé

$T_3 = T_{sat@1000kPa} = \boxed{179.88°C}$

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{2266.44 - 762.51}{2014.6} = \boxed{0.7465}$$

Solution alternative : voir pages 4 et 5

b) $\dot{Q}_{ea} = ?$:



i) <u>conservation de la masse :</u>

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

ii) <u>bilan d'énergie (1ère loi)</u> : $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}_1 \theta_1 + \dot{m}_2 \theta_2 = \dot{m}_3 \theta_3 + \dot{Q}_{ea}$$

$$\dot{m}_1 \theta_1 + \dot{m}_2 \theta_2 = (\dot{m}_1 + \dot{m}_2)\theta_3 + \dot{Q}_{ea}$$

$$\dot{Q}_{ea} = \dot{m}_1(\theta_1 - \theta_3) + \dot{m}_2(\theta_2 - \theta_3)$$

$$= \dot{m}_1 (h_1 - h_3 + \cancelto{\simeq 0}{\Delta e_{p31}} + \cancelto{\simeq 0}{\Delta e_{c31}})$$

$$+ \dot{m}_2 (h_2 - h_3 + \cancelto{\simeq 0}{\Delta e_{p32}} + \cancelto{\simeq 0}{\Delta e_{c32}})$$

$$\dot{Q}_{ea} = \dot{m}_1 (h_1 - h_3) + \dot{m}_2 (h_2 - h_3)$$

$$= (3.876 \ \tfrac{kg}{s})(3051.6 - 2266.44)\ \tfrac{kJ}{kg}$$

$$+ (1 \ \tfrac{kg}{s})(419.17 - 2266.44)\ \tfrac{kJ}{kg}$$

$$\boxed{\dot{Q}_{ea} = 1196.0 \ kW}$$

<u>Solution alternative</u> : voir page 4

c) $\vartheta_3 = ?$: $\dot{m}_3 = \dfrac{\vartheta_3 A_3}{v_3} = \dot{m}_1 + \dot{m}_2$

$$\vartheta_3 = \frac{(\dot{m}_1 + \dot{m}_2)\, v_3}{A_3}$$

$\Rightarrow v_3 = ?$: $\left.\begin{array}{l} P_3 = 1000 \ kPa \\ x_3 = 0.7465 \end{array}\right\}$ Table A-5 : pour $P_{sat} = 1000 \ kPa$

$v_f = 0.001127 \ \frac{m^3}{kg}$

$v_g = 0.19436 \ \frac{m^3}{kg}$

$$v_3 = v_f + x_3(v_g - v_f)$$

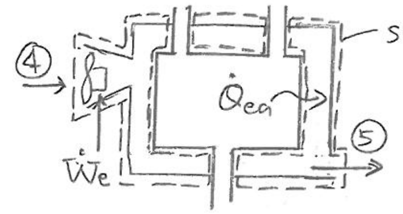
$$= 0.001122 + (0.7465)(0.19436 - 0.001127)$$

$$v_3 = 0.14538 \ m^3/kg$$

$$\vartheta_3 = \frac{(3.876 + 1)\ kg/s\ (0.14538\ m^3/kg)}{(0.1\ m^2)} = \boxed{7.089 \ \frac{m}{s}}$$

Solution alternative pour parties (a) & (b)

b) $\dot{Q}_{ea} = ?$:



système = chemise de refroidissement (incluant ventilateur)

i) conservation de la masse

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_4 = \dot{m}_5 = \dot{m}_a$$

ii) bilan d'énergie (1ère loi) : $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}_4\theta_4 + \dot{W}_e + \dot{Q}_{ea} = \dot{m}_5\theta_5$$

$$\dot{Q}_{ea} = \dot{m}_a(\theta_5 - \theta_4) - \dot{W}_e$$

$$= \dot{m}_a\left(h_5 - h_4 + \frac{\mathcal{V}_5^2 - \mathcal{V}_4^2}{2} + \cancel{\Delta e_{p45}}^{\simeq 0}\right) - \dot{W}_e$$

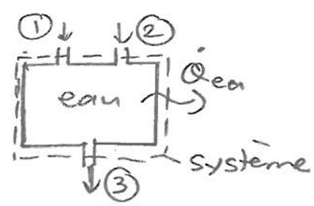
$$= \dot{m}_a\left[C_p(T_5 - T_4) + \frac{\mathcal{V}_5^2 - \mathcal{V}_4^2}{2}\right] - \dot{W}_e$$

$$= \left(30\,\frac{kg}{s}\right)\left[\left(1.005\,\frac{kJ}{kg \cdot K}\right)(60-20)°C \cdot \frac{K}{°C} + \frac{(7^2 - 3^2)}{2}\,\frac{m^2}{s^2} \times \frac{1\,kJ}{10^3\,\frac{m^2}{s^2}}\right]$$

$$- 10.6\,kW$$

$$\boxed{\dot{Q}_{ea} = 1196.0\,kW}$$

a) $T_3, x_3 = ?$: On utilise la réponse de (b) pour trouver h_3



i) conservation de la masse

$$\Sigma\dot{m}_{in} = \Sigma\dot{m}_{out}$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

ii) bilan d'énergie (1ère loi) : $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}_1\theta_1 + \dot{m}_2\theta_2 = \dot{m}_3\theta_3 + \dot{Q}_{ea}$$

$$\dot{m}_1\theta_1 + \dot{m}_2\theta_2 = (\dot{m}_1 + \dot{m}_2)\theta_3 + \dot{Q}_{ea}$$

$$\dot{m}_1(\theta_3 - \theta_1) + \dot{m}_2(\theta_3 - \theta_2) = -\dot{Q}_{ea}$$

$$\dot{m}_1(h_3 - h_1 + \cancel{\Delta e_{c13}}^{\simeq 0} + \cancel{\Delta e_{p13}}^{\simeq 0})$$

$$+ \dot{m}_2(h_3 - h_2 + \cancel{\Delta e_{c23}}^{\simeq 0} + \cancel{\Delta e_{p23}}^{\simeq 0}) = -\dot{Q}_{ea}$$

$$(\dot{m}_1 + \dot{m}_2)h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{Q}_{ea}$$

$$h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{Q}_{ea}}{\dot{m}_1 + \dot{m}_2}$$

$$= \frac{(3.876\ \frac{kg}{s})(3051,6\ \frac{kJ}{kg}) + (1\ \frac{kg}{s})(419.17\ \frac{kJ}{kg}) - (1196,0\ \frac{k}{s}}{(3.876 + 1)\ \frac{kg}{s}}$$

$$h_3 = 2266,44\ kJ/kg$$

$P_3 = 1000\ kPa$

$h_3 = 2266,44\ \frac{kJ}{kg}$ } mélange liquide-vapeur saturé

$$T_3 = T_{sat@1000\,kPa} = \boxed{179,88°C}$$

$$x_3 = \frac{h_3 - h_{f@1000kPa}}{(h_g - h_f)_{@1000kPa}} = \boxed{0,7465}$$