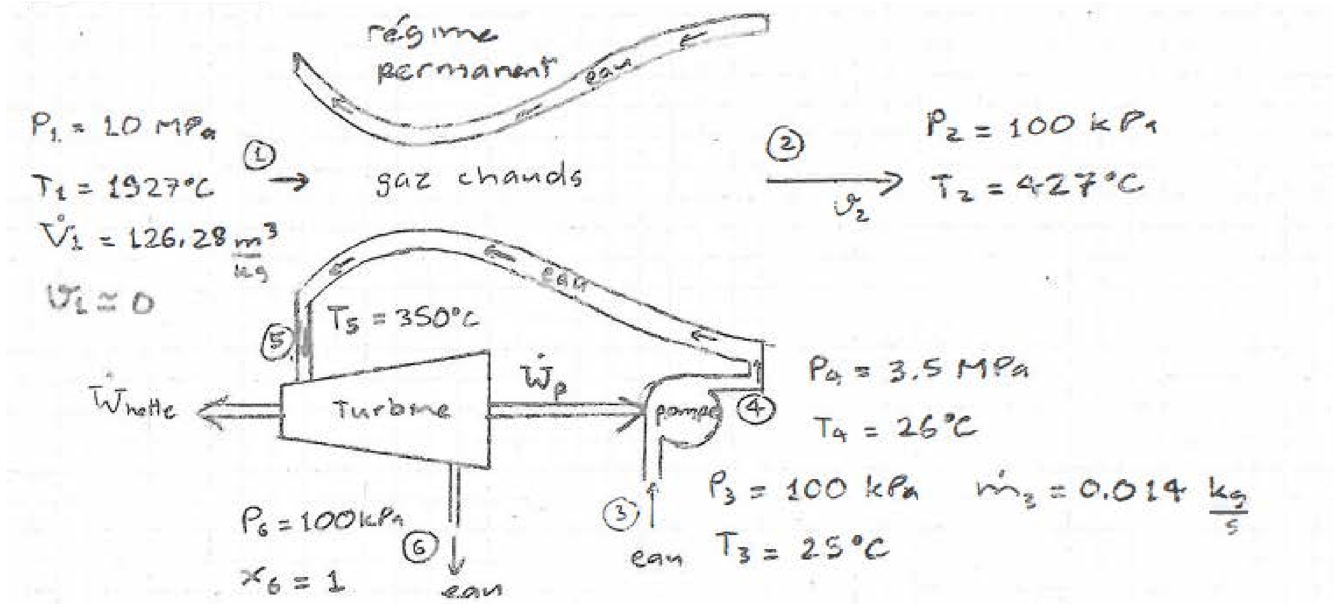
MEC1210 Automne 2024, TD3: Problème à faire à la maison (solutionnaire)



a) $\dot{W}_p = ?$

b) $\dot{W}_{nette} = ?$

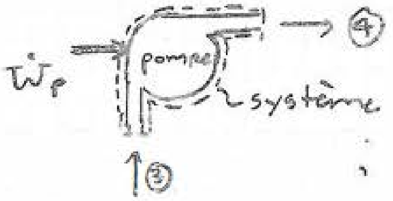
c) $\mathcal{V}_2 = ?$

d) $\eta = \frac{\dot{W}_{nette}}{\dot{Q}_{eau}} = ?$

- gaz chauds → air comme gaz parfait avec C_p, C_v variables $R = 0,287 \frac{kPa \cdot m^3}{kg \cdot K}$
- $\dot{Q}_{ext} \simeq 0$
- $\Delta P_{chemise} \simeq 0 \quad (P_5 = P_4)$
- $\Delta E_{c\,eau} \simeq 0$, $\Delta E_{p\,eau,\,gaz\,chauds} \simeq 0$

<u>Suppositions additionnelles</u> : aucune

a) $\dot{W}_p = ?$:



<u>conservation de la masse</u>

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_e$$

<u>bilan d'énergie (1ère loi)</u>: $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}_3 \theta_3 + \dot{W}_p = \dot{m}_4 \theta_4$$

$$\dot{W}_p = \dot{m}_e(\theta_4 - \theta_3)$$

$$= \dot{m}_e(h_4 - h_3 + \cancel{\Delta e_{c\,eau_{34}}}^{\,0} + \cancel{\Delta e_{p\,eau_{34}}}^{\,0})$$

$$\dot{W}_p = \dot{m}_e(h_4 - h_3)$$

$\Rightarrow h_3 = ?$: $\left.\begin{matrix} P_3 = 100\,kPa \\ T_3 = 25°C \end{matrix}\right\}$ $T_3 < T_{sat@100kPa} \overset{A-5}{=} 99,61°C$ ③ liquide comprimé

$$h_3 \simeq h_{f@25°C} \overset{\text{Table A-4}}{=} 104.83 \text{ kJ/kg}$$

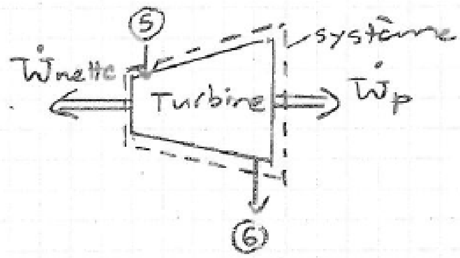
$\Rightarrow h_4 = ?$: $\left.\begin{array}{l} P_4 = 3500 \text{ kPa} \\ T_4 = 26°C \end{array}\right\}$ $T_4 < T_{sat@3500kPa} = 242{,}56°C$ ④ liquide comprimé

$$h_4 \simeq h_{f@26°C} = 109{,}012 \ \frac{kJ}{kg}$$

(table A-4, interpol.)

$$\dot{W}_p = \left(0.014 \frac{kg}{s}\right)(109.012 - 104.83) \frac{kJ}{kg} = \boxed{0.058548 \text{ kW}}$$

b) $\dot{W}_{nette} = ?$:



conservation de la masse

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_6 = \dot{m}_5 \left(= \dot{m}_4 = \dot{m}_3 = \dot{m}_e\right)$$

conservation de mass sur chemise

Bilan d'énergie (1ère loi) : $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}_5 \theta_5 = \dot{m}_6 \theta_6 + \dot{W}_p + \dot{W}_{nette}$$

$$\dot{W}_{nette} = \dot{m}_e(\theta_5 - \theta_6) - \dot{W}_p$$

$$= \dot{m}_e\left(h_5 - h_6 + \cancel{\Delta e_{cin\,65}}^{\to 0} + \cancel{\Delta e_{pot\,65}}^{\to 0}\right) - \dot{W}_p$$

$$\dot{W}_{nette} = \dot{m}_e(h_5 - h_6) - \dot{W}_p$$

$\Rightarrow h_5 = ?$: $\left.\begin{array}{l} P_5 = P_4 = 3.5 \text{ MPa} \\ T_5 = 350°C \end{array}\right\}$ $T_5 > T_{sat@3500kPa}$ ⑤ vapeur surch.

Table A-6 : $h_5 = 3104.9$ kJ/kg

$\Rightarrow h_6 = ?$: $\left.\begin{array}{l} P_6 = 100 \text{ kPa} \\ x_6 = 1 \end{array}\right\}$ $h_6 = h_{g@100kPa} \overset{A-5}{=} 2675.0 \frac{kJ}{kg}$

$$\dot{W}_{nette} = \left(0.014 \frac{kg}{s}\right)(3104.9 - 2675.0) \frac{kJ}{kg} - 0.058548 \frac{kJ}{s}$$

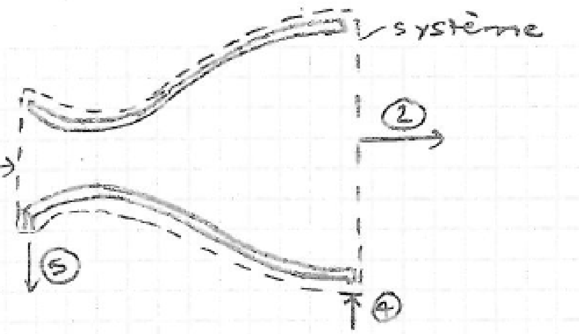
$\boxed{\dot{W}_{nette} = 5.960 \text{ kW}}$ (solution alternative : p. 4)

c) $V_2 = ?$: conservation de la masse : $\dot{m}_{in} = \dot{m}_{out}$

système = chemise : $\dot{m}_5 = \dot{m}_4 (= \dot{m}_3 = \dot{m}_e)$

système = tuyère : $\dot{m}_2 = \dot{m}_1$

bilan d'énergie (1ère loi) :



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1\theta_1 + \dot{m}_4\theta_4 = \dot{m}_2\theta_2 + \dot{m}_5\theta_5$$

$$\dot{m}_1(\theta_2-\theta_1) = \dot{m}_e(\theta_4-\theta_5)$$

$$\dot{m}_1\left(h_2-h_1 + \frac{v_2^2-\cancel{v_1^2}^{\,?\approx 0}}{2} + \cancel{\Delta e_{p\,gaz\,12}}^{\,0}\right) = \dot{m}_e\left(h_4-h_5 + \cancel{\Delta e_{c\,eau\,54}}^{\,0} + \cancel{\Delta e_{p\,eau\,54}}^{\,0}\right)$$

$$\dot{m}_1\frac{v_2^2}{2} = \dot{m}_1(h_1-h_2) - \dot{m}_e(h_5-h_4)$$

$$v_2 = \sqrt{2\left[(h_1-h_2) - \frac{\dot{m}_e}{\dot{m}_1}(h_5-h_4)\right]}$$

$$\Rightarrow \dot{m}_1 = \frac{\dot{V}_1}{v_1} = \frac{\dot{V}_1}{RT_1/P_1} = \frac{P_1\dot{V}_1}{RT_1} = \frac{(10\,000\ kPa)(126.28\ \frac{m^3}{kg})}{\left(0.287\ \frac{kPa\cdot m^3}{kg\cdot K}\right)(1927+273)K}$$

$$\dot{m}_1 = 2000\ kg/s$$

$\Rightarrow h_1, h_2 = ?$: Table A-17 :

$$h_1 = h(T_1 = (1927+273)K = 2200K) = 2503.2\ \frac{kJ}{kg}$$

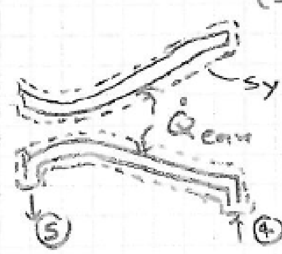
$$h_2 = h(T_2 = (427+273)K = 700K) = 713{,}27\ \frac{kJ}{kg}$$

$$v_2 = \sqrt{2\left[(2503.2-713.27)\frac{kJ}{kg} - \frac{(0.014\ kg/s)}{(2000\ kg/s)}(3104.9-105.012)\frac{kJ}{kg}\right]\times\frac{10^3\ m^2/s^2}{kJ/kg}}$$

$$\boxed{v_2 = 1892.04\ m/s}$$

(sol'n alternative: p.4)

d) $\eta = \dfrac{\dot{W}_{nette}}{\dot{Q}_{eau}}$ $\Rightarrow \dot{Q}_{eau} = ?$:



système = chemise

cons. de la masse

$$\dot{m}_4 = \dot{m}_5\ (= \dot{m}_e)$$

1ère loi ($\dot{E}_{in} = \dot{E}_{out}$) : $\dot{m}_4\theta_4 + \dot{Q}_{eau} = \dot{m}_5\theta_5$

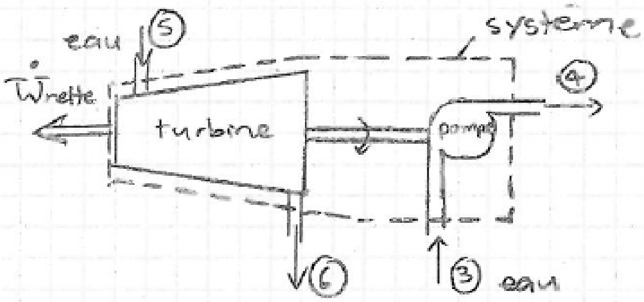
$$\dot{Q}_{eau} = \dot{m}_e(\theta_5-\theta_4) = \dot{m}_e\left(h_5-h_4 + \cancel{\Delta e_{c\,eau\,45}}^{\,0} + \cancel{\Delta e_{p\,eau\,45}}^{\,0}\right)$$

$$\dot{Q}_{eau} = \dot{m}_e(h_5-h_4)$$

$$\eta = \frac{\dot{W}_{nette}}{\dot{m}_e(h_5-h_4)} = \frac{(5{,}560)\ kJ/s}{(0.014\ \frac{kg}{s})(3104.9-105.012)\frac{kJ}{kg}} = \boxed{0.1421}$$

Solutions alternatives

partie b)



Conservation de masse

$$\dot{m}_{in} = \dot{m}_{out}$$

$$(\dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_6 = \dot{m}_e)$$

Bilan d'énergie (1ère loi) : $\dot{E}_{in} = \dot{E}_{out}$

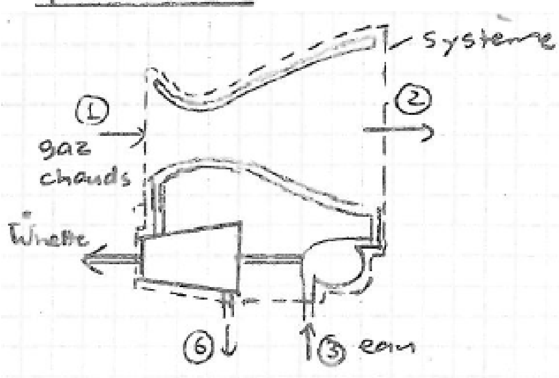
$$\dot{m}_3\theta_3 + \dot{m}_5\theta_5 = \dot{m}_4\theta_4 + \dot{m}_6\theta_6 + \dot{W}_{nette}$$

$$\dot{W}_{nette} = \dot{m}_e(\theta_5 - \theta_6) - \dot{m}_e(\theta_4 - \theta_3)$$

$$\dot{W}_{nette} = \dot{m}_e(h_5 - h_6) - \underbrace{\dot{m}_e(h_4 - h_3)}_{=\dot{W}_{pompe}}$$

$\Rightarrow$ même éqn. que sur p. 2

partie c)



Conservation de masse: $\dot{m}_{in} = \dot{m}_{out}$

système = chemise + turbine-pompe :

$$\dot{m}_6 = \dot{m}_3 = \dot{m}_e$$

système = tuyère ; $\dot{m}_2 = \dot{m}_1$

Bilan d'énergie (1ère loi) : $\dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}_1\theta_1 + \dot{m}_3\theta_3 = \dot{m}_2\theta_2 + \dot{m}_6\theta_6 + \dot{W}_{nette}$$

$$\dot{m}_1(\theta_2 - \theta_1) = \dot{m}_e(\theta_3 - \theta_6) - \dot{W}_{nette}$$

$$\dot{m}_1\left(h_2 - h_1 + \frac{v_2^2 - v_1^2}{2} + \cancelto{0}{\Delta e_{p_{12}}}\right) = \dot{m}_e\left(h_3 - h_6 + \cancelto{0}{\Delta e_{c_{63}}} + \cancelto{0}{\Delta e_{p_{63}}}\right) - \dot{W}_{nette}$$

$$\frac{1}{2}\dot{m}_1 v_2^2 = \dot{m}_1(h_1 - h_2) - \dot{m}_e(h_6 - h_3) - \dot{W}_{nette}$$

$$v_2 = \sqrt{2\left[(h_1 - h_2) - \frac{\dot{m}_e}{\dot{m}_1}(h_6 - h_3) - \frac{\dot{W}_{nette}}{\dot{m}_1}\right]}$$

$\Rightarrow \dot{m}_1 = \frac{\dot{V}_1}{v_1} = \frac{P_1 \dot{V}_1}{RT_1} = (\text{calcul}) = 2000\ \text{kg/s}$

$\Rightarrow h_1 = 2503.2$ $\frac{kJ}{kg}$ (Table A-17)
$h_2 = 713.27$

$\Rightarrow h_3 \simeq 104.83$ kJ/kg (partie a)

$\Rightarrow h_6 = 2675,0$ kJ/kg (partie b)

$\Rightarrow \dot{W}_{nette} = 5.960$ kW (partie b)

$$v_2 = \sqrt{2\left[(2503,2 - 713,27)\frac{kJ}{kg} - \frac{0.014\ kg/s}{2000\ kg/s}(2675.0 - 104,83)\frac{kJ}{kg} - \frac{5.960\ \frac{kJ}{s}}{2000\ kg/s}\right] \times \frac{10^3\ m^2/s^2}{1\ kJ/kg}}$$

$v_2 = 1892.04$ m/s