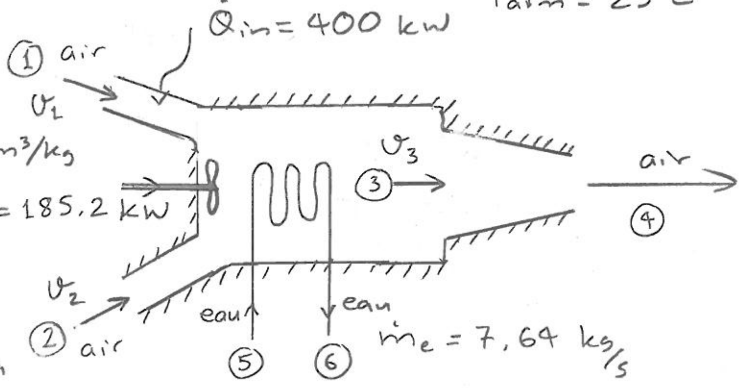
MEC1210 Automne 2024, TD4, Groupe 1: Problème à faire en classe (solutionnaire)

$\dot{Q}_{in} = 400$ kW $\quad T_{atm} = 25°C$

$P_1 = 133.5$ kPa
$T_1 = -3°C$
$\dot{V}_1 = 55.5607\ m^3/kg$
$\dot{W}_h = 185.2$ kW



$P_4 = 100$ kPa
$T_4 = 82°C$

$\dot{m}_e = 7,64$ kg/s

$P_2 = 133.5$ kPa
$T_2 = 25°C$
$\dot{m}_2 = 101,69$ kg/s

$P_5 = 1$ MPa $\quad T_6 = 100°C$
$T_5 = 200°C$

$\upsilon_1 = \upsilon_2 = \upsilon_3 \simeq 10$ m/s

a) $T_3 = ?$

b) $\eta_N = ?$

c) $\dot{S}_{gen\ totale} = ?$

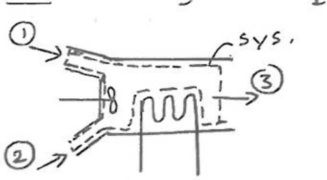
d) Diagramme T-S pour eau & air

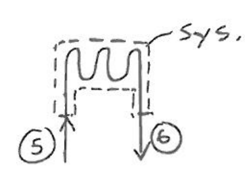
- régime permanent
- air → gaz parfait avec c_p, c_v variables, $R = 0.287$ kPa·m³/kg·K
- $\Delta e_{c\,eau}, \Delta e_{p\,eau}, \Delta e_{p\,air} \simeq 0$
- $\Delta P_{air,\,eau} \simeq 0$ à travers échangeur de chaleur ($P_3 \simeq P_1 = P_2$, $P_6 \simeq P_5$)

Suppositions additionnelles : les vitesses sont des vitesses moyennes

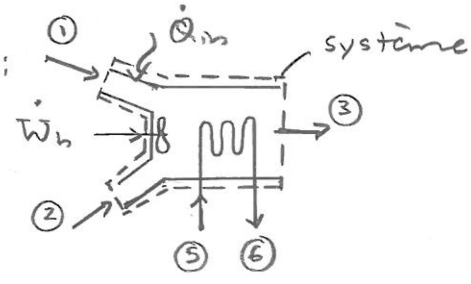
a) $T_3 = ?$: i) Conservation de la masse : $\sum \dot{m}_{in} = \sum \dot{m}_{out}$

(air) : $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ $\qquad$ (eau) : $\dot{m}_5 = \dot{m}_6 = \dot{m}_e$





ii) Bilan d'énergie (1ère loi) :



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_1\theta_1 + \dot{m}_2\theta_2 + \dot{m}_5\theta_5 + \dot{W}_h + \dot{Q}_{in} = \dot{m}_3\theta_3 + \dot{m}_6\theta_6$$

$$\dot{m}_1\theta_1 + \dot{m}_2\theta_2 + \dot{m}_e\theta_5 + \dot{W}_h + \dot{Q}_{in} = (\dot{m}_1+\dot{m}_2)\theta_3 + \dot{m}_e\theta_6$$

$$\dot{m}_1(\theta_3-\theta_1) + \dot{m}_2(\theta_3-\theta_2) = \dot{m}_e(\theta_5-\theta_6) + \dot{W}_h + \dot{Q}_{in}$$

$$\dot{m}_1\left(h_3-h_1 + \cancel{\frac{v_3^2-v_1^2}{2}}^{\,0} + \cancel{\Delta e_{p13}}^{\,0}\right) + \dot{m}_2\left(h_3-h_2 + \cancel{\frac{v_3^2-v_2^2}{2}}^{\,0} + \cancel{\Delta e_{p23}}^{\,0}\right)$$

$$= \dot{m}_e\left(h_5-h_6 + \cancel{\Delta e_{c\,eau\,65}}^{\,0} + \cancel{\Delta e_{p\,eau\,65}}^{\,0}\right) + \dot{W}_h + \dot{Q}_{in}$$

$$(\dot{m}_1+\dot{m}_2)h_3 = \dot{m}_1h_1 + \dot{m}_2h_2 + \dot{m}_e(h_5-h_6) + \dot{W}_h + \dot{Q}_{in}$$

$$h_3 = \frac{\dot{m}_1h_1 + \dot{m}_2h_2 + \dot{m}_e(h_5-h_6) + \dot{W}_h + \dot{Q}_{in}}{\dot{m}_1+\dot{m}_2}$$

$$\Rightarrow \dot{m}_1 = \frac{\dot{V}_1}{v_1} = \dot{V}_1\left(\frac{P_1}{RT_1}\right) = \frac{(55.5607\ \frac{m^3}{s})(133.5\ kPa)}{(0.287\ \frac{kPa\cdot m^3}{kg\cdot k})(-3+270)k} = 95.72\ \frac{kg}{s}$$

$$\Rightarrow h_1 = h(T_1 = -3°C = 270k) \overset{A-17}{=} 270.11\ kJ/kg \quad (s_1^{\circ} \overset{A-17}{=} 1.59634\ kJ/kg\cdot k)$$

$$\Rightarrow h_2 = h(T_2 = 25°C = 298k) = 298.18\ kJ/kg \quad (s_2^{\circ} \overset{A-17}{=} 1.69528\ kJ/kg\cdot k)$$

$\Rightarrow h_5 = ?$: $P_5 = 1\,MPa$, $T_5 = 200°C$ } $T_5 > T_{sat@1MPa}$ vap. surchauffée $\xrightarrow{\text{Table A-6}}$ $h_5 = 2828.3\ \frac{kJ}{kg}$

$\left(s_5 = 6.6956\ \frac{kJ}{kg\cdot k}\right)$

$\Rightarrow h_6 = ?$: $P_6 = P_5 = 1\,MPa$, $T_6 = 100°C$ } $T_6 < T_{sat@1MPa}$ liq. comprimé

$h_6 \simeq h_{f@100°C} \overset{A-4}{=} 419.17\ kJ/kg$

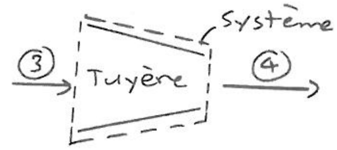
$(s_6 \simeq s_{f@100°C} = 1.3072\ kJ/kg\cdot k)$

$$h_3 = \frac{(95.72\ \frac{kg}{s})(270.11\ \frac{kJ}{kg}) + (101.69\ \frac{kg}{s})(298.18\ \frac{kJ}{kg}) + (7.64\ \frac{kg}{s})(2828.3-419.17)\frac{kJ}{kg} + 185.2\,kW + 400\,kW}{(95.72+101.69)\ kg/s}$$

$h_3 = 380.77\ \frac{kJ}{kg}$ $\xrightarrow{\text{Table A-17}}$ $\boxed{T_3 = 380k = 107°C}$

b) $\eta_N = ?$: $\eta_N = \dfrac{e_{c4}}{e_{c4,isen}} = \dfrac{\vartheta_4^2/2}{\vartheta_{4s}^2/2} = \dfrac{\vartheta_4^2}{\vartheta_{4s}^2}$

$\Rightarrow \vartheta_4, \vartheta_{4s} = ?$:



<u>conservation de la masse:</u>

$\dot{m}_3 = \dot{m}_4$

<u>bilan d'énergie (1ère loi)</u> : $\dot{E}_{in} = \dot{E}_{out}$

$\dot{m}_3 \theta_3 = \dot{m}_4 \theta_4$

$\cancel{\dot{m}_3}(\theta_3 - \theta_4) = 0$

$h_3 - h_4 + \dfrac{\vartheta_3^2 - \vartheta_4^2}{2} + \cancel{\Delta e_{p43}}^{\,0} = 0$

$\vartheta_4 = \sqrt{\vartheta_3^2 + 2(h_3 - h_4)}$

Similairement,

$\vartheta_{4s} = \sqrt{\vartheta_3^2 + 2(h_3 - h_{4s})}$

$\rightarrow h_4 = h(T_4 = 82°C = 355\,K) \overset{A\text{-}17}{\underset{(interp.)}{=}} 355.535 \dfrac{kJ}{kg}$

$\left(s_4^o = s^o(T_4 = 355\,K) \overset{A\text{-}17}{=} 1.871255 \dfrac{kJ}{kg \cdot K} \right)$

$\rightarrow h_{4s} = ?$: $\dfrac{P_{r4s}}{P_{r3}} = \dfrac{P_4}{P_3}$

$P_{r4s} = P_{r3}\left(\dfrac{P_4}{P_3}\right) \rightarrow T_3 = 380\,K$, $P_{r3} \overset{A\text{-}17}{=} 3.176$

$P_{r4s} = (3.176)\left(\dfrac{100\,kPa}{133.5\,kPa}\right)$

$P_{r4s} = 2.379 \xrightarrow{A\text{-}17} h_{4s} = 350.49 \dfrac{kJ}{kg}$

$\vartheta_4 = \sqrt{\left(10 \frac{m}{s}\right)^2 + 2\,(380.77 - 355.535) \frac{kJ}{kg} \times \frac{10^3\, m^2/s^2}{kJ/kg}}$

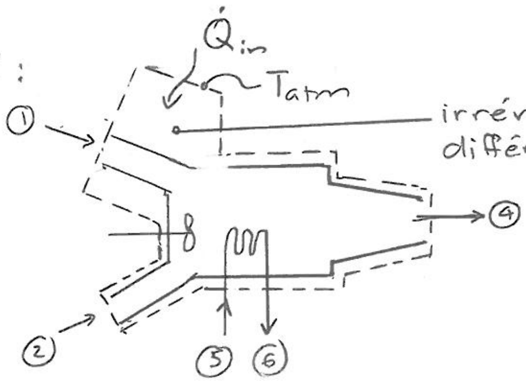
$\vartheta_4 = 224.878$ m/s

$$\vartheta_{4s} = \sqrt{\left(10\,\frac{m}{s}\right)^2 + 2(380.77 - 350.49)\,\frac{kJ}{kg} \times \frac{10^3\,m^2/s^2}{kJ/kg}}$$

$$\vartheta_{4s} = 246.293 \text{ m/s}$$

$$\eta_N = \frac{(224.878 \text{ m/s})^2}{(246.293 \text{ m/s})^2} = \boxed{0.833}$$

c) $\dot{S}_{gen,\,totale} = ?$:



irréversibilité due à un Q à travers une différence de température, incluse dans système pour que $\dot{S}_{gen,\,sys} = \dot{S}_{gen,\,totale}$

$$\dot{m}_4 = \dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

(partie b) (partie a)

<u>bilan d'entropie (2ème loi)</u> :

$$\dot{S}_{sys} = \int_{frontière} \frac{\dot{Q}}{T} + \sum \dot{m}_{in} s_{in} - \sum \dot{m}_{out} s_{out} + \dot{S}_{gen}$$

(régime permanent) $\dot{S}_{sys} = 0$

$$0 = \frac{\dot{Q}_{in}}{T_{amb}} + \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{m}_5 s_5 - \dot{m}_4 s_4 - \dot{m}_6 s_6 + \dot{S}_{gen,\,totale}$$

$$0 = \frac{\dot{Q}_{in}}{T_{amb}} + \dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{m}_e s_5 - (\dot{m}_1 + \dot{m}_2) s_4 - \dot{m}_e s_6 + \dot{S}_{gen,\,totale}$$

$$\dot{S}_{gen,\,totale} = \dot{m}_1 (s_4 - s_1) + \dot{m}_2 (s_4 - s_2) + \dot{m}_e (s_6 - s_5) - \frac{\dot{Q}_{in}}{T_{amb}}$$

$\Rightarrow s_4 - s_1 = s_4^\circ - s_1^\circ - R \ln \frac{P_4}{P_1}$ $\quad \rightarrow s_1^\circ = 1.59634$ (partie a)

$s_4^\circ = 1.871255$ (partie b)

$$= (1.871255 - 1.59634) - 0.287 \ln\left(\frac{100}{133.5}\right)$$

$$= 0.357838 \text{ kJ/kg·K}$$

$\Rightarrow s_4 - s_2 = s_4^\circ - s_2^\circ - R \ln \frac{P_4}{P_2}$ $\quad \rightarrow s_2^\circ = 1.69528$ (partie a)

$$= (1.871255 - 1.69528) - (0.287) \ln\left(\frac{100}{133.5}\right)$$

$$s_4 - s_2 = 0.258898 \text{ kJ/kg·K}$$

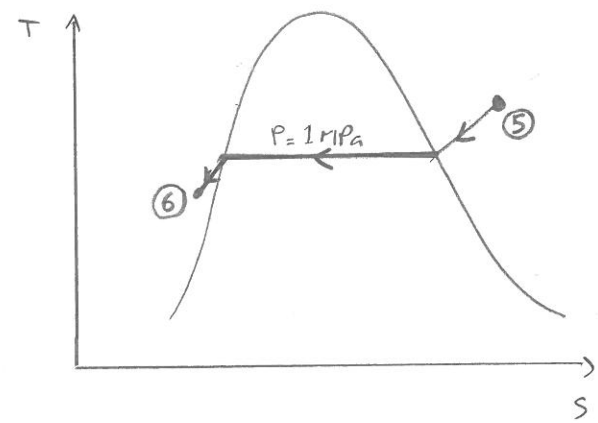
$$\Rightarrow s_5 = 6.6956 \text{ kJ/kg·K} \quad \text{(partie a)}$$

$$\Rightarrow s_6 = 1.3072 \text{ kJ/kg·K} \quad \text{(partie a)}$$

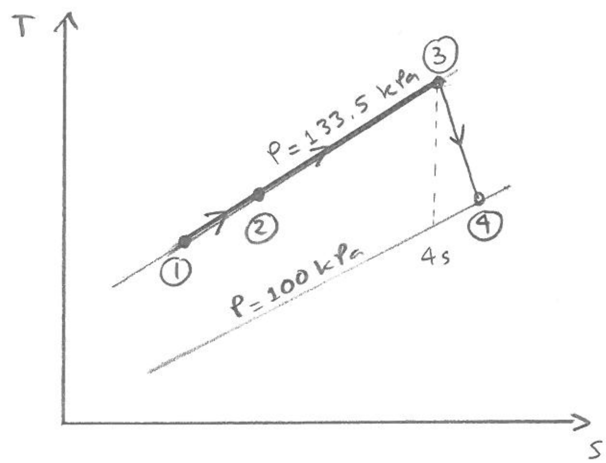
$$\dot{S}_{gen\ totale} = \left(95.72 \tfrac{kg}{s}\right)\left(0.357838 \tfrac{kJ}{kg·K}\right) + \left(101.69 \tfrac{kg}{s}\right)\left(0.258898 \tfrac{kJ}{kg·K}\right) + \left(7.64 \tfrac{kg}{s}\right)(1.3072 - 6.6956)\tfrac{kJ}{kg·K} - \frac{400 \text{ kJ/s}}{(25+273) \text{ K}}$$

$$\boxed{\dot{S}_{gen\ totale} = 18.07 \ \frac{kW}{K}}$$

d)



eau



air