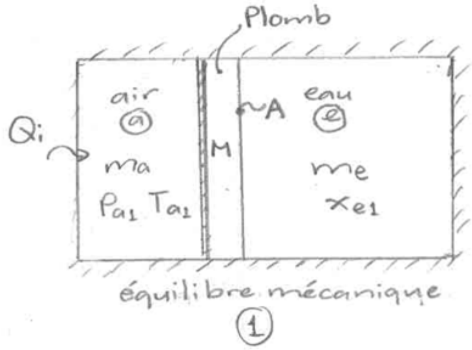
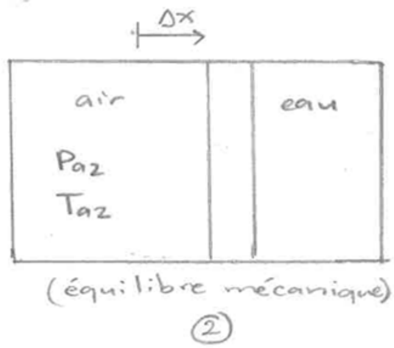
MEC1210 Automne 2024, TD2: Problème à faire en classe (solutionnaire)





$P_{a1} = 100$ kPa $\quad x_{e1} = 1.0$ (vap. sat.)

$T_{a1} = 20°C$

$m_a = 0.04$ kg $\quad m_e = 0.04$ kg

$P_{a2} = 200$ kPa

$T_{a2} = 600°C$

$M = 2$ kg

$A = 0.1\ m^2$

- air → gaz parfait, C_p, C_v cst.

  $R_a = 0.287$ kJ/kg·K

  $C_{pa} = 1.045$

- Plomb : incomp.

  $C_{(Pb)} = 0.128 \frac{kJ}{kg \cdot K}$

  $T_p = T_e$

- $\Delta U_{isolant} \simeq 0$
- $Q_{air \to eau} = 0$
- piston sans friction

a) V_{a1}, $\Delta x = ?$

b) $T_{e1}, T_{e2} = ?$

c) $W_{b\ par\ air} = ?$

d) $Q_i = ?$

a) V_{a1}, $\Delta x = ?$: air, gaz parfait → $P_a V_a = m_a R_a T_a$

i) $$V_{a1} = \frac{m_a R_a T_{a1}}{P_{a1}} = \frac{(0.04\ kg)(0.287\ \frac{kPa \cdot m^3}{kg \cdot K})(20+273)K}{100\ kPa}$$

$$\boxed{V_{a1} = 0.03364\ m^3}$$

ii) $\Delta x = ?$: $V_{a2} - V_{a1} = A\Delta x$

$$\Delta x = \frac{V_{a2} - V_{a1}}{A} \Rightarrow V_{a2} = \frac{m_a R_a T_{a2}}{P_{a2}}$$

$$= \frac{(0.04)(0.287)(600+273)}{200}$$

$$V_{a2} = 0.05011\ m^3$$

$$\Delta x = \frac{(0.05011 - 0.03364)\ m^3}{(0.1)\ m^2} = \boxed{0.16474\ m}$$

b) i) $T_{e1} = ?$: $P_{e1} = P_{a1} = 100 \text{ kPa}$ (équilibre mécanique sur piston horizontal libre)

$x_{e1} = 1.0$ (vap. sat.)

$T_{e1} = T_{sat@100\,kPa} \overset{A\text{-}5}{=} \boxed{99.61\,°C}$

ii) $T_{e2} = ?$: $P_{e2} = P_{a2} = 200 \text{ kPa}$ (équilibre mécanique)

$$v_{e2} = \frac{V_{e2}}{m_e} = \frac{V_{e1} - A\Delta x}{m_e} = \frac{m_e v_{e1} - A\Delta x}{m_e} = v_{e1} - \frac{A\Delta x}{m_e}$$

$\Rightarrow v_{e1} = ?$: $\left.\begin{array}{l} P_{e1} = 100 \text{ kPa} \\ x_{e1} = 1.0 \end{array}\right\}$ $v_{e1} = v_{g@100\,kPa} \overset{A\text{-}5}{=} 1.6941 \frac{m^3}{kg}$

$$v_{e2} = (1.6941) - \frac{(0.1)(0.16474)}{0.04} = 1.28225 \frac{m^3}{kg}$$

phase = ? : Table A-5 : à $P_{sat} = 200 \text{ kPa}$: $v_f = 0.001061 \frac{m^3}{kg}$

$v_g = 0.88578$

$\downarrow$

vapeur surchauffée $\leftarrow$ $v_{e2} > v_g$

$\downarrow$

Table A-6 : $P_{e2} = 0.20 \text{ MPa}$

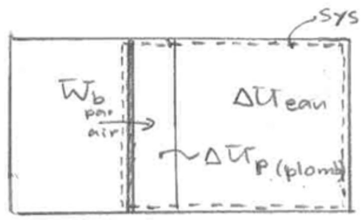
$v_{e2} = 1.28225 \frac{m^3}{kg}$

interpolation :

$$T_{e2} = 250 + \frac{(300-250)(1.28225 - 1.19890)}{(1.31623 - 1.19890)}$$

$\boxed{T_{e2} = 285.52\,°C}$

c) $W_{b\,\text{par air}} = ?$: Comme on ne connait pas comment la pression de l'air varie avec son volume dans ce cas, on ne peut pas obtenir W_b par $\int_{1}^{2} P\,dV$. Il faut donc utiliser la 1ère loi de la thermodynamique en choisissant un système où W_b est le seul inconnu:



1ère loi: $\Delta E_{sys} = E_{in} - E_{out}$

$$\Delta U_{eau} + \Delta U_p + \underbrace{\cancel{\Delta U}_{\text{isolant sur piston}}}_{\to 0} + \underbrace{\cancel{\Delta E_c}}_{\simeq 0} + \underbrace{\cancel{\Delta E_p}}_{=0} = W_{b\,\text{par air}}$$

$$W_{b\,\text{par air}} = m_e(u_{e2} - u_{e1}) + M(u_{p2} - u_{p1})$$

$\Rightarrow u_{e1} = ?$: $\left.\begin{matrix} P_{e1} = 100\text{ kPa} \\ x_{e1} = 1.0 \end{matrix}\right\}$ $u_{e1} = u_g @ 100\text{ kPa} \underset{(A-5)}{=} 2505.6\ \frac{kJ}{kg}$

$\Rightarrow u_{e2} = ?$: $\left.\begin{matrix} P_{e2} = 200\text{ kPa} \\ v_{e2} = 1.28225\ \frac{m^3}{kg} \end{matrix}\right\}$ Table A-6 : interpol. $u_{e2} = 2786.4\ \frac{kJ}{kg}$

$$\Rightarrow u_{p2} - u_{p1} = C_{(Pb)}(T_{p2} - T_{p1})$$
$$= C_{(Pb)}(T_{e2} - T_{e1})$$
$$= (0.128)(285.52 - 99.61)$$
$$= 23.80\ \frac{kJ}{kg}$$

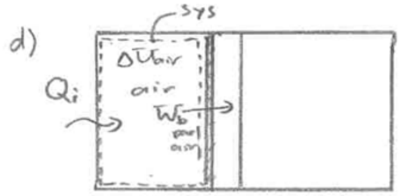
Note : incomp. $C_{P(Pb)} = C_{v(Pb)} = C_{(Pb)}$

Note : $\Delta T\,[°C] = \Delta T\,[K]$

$$W_{b\,\text{par air}} = (0.04\text{ kg})(2786.4 - 2505.6)\frac{kJ}{kg} + (2\text{ kg})\left(23.80\ \frac{kJ}{kg}\right)$$

$$\boxed{W_{b\,\text{par air}} = 58.8\ \text{kJ}}$$

d)



1ère loi : $\Delta E_{sys} = E_{in} - E_{out}$

$$\Delta U_{air} + \underbrace{\cancel{\Delta E_c}}_{=0} + \underbrace{\cancel{\Delta E_p}}_{=0} = Q_i - W_{b\,\text{par air}}$$

$$Q_i = m_a(u_{a2} - u_{a1}) + W_{b\,\text{par air}}$$

$$Q_i = m_a C_{va}(T_{a2} - T_{a1}) + W_{b\,\text{par air}}$$

$$\Rightarrow C_{va} = C_{pa} - R_a = 1.045 - 0.287$$

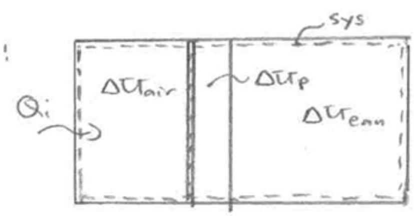
$$C_{va} = 0.758 \ \frac{kJ}{kg \cdot K}$$

$$Q_i = (0.04\,kg)\left(0.758\,\frac{kJ}{kg\cdot k}\right)\underbrace{(600-20)}_{}k + 58.83\,kJ$$

$\Delta T\,[°C] = \Delta T\,[K]$

$$\boxed{Q_i = 76.4\ kJ}$$

Alternatif :



1ère loi : $\Delta E_{sys} = E_{in} - E_{out}$

$$\Delta U_{air} + \Delta U_{eau} + \Delta U_p + \underbrace{\cancel{\Delta U_{\text{isolant sur piston}}}}_{=0} + \underbrace{\cancel{\Delta E_c}}_{=0} + \underbrace{\cancel{\Delta E_p}}_{=0} = Q_i$$

$$Q_i = m_a(u_{a2} - u_{a1}) + \boxed{m_e(u_{e2} - u_{e1}) + M(u_{p2} - u_{p1})} \simeq W_{b\,\text{par air}} \ (\text{voir partie c})$$

$$Q_i = m_a C_{va}(T_{a2} - T_{a1}) + m_e(u_{e2} - u_{e1}) + M C_{(Pb)}(T_{e2} - T_{e1})$$

$$Q_i = 76.4\,kJ$$

Bonus : partie (d) avec air comme gaz parfait à C_p, C_v variables

$$Q_i = m_a(u_{a2} - u_{a1}) + W_{b\,\text{par air}}$$

$\Rightarrow$ Table A-17 : $T_{a1} = 20°C = 293\,k \xrightarrow{\text{interp.}} u_{a1} = 209.058 \ \frac{kJ}{kg}$

$T_{a2} = 600°C = 873\,k \longrightarrow u_{a2} = 652.1575$

$$Q_i = (0.04\,kg)(652.1575 - 209.058)\frac{kJ}{kg} + 58.83\,kJ = \boxed{76.55\ kJ}$$