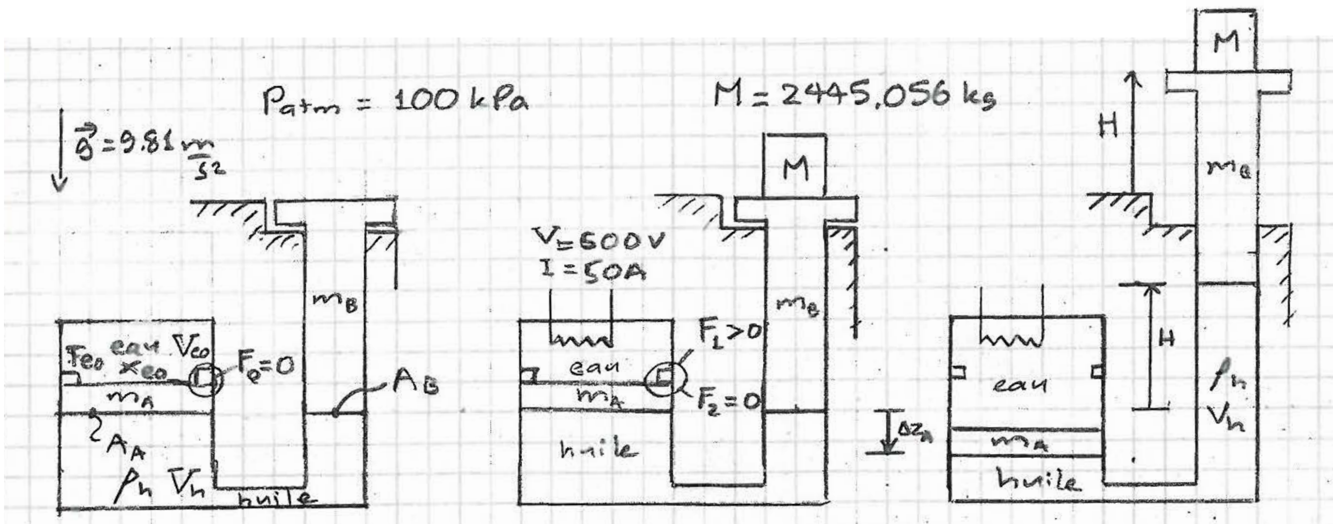


état ⓪ — états ① ② (M sur le point de monter) — état ③

État ⓪ :
$T_{eo} = 130°C$
$x_{eo} = 0.38148$
$V_{eo} = 0.3943\ m^3$

États ① ② :
$T_{e1} = T_{eo}$
$x_{e1} = x_{eo}$
$(P_{e1} = P_{eo})$
$V_{e1} = V_{e2} = V_{eo}$

État ③ :
$H = 5.2691\ m$

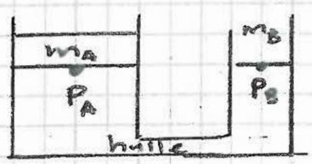
$m_A = 1300\ kg$
$A_A = 0.5\ m^2$
$A_B = 0.05\ m^2$
$V_h = 0.749\ m^3$

a) $m_B = ?$
b) $P_{e2}, T_{e2}, P_{e3}, T_{e3} = ?$
c) Diagramme P-v (eau)
d) $W_{\text{par eau}} = ?$
e) $\Delta t = ?$ (alimentation élec.)
f) $\Delta z_{cg,h} = ?$

- huile → fluide incompressible avec $\rho_h = 879.37\ \frac{kg}{m^3}$
- éléments solides → isolant thermique parfait ($Q = 0$)
- friction/viscosité négligeable
- $\Delta U_{\text{solides/huile}} \simeq 0$
- $\Delta E_{p\,eau} \simeq 0$

Suppositions additionnelles : aucune

a) $m_B = ?$: Considérons la pression sur les surfaces inférieures du piston A (P_A) et du piston B (P_B).

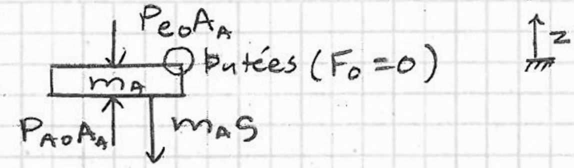


$$P_A = P_B + \rho_h g h$$
où $h = 0$ pour ⓪, ① et ②

Donc : $P_A = P_B$ pour ⓪, ① et ②

$$P_{A0} = P_{B0}$$

$\Rightarrow P_{A0} = ?$:

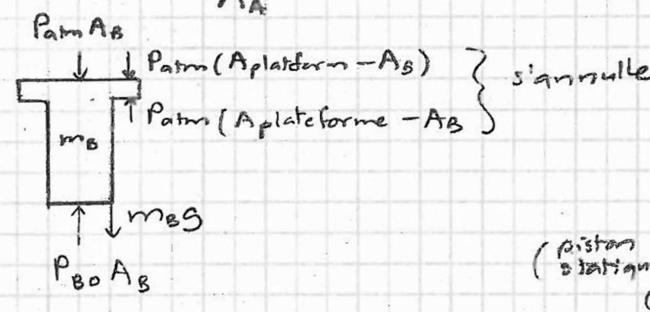


(piston statique)

$$\sum F_z = P_{A0}A_A - P_{e0}A_A - m_A g = m_A \cancel{a_A}$$

$$P_{A0} = P_{e0} + \frac{m_A g}{A_A}$$

$\Rightarrow P_{B0} = ?$:



(piston statique)

$$\sum F_z = P_{B0}A_B - P_{atm}A_B - m_B g = m_B \cancel{a_B}$$

$$P_{B0} = P_{atm} + \frac{m_B g}{A_B}$$

$$P_{e0} + \frac{m_A g}{A_A} = P_{atm} + \frac{m_B g}{A_B}$$

$$\frac{m_B g}{A_B} = P_{e0} - P_{atm} + \frac{m_A g}{A_A}$$

$$m_B = (P_{e0} - P_{atm})\frac{A_B}{g} + m_A \frac{A_B}{A_A}$$

$\Rightarrow P_{e0} = ?$: $\left.\begin{array}{l} T_{e0} = 130\,°C \\ x_{e0} = 0.38148 \end{array}\right\}$ phase$_0$ = mélange saturé

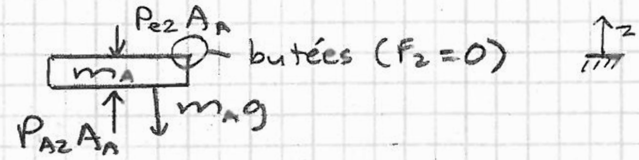
$$P_{e0} = P_{sat@130°C} \overset{\text{Table A-4}}{=} 270.28\ kPa$$

$$m_B = (270.28 - 100)\ kPa \frac{(0.05\,m^2)}{9.81\,m/s^2} \times \frac{10^3\ kg\cdot m/s^2/m^2}{1\ kPa} + (1300\ kg)\left(\frac{0.05\,m^2}{0.5\,m^2}\right)$$

$$\boxed{m_B = 997.89\ kg}$$

b) i) $P_{e2}, T_{e2} = ?$: $P_{A2} = P_{B2}$ (voir partie a)

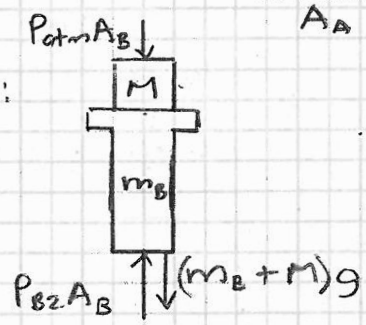
$\Rightarrow P_{A2} = ?$;



$$\sum F_z = P_{A2}A_A - P_{e2}A_A - m_A g = m_A \cancel{a_A}^{\,0}$$

$$P_{A2} = P_{e2} + \frac{m_A g}{A_A}$$

$\Rightarrow P_{B2} = ?$:



$$\sum F_z = P_{B2}A_B - P_{atm}A_B - (m_B + M)g = (m_B + M)\cancel{a_B}^{\,0}$$

$$P_{B2} = P_{atm} + \frac{(m_B + M)g}{A_B}$$

$$P_{e2} + \frac{m_A g}{A_A} = P_{atm} + \frac{(m_B + M)g}{A_B}$$

$$P_{e2} = P_{atm} + \frac{(m_B + M)g}{A_B} - \frac{m_A g}{A_A}$$

$$= 100\ kPa + \left[\frac{(997.89 + 2445.056)\ kg\ (9.81\ m/s^2)}{0{,}05\ m^2} - \frac{(1300\ kg)(9.81\ m/s^2)}{0.5\ m^2}\right] \times \frac{1\ kPa}{10^3\ \frac{kg \cdot m/s^2}{m^2}}$$

$$\boxed{P_{e2} = 750\ kPa}$$

Table A-4

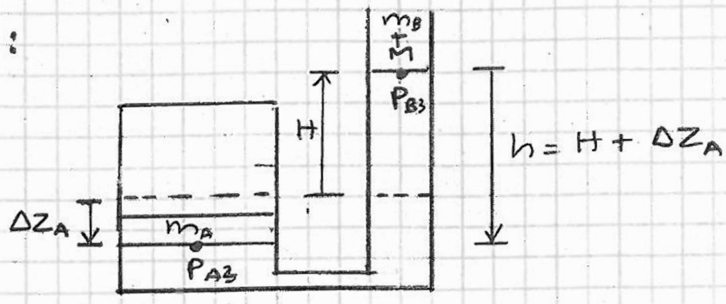
$$v_{e2} = \frac{V_{e2}}{m_e} = \frac{V_{e0}}{m_e} = v_{e0} = v_{f@130°C} + x_{e0}(v_g - v_f)_{@130°C}$$

$$= 0{,}001070 + (0{,}38148)(0{,}66808 - 0.001070)$$

$$v_{e_2} = 0{,}25552 \ \frac{m^3}{kg} = v_{g@750kPa} \quad \text{(table A-5)}$$

$$\text{phase}_2 = \text{vap. saturé} \rightarrow T_{e2} = T_{g@750kPa} = \boxed{167.75\,°C}$$

ii) $P_{e_3}, T_{e_3} = ?$:



$$P_{A3} = P_{B3} + \rho_h g h = P_{B3} + \rho_h g (H + \Delta Z_A)$$

$\Rightarrow \Delta Z_A = ?$: conservation de masse de l'huile : $\Delta m_{h_{\text{gauche}}} = \Delta m_{h_{\text{droite}}}$

$$\cancel{\rho_h} A_A \Delta Z_A = \cancel{\rho_h} A_B H$$

$$A_A \Delta Z_A = A_B H$$

$$\Delta Z_A = \frac{A_B}{A_A} H$$

$$P_{A3} = P_{B3} + \rho_h g H \left(1 + \frac{A_B}{A_A}\right)$$

$\Rightarrow$ Similairement au raisonnement de la partie b) i) :

$$P_{A3} = P_{e3} + \frac{m_A g}{A_A}$$

$$P_{B3} = P_{atm} + \frac{(m_B + M) g}{A_B}$$

$$P_{e3} + \frac{m_A g}{A_A} = P_{atm} + \frac{(m_B + M) g}{A_B} + \rho_h g H \left(1 + \frac{A_B}{A_A}\right)$$

$$P_{e3} = \underbrace{P_{atm} + \frac{(m_B + M) g}{A_B} - \frac{m_A g}{A_A}}_{= P_{e2}} + \rho_h g H \left(1 + \frac{A_B}{A_A}\right)$$

$$P_{e_3} = 750\,kPa + \left(879.37\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)(5.2691\,m)\left(1 - \frac{0.05}{0.5}\right) \times \frac{1\,kPa}{10^3\,\frac{kg\cdot m/s^2}{m^2}}$$

$$\boxed{P_{e_3} = 800\,kPa}$$

$$v_{e_3} = \frac{V_{e_3}}{m_e} = \frac{V_{e0} + A_A \Delta z_A}{m_e} = v_{e0} + \frac{A_A}{m_e}\frac{A_B}{A_A} H$$

$$v_{e_3} = v_{e0} + \frac{A_B H}{m_e}$$

$$\Rightarrow m_e = \frac{V_{e0}}{v_{e0}} = \frac{0.3943\,m^3}{0.25552\,\frac{m^3}{kg}} = 1.54313\,kg$$

$$v_{e_3} = 0.25552\,\frac{m^3}{kg} + \frac{(0.05\,m^2)(5.2691\,m)}{(1.54313\,kg)}$$

$$v_{e_3} = 0.42625\,\frac{m^3}{kg} > v_{g@800kPa} \underset{A-5}{=} 0.24035\,\frac{m^3}{kg}$$

phase$_3$ = vapeur surchauffée

Table A-6 : pour $P_{e_3} = 0.8\,MPa$, $v_{e_3} = 0.42625\,\frac{m^3}{kg}$

interpolation : $\boxed{T_{e_3} = 471.08\,°C}$

$\left(u_{e_3} = 3078.48\,\frac{kJ}{kg}\right)$

c) Diagramme P-v pour eau

états ⓪ = ① : mélange saturé @ $P = P_{sat@130°C} = 270.28\,kPa$

état ② : vapeur saturée @ $P = 750\,kPa$

état ③ : vapeur surchauffée @ $P = 800\,kPa$

évolution ① → ② : volume constant

évolution ② → ③ : comment P varie avec v ?

partie b)ii) a montré que

$$P_{e_3} = P_{e_2} + \rho_h g H \left(1 + \frac{A_B}{A_A}\right)$$

Si on remplace H par $0 < z < H$ pour un état entre ② et ③, on a :

$$P_e = P_{e_2} + \rho_h g z \left(1 + \frac{A_B}{A_A}\right)$$

$$\Rightarrow \bar{V}_e - \bar{V}_{e2} = A_B z$$ (augmentation du volume de l'eau = augmentation du volume d'huile dans cylindre B)

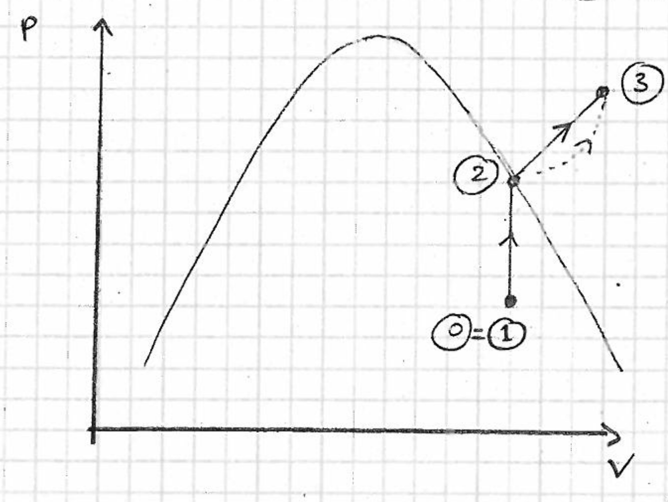
$$z = \frac{\bar{V}_e - \bar{V}_{e2}}{A_B}$$

$$z = \frac{m_e (v_e - v_{e2})}{A_B}$$

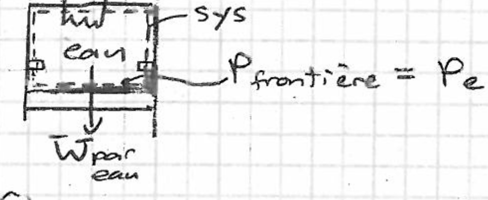
$$P_e = P_{e2} + \frac{\rho_h g m_e}{A_B}(v_e - v_{e2})$$

$$P_e = \underbrace{\left(P_{e2} - \frac{\rho_h g m_e v_{e2}}{A_B}\right)}_{\text{const.}} + \underbrace{\left(\frac{\rho_h g m_e}{A_B}\right)}_{\text{const.}} v_e$$

$P_e = f(v_e)$ est une fonction linéaire entre ② et ③



d) $W_{\substack{par\\eau}} = ?$: système $\Rightarrow$



$$W_{\substack{par\\eau}} = W_{\substack{b\\par\\sys}} = \int_{①}^{③} P\,d\bar{V}$$

$$= \int_{①}^{②} P_e \cancel{d\bar{V}_e}^{\,0\ (\bar{V}\text{const})} + \int_{②}^{③} P_e\,d\bar{V}_e$$

$$\Rightarrow \text{②} \to \text{③} : \; P_e = P_{e2} + \rho_h g z \left(1 + \frac{A_B}{A_A}\right) \quad (\text{partie c})$$

$$\cancel{\rho_h} dV_e = \cancel{\rho_h} A_B dz \quad \left(\begin{array}{c}\text{conservation de} \\ \text{masse d'huile}\end{array}\right)$$

$$W_{\substack{par \\ eau}} = \int_0^H \left[P_{e2} + \rho_h g z \left(1 + \frac{A_B}{A_A}\right)\right](A_B dz)$$

$$= \int_0^H P_{e2} A_B \, dz + \rho_h g A_B \left(1 + \frac{A_B}{A_A}\right) \int_0^H z \, dz$$

$$= P_{e2} A_B H + \rho_h g A_B \left(1 + \frac{A_B}{A_A}\right) \frac{H^2}{2}$$

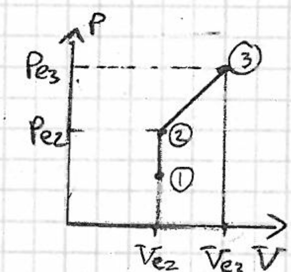
$$= (750 \text{ kPa})(0.05 \text{ m}^2)(5.2691 \text{ m}) \times \frac{1 \text{ kN/m}^2}{1 \text{ kPa}} \times \frac{\text{kJ}}{\text{kN}\cdot\text{m}}$$

$$+ \left(879.37 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(0.05 \text{ m}^2)\left(1 + \frac{0.05}{0.5}\right)\frac{(5.2691 \text{ m})^2}{2}$$

$$\times \frac{1 \text{ kN}}{10^3 \text{ kg}\cdot\text{m/s}^2} \times \frac{\text{kJ}}{\text{kN}\cdot\text{m}}$$

$$\boxed{W_{\substack{par \\ eau}} = 204{,}178 \text{ kJ}}$$

Solution alternative (plus simple) :



$$W_{\substack{par \\ eau}} = \text{aire sous la courbe } P\text{-}V$$

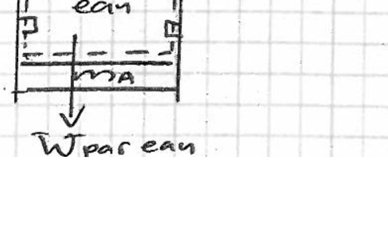
$$= \left(\frac{P_{e2} + P_{e3}}{2}\right) A_B H$$

$$A_B H = A_A \Delta z_A$$

$$= \left(\frac{750 + 800}{2}\right) \text{kPa} \, (0.05 \text{ m}^2)(5.2691 \text{ m}) \times \frac{\text{kN/m}^2}{\text{kPa}} \times \frac{\text{kJ}}{\text{kN}\cdot\text{m}}$$

$$W_{\substack{par \\ eau}} = 204{,}178 \text{ kJ}$$

e) $\Delta t = ?$: $W_{élec} = VI\Delta t$ (il faut donc trouver le travail électrique)



1ère loi :

$$\Delta E_{sys_{13}} = E_{in_{13}} - E_{out_{13}}$$

$$(\Delta U_{solides} = 0)$$

$$\Delta U_{eau,13} + \cancel{\Delta U_{\substack{résistance\\butées\\13}}}^{0} + \cancel{\Delta E_{c,sys}}^{0} + \cancel{\Delta E_{p,sys}}^{0} = W_{élec} - W_{par\ eau}$$

$$m_e(u_{e3} - u_{e1}) = VI\Delta t - W_{par\ eau}$$

$$\Delta t = \frac{m_e(u_{e3} - u_{e1}) + W_{par\ eau}}{VI}$$

$$\overset{①=⓪}{\Rightarrow u_{e1} = u_{e0}} = u_{f@130°C} + x_{e0}\, u_{fg@130°C} \quad \begin{pmatrix}\text{Table}\\ \text{A-4}\end{pmatrix}$$

$$= 546.10 + (0.38148)(1953.4)$$

$$= 1306.54 \text{ kJ/kg}$$

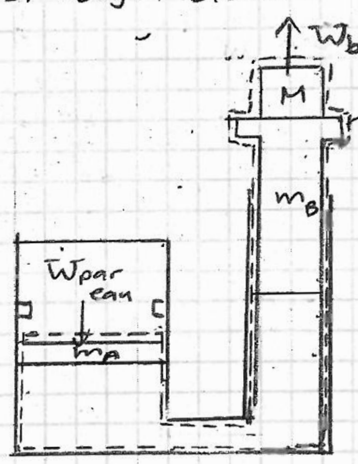
$$\Rightarrow u_{e3} = 3078.48 \text{ kJ/kg} \quad \text{(voir partie b)}$$

$$\Delta t = \frac{(1.54313 \text{ kg})(3078.48 - 1306.54)\frac{\text{kJ}}{\text{kg}} + 204.178 \text{ kJ}}{(600 V)(50 A) \times \frac{1 J}{1 V \cdot A \cdot s} \times \frac{1 kJ}{10^3 J}}$$

$$\boxed{\Delta t = 97.95 \text{ s}}$$

f) $\Delta Z_{cg,h} = ?$: $\Delta E_{ph} = m_h g \Delta Z_{cg,h} = \rho_h V_h g \Delta Z_{cg,h}$

Il s'agit donc de trouver ΔE_p de l'huile



système = piston A + huile + piston B + M

$$W_b = \int_①^③ P\,dV = \int_0^H P_{atm}(A_B\,dz)$$

$$= P_{atm} A_B H$$

$\underline{1^{ère}\ loi}$: $\Delta E_{sys,13} = E_{in,13} - E_{out,13}$

$$\underbrace{\cancel{\Delta U}^{\,0\ (\Delta U_{\substack{solides\\huile}}=0)}}_{\substack{pistons\\plateforme\\M\ \&\ huile\\13}} - \underbrace{\Delta E_p}_{\substack{piston A\\13}} + \underbrace{\Delta E_p}_{\substack{piston B\\plateforme\\M\ 13}} + \underbrace{\Delta E_p}_{\substack{huile\\13}} + \underbrace{\cancel{\Delta E_c}^{\,0}}_{\substack{sys\\13}} = W_{\substack{par\\eau}} - W_b$$

$$-m_A g \Delta z_A + (m_B + M) g H + \rho_h V_h g \Delta z_{cg,h} = W_{\substack{par\\eau}} - P_{atm} A_B H$$

$$\Rightarrow \Delta z_A = \frac{A_B}{A_A} H$$

$$\rho_h V_h g \Delta z_{cg,h} = m_A g \frac{A_B}{A_A} H - (m_B + M) g H + W_{\substack{par\\eau}} - P_{atm} A_B H$$

$$= W_{\substack{par\\eau}} - \underbrace{\left[P_{atm} + \frac{(m_B + M) g}{A_B} - \frac{m_A g}{A_A} \right]}_{= P_{e2}} A_B H$$

$$= W_{\substack{par\\eau}} - P_{e2} A_B H$$

$$\Delta z_{cg,h} = \frac{W_{\substack{par\\eau}} - P_{e2} A_B H}{\rho_h V_h g}$$

$$\Delta z_{cg,h} = \frac{204{,}178\ \text{kJ} - (750\ \text{kPa})(0.05\ \text{m}^2)(5.2691\ \text{m}) \times \frac{1\ \text{kJ}}{1\ \text{kPa}\cdot\text{m}^3}}{(879{,}37\ \frac{\text{kg}}{\text{m}^3})(0{,}749\ \text{m}^3)(9.81\ \frac{\text{m}}{\text{s}^2}) \times \frac{1\ \text{kJ}}{10^3\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}}}$$

$$\boxed{\Delta z_{cg,h} = 1{,}0194\ \text{m}}$$