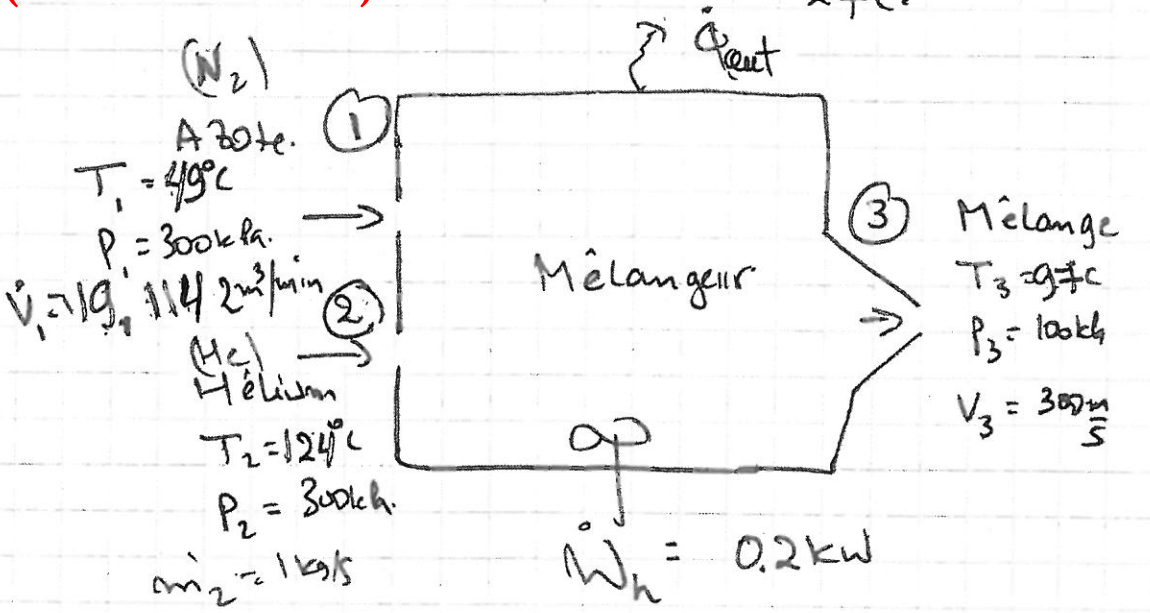
MEC1210: TD6 (Solutionnaire)



Hypothèses :

- $V_1 \simeq V_2 \simeq 0 m/s$
- Régime Permanent
- $\Delta E_p \simeq 0$

a) $\dot{Q}_{out}$?

b) Y_{He}, Y_{N_2} ?

d) $\dot{S}_{gen}$

$M_{N_2} = 28\frac{kg}{kmol}$; $C_{P_{N_2}} = 1,039\frac{kJ}{kg.K}$; $R_{N_2} = 0.2968\frac{kJ}{kg.K}$

$M_{He} = 4\frac{kg}{kmol}$; $C_{P_{He}} = 5.1926\frac{kJ}{kg.K}$; $R_{He} = 2.0769\frac{kJ}{kg.K}$

indice 1 N_2
indice 2 He
indice 3 mélange.

a) $\dot{Q}_{out}$?

Système tout :

Bilan de masse. $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Bilan d'énergie : $\dot{m}_1\theta_1 + \dot{m}_2\theta_2 + \dot{W}_h = \dot{m}_3\theta_3 + \dot{Q}_{out}$

$$\dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_h = \dot{m}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right) + \dot{Q}_{out}$$

Or : $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$$h_3 = h_m = \sum m_i h_i = \frac{\dot{m}_1}{\dot{m}_3} h_{N_2,3} + \frac{\dot{m}_2}{\dot{m}_3} h_{He,3}$$

après arrangement :

$$\dot{m}_1 \left(h_1 - h_3 - \frac{V_3^2}{2} \right)_{N_2} + \dot{m}_2 \left(h_2 - h_3 - \frac{V_3^2}{2} \right)_{He} + \dot{W}_h = \dot{Q}_{out}$$

$$\dot{Q}_{out} = \dot{m}_1 \left(\left(c_p (T_1 - T_3) - \frac{V_3^2}{2} \right)_{N_2} + \dot{m}_2 \left(c_p (T_2 - T_3) - \frac{V_3^2}{2} \right)_{He} + \dot{W}_h \right.$$

$\dot{m}_2 = 1$ kg/s

$P_1 \dot{V}_1 = \dot{m}_1 R T_1 \Rightarrow \boxed{\dot{m}_1 = \frac{P_1 \dot{V}_1}{R T_1}}$

$$\dot{m}_1 = \frac{300 \cdot 19,1142/60}{0.2968 \cdot (49 + 273)} = 1 \text{ kg/s.} \qquad \boxed{\dot{m}_1 = 1 \text{ kg/s}}$$

$$\dot{Q}_{out} = 1 \left(1.039 (49 - 97) - \frac{300^2}{2} \cdot \frac{1}{1000} \right) + 1 \left(5.1926 (124 - 97) - \frac{300^2}{2} \cdot \frac{1}{1000} \right) + 0.2$$

$$\boxed{\dot{Q}_{out} = 0.5282 \text{ kW}}$$

b) Y_{N_2}, Y_{He} ?

$Y_{N_2} = \frac{\dot{N}_{N_2}}{\dot{N}_m}$, $Y_{He} = \frac{\dot{N}_{He}}{\dot{N}_m}$ avec $\dot{N}_m = \dot{N}_{N_2} + \dot{N}_{He}$

$$\dot{N}_{N_2} = \frac{\dot{m}_{N_2}}{M_{N_2}} = \frac{1}{28} = 0.0357 \text{ kmol/s}$$

$$\dot{N}_{He} = \frac{\dot{m}_{He}}{M_{He}} = \frac{1}{4} = 0.25 \text{ kmol/s}$$

$$\dot{N}_m = 0.0357 + 0.25 = 0.2857 \text{ kmol/s}$$

D'où $Y_{N_2} = \frac{0.0357}{0.2857} = 0.125$ $\boxed{Y_{N_2} = 12.5\,\%}$

$Y_{He} = \frac{0.25}{0.2857} = 0.875$ $\boxed{Y_{He} = 87.5\,\%}$

$Y_{N_2} + Y_{He} = 1$ OK

c) $\dot{S}_{gen} = ?$

Système tout : $\dot{S}_{sy} = \int \frac{\dot{Q}}{T} + \sum_{in} \dot{m} s - \sum_{out} \dot{m} s + \dot{S}_{gen} = 0$

$\Rightarrow \dot{S}_{gen} = -\frac{\dot{Q}_{out}}{T} + \sum_{out} \dot{m} s - \sum_{in} \dot{m} s$ (Régime Permanent)

$= + \frac{\dot{Q}_{out}}{T} + \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$

Or $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ et $s_3 = s_m = \frac{\dot{m}_1}{\dot{m}_3} s_{1,3} + \frac{\dot{m}_2}{\dot{m}_3} s_{2,3}$

D'où $\dot{S}_{gen} = \frac{\dot{Q}_{out}}{T} + \dot{m}_1 (s_3 - s_1)_{N_2} + \dot{m}_2 (s_3 - s_2)_{He}$

$(s_3 - s_1)_{N_2} = C_{p_1} \ln \frac{T_3}{T_1} - R_1 \ln \frac{P_{1,3}}{P_1} = C_{p_1} \ln \frac{T_3}{T_1} - R_1 \ln \left(\frac{Y_{N_2} P_3}{P_1} \right)$

$= 1,039 \ln \left(\frac{97+273}{49+273} \right) - 0.2968 \cdot \ln \left(\frac{0.125 \cdot 100}{300} \right) = 1,088 \frac{kJ}{kg.K}$

$(s_3 - s_2)_{He} = C_{p_2} \ln \frac{T_3}{T_2} - R_2 \ln \frac{P_{2,3}}{P_1} = C_{p_2} \ln \frac{T_3}{T_2} - R_2 \ln \frac{Y_{He} P_3}{P_2}$

$= 5.1926 \cdot \ln \left(\frac{97+273}{124+273} \right) - 2,0769 \cdot \ln \left(\frac{0.875 \cdot 100}{300} \right) = 2.193 \frac{kJ}{kg.K}$

D'où $\dot{S}_{gen} = \frac{0.5282}{27+273} + 1 (1,088) + 1 \cdot (2.193) = 3.283 \frac{kW}{K}$

$\boxed{\dot{S}_{gen} = 3,283 \frac{kW}{K}}$