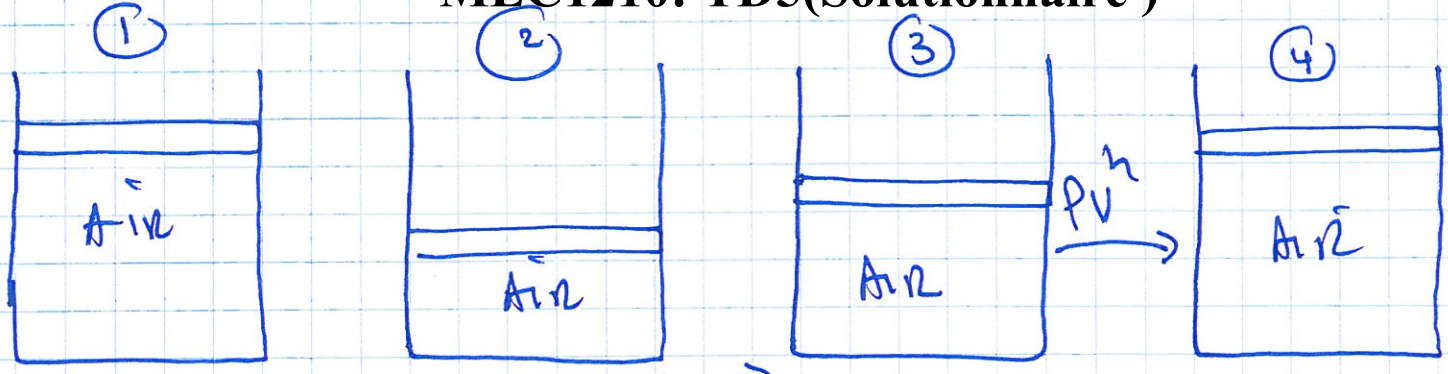
MEC1210: TD5(Solutionnaire)



$P_1 = 100 kPa$
$T_1 = 300 K$

$S_2 = S_1$ $\quad q_{in} = 954,7 \frac{kJ}{kg}$ $\quad P_3 = P_2$

$V_2 = \frac{V_1}{11,63379}$

$V_4 = V_1$

$n = 1,334877$

Hypothèse:

Air gaz parfait à C_p et C_v = cstes

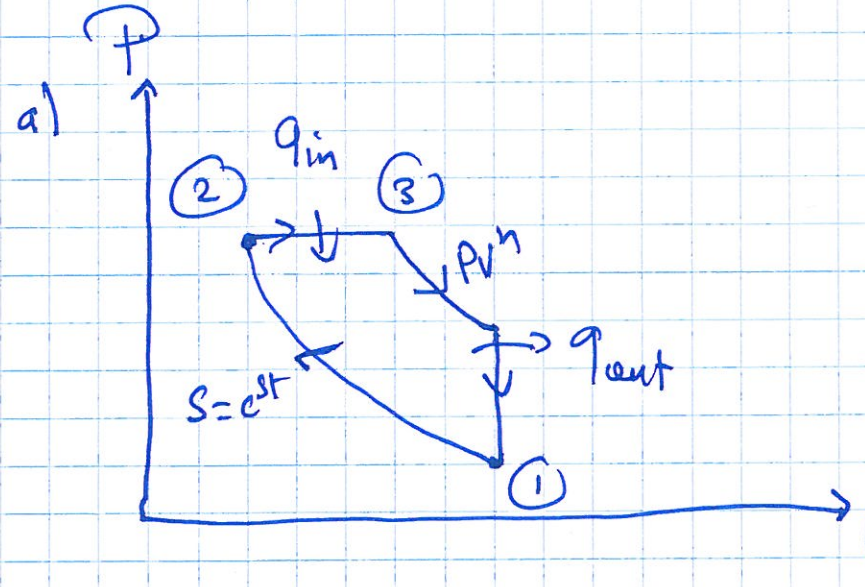
$C_v = 0,718 \ kJ/kg.K$

$R = 0,287 \ kJ/kg.K$

$C_p = C_v + R = 1,005 \ kJ/kg.K$

$K = C_p/C_v = 1,3997$

a)



b) (1) $P_1 = 100 kPa$, $T_1 = 300 K$, $P_1 v_1 = RT_1$

$$\Rightarrow v_1 = \frac{RT_1}{P_1} = \frac{0,287 \cdot 300}{100}$$

$$\boxed{v_1 = 0,861 \ m^3/kg}$$

② $V_2 = \frac{V_1}{11,63379}$ ou $v_2 = \frac{v_1}{11,63379}$ $\Rightarrow v_2 = \frac{0.861}{11,63379} = 0.074 \frac{m^3}{kg}$

$T_1 V_1^{k-1} = T_2 V_2^{k-1} \Rightarrow \boxed{T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{k-1}}$

$T_2 = 300 \left(11,63379\right)^{(1,3997-1)} = 800 K$ $\boxed{T_2 = 800K}$

$P_2 V_2 = R T_2 \Rightarrow \boxed{P_2 = \frac{R T_2}{V_2}}$ $P_2 = \frac{0287 \cdot 800}{0.074} = 3102.7 kPa$

$\boxed{P_2 = 3102.7 kPa}$

③ $\boxed{P_3 = P_2 = 3102.7 kPa}$

1er loi $\Delta U_{23} = Q_{23} - W_{33}$ (kJ)

$\Delta u_{23} = q_{23} - w_{23} = q_{23} - P(v_3 - v_2)$ (kJ/kg)

$u_3 - u_2 + P(v_3 - v_2) = q_{23}$

$h_3 - h_2 = c_p (T_3 - T_2) = q_{23}$

$\boxed{T_3 = \frac{q_{23}}{c_p} + T_2}$ $T_3 = \frac{954.75}{1,005} + 800 = 1750K$

$\boxed{T_3 = 1750K}$

④ $V_4 = V_1 = 0.861 m^3/kg.$

Polytropique 3-4 $\Rightarrow P_4 V_4^n = P_3 V_3^n \Rightarrow$

$P_4 = P_3 \cdot \left(\frac{V_3}{V_4}\right)^n$

$P_3 V_3 = RT_3 \Rightarrow \boxed{V_3 = \frac{RT_3}{P_3}}$ $\quad V_3 = \frac{0,287 \cdot 1710}{3102,7} = 0,16188 \frac{m^3}{kg}$

$V_3 = 0,16188\ m^3/kg.$

D'où $P_4 = 3102,7 \left(\frac{0,16188}{0,861}\right)^{1,334877} = 333,33\ kPa.$

$\boxed{P_4 = 333,33\ kPa}$ $\quad P_4 V_4 = RT_4 \Rightarrow \boxed{T_4 = \frac{P_4 V_4}{R}}$

$T_4 = \frac{333,33 \cdot 0,861}{0,287} = 1000K.$ $\quad \boxed{T_4 = 1000K}$

c) W_{12}, W_{23}, W_{34}, W_{41}

1-2 isentropique, 1ère loi $\Delta U_{12} = \cancel{Q_{12}}^{0} - W_{12}$ (Adiabatique)

$\boxed{W_{12} = U_1 - U_2 = C_v (T_1 - T_2)}$

$W_{12} = 0,718 (300 - 800) = \ominus 359\ kJ/kg.$ Sur l'air

Alternatif : $W_{12} = \frac{P_2 V_2 - P_1 V_1}{1-k} = \frac{3102,7 \cdot 0,074 - 100 \cdot 0,861}{1 - 1,3997} = \ominus 359 \frac{kJ}{kg}$

2-3 $P = C^{ste}$

$\boxed{W_{23} = P_3 (V_3 - V_2)}$

$W_{23} = 3102,7 (0,16188 - 0,074) = 272,66\ kJ/kg$ Par l'air

3-4 Polytropique

$W_{34} = \frac{P_4 V_4 - P_3 V_3}{1-n} = \frac{333,33\ 0,861 - 3102,7 \cdot 0,16188}{1 - 1,334877} = \oplus 642,83 \frac{kJ}{kg}$

$W_{34} = \oplus 642,83\ kJ/kg$ Par l'air

(4-1) volume constant $W_{41} = 0$

d) q_{12}, q_{23}, q_{34}, q_{41}.

$q_{12} = 0$ (adiabatique)

$q_{23} = 954{,}75$ KJ/Kg (donnée) à l'air

(3-4) 1ère loi $\Delta u_{34} = q_{34} - w_{34}$

$$q_{34} = \Delta u_{34} + w_{34} = C_v (T_4 - T_3) + w_{34}$$

$q_{34} = 0{,}718 (1000 - 1750) + 642{,}83 = 104{,}33 \frac{KJ}{Kg}$ à l'air

(4-1) 1ère loi $\Delta u_{41} = q_{41} - w_{41}$ ($V = cst$)

$$\Rightarrow q_{41} = \Delta u_{41} + w_{41} = C_v (T_1 - T_4) + \cancel{w_{41}}^{0}$$

$q_{41} = 0{,}718 (300 - 1000) = -502{,}6 \frac{KJ}{Kg}$ Par l'air

e) $\eta_{th} = ?$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{w_{12} + w_{23} + w_{34} + w_{41}}{q_{23} + q_{34}}$$

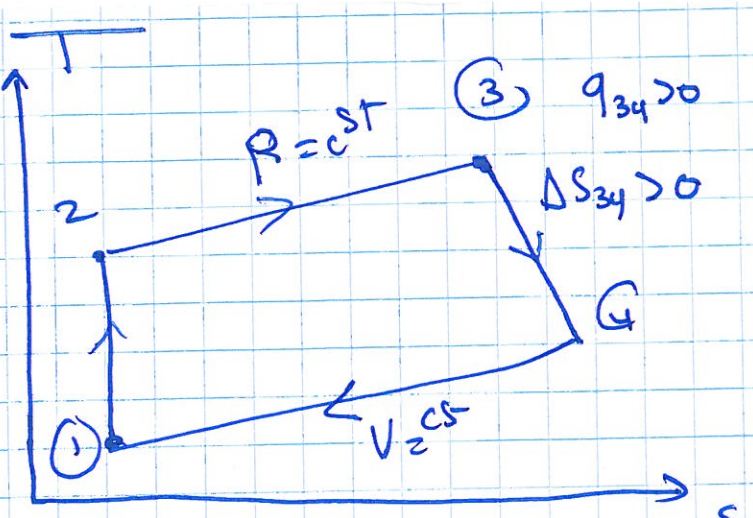
$$\eta_{th} = \frac{-359 + 272{,}66 + 642{,}83 + 0}{954{,}75 + 104{,}33} = 52{,}54\%$$

$\eta_{th} = 52{,}54\%$

Alternatif : $\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{q_{41}}{q_{23} + q_{34}} = 1 - \frac{502{,}6}{954{,}75 + 104{,}33}$

$\eta_{th} = 52{,}54\%$

f)



③ $q_{34} > 0$

$\Delta S_{34} > 0$

$$\Delta S_{34} = C_p \ln \frac{T_4}{T_3} - R \ln \frac{P_4}{P_3}$$

$$= 1{,}005 \cdot \ln \frac{1000}{780} - 0{,}287 \ln \frac{333{,}33}{302{,}7}$$

$$= 0.078 \ \frac{kJ}{kg.K} > 0$$

$$S_4 > S_3$$

Refaire b) avec $\overline{C_p}$ et C_v variables

① $P_1 = 100\ kPa$, $T_1 = 300K$ $\longrightarrow$ A17 $v_{r_1} = 621{,}2$, $v_1 = \frac{RT_1}{P_1} = 0{,}861$

$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r_1}} \Rightarrow v_{r2} = \frac{v_2}{v_1} v_{r_1} = \frac{621{,}2}{11{,}63333} = 53.38$$

T	v_r	h
760	55.54	778.18
X	53,38	h
780	51,64	800,03

$\Rightarrow$ **$X = T_2 = 770.99K$**

Interpolation

$h_2 = 790.19\ kJ/kg$

$$\frac{P_1 v_1}{RT_1} = \frac{P_2 v_2}{RT_2} \Rightarrow \boxed{P_2 = P_1 \cdot \frac{v_1}{v_2} \cdot \frac{T_2}{T_1}}$$

$$P_2 = 100 \cdot 11.63333 \cdot \frac{770.99}{300}$$

$$P_2 = 2989.85\ kPa$$

③ 1ère loi 2-3

$$\Delta u_{23} = q_{23} - w_{23} \Rightarrow \Delta u_{23} = q_{23} - P(v_3 - v_2)$$

$$q_{23} = u_3 - u_2 + Pv_3 - Pv_2 = h_3 - h_2 \Rightarrow \boxed{h_3 = q_{23} + h_2}$$

$$h_3 = 954,75 + 790,19 = 1744,94 \text{ kJ/kg}.$$

$$h_3 = 1744,94 \frac{\text{kJ}}{\text{kg}} \xrightarrow[\text{interpolation}]{\text{A17}} \boxed{T_3 = 1589,81 \text{ K}}$$

$$\boxed{P_3 = P_2 = 2989,85 \text{ kPa}}$$

④ $P_3 v_3^{\gamma} = P_4 v_4^{\gamma} \Rightarrow P_4 = P_3 \left(\frac{v_3}{v_4}\right)^{\gamma}$ avec $\gamma = 1,334877$

OR $v_4 = v_1 = 0,861 \text{ m}^3/\text{kg}$

$$P_3 v_3 = RT_3 \Rightarrow \boxed{v_3 = \frac{RT_3}{P_3}} = \frac{0,287 \cdot 1589,81}{2989,85} = 0,15261 \frac{\text{m}^3}{\text{kg}}$$

$$P_4 = 2989,85 \cdot \left(\frac{0,15261}{0,861}\right)^{1,334877} = 296,89 \text{ kPa}$$

$$\boxed{P_4 = 296,89 \text{ kPa}}$$

$$P_4 v_4 = RT_4 \Rightarrow \boxed{T_4 = \frac{P_4 v_4}{R}}$$

$$T_4 = \frac{296,89 \cdot 0,861}{0,287} = 890,67 \text{ K} \qquad \boxed{T_4 = 890,67 \text{ K}}$$