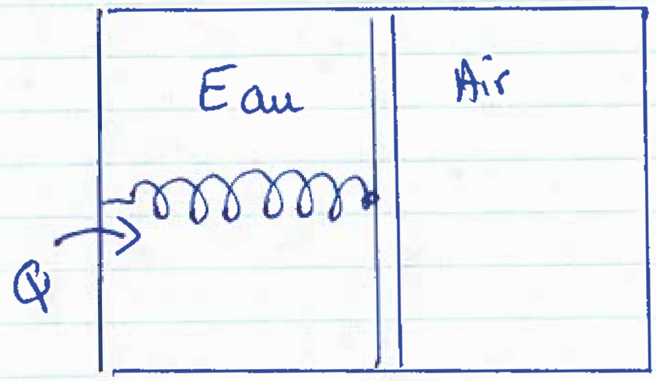
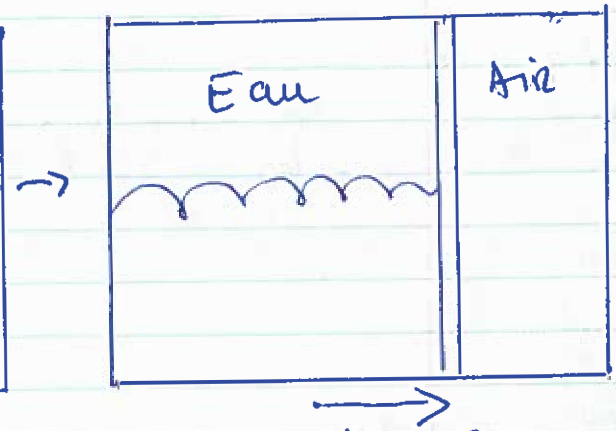
MEC 1210, TD2 (SOLUTIONNAIRE)





$M_e = 1\,kg$ initial

$\Delta X = 6{,}453\,cm$

final.

Eau $\begin{cases} T_{e_1} = 130°C \\ X_{e_1} = 0.7 \end{cases}$

Eau $\begin{cases} \\ \end{cases}$

Air $\begin{cases} V_{a_1} = 0{,}051\,m^3 \\ T_{a_1} = 27°C \end{cases}$

Air $\begin{cases} P_{a_2} = 400\,kPa. \\ \end{cases}$

Hypothèses:

$K = 154{,}967\,kN/m$

$A = 0.1\,m^2$

Évolutions quasi-statiques

$\Delta E_p, \Delta E_c = 0$ (Eau et Air)

Air gaz parfait à C_p et C_v = Constantes

$C_p = 1{,}005\,kJ/kg.K$, $R = 0{,}287\,kJ/kg.K$.

a) $M_a = ?$

b) T_{a_2}, T_{e_2} : ?

c) W_b sur l'air

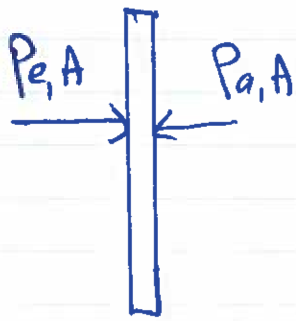
d) $Q = ?$

a) $M_a = ?$

Air gaz parfait $\Rightarrow P_{a_1} V_{a_1} = M_a R T_{a_1} \Rightarrow$ $\boxed{M_a = \frac{P_{a_1} V_{a_1}}{R T_{a_1}}}$

$P_{a_1} = ?$

à l'état ①



$\Sigma F = 0$

$\Rightarrow P_{e_1} A - P_{a_1} A = 0$

$\Rightarrow \boxed{P_{e_1} = P_{a_1}}$

état ① eau mélange saturé $\Rightarrow$ $\boxed{P_{e_1} = P_{sat @ 130°C} = 270.28 \text{ kPa}}$

$\boxed{P_{a_1} = 270.28 \text{ kPa}}$

$$M_a = \frac{270{,}28 \cdot 0{,}051}{0{,}287(27+273)} \simeq 0{,}16 \text{ kg}$$

$\boxed{M_a = 0{,}16 \text{ kg}}$

b) T_{a_2}, T_{e_2}

Air gaz parfait $P_{a_2} V_{a_2} = M_a R T_{a_2}$

$\Rightarrow$ $\boxed{T_{a_2} = \frac{P_{a_2} V_{a_2}}{M_a R}}$

$\boxed{V_{a_2} = V_{a_1} - A \Delta x}$

$V_{a_2} = 0{,}051 - 0{,}1 \cdot 0.06453 =$

$\boxed{V_{a_2} = 0{,}044547 \text{ m}^3}$

$$T_{a_2} = \frac{400 \cdot 0{,}044547}{0{,}16 \cdot 0{,}287} \simeq 388 \text{ K}$$

$\boxed{T_{a_2} = 388 \text{ K}}$

$T_{e_2} = ?$ (Il faut deux variables)

$v_{e_2} = \frac{V_{e_2}}{M_e}$, $V_{e_2} = V_{e_1} + A \cdot \Delta x$

$V_{e_1} = M_e \, v_{e_1} = M_e \left(v_f + x_{e_1} \cdot (v_g - v_f) \right)$

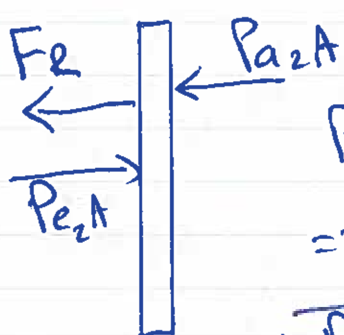
$\xrightarrow[130°C.]{A4}$ $V_{e_1} = 1 \left(0{,}001070 + 0{,}7 \left(0{,}66808 - 0{,}001070 \right) \right)$

$= 0{,}467977 \, m^3$

$V_{e_2} = 0{,}467977 + 0{,}1 \cdot 0{,}06453 = 0{,}47443 \, m^3$

$v_{e_2} = \frac{V_{e_1}}{M_e} = \frac{0{,}47443}{1} = 0{,}47443 \, m^3/kg$

état (2)



$\Sigma F = 0$

$P_{e_2} A - P_{a_2} A - F_r = 0$

$\Rightarrow P_{e_2} = P_{a_2} + \frac{F_r}{A}$

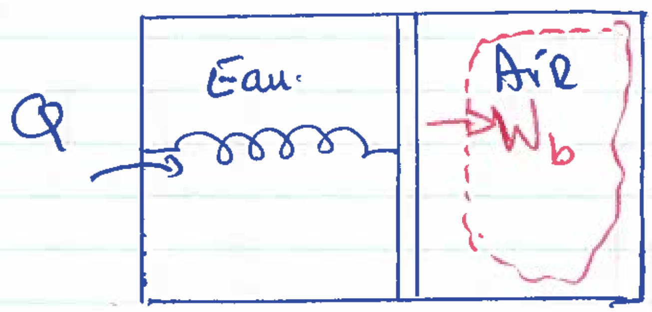
$\boxed{P_{e_2} = P_{a_2} + \frac{k \Delta x}{A}}$

$P_{e_2} = 400 + \frac{154.967 \cdot 0{,}06453}{0{,}1} \simeq 500 \, kPa$ $\boxed{P_{e_2} = 500 \, kPa}$

(2) $\begin{cases} P_{e_2} = 500 \, kPa \\ v_{e_2} = 0{,}47443 \, \frac{m^3}{kg} \end{cases}$ $\xrightarrow{A5}$ $v_{e_2} > v_g \Rightarrow$ Valeur surchauffée

$\xrightarrow{A6}$ $\boxed{T_{e_2} = 250°C}$

c) $W_{sur.Air}$



On ne peut pas calculer le travail fait sur l'air avec la formulation $\int P dV$, car on ne connait pas comment évolue P avec le volume V.

On passe alors par la première loi en choisissant l'air comme système.

Système Air

1ère loi $\Delta U_a + \cancel{\Delta E_c} + \cancel{\Delta E_p} = |E_{in}| - |E_{out}|$

$\Delta U_a = W_b \Rightarrow M_a C_v (T_{a_2} - T_{a_1}) = W_b$

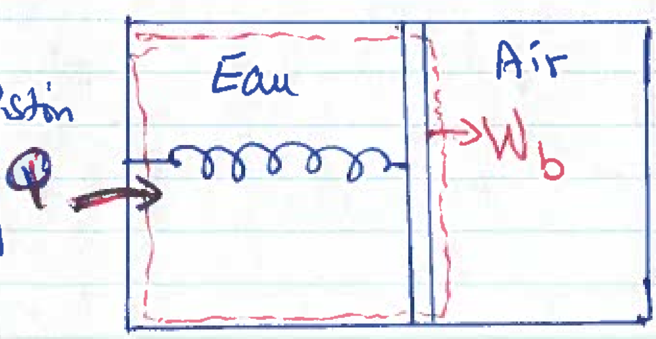
$$\boxed{M_a (C_p - R)(T_{a_2} - T_{a_1}) = W_b}$$

$W_b = 0.16 \cdot (1{,}005 - 0{,}287)(388 - 300) \simeq 10{,}11 \text{ KJ}$

$$\boxed{W_b = 10.11 \text{ KJ}}$$

d) $Q = ?$

Système Eau + Ressort + Piston



$\Delta U + \cancel{\Delta E_c} + \Delta E_p = |E_{in}| - |E_{out}|$

$\Delta U_e + \frac{1}{2} K (\Delta x)^2 = Q - W_b$

$$M_e(u_{e_2}-u_{e_1})+\frac{1}{2}k(\Delta x)^2+W_b=Q$$

$\xrightarrow[130°C]{A4}$ $u_{e_1}=u_{f_1}+x_{e_1}(u_{fg})$

$u_{e_1}=546.10+0{,}7\cdot1993.4=1941.48$ kJ/kg

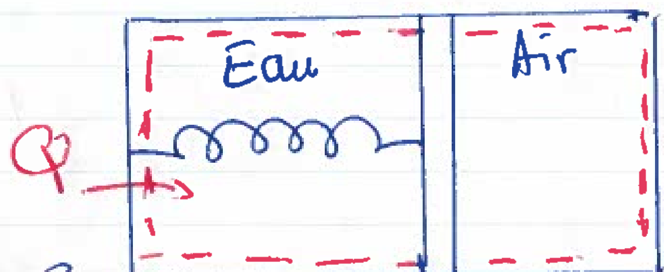
$$u_{e_1}=1941.48 \text{ kJ/kg}$$

état (2) $\xrightarrow{A6}$ $u_{e_2}=2723{,}8$ kJ/kg.

$Q=1\cdot(2723{,}8-1941{,}48)+\frac{1}{2}151967\cdot0{,}06453^2+10{,}11$

$$Q=792.753 \text{ kJ}$$

Solution alternative.
Système tout



$\Delta U_e+\Delta U_a+\Delta Ep_{air}+\Delta Ep_{eau}+\Delta Ec_{eau}+\Delta Ec_{air}+\Delta Ep_e=|E_{in}|-|E_{out}|$

$\Delta U_e+\Delta U_a+\frac{1}{2}k\Delta x^2=Q$

$M_e(u_{e_2}-u_{e_1})+M_a C_v(T_{a_2}-T_{a_1})+\frac{1}{2}k(\Delta x)^2=Q$

$C_v = C_p - R.$

$Q=1(2723{,}8-1941{,}48)+0{,}16(1{,}005-0{,}287)(388-300)+\frac{1}{2}154{,}967\cdot0{,}06453^2$

$$Q=792{,}753 \text{ kJ}$$