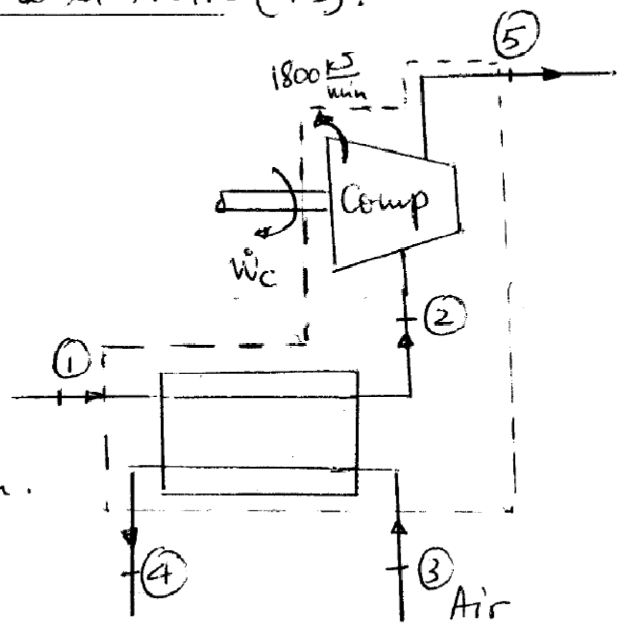
MEC1210:TD4(Solutionnaire)

① Température à l'entrée de la turbine (T5):

_hypothèse :

- Régime permanent
- pas de variation d'énergie cinétique et potentielle pour l'air et l'eau.

- pas de perte de pression dans l'échangeur de chaleur.
- Air est un gaz parfait à Cp et Cv variables !



syst = Échangeur de chaleur + compresseur.

Bilan d'énergie :

$$\dot{W}_{in} + \cancel{\dot{Q}_{in}}^{0} + \sum_{in} \dot{m}\theta = \cancel{\dot{W}_{out}}^{0} + \dot{Q}_{out} + \sum_{out} \dot{m}\theta$$

$$\theta = h + \cancel{\frac{V^2}{2}}^{0} + \cancel{gz}^{0} = h$$

$$\dot{W}_{comp} + \dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{Q}_{out} + \dot{m}_5 h_5 + \dot{m}_4 h_4 .$$

Bilan de masse : $\dot{m}_1 = \dot{m}_5 = \dot{m}_e$
$\dot{m}_3 = \dot{m}_4 = \dot{m}_a$.

Donc : $\dot{W}_c + \dot{m}_a(h_3 - h_4) - \dot{Q}_{out} = \dot{m}_e(h_5 - h_1)$

$$h_5 = h_1 + \frac{\dot{m}_a}{\dot{m}_e}(h_3 - h_4) + \frac{\dot{W}_c - \dot{Q}_{out}}{\dot{m}_e}$$

h_1 ?

$$\begin{cases} P_1 = 2\,MPa \\ x_1 = 0 \end{cases} \longrightarrow \begin{cases} v_1 = v_{f@2MPa} = 0{,}001177\ m^3/kg. \\ s_1 = s_{f@2MPa} = 2{,}4467\ kJ/kg.K \\ h_1 = h_{f@2MPa} = 908{,}47\ kJ/kg. \end{cases}$$

$$\dot{m}_e = \frac{\dot{V}e}{v_1} = \frac{8{,}239\ 10^{-3}\left(\frac{m^3}{s}\right)}{0{,}001177\left(m^3/kg\right)} = 7\ kg/s.$$

$\dot{m}a$?

$$P_3\dot{V}_3 = \dot{m}a\,R\,T_3 \longrightarrow \dot{m}a = \frac{P_3\dot{V}_3}{R\,T_3}$$

$$\dot{m}a = \frac{(100\,kPa)\left(136{,}1815\ \frac{m^3}{s}\right)}{\left(0{,}287\ \frac{kJ}{kg.K}\right)(457+273)} = 65\ kg/s.$$

h_3 ? h_4 ?

Air est un gaz parfait à C_p et C_v variable $\longrightarrow$ A17.

$$T_3 = 457+273 = 730\,K \xrightarrow{A17} \begin{cases} h_3 = 745{,}62\ kJ/kg. \\ s_3^o = 2{,}61803\ kJ/kg.K \end{cases}$$

$$T_4 = 247+273 = 520\,K \xrightarrow{A17} \begin{cases} h_4 = 523{,}63\ kJ/kg. \\ s_4^o = 2{,}25997\ kJ/kg.K. \end{cases}$$

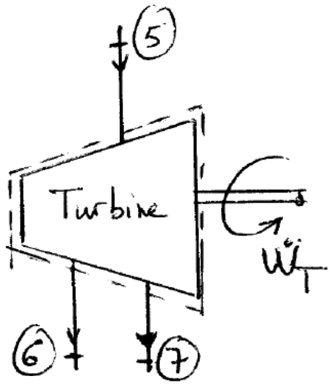
$$h_5 = 908{,}47 + \frac{(65)}{(7)}\left[745{,}62 - 523{,}63\right] + \frac{548{,}66 - \frac{1800}{60}}{(7)}$$

$$h_5 = (908{,}47) + 2061{,}3357 + 74{,}0942 = 3043{,}9\ \frac{kJ}{kg}.$$

$$\text{état } 5 \begin{cases} P_5 = 6\,MPa \longrightarrow h_g = 2784{,}6\ kJ/kg \\ h_5 = 3043{,}9\ kJ/kg \text{ et : } h_5 > h_g \longrightarrow \text{Vapeur surchauffée.} \end{cases}$$

$\underrightarrow{A6}$ $\boxed{T_5 = 350°C}$

② puissance de la turbine $(\dot{W}_T)$:



Bilan d'énergie :

$$\dot{W}_{in}^{\nearrow 0} + \dot{Q}_{in}^{\nearrow 0} + \sum_{in} \dot{m} E = \dot{W}_{out} + \dot{Q}_{out}^{\nearrow 0} + \sum_{out} \dot{m} E$$

$$E = h + \frac{V^2}{2} + gz$$

$$\dot{m}_5 h_5 = \dot{m}_6 h_6 + \dot{m}_7 h_7 + \dot{W}_T$$

syst = turbine .

Bilan de masse :

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\dot{m}_5 = \dot{m}_e = \dot{m}_6 + \dot{m}_7 = 0,15\,\dot{m}_e + 0,85\,\dot{m}_e$$

$$\dot{m}_e h_5 = 0,15\,\dot{m}_e h_6 + 0,85\,\dot{m}_e h_7 + \dot{W}_T$$

$$\dot{W}_T = \dot{m}_e (h_5 - 0,15 h_6 - 0,85 h_7)$$

h_6 ? h_7 ?

état ⑥ $\begin{cases} P_6 = 1,2\,MPa \longrightarrow T_{sat@1,2MPa} = 187,96\,°C \\ T_6 = 200°C \longrightarrow T_6 > T_{sat} \longrightarrow \text{vapeur surchauffée} \end{cases}$

$(A6) \longrightarrow \begin{cases} h_6 = 2816,1 \ kJ/kg . \\ s_6 = 6,5909 \ kJ/kg \cdot K . \end{cases}$

État ⑦ $\begin{cases} P_7 = 100kPa \\ x_7 = 0,9 \end{cases} \longrightarrow \begin{cases} h_7 = h_{f@100kPa} + x_7\, h_{fg@100kPa} \\ h_7 = 417,51 + (0,9)(2257,5) = 2449,26 \ \frac{kJ}{kg} . \\ s_7 = 1,3028 + (0,9)(6,0562) \\ s_7 = 6,75338 \ kJ/kg \cdot K . \end{cases}$

$$\dot{W}_T = \left(7\,\tfrac{kg}{s}\right)\left[3043{,}9 - (0{,}15)(2816{,}1) - (0{,}85)(2449{,}26)\right]$$

$$= \underset{(kg/s)}{(7)}\left[3043{,}9 - \underset{(kJ/kg)}{422{,}415 - 2081{,}871}\right] = 3777{,}3\ kW$$

$$\boxed{\dot{W}_T = 3{,}77\ MW}$$

③ Taux de production totale d'entropie.

Bilan d'entropie pour un syst ouvert: $\boxed{\text{syst} = \text{tout}}$

$$\dot{S}_{syst} = \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \int \frac{\dot{Q}}{T} + \dot{S}_{gén} = 0 \quad (\text{Régime permanent})$$

$$\longrightarrow \dot{S}_{gén} = \sum_{out} \dot{m}s - \sum_{in} \dot{m}s - \int \frac{\dot{Q}}{T}$$

$$= (\dot{m}_7 s_7 + \dot{m}_6 s_6 + \dot{m}_4 s_4) - (\dot{m}_1 s_1 + \dot{m}_3 s_3) - \frac{-\frac{1800}{60}(kW)}{(25+273)}$$

Bilan de masse :

$\dot{m}_1 = \dot{m}_e$; $\dot{m}_7 = 0{,}85\,\dot{m}_e$; $\dot{m}_6 = 0{,}15\,\dot{m}_e$; $\dot{m}_3 = \dot{m}_4 = \dot{m}_a$.

$$\dot{S}_{gén} = \dot{m}_e(0{,}85\,s_7 + 0{,}15\,s_6 - s_1) + \dot{m}_a(s_4 - s_3) + \frac{30}{298}$$

$$s_4 - s_3 = s_4^\circ - s_3^\circ - \cancel{R \ln \frac{P_4}{P_3}} = s_4^\circ - s_3^\circ$$

$$\dot{S}_{gén} = \left(7\,\tfrac{kg}{s}\right)\left[(0{,}85)(6{,}75338) + (0{,}15)(6{,}5909) - 2{,}4467\right] +$$

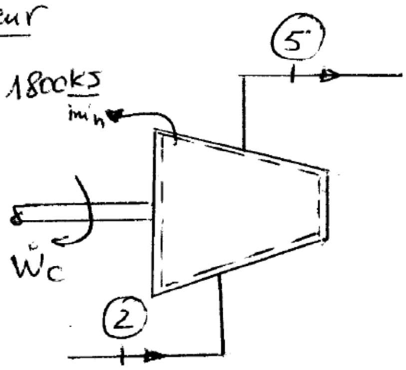
$$+ (65)\left[2{,}25997 - 2{,}61803\right] + 0{,}1$$

$$= (7)(5{,}7403 + 0{,}9886 - 2{,}4467) + (-23{,}2739) + 0{,}1$$

$$\boxed{\dot{S}_{gén} = 6{,}8\ \frac{kW}{K}}$$

Solution Alternative pour ①:

Bilan d'énergie : syst = Compresseur



$$\dot W_{in} + \cancelto{0}{\dot Q_{in}} + \sum_{in} \dot m \theta = \cancelto{0}{\dot W_{out}} + \dot Q_{out} + \sum_{out} \dot m \theta .$$

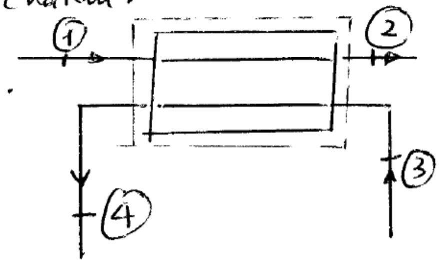
$$\theta = h + \frac{V^2}{2} + gz = h$$

$$\rightarrow \dot W_C + \dot m_2 h_2 = \dot Q_{out} + \dot m_5 h_5 .$$

$$h_5 = h_2 + \frac{\dot W_C - \dot Q_{out}}{\dot m_e} \quad (\text{car} : \dot m_2 = \dot m_5 = \dot m_e).$$

$$h_2 = ?$$

Bilan d'énergie : syst = Échangeur de chaleur.



$$\cancelto{0}{\dot W_{in}} + \cancelto{0}{\dot Q_{in}} + \sum_{in} \dot m \theta = \cancelto{0}{\dot W_{out}} + \cancelto{0}{\dot Q_{out}} + \sum_{out} \dot m \theta .$$

$$\dot m_1 h_1 + \dot m_3 h_3 = \dot m_2 h_2 + \dot m_4 h_4$$

Bilan de masse : $\begin{cases} \dot m_1 = \dot m_2 = \dot m_e \\ \dot m_3 = \dot m_4 = \dot m_a . \end{cases}$

$$h_2 = h_1 + \frac{\dot m_a}{\dot m_e}(h_3 - h_4) = 908{,}47 + \frac{(65)}{(7)}\left[745{,}62 - 523{,}63\right]$$

$$h_2 = 2969{,}805 \text{ kJ/kg} .$$

$$h_5 = 2969{,}805 + \frac{548{,}66 - \frac{1800}{60}}{(7)} = 2969{,}805 + 74{,}094$$

$$h_5 = 3043{,}9 \text{ kJ/kg} .$$

état⑤ $\begin{cases} P_5 = 6\,MPa \longrightarrow h_{g@6MPa} = 2784{,}6 \text{ kJ/kg} \\ h_5 = 3043{,}9 \text{ kJ/kg} > h_g \longrightarrow \text{Vap. surchauffée} \end{cases}$

$$\longrightarrow \boxed{T_5 = 350°C}$$