



Turbomachinery Lecture Series
Gas Turbine Engine Design & Development

PART 2

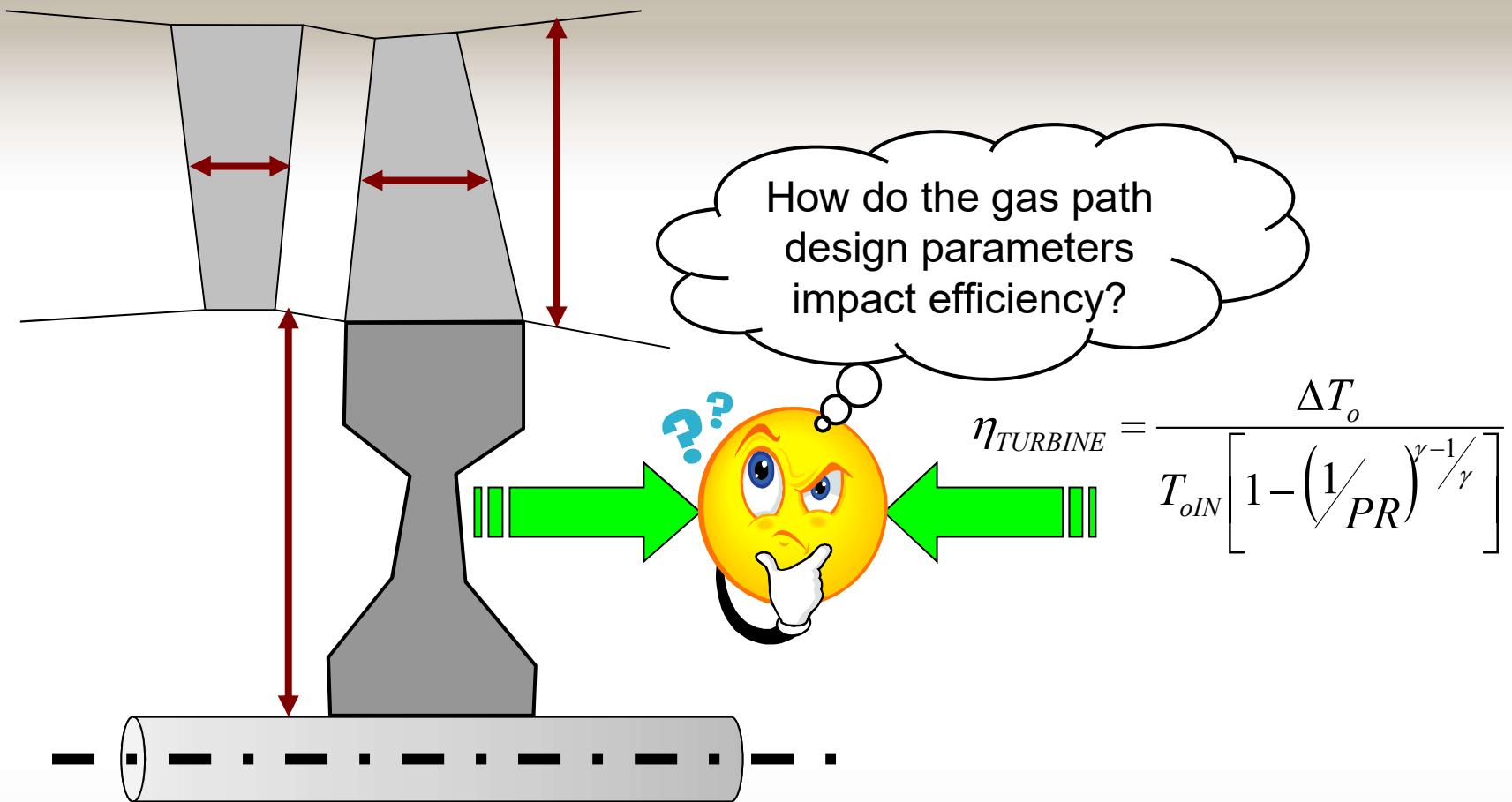


“A mean line efficiency prediction method is the sum of a large number of loss components. While some of them may prove to be quantitatively imperfect, the manner in which they are combined may cause errors to cancel. The final proof of a loss system must be its ability to correctly predict the efficiencies of well documented turbines [or compressors]”- **Kacker & Okapuu**

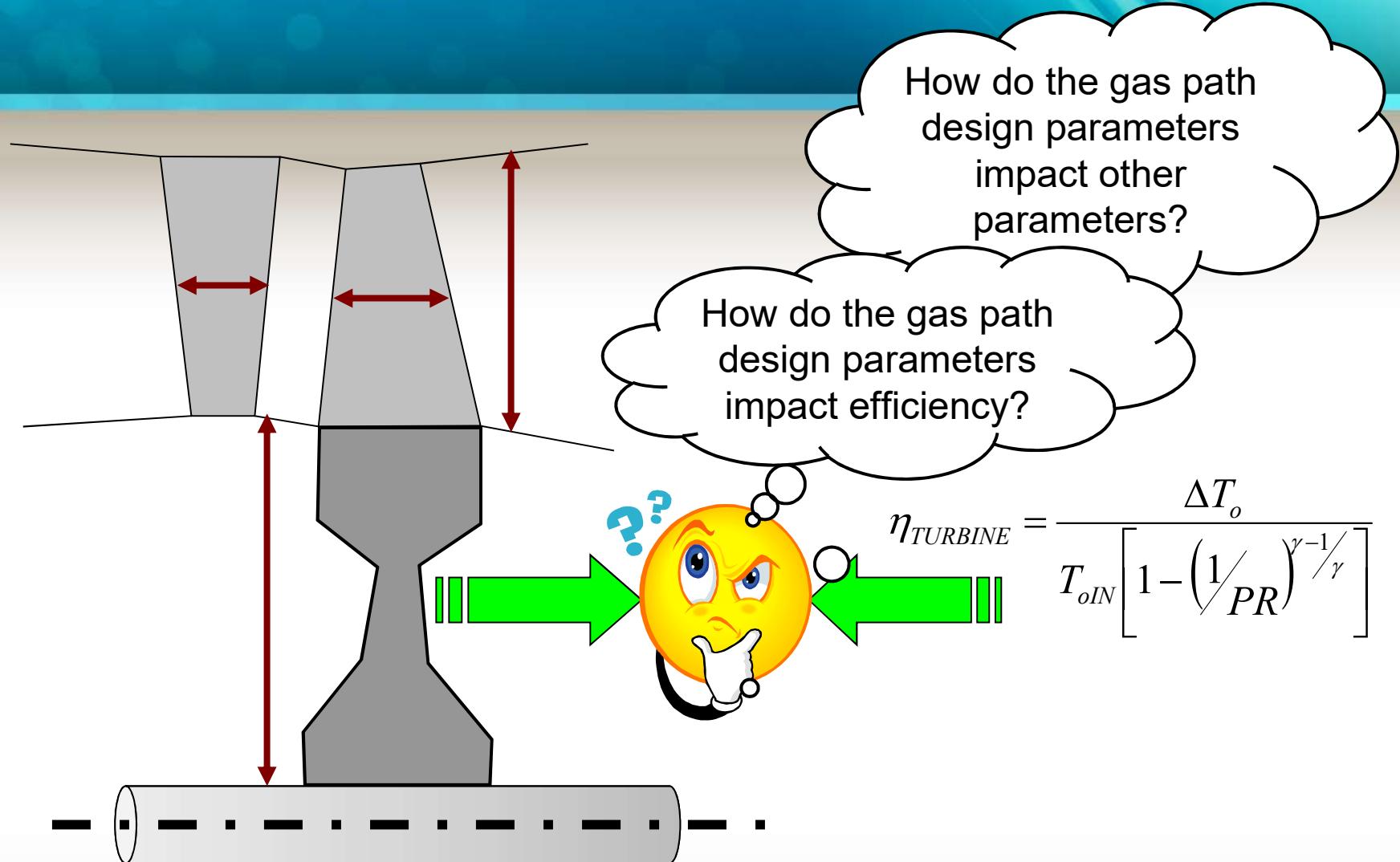
Aerodynamics

TURBINES

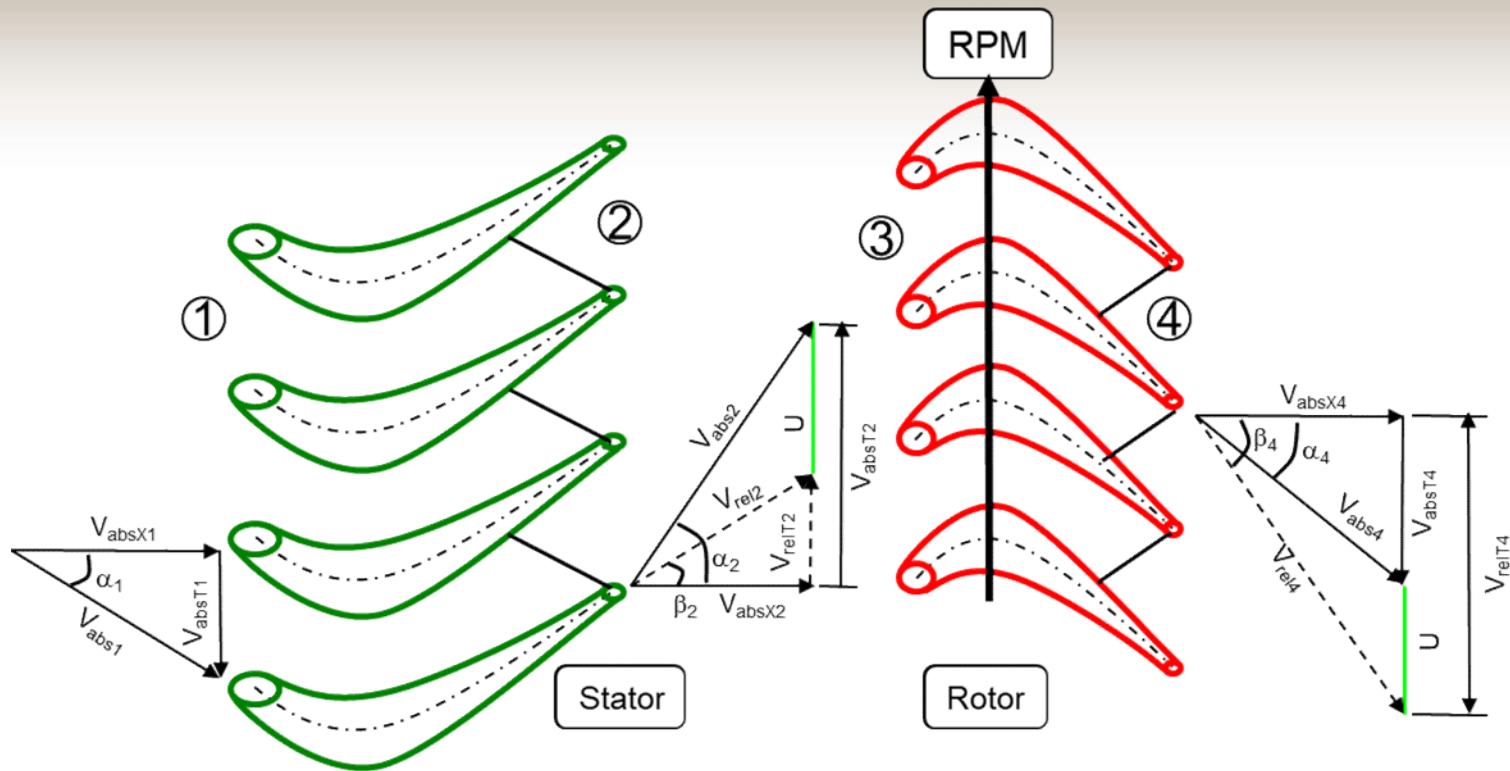
Aerodynamics and loss modeling



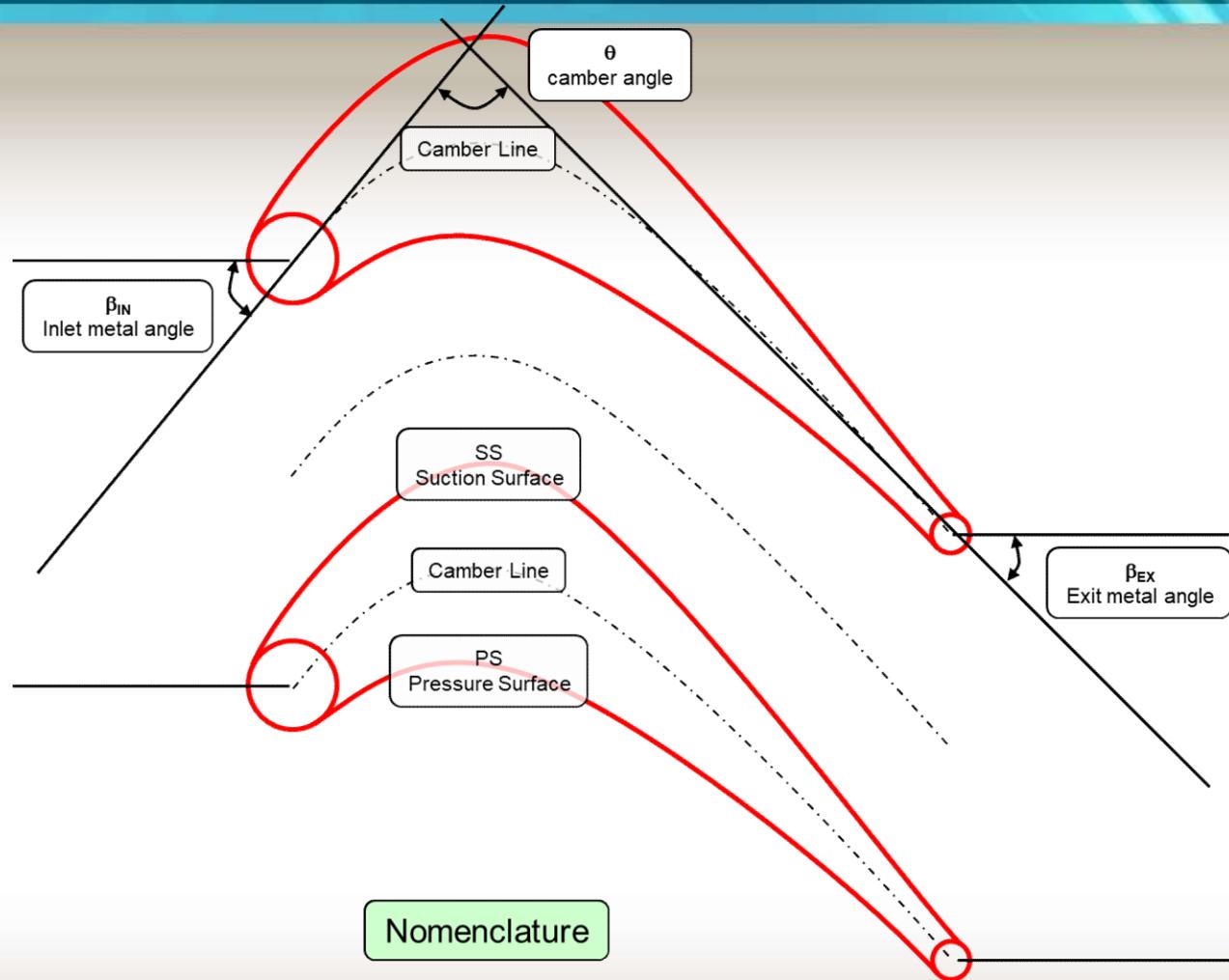
Aerodynamics and loss modeling



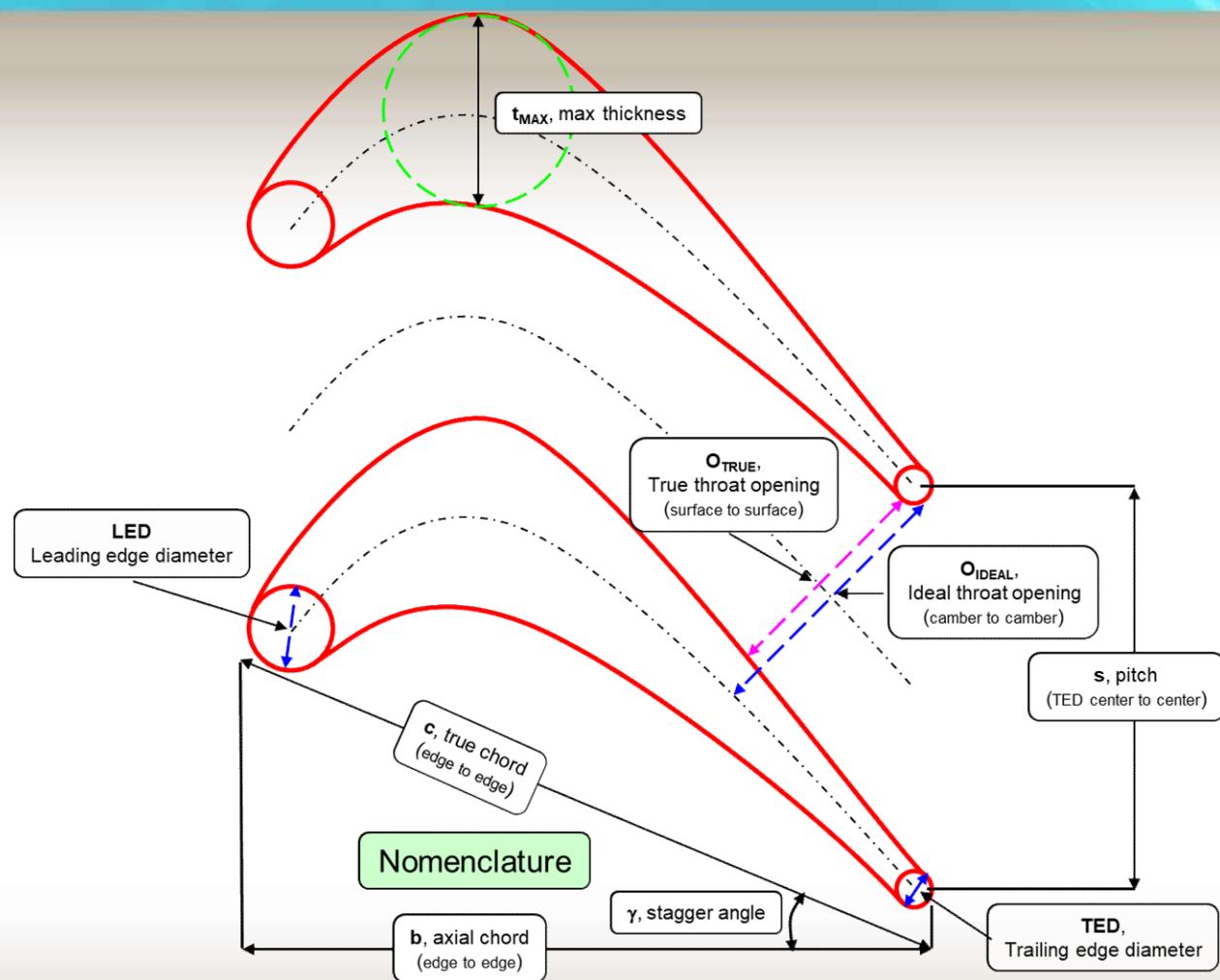
Turbine Stage



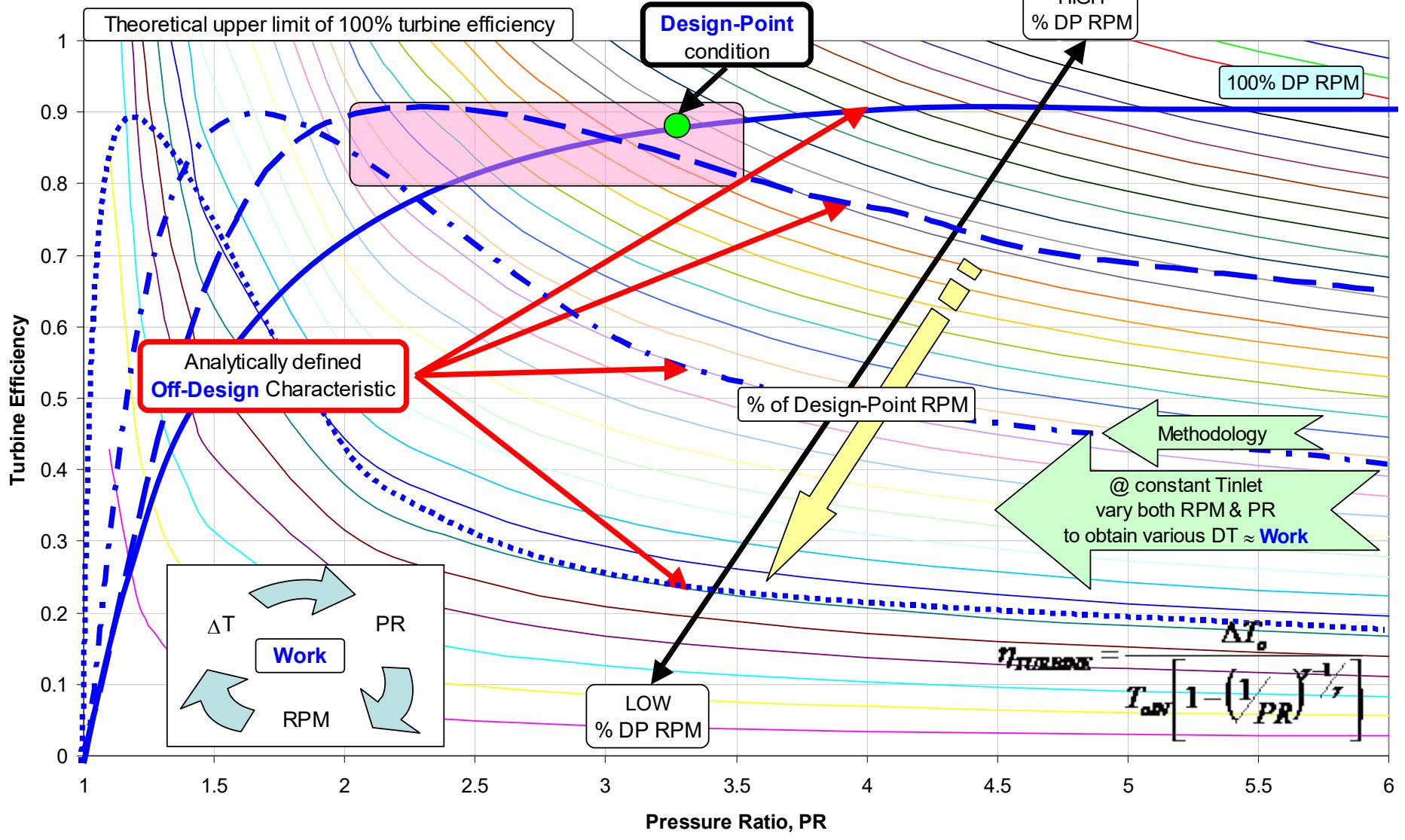
Turbine Blade Design



Turbine Blade Design



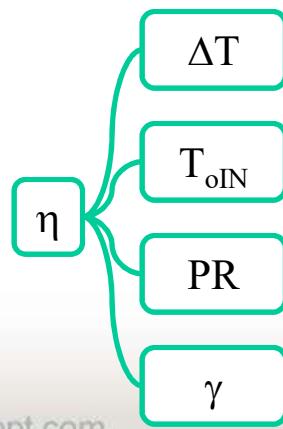
Turbine Efficiency & the Physical Design Space



$$\eta_{TURBINE} = \frac{\Delta T_o}{T_{oIN} \left[1 - \left(\frac{1}{PR} \right)^{1/\gamma} \right]}$$

Turbine Efficiency

After much algebraic manipulation

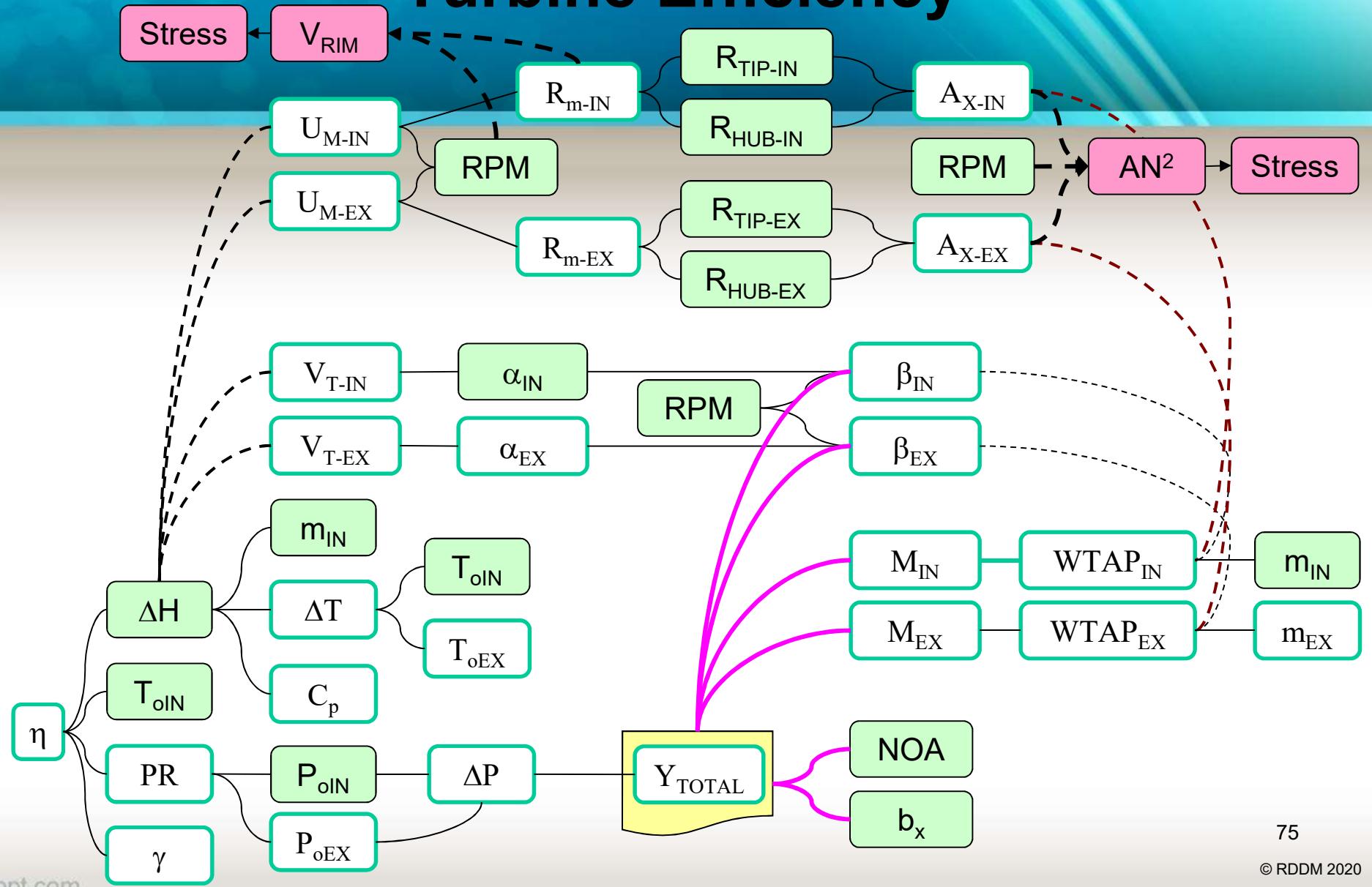


$$\Delta H = \dot{m} (C_p T_{oIN} - C_p T_{oEX}) = \dot{m} C_p (T_{oIN} - T_{oEX}) = \dot{m} C_p \Delta T$$

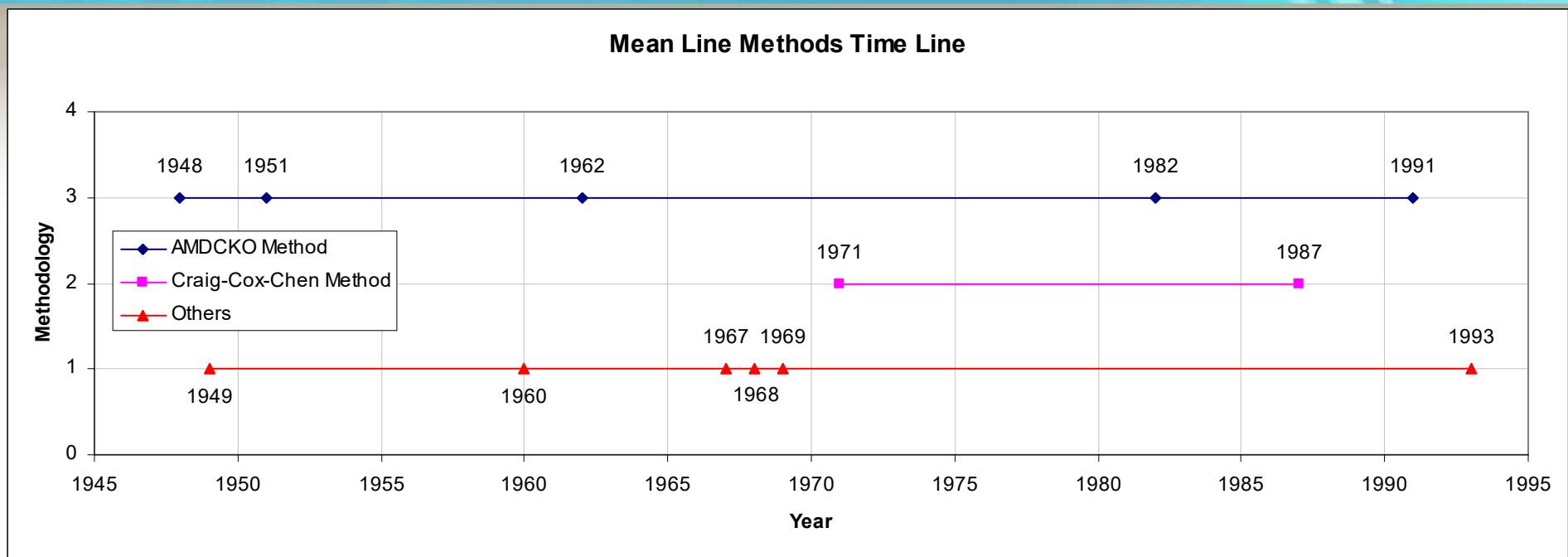
$$(C_p T_{oIN} - C_p T_{oEX}) = (V_{TANG-IN} U_{IN} - V_{TANG-EX} U_{EX}) = [V_{TANG-IN}(\omega R_{IN}) - V_{TANG-EX}(\omega R_{EX})]$$

$$\eta_{TURBINE} = \frac{\Delta T_o}{T_{oIN} \left[1 - \left(\frac{1}{PR} \right)^{\gamma-1/\gamma} \right]}$$

Turbine Efficiency

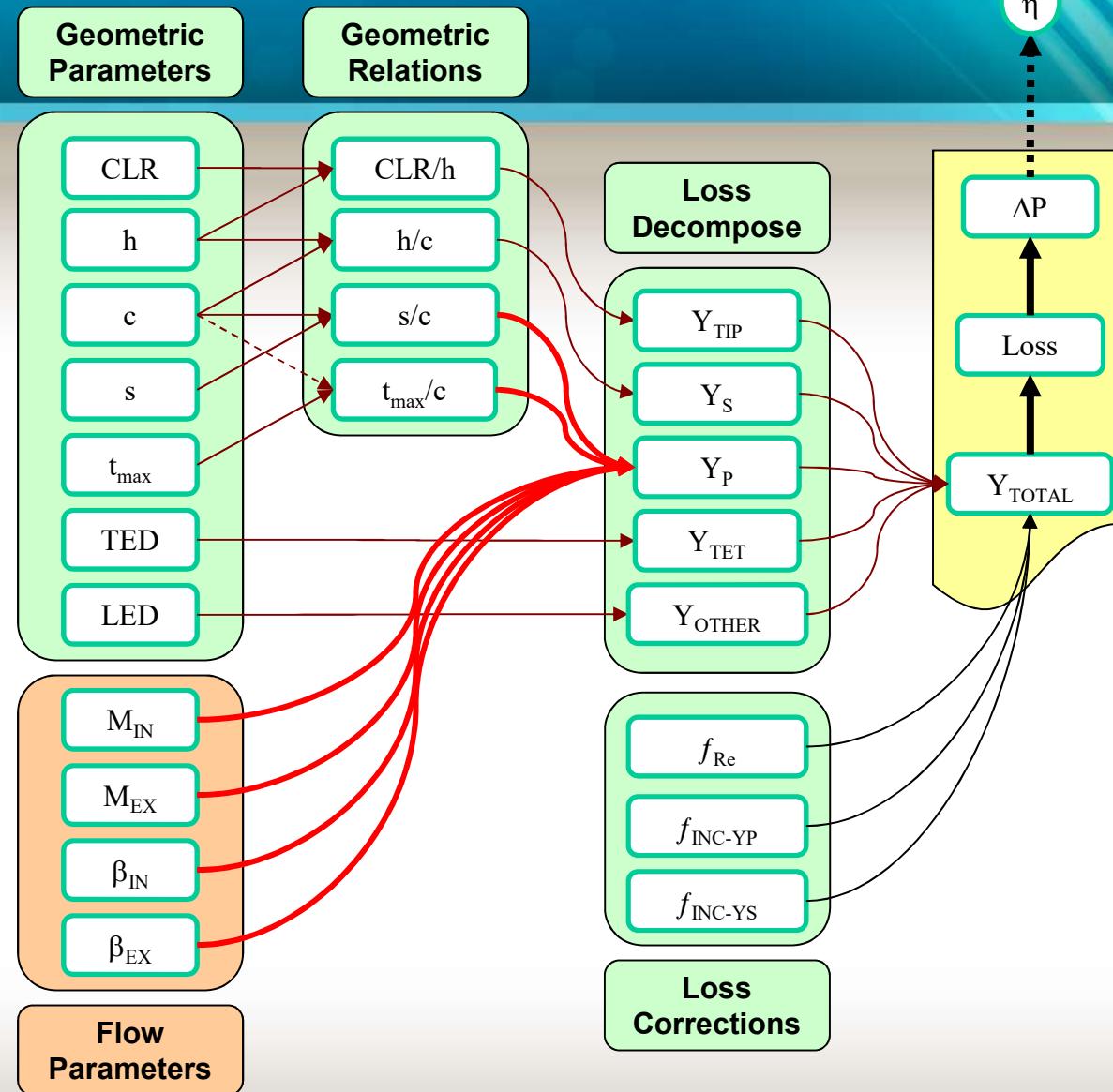


Turbine Loss Models



- Ainley & Matheison (AM)** $Y_{TOTAL} = (Y_P + Y_S + Y_{TC})Y_{TE}$ (1951) 1945 Zweifel
- Dunhum & Came (DC)** $Y_{TOTAL} = [(Y_P + Y_S)REFAC + Y_{TC}]Y_{TE}$ (1970) $\therefore \psi_T = 2 \cdot \cos^2 \beta_2 [\tan \beta_2 + \tan \beta_1] \cdot \frac{s}{b_x}$
- Kacker & Okapuu (KO)** $Y_{TOTAL} = Y_P f_{(Re)} + Y_S + Y'_{TE} + Y_{TC}$ (1982)

Turbine Loss Decomposition



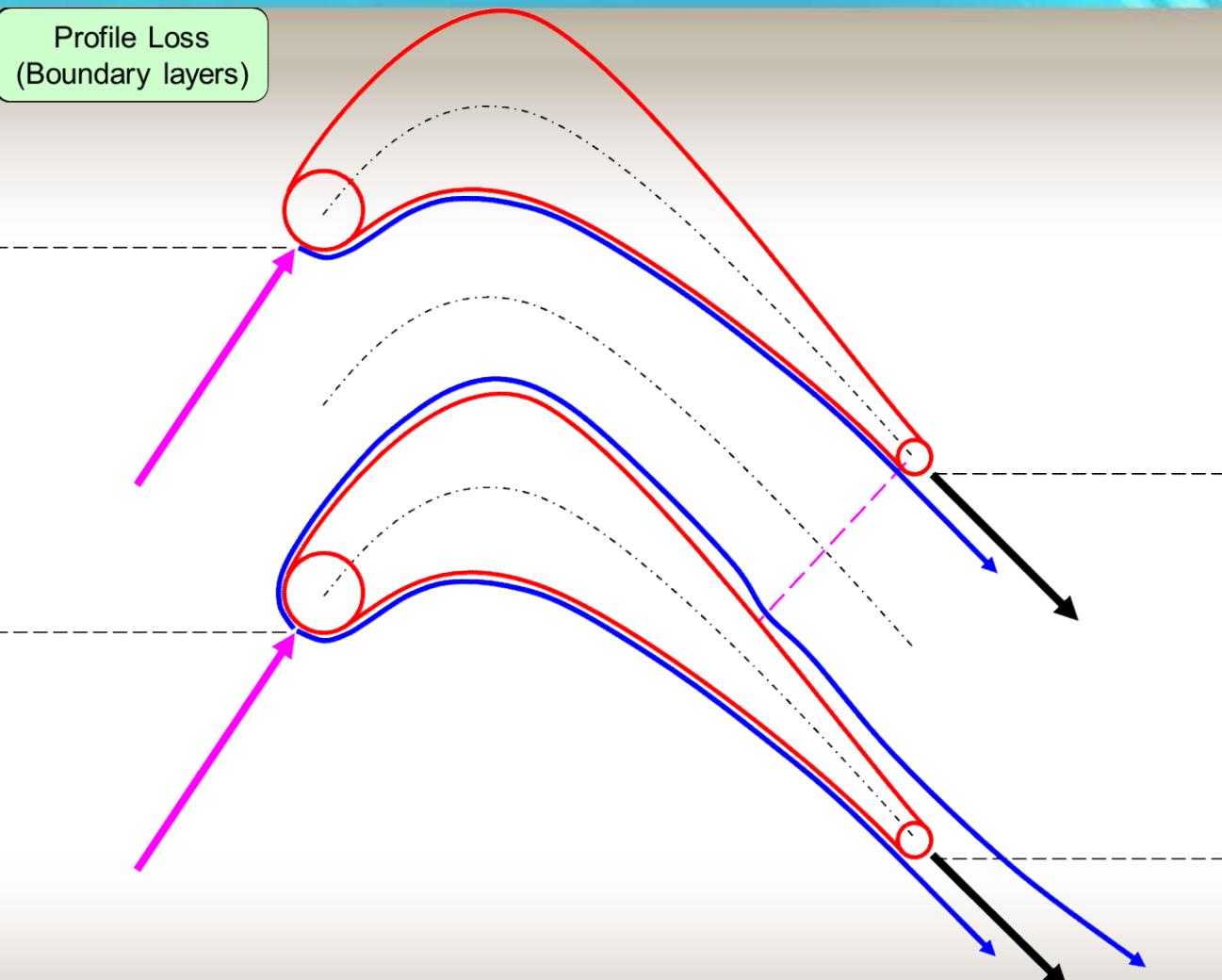
Turbine Loss Decomposition

Turbine losses can be classically decomposed into the following loss components

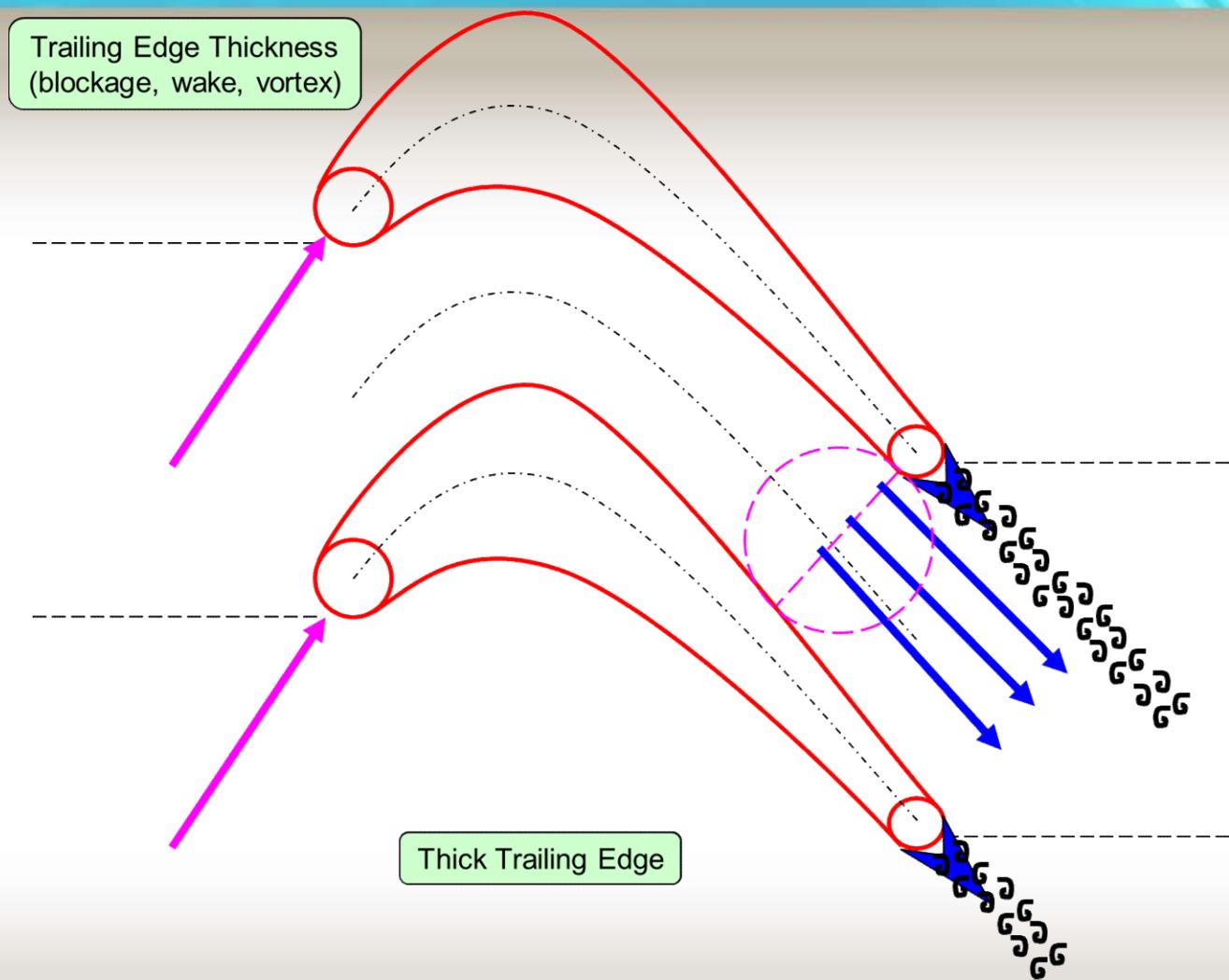
- Profile
- Secondary
- Trailing Edge
- Tip Clearance
- Other
- Y_P
- Y_S
- Y_{TET}
- Y_{TIP}
- Y_{OTHER}

Both geometric parameters and flow values have an impact on loss.

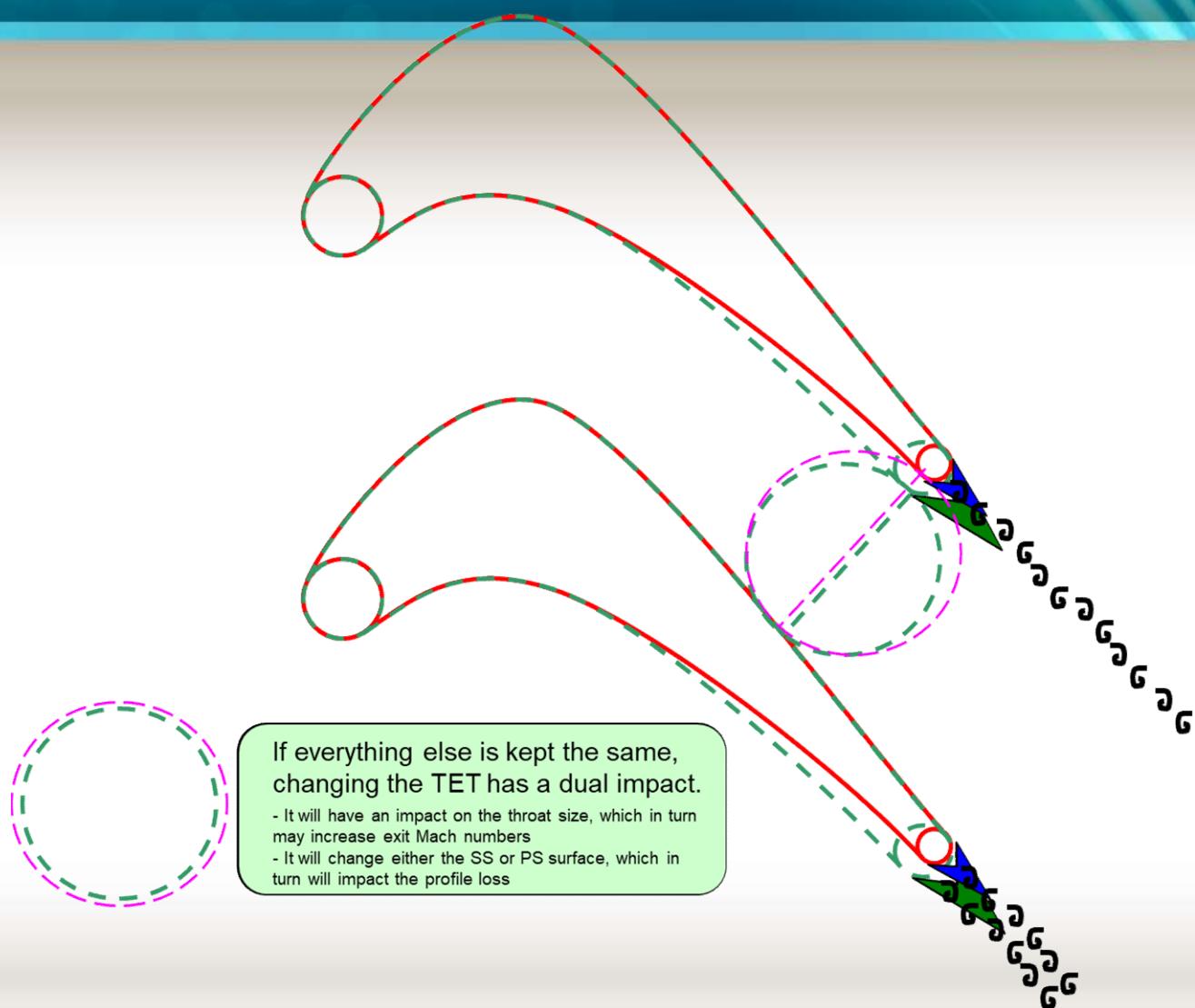
Turbine Losses



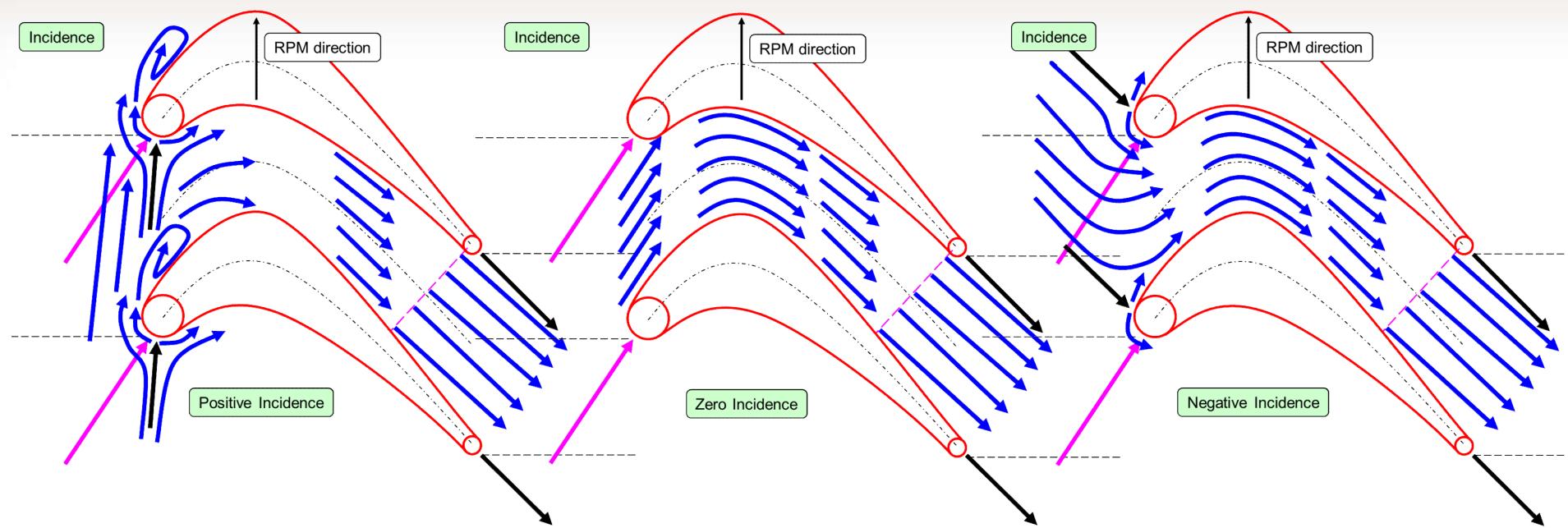
Turbine Losses



Turbine Losses



Turbine Losses

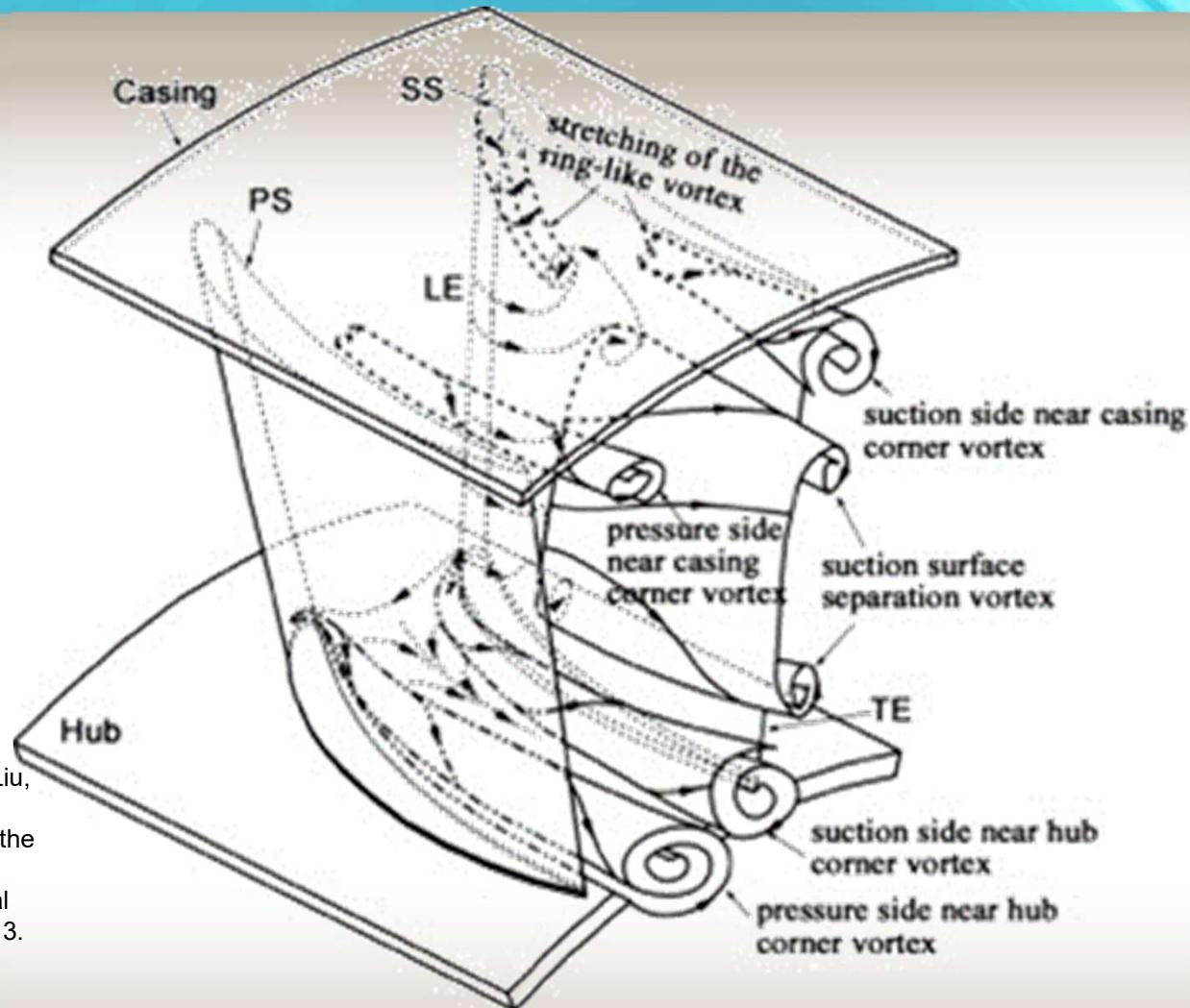




Aerodynamics

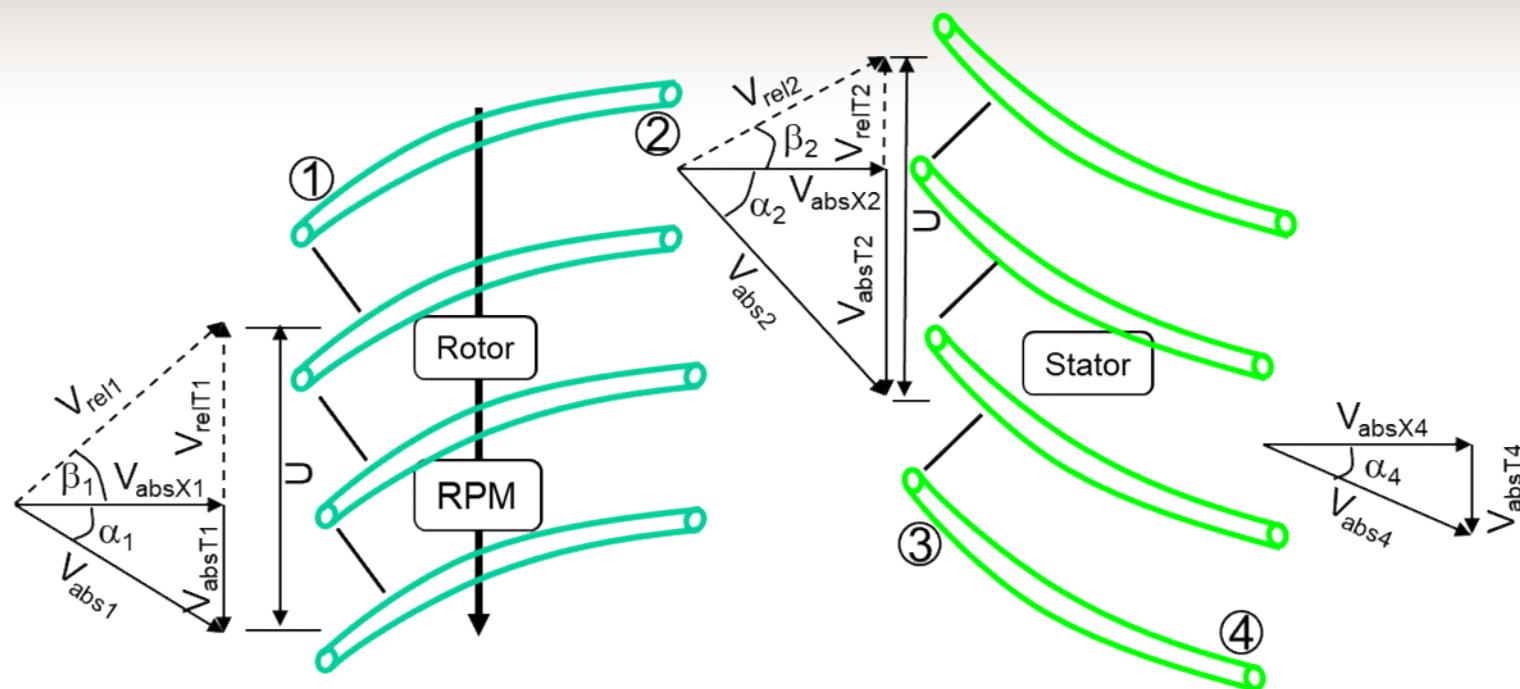
COMPRESSORS

Compressor Loss Mechanisms

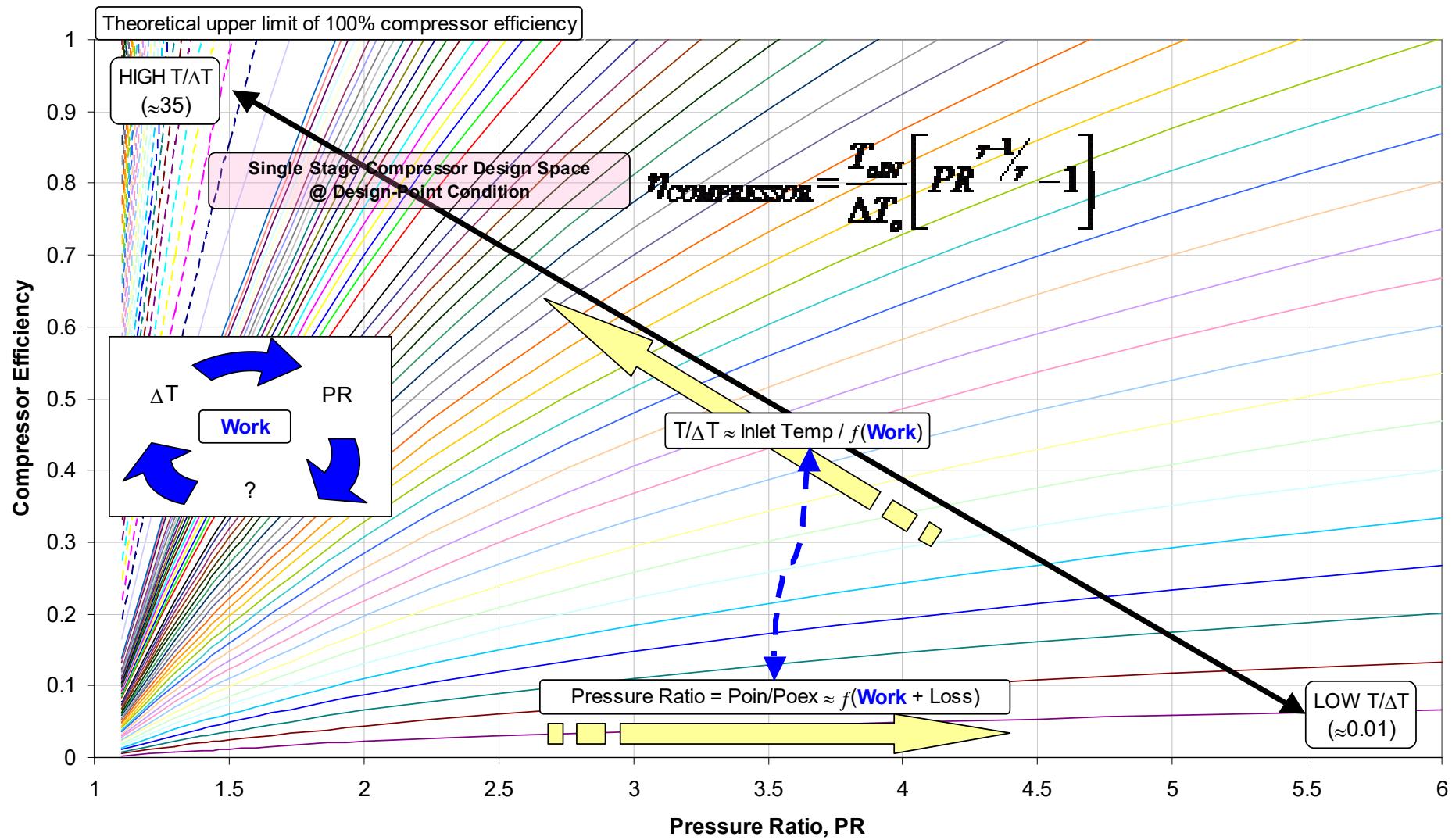


REF: Yu, X., Z. Zhang, and B. Liu, The evolution of the flow topologies of 3D separations in the stator passage of an axial compressor stage. Experimental Thermal and Fluid Science, 2013. 44: p. 301-311.

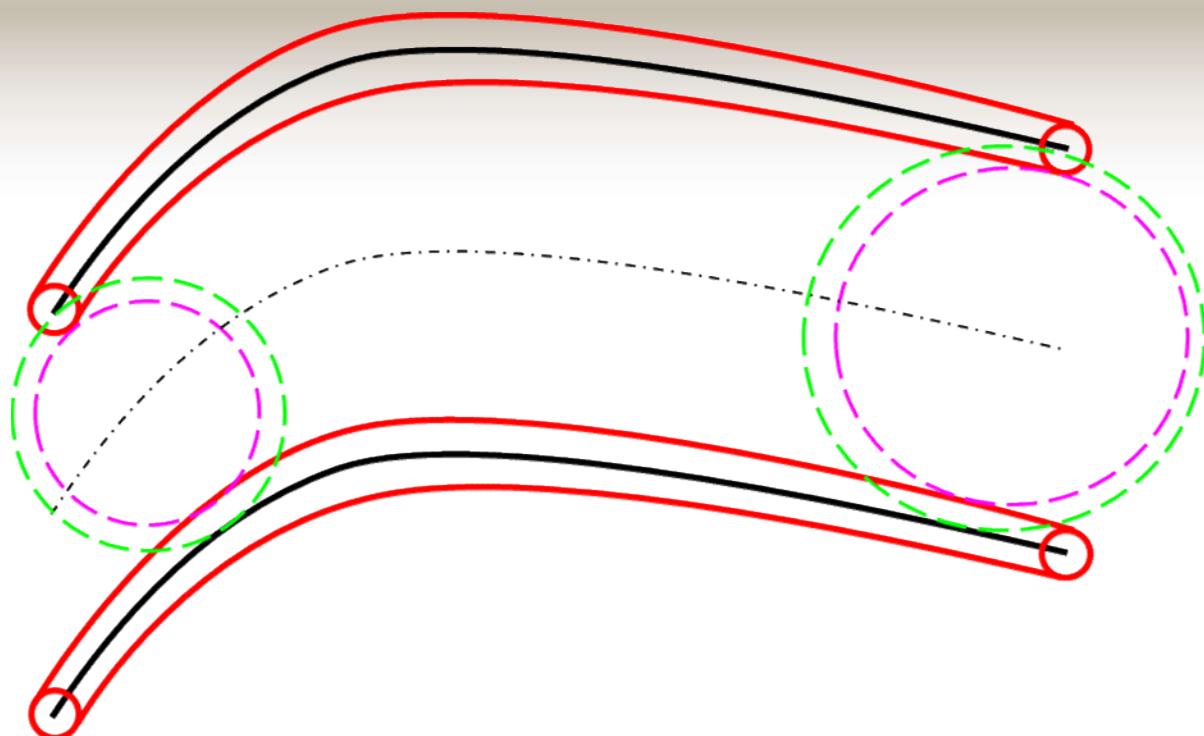
Compressor Stage



Compressor Efficiency & the Physical Design Space



Compressor Blade Design



Thin vs Thick Airfoil Assumption

Compressor mean-line model

"Bladed mass flow equation"

$$\frac{\dot{m}_0 \sqrt{T_{\infty}}}{P_{\infty} C_D A_0 \cos(\alpha_0)} = \frac{M \sqrt{\gamma / R_g}}{\left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{(\gamma+1)/2(\gamma-1)}}$$

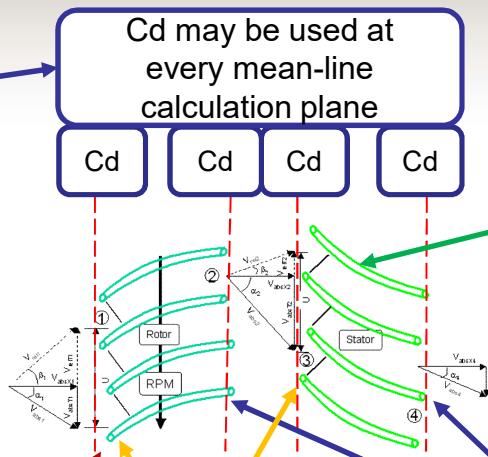
C_d may be used at every mean-line calculation plane

C_d C_d C_d C_d

Blockage factor used to mimic Boundary layer thickness

Blockage factor used to mimic Airfoil thickness

Blockage factor used as "go-to" parameter for PR match.



Actual airfoil thickness ignored

Literature shows mainly using idealized thin cambered airfoil

Constant tip clearance

Literature shows that tip clearance remains fixed in a ML model

Incidence at LE

Directly obtained from ML solution

Deviation at TE

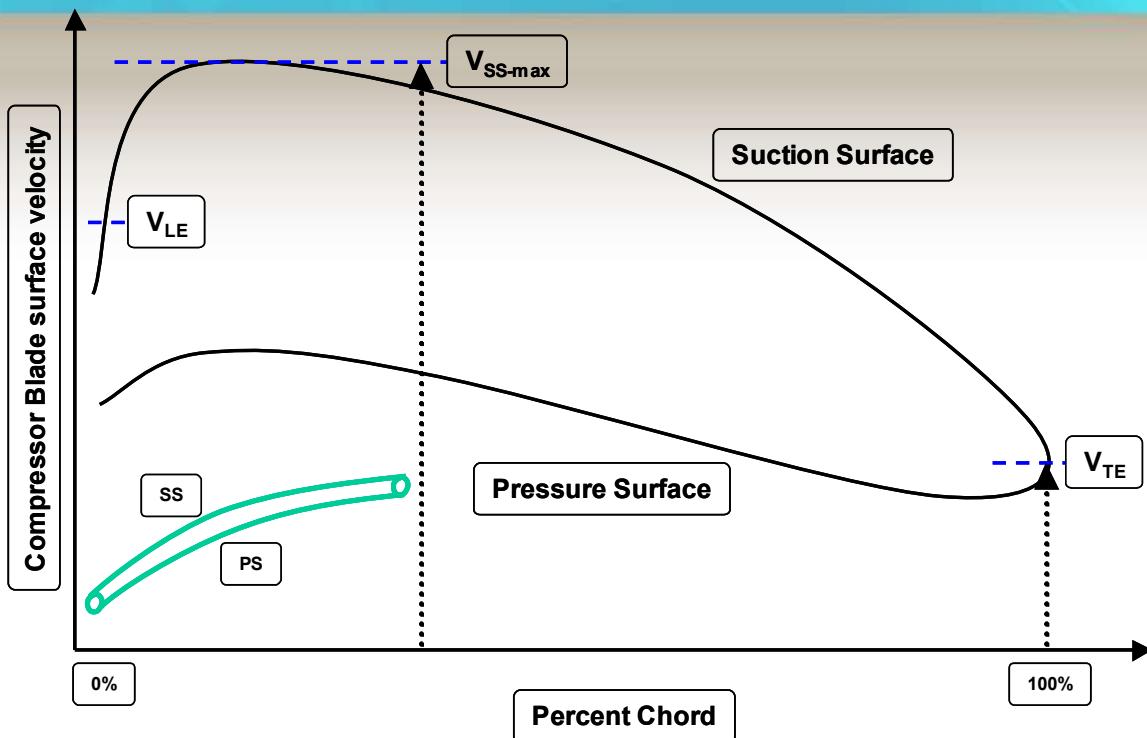
Literature has shown widespread use of Carter's rule
Remains constant through OD conditions

Compressor loss models

Author	Formula
Lieblein (1953)	$\omega_{TOTAL} = \omega_p$
Koch & Smith (1976)	$\omega_{total} = \omega_{P&TE} + \omega_{EW&TC} + \omega_{shock} + \omega_{PS-shrouds}$ <i>(interpreted formula from reference)</i>
Barbosa (1987)	$\omega_{TOTAL} = \omega_p f_{Re} + \omega_s + \omega_{SH} + \omega_{COR}$
Wright & Miller (1991)	$\omega_{total} = (\omega_p _{Re=10^6} + \omega_{EW&TC})f(Re) + \omega_{shock}$ <i>(interpreted formula from reference)</i>
Bloch, Copenhaver, O'Brien (1997)	$\bar{\omega} = \omega_{profile} + \omega_{shock}$
Cahill (1997)	$\omega_{TOTAL} = \omega_p + \omega_s + \omega_{shock}$
Lynette Smith (1999)	$\omega = (\varpi_{min} + \varpi_M) \left[1 + \left(\frac{i - i_{min}}{W} \right)^2 \right]$
Ramakdawala (2001)	$\omega_{TOTAL} = \omega_{Profile} + \omega_{EW&TC} + \omega_{Shock}$
Boyer (2001)	$\omega = (\varpi_{min} + \varpi_M + \varpi_{tip} + \varpi_{hub}) \left[1 + \left(\frac{i - i_{min}}{W} \right)^2 \right]$
van Antwerpen (2007)	$\omega = \omega_p^* \left(\frac{\omega}{\omega_i} \right)_{inc} \left(\frac{\omega}{\omega_i} \right)_{Re} \left(\frac{\omega}{\omega_i} \right)_{Ma} + \omega_s \left(\frac{\omega}{\omega_i} \right)_{Re} + \omega_a, \text{ for } i > i_{min}$ $\omega = \omega_p^* \left(f \left(\frac{\omega}{\omega_i}, \Phi \right) \right)_{inc} \left(\frac{\omega}{\omega_i} \right)_{Re} + \omega_s \left(\frac{\omega}{\omega_i} \right)_{Re} + \omega_a, \text{ for } i < i_{min}$
Falck (2008)	$\omega_{TOTAL} = \omega_p + \omega_{ew}$
Veres (2009)	$\omega_{TOTAL} = \omega_p + \omega_{shock}$
Benini (2010)	$\zeta = \zeta_{(M=0)} \chi_R \chi_M + \zeta_{shock} + \zeta_S + \zeta_\delta + K_M (i - i_{ref})^2$

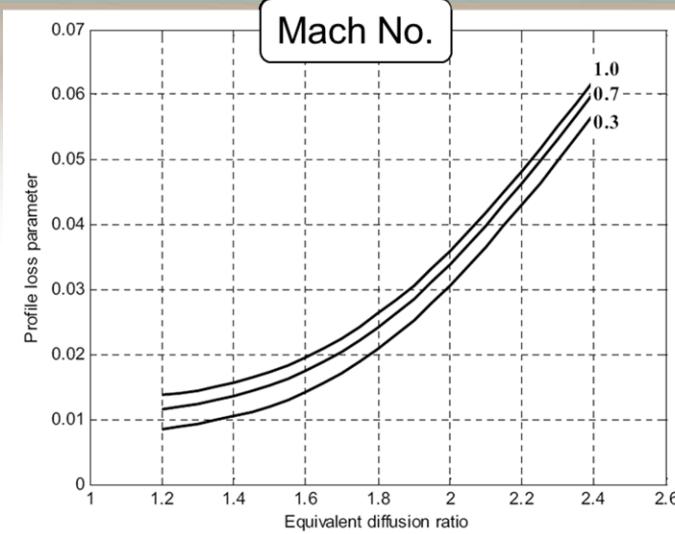
Other references of interest

- Steinke
- Howell & Calvert
- Schobeiri
- Denton
- AGARD
- Etc ...



$$\text{de Haller No.} = \left(\frac{V_{rel2}}{V_{rel1}} \right)_{\text{rotor}} \cdot \left(\frac{V_{abs4}}{V_{abs3}} \right)_{\text{stator}} \quad DF = \frac{V_{SS\text{-max}} - V_{TE}}{V_{LE}} \quad DR = \frac{V_{SS\text{-max}}}{V_{TE}} \quad V_{rel,SS\text{-max}} = V_{rel,LE} + \frac{\Delta V_{rel,T}}{2} \frac{s}{c}$$

Compressor loss

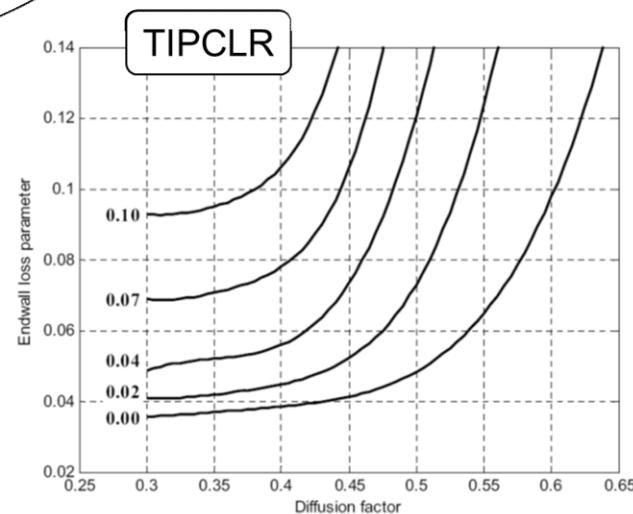


$$\omega_p \cdot 0.5 \frac{V_1^2}{V_2^2} \cos(\alpha_2) = f(M_1, D_{eq})$$

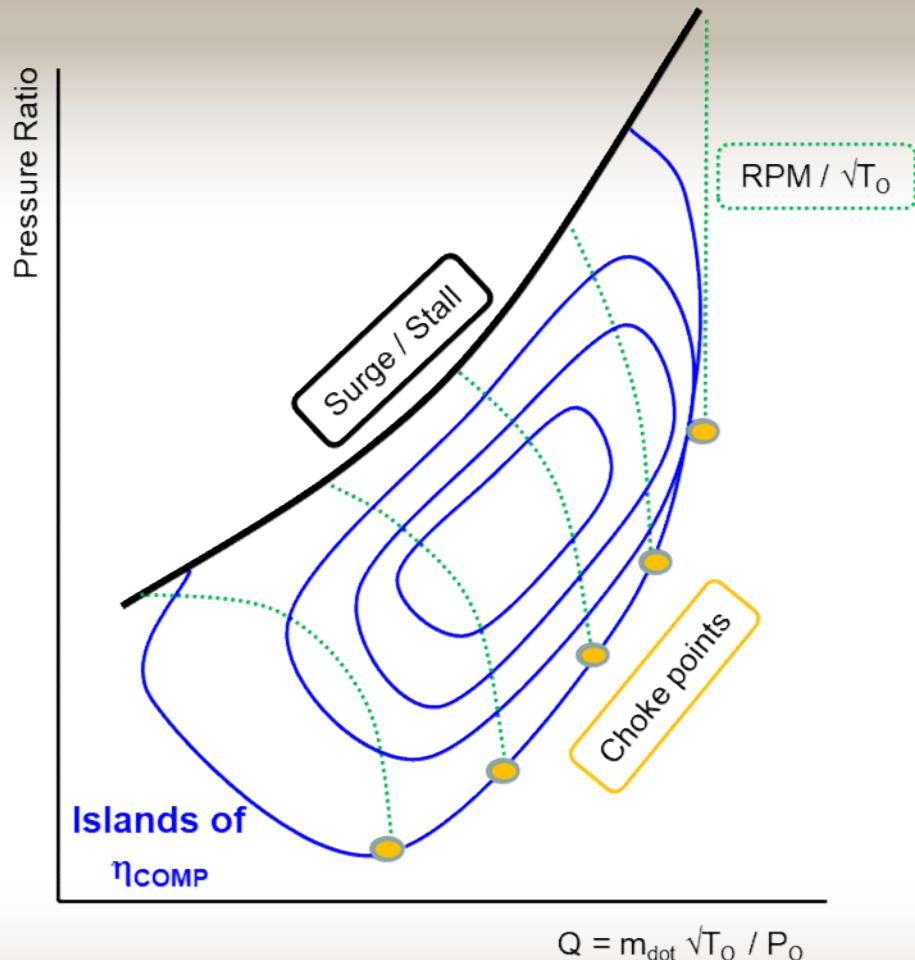
$$\omega_{ew} \frac{h V_1^2}{c V_2^2} = f\left(\frac{\varepsilon}{c}, DF\right)$$

$\omega = \omega_{ew} + \omega_p$

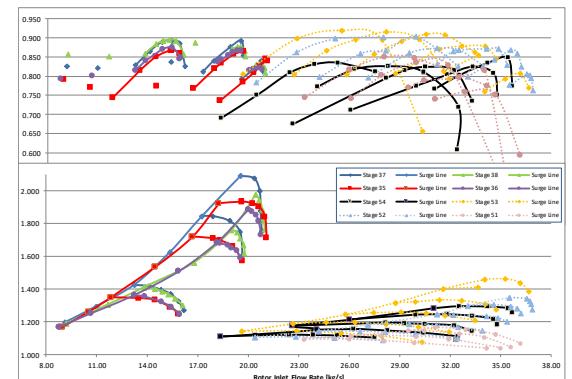
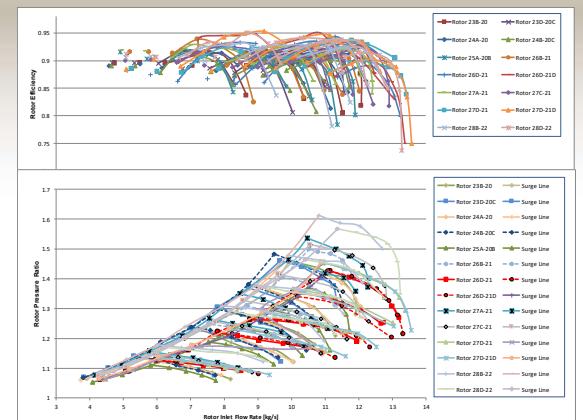
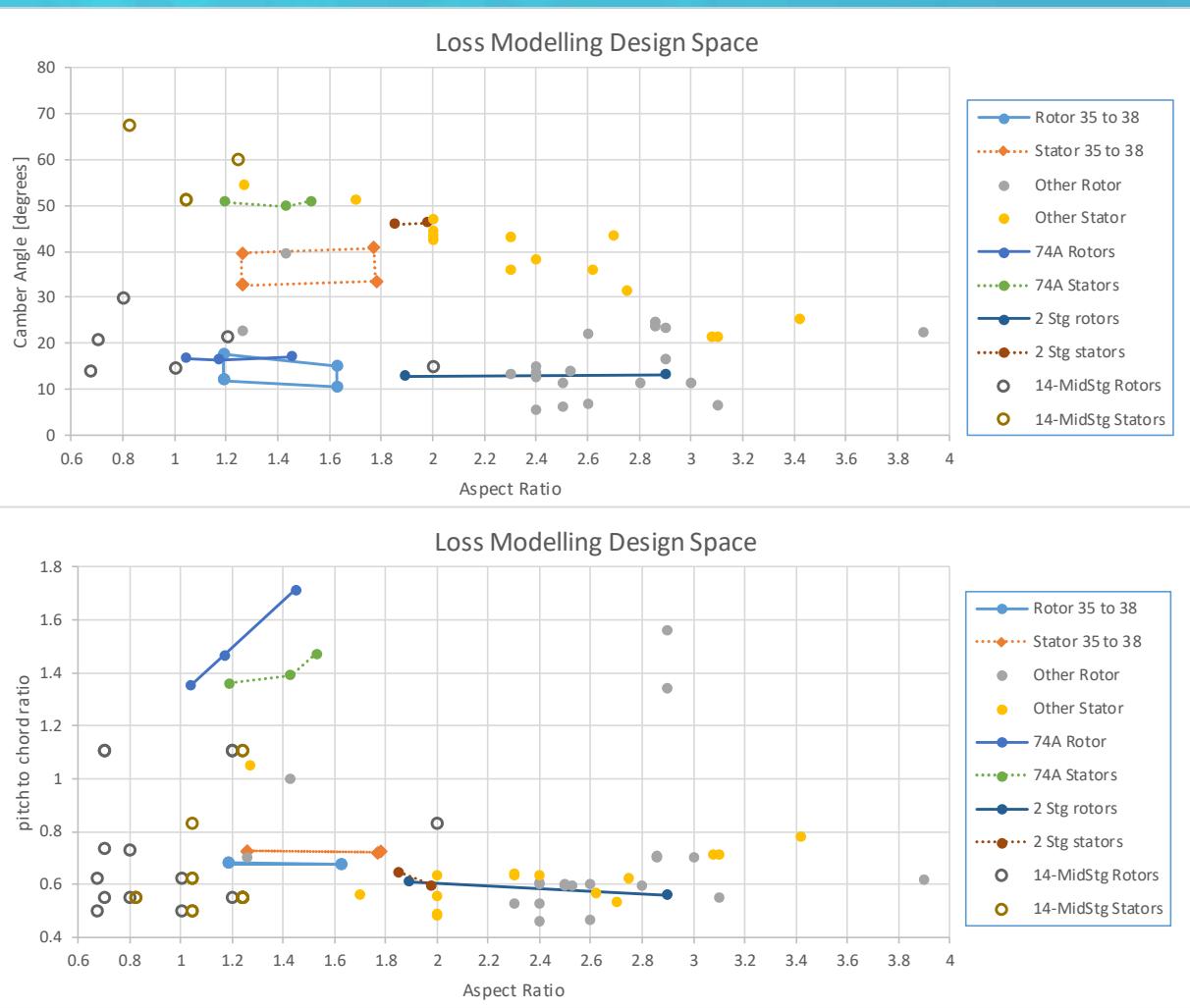
$\omega = \frac{\Delta p_0}{p_{01} - p_1}$



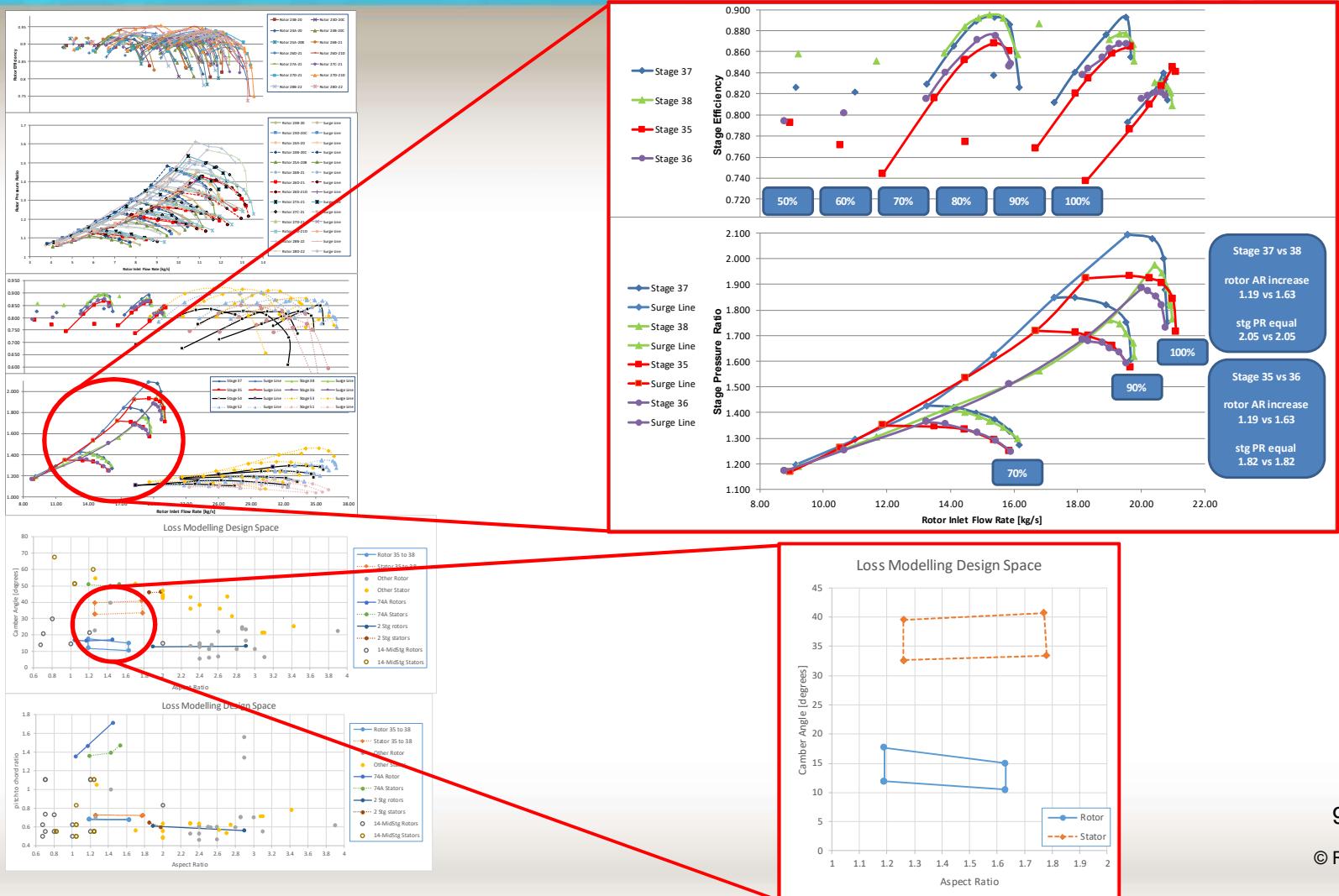
Compressor Performance Chart



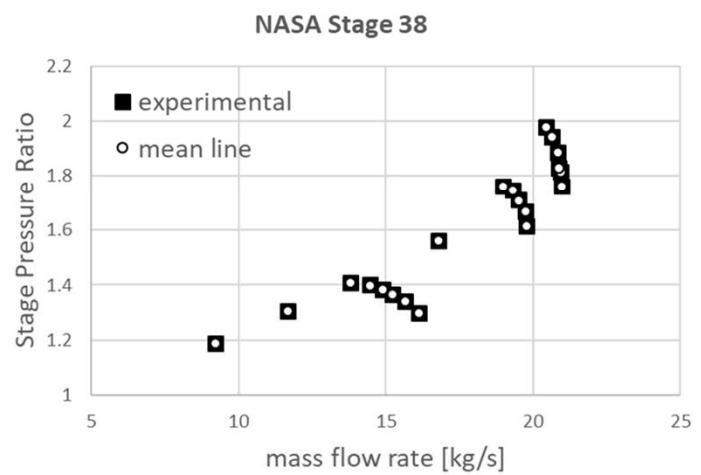
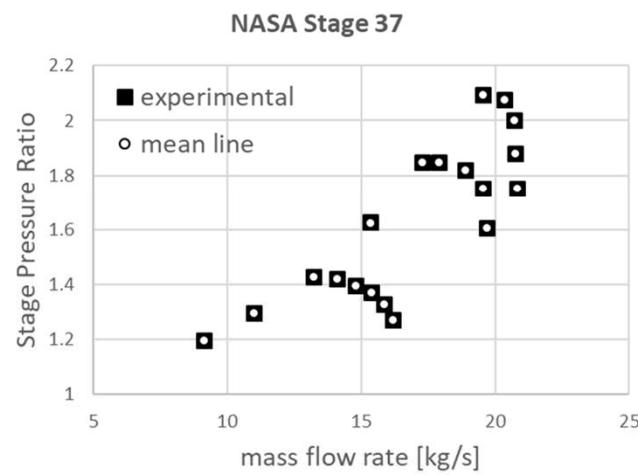
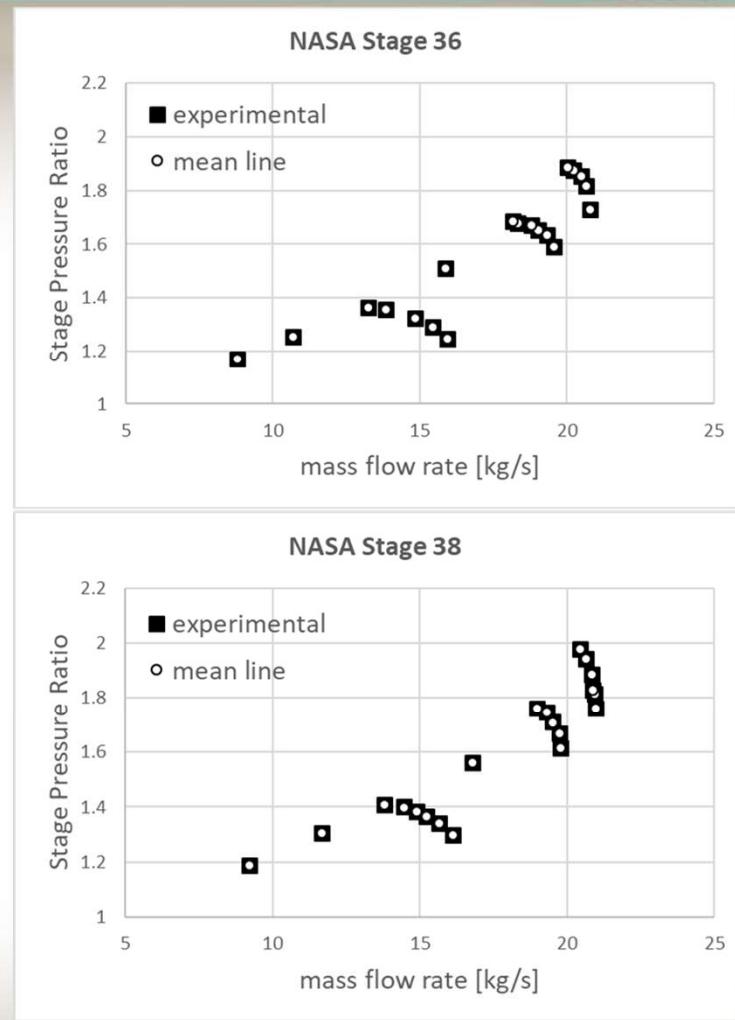
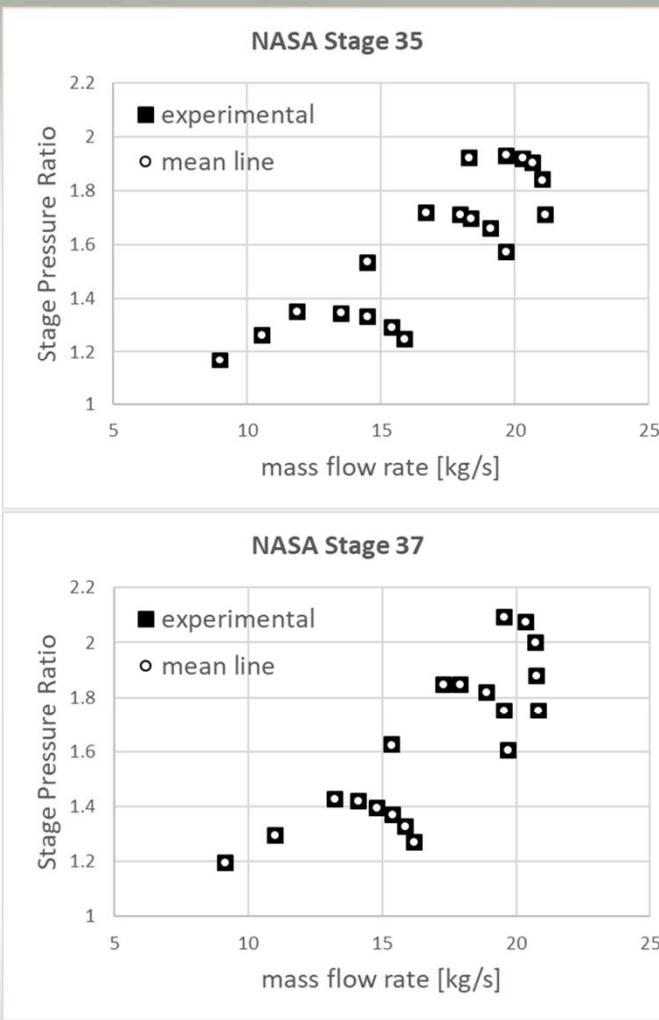
Compressor Data



Compressor Loss Modeling



MDIDS-GT modeling



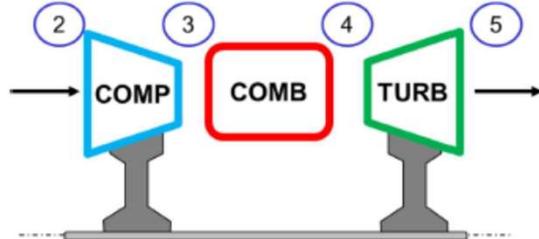


Calculations

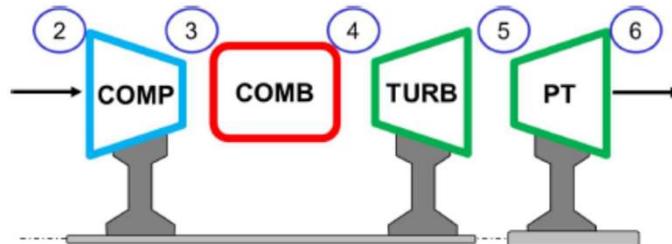
PERFORMANCE

Cycle Calculation

A. Compte tenu de la section "core" de turbine à gaz suivant, et les données liées, déterminer les températures et les pressions des entrées et des sorties, puis le taux de dilution (fuel-air ratio), et le SFC.



B. Si une turbine de puissance ("power turbine") est ajoutée, trouver les températures, les pressions, et le SFC du moteur suivant.



Air froide	Air chaude	Carburant
$\gamma = 1.4$ $C_p = 1004.5 \text{ J / kg K}$ $R_{\text{gas}} = 287 \text{ J / kg K}$	$\gamma = 1.33$ $C_p = 1156.7 \text{ J / kg K}$ $R_{\text{gas}} = 287 \text{ J / kg K}$	$Q = 46.2 \times 10^6 \text{ J / kg K}$ $C_p = 2.01 \times 10^3 \text{ J / kg K}$ $\eta_{\text{COMB}} = 100\%$
Ambient	Compresseur	Turbine
$T_{\text{amb}} = 288 \text{ K}$ $P_{\text{amb}} = 101.3 \text{ kPa}$	$PR_{\text{COMP}} = 3.25$ $\eta_{\text{COMP}} = 81.35\%$	$T_{04} = 1175 \text{ K}$ $\eta_{\text{HPT}} = 83.75\%$ $\eta_{\text{PT}} = 92.25\%$
		Nozzle
	$SFC = \frac{f}{\sum \Delta h}$	$\eta_{\text{nozzle}} = 100\%$

Simplified Performance Calculation

Intake	$T_{T2}/T_{T0} = 1$ 0.4 < K < 0.7	$\eta_{INLET} = P_{T2}/P_{T1}$ $\eta_{RAM} = \frac{P_{T2} - P_{S1}}{P_{T1} - P_{S1}}$ $D_{SPILL} = K \cdot [\dot{m}_1 (V_{abs1} - V_{abs0}) + A_1 (P_{S1} - P_{S0})]$
Compressor	$H = \dot{m} C_p \Delta T_{T23} = \dot{m} C_p (T_{T2} - T_{T3})$	$\eta_C = \frac{T_{T2}}{\Delta T_{T23}} \left[PR_{32}^{\frac{(\gamma-1)}{\gamma}} - 1 \right] = \frac{T_{T2}}{(T_{T3} - T_{T2})} \left[\left(\frac{P_{T3}}{P_{T2}} \right)^{\frac{(\gamma-1)}{\gamma}} - 1 \right]$
Combustion	$\frac{T_{T4}}{T_{T3}} = \frac{\left(1 + f \cdot \eta_{COMB} \cdot \frac{Q}{(C_p T_{T3})} \right)}{(1+f)}$	-
Turbine	$H = \dot{m} C_p \Delta T_{T45} = \dot{m} C_p (T_{T4} - T_{T5})$	$\eta_T = \frac{\Delta T_{T45}}{T_{T4} \left[1 - \left(\frac{1}{PR_{45}} \right)^{\frac{(\gamma-1)}{\gamma}} \right]} = \frac{(T_{T4} - T_{T5})}{T_{T4} \left[1 - \left(\frac{1}{P_{T4}/P_{T5}} \right)^{\frac{(\gamma-1)}{\gamma}} \right]}$
Exhaust	$T_{T8}/T_{T5} = 1$ $V_{abs8} = \sqrt{2 \cdot \eta_{NOZZLE} \cdot C_p \cdot T_{T8} \left[1 - \left(\frac{1}{NPR} \right)^{\frac{(\gamma-1)}{\gamma}} \right]}$	$NPR = P_{T8}/P_{S8} = P_{T8}/P_{S0}$ $\eta_{NOZZLE} = \frac{T_{T5} - T_{S8}}{T_{T5} \left[1 - \left(\frac{P_{S8}}{P_{T5}} \right)^{\frac{(\gamma-1)}{\gamma}} \right]}$
Mechanical or Parasitic Loss	$WORK_{COMP} = \frac{1}{\eta_m} C_p \cdot \Delta T_{32}$	$\frac{1}{\eta_{para}}$
Thrust	$Thrust = (1+f) \cdot (\dot{m}_{IN} - \dot{m}_{BLEED}) V_{EX} - \dot{m}_{IN} V_{IN} + A_{EX} (P_{EX} - P_{IN})$	$V_{EX} = V_{abs8} = \sqrt{2 \cdot \eta_{NOZZLE} \cdot C_p \cdot T_{T8} \left[1 - \left(\frac{1}{NPR} \right)^{\frac{(\gamma-1)}{\gamma}} \right]}$

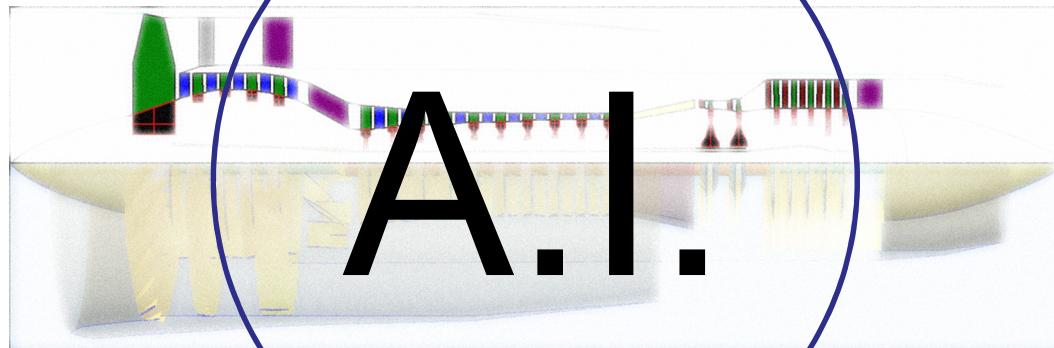


Goal of the future

TWO MAIN GOALS

“Hey Google ...”

“I need a gas turbine that develops a thrust of 63,500 lbf with an SFC of 0.20 at take-off”



“Change the turbine disk material, use bleed air from the stator leading edge, and run full 3D please”

Or ...



Any questions?

