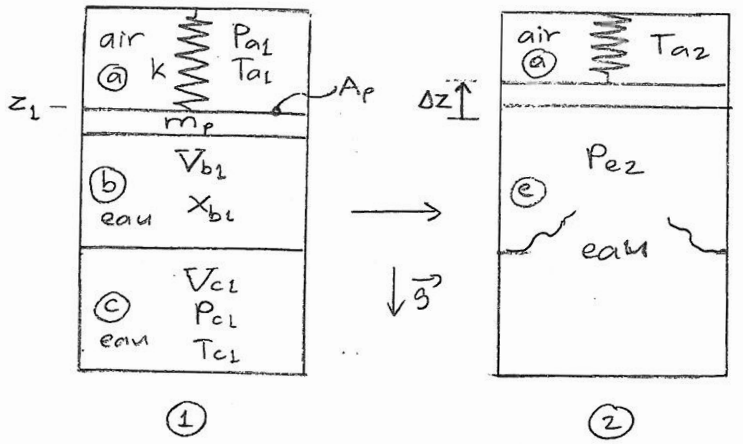
MEC1210 - Automne 2024: Mini-contrôle 1 (Solutionnaire)



$m_p = 16.3099\ kg$
$A_p = 0.016\ m^2$
$k = 64\ kN/m$

- Cylindre vertical, fixe, rigide
- Piston sans friction
- Isolant thermique parfait pour parois/piston (Q_{ext}, $Q_{air-eau} = 0$)
- Piston stationnaire à ① & ②
- $\Delta \bar{U}_{piston, ressort, membrane} = 0$
- air → gaz parfait à c_p, c_v variables avec $R = 0,287\ \frac{kPa \cdot m^3}{kg \cdot K}$
- $\Delta E_{p\ air, eau} \simeq 0$

Suppositions additionnelles:
- mélange d'eau uniforme

État ①:
$P_{a1} = 50\ kPa$
$T_{a1} = 25°C$
$z_1 = 1\ cm = 0.01\ m$
$\bar{V}_{b1} = \bar{V}_{c1} = 0.004\ m^3$
$x_{b1} = 0.883$
$P_{c1} = 600\ kPa$
$T_{c1} = 350°C$

État ②:
$T_{a2} = 52°C$
$\Delta z = 4.582\ cm = 0.04582\ m$
$P_{e2} = 300\ kPa$

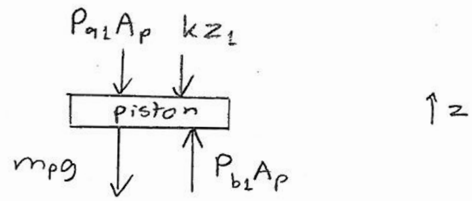
a) m_{b1}, $phase_{b1}$, m_{c1}, $phase_{c1}$ = ?

b) P_{a2}, T_{e2} = ?

c) $\bar{W}_{sur\ piston}$ = ?

d) m_a = ? (via 1ère loi) & $\bar{V}_{a1}$ = ?

a) i) m_{b1}, $phase_{b1}$ = ? : P_{b1} = ?



$$\sum F_z = P_{b1}A_p - P_{a1}A_p - m_p g - kz_1 = m_p \cancel{a_p}^{\,0} \quad \text{(piston stationnaire)}$$

$$P_{b1} = P_{a1} + \frac{m_p g}{A_p} + \frac{k z_1}{A_p}$$

$$= 50\ kPa + \frac{(16.3099\ kg)(9.81\ m/s^2)}{(0.016\ m^2)} \times \frac{1\ kPa}{10^3\ \frac{kg \cdot m/s^2}{m^2}}$$

$$+ \frac{(64\ kN/m)(0.01\ m)}{(0.016\ m^2)} \times \frac{kPa}{kN/m^2}$$

$$P_{b1} = 100\ kPa$$

$$\left.\begin{array}{l} P_{b1} = 100\,kPa \\ x_{b1} = 0{,}883 \end{array}\right\}$$ $\boxed{phase_{b1} = \text{mélange liquide-vapeur saturé}}$ $(x_{b1} < 1)$

Table A-5 : pour $P = 100\,kPa$, $v_f = 0.001043\ m^3/kg$, $v_g = 1.6941$

$\left(\begin{array}{l} u_f = 417.40\ kJ/kg \\ u_{fg} = 2088.2 \end{array} \right)$

$$v_{b1} = v_f + x_{b1}(v_g - v_f) = 0.001043 + 0.883(1.6941 - 0.001043)$$

$$v_{b1} = 1.4960\ m^3/kg \qquad (u_{b1} = u_f + x_{b1}u_{fg} = 2261.28\ \frac{kJ}{kg})$$

$$m_{b1} = \frac{V_{b1}}{v_{b1}} = \frac{0.004\ m^3}{1.4960\ m^3/kg} = \boxed{2.6738 \times 10^{-3}\ kg}$$

ii) m_{c1}, $phase_{c1} = ?$: $\left.\begin{array}{l} P_{c1} = 600\ kPa \\ T_{c1} = 350\,°C \end{array}\right\}$ $T_{c1} > T_{sat@600kPa} \overset{A-5}{=} 158.83\,°C$

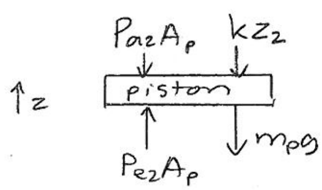
$\boxed{phase_{c1} = \text{vapeur surchauffée}}$

Table A-6 : $v_{c1} = 0.47428\ m^3/kg$

$(u_{c1} = 2881.6\ kJ/kg)$

$$m_{c1} = \frac{V_{c1}}{v_{c1}} = \frac{0.004\ m^3}{0.47428\ m^3/kg} = \boxed{8.4338 \times 10^{-3}\ kg}$$

b) i) $P_{a2} = ?$:



(piston stationnaire)

$$\sum F_z = P_{e2}A_p - P_{a2}A_p - m_p g - kz_2 = m_p \cancel{a_p}^{\,0}$$

$$P_{a2} = P_{e2} - \frac{m_p g}{A_p} - \frac{kz_2}{A_p} \qquad \Rightarrow z_2 = z_1 + \Delta z = (1 + 4.582)\,cm$$

$$= 5.582\ cm = 0.05582\ m$$

$$P_{a2} = 300\ kPa - \frac{(16.3099\,kg)(9.81\,\frac{m}{s^2})}{(0.016\ m^2)} \times \frac{1\,kPa}{10^3\,\frac{kg \cdot m/s^2}{m^2}} - \frac{(64\,\frac{kN}{m})(0.05582\,m)}{(0.016\,m^2)} \times \frac{kPa}{kN/m^2}$$

$$\boxed{P_{a2} = 66.72\ kPa}$$

ii) $T_{e2} = ?$: $P_{e2} = 300\,kPa$

$$v_{e2} = \frac{V_{e2}}{m_e} = \frac{V_{b1} + V_{c1} + A_p\Delta z}{m_{b1} + m_{c1}}$$

$$= \frac{(0.004 + 0.004)\,m^3 + (0.016\,m^2)(0.04582\,m)}{(2.6738 + 8.4338) \times 10^3\ kg}$$

$$v_{e2} = 0.78623\ m^3/kg > v_{g@300kPa} \overset{A-5}{=} 0.60582\ m^3/kg$$

(e2) vapeur surchauffée

Table A-6 : pour $P_{e2} = 0{,}3\,MPa$ et $v_{e2} = 0{,}78623\ m^3/kg$

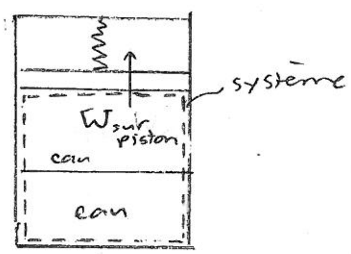
interpolation donne

$$T_{e2} = 200 + \frac{(250-200)}{(0{,}79645 - 0{,}71643)}(0{,}78623 - 0{,}71643)$$

$$\boxed{T_{e2} = 243{,}61\ °C}$$

(similairement : $u_{e2} = 2718{,}95\ kJ/kg$)

c) $W_{\substack{sur\\piston}} = ?$:



1ère loi : $\Delta E_{sys_{12}} = E_{in_{12}} - E_{out_{12}}^{\to 0}$

$$\Delta U_{eau} + \cancel{\Delta U_{membrane}}^{\to 0} + \cancel{\Delta E_{c\ \substack{eau\\membrane}}}^{\to 0} + \cancel{\Delta E_{p\ \substack{eau\\membrane}}}^{\to 0} = -W_{\substack{sur\\piston}}$$

$$\overbrace{(m_{b1}+m_{c1})u_{e2}}^{U_{e2}} - \overbrace{(m_{b1}u_{b1} + m_{c1}u_{c1})}^{U_{e1}} = -W_{\substack{sur\\piston}}$$

$$W_{\substack{sur\\piston}} = m_{b1}(u_{b1} - u_{e2}) + m_{c1}(u_{c1} - u_{e2})$$

$\Rightarrow u_{b1} = 2261{,}28\ kJ/kg$ (voir partie a)

$\Rightarrow u_{c1} = 2881{,}6\ kJ/kg$ (voir partie a)

$\Rightarrow u_{e2} = 2718{,}95\ kJ/kg$ (voir partie b)

$\Rightarrow m_{b1} = 2{,}6738 \times 10^{-3}\ kg$

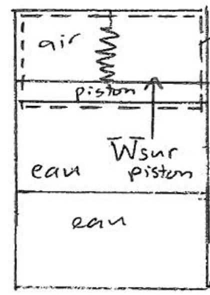
$\Rightarrow m_{c1} = 8{,}4338 \times 10^{-3}\ kg$

$$W_{\substack{sur\\piston}} = (2{,}6738\times10^{-3}\ kg)(2261{,}28 - 2718{,}95)\frac{kJ}{kg} + (8{,}4338\times10^{-3}\ kg)(2881{,}6 - 2718{,}95)\frac{kJ}{kg}$$

$$\boxed{W_{\substack{sur\\piston}} = 0{,}14804\ kJ}$$

<u>Note</u> : $W_{\substack{sur\\piston}}$ ne peut pas être calculé par $\int_{1}^{2} P\,dV$ car on ne connaît pas comment P à la surface inférieure du piston varie avec le volume (déplacement du piston) surtout que le processus ① → ② n'est pas nécessairement quasi-statique dans ce cas-ci.

d) i) $m_a = ?$ (par la $1^{ère}$ loi)



$1^{ère}$ loi:

$$\Delta E_{sys_{12}} = E_{in_{12}} - E_{out_{12}}$$

$$\Delta U_{air} + \cancel{\Delta U_{\substack{ressort\\piston}}}^{\,0} + \cancel{\Delta E_{c_{\substack{air\\ressort\\piston}}}}^{\,0} + \cancel{\Delta E_{p_{air}}}^{\,0} + \Delta E_{p_{ressort}} + \Delta E_{p_{piston}} = W_{\substack{sur\\piston}}$$

$$m_a(u_{a2} - u_{a1}) + \left(\frac{k}{2} z_2^2 - \frac{k}{2} z_1^2\right) + m_p g \Delta z = W_{\substack{sur\\piston}}$$

$$m_a(u_{a2} - u_{a1}) = W_{\substack{sur\\piston}} - \frac{k}{2}(z_2^2 - z_1^2) - m_p g \Delta z$$

$$m_a = \frac{W_{\substack{sur\\piston}} - \frac{k}{2}(z_2^2 - z_1^2) - m_p g \Delta z}{u_{a2} - u_{a1}}$$

$$\Rightarrow W_{\substack{sur\\piston}} = 0.14804 \text{ kJ}$$

$$\Rightarrow u_{a1} = u(T_{a1} = 25°C = 298\,k) \overset{A-17}{=} 212.64 \ \frac{kJ}{kg}$$

$$\Rightarrow u_{a2} = u(T_{a2} = 52°C = 325\,k) \overset{A-17}{=} 232.02$$

$$m_a = \frac{0.14804 \text{ kJ} - \frac{1}{2}\left(64 \frac{kN}{m}\right)(0.05582^2 - 0.01^2)\, m^2 \times \frac{1\,kJ}{kN \cdot m} - (16.3099\,kg)\left(9.81 \frac{m}{s^2}\right)(0.04582\,m) \times \frac{1\,kJ}{10^3\,kg \cdot \frac{m^2}{s^2}}}{(232.02 - 212.64)\ kJ/kg}$$

$$\boxed{m_a = 2.2807 \times 10^{-3}\ kg}$$

ii) $V_{a1} = ?$: $P_{a1} V_{a1} = m_a R T_{a1}$

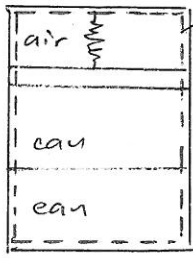
$$V_{a1} = \frac{m_a R T_{a1}}{P_{a1}} = \frac{(2.2807 \times 10^{-3}\,kg)\left(0.287 \frac{kPa \cdot m^3}{kg \cdot k}\right)(25 + 273)k}{(50\,kPa)}$$

$$\boxed{V_{a1} = 0.0039\ m^3}$$

Partie d) Solutions alternative

Solution I (acceptable)

i)



système

$$\underline{1^{\text{ère}}\text{ loi}}: \quad \Delta E_{sys_{12}} = E_{in_{12}} - E_{out_{12}}$$

$$\Delta U_{air} + \Delta U_{eau} + \cancel{\Delta U_{\substack{piston\\ ressort\\ membrane}}}^{\,0} + \cancel{\Delta E_{c_{sys}}}^{\,0} + \cancel{\Delta E_{p_{\substack{air\\ eau\\ membrane}}}}^{\,0}$$

$$+ \Delta E_{p_{ressort}} + \Delta E_{p_{piston}} = 0$$

$$m_a(u_{a2}-u_{a1}) + \left[(m_{b1}+m_{c1})u_{e2} - (m_{b1}u_{b1} + m_{c1}u_{c1})\right] + \frac{k}{2}(z_2^2 - z_1^2) + m_p g \Delta z = 0$$

$$m_a = \frac{\overbrace{\left[m_{b1}(u_{b1}-u_{e2}) + m_{c1}(u_{c1}-u_{e2})\right] - \frac{k}{2}(z_2^2-z_1^2) - m_p g \Delta z}^{= W_{sur\ piston}}}{u_{a2}-u_{a1}}$$

⋮

(même réponse que sur page 4)

ii) $V_{a1} = m_a R T_{a1} / P_{a1}$ (même que page 4)

Solution II (non acceptable car $1^{\text{ère}}$ loi pas utilisée tel que demandé)

(max : 1.75pts)

ii) $$\frac{P_{a1} V_{a1}}{T_{a1}} = m_a R = \frac{P_{a2} V_{a2}}{T_{a2}} \quad \Rightarrow \quad V_{a2} = V_{a1} - A_p \Delta z$$

$$\frac{P_{a1} V_{a1}}{T_{a1}} = \frac{P_{a2}(V_{a1} - A_p \Delta z)}{T_{a2}}$$

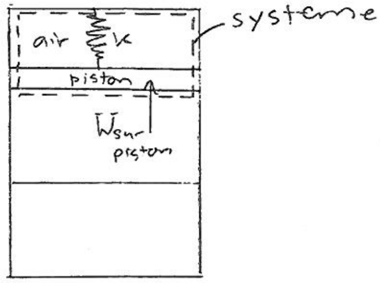
Isoler V_{a1} donne : $$V_{a1} = \frac{A_p \Delta z}{1 - \frac{P_{a1}}{P_{a2}} \frac{T_{a2}}{T_{a1}}} = \frac{(0{,}016\text{ m})(0{,}04582\text{ m}^2)}{1 - \left(\frac{50\text{ kPa}}{66{,}72\text{ kPa}}\right)\left(\frac{52+273}{25+273}\right)\frac{\text{k}}{\text{k}}}$$

$$V_{a1} = 0.0040\text{ m}^3$$

i) $$m_a = \frac{P_{a1} V_{a1}}{R T_{a1}} = \frac{(50\text{ kPa})(0.0040\text{ m}^3)}{\left(0.287\ \frac{\text{kPa}\cdot\text{m}^3}{\text{kg}\cdot\text{k}}\right)(25+273)\text{k}} = 2{,}3459 \times 10^{-3}\text{ kg}$$

(Note : Petites différences dans valeurs de V_{a1} & m_a versus p.4, causées par les interpolations et arrondissement de la méthode utilisant la $1^{\text{ère}}$ loi.)

Partie c) Solution alternative



$\underline{1^{ère}\ loi}$: $\Delta E_{sys_{12}} = E_{in_{12}} - E_{out_{12}}$

$$\Delta U_{air} + \cancel{\Delta U_{\substack{piston\\ressort}}}^{\,0} + \cancel{\Delta E_{c_{\substack{air\\ressort\\piston}}}}^{\,0} + \cancel{\Delta E_{p_{air}}}^{\,0} + \Delta E_{p_{ressort}}$$

$$+ \Delta E_{p_{piston}} = + W_{\substack{sur\\piston}}$$

$$m_a(u_{a2} - u_{a1}) + \frac{k}{2}(z_2^2 - z_1^2) + m_p g \Delta z = W_{\substack{sur\\piston}}$$

$$W_{\substack{sur\\piston}} = m_a(u_{a2} - u_{a1}) + \frac{k}{2}(z_2^2 - z_1^2) + m_p g \Delta z$$

$$\Rightarrow m_a = \frac{P_{a1} V_{a1}}{R T_{a1}} \quad \rightarrow \quad V_{a1} = \frac{A_p \Delta z}{1 - \frac{P_{a1}}{P_{a2}} \frac{T_{a2}}{T_{a1}}}$$

$$V_{a1} = 0.0040\ m^3$$

voir bas de la page 5

$$m_a = 2.3459 \times 10^{-3}\ kg$$

$$\Rightarrow u_{a1} = u(T_{a1} = 298\,K) \overset{A-17}{=} 212,64\ kJ/kg$$

$$\Rightarrow u_{a2} = u(T_{a2} = 325\,K) = 232,02\ kJ/kg$$

$$W_{\substack{sur\\piston}} = (2.3459 \times 10^{-3}\ m^3)(232,02 - 212.64)\frac{kJ}{kg} + \frac{(64\,\frac{kN}{m})}{2}(0,05582^2 - 0.01^2)\,m^2$$

$$+ (16.3095\ kg)(9.81\ \frac{m}{s^2})(0.04582\ m) \times 1\,kJ / 10^3\ kg \cdot \frac{m^2}{s^2}$$

$$W_{\substack{sur\\piston}} = 0.14930\ kJ$$