

18. Montrez que toute équation à variables séparables

$$M(x) + N(y)y' = 0$$

est également exacte.

L'EDO est exact si $M_y = N_x$:

$$N_y(x) = N_x(y) = 0$$

19. $x^2y^3 + x(1+y^2)y' = 0, \quad \mu(x, y) = 1/xy^3$

a) Test

$$\left. \begin{array}{l} M_y = 3x^2y^2 \\ N_x = 1+y^2 \end{array} \right\} M_y \neq N_x \Rightarrow \text{pas exact.}$$

Analysons $\mu M + \mu N_y = \frac{x^2y^3}{xy^3} + \frac{x(1+y^2)}{xy^3}y' = 0$

$$\Rightarrow x + \frac{1+y^2}{y^3}y' = 0$$

$$M_y = N_y = 0 \Rightarrow \text{EDO exact.}$$



b) Résolution

$$\frac{\partial U}{\partial x} = M \quad \text{et} \quad \frac{\partial U}{\partial y} = N \quad \Rightarrow \quad U(x, y) = \frac{x^3}{3} + \ln|y| - \frac{2}{y^2} = C$$

$$\begin{aligned} \Rightarrow U &= \int M(x, y) dx + k(y) \\ &= \int x dx + k(y) \\ &= \frac{x^2}{2} + k(y) \end{aligned}$$

$$\Rightarrow \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left(\frac{x^2}{2} + k(y) \right) + k'(y)$$

$$= k'(y) = \frac{1+y^2}{y^3} \quad \xrightarrow{N(y,y)}$$

$$\Rightarrow k(y) = \int \frac{1+y^2}{y^3} dy$$

$$= \int y^{-3} dy + \int \frac{1}{y} dy$$

$$= -\frac{2}{y^2} + \ln|y|$$

EDO à VS?

$$x + \frac{1+y^2}{y^3}y' = 0$$

$$\int x dx + \int \frac{1+y^2}{y^3} dy = C$$

$$\Rightarrow \frac{x^2}{2} + \ln|y| - \frac{2}{y^2} = C$$

