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MRI and Compressed Sensing
MR Basics

After this course...

Slide courtesy of Elfar Adalsteinsson
MRI Instrumentation
Variations in magnet design

Slide courtesy of Ives Levesque
The main magnet

- Superconducting solenoid magnet
- Liquid helium cooled
- Many km's of wire
- \( \sim 0.5 - 3.0 \) tesla
- Up to 11.7 T for human research
- Self-shielded
- Shim coils

Slide courtesy of Ives Levesque
Gradients

- Adds linear spatial variation to $B_0$ magnetic field (millitesla / meter)
- Fast-switching, short ramp times
- High currents
  - Water cooling
- Bioeffects
RF transmission system ($B_1$)

- Components: signal generator, amplifier, coil
- Time-varying magnetic field $B_1$
- Max. amplitude in $\mu$T: $\sim 100,000$ less than $B_0$
- RF pulses for excitation, inversion, refocusing, etc.
Signal reception

- Coil, amplifiers, demodulator, digitizer
- Match coil to object size
- Combine coils into arrays

Slide courtesy of Ives Levesque
MRI Physics
MRI basics

\[ B_0 \]
Magnetic Resonance

\[ B_0 \]

**Larmor frequency**

\[ \omega = \gamma B_0 \]

**Gyromagnetic ratio**

42.58 MHz/T
Excitation

$B_0$  

$B_1$  

RF pulse
**MR Sequence**

- **B₀ relaxation**
- **T₁ relaxation**
- **T₂ relaxation**

- **RF**
- **TR**
- **M_xy**
- **M_z**

- **B₁**

Diagram showing the sequence of events in an MR sequence, including the application of RF pulses and the effect on the magnetic field (B₀) and the relaxation processes (T₁ and T₂).
MR pulse sequence
MR pulse sequence
MR pulse sequence

$B_0$  

$T_1$ contrast $T_2$ contrast
Spatial localization (k-space)
Free Precession

\[ \omega = \gamma B_0 \]

- Larmor frequency
- Gyromagnetic ratio: 42.58 MHz/T
Free Precession

\[ \omega = \gamma B_0 \]

Larmor frequency

Gyromagnetic ratio
42.58 Mhz/T
Free Precession

\[ \omega = \gamma B_0 \]

Larmor frequency

Gyromagnetic ratio
42.58 Mhz/T

\( B_0 \)
Gradients and the Fourier Transform

\[ B_0 + G_x x \]

Larmor frequency

Gyromagnetic ratio

42.58 MHz/T

Signal

Image

FT
Fourier Representations

Figure 4.5 Constructing a complex waveform or image from simpler components. Any data, no matter how complex, can be constructed from simpler components. Shown in (A) are three sine waves, each with a different frequency. When combined, they form the waveform at right. By combining more and more sine waves of different frequencies and phases, very complex waveforms can be created, such as that of music. The same principle holds for two-dimensional data (B), except that here the components are gratings of particular spatial frequencies (i.e., how closely spaced are the bars), phases, and angles. By combining a very large number of these gratings, complex images can be created, such as those used in MRI. Shown in (C) is the $k$-space plot of the summed image; the individual gratings are associated with the three bright pixels.
2D FT

Image space $\approx \sum \text{ Fourier Transform }$ k-space

Slide courtesy of Elfar Adalsteinsson
K-space

\[ G_x \]

\[ G_y \]

Spin position

\[ B_0 + G_y Y \]

\[ B_0 + G_x X \]
K-space

\[ s(t) = \int_{x} m(x) e^{-i2\pi k(t)x} \, dx \]

\[ k(t) = \frac{\gamma}{2\pi} \int_{0}^{t} G(t) \, dt \]
Low Spatial Frequencies

High Spatial Frequencies

Full

1/2

1/4

1/8

3/4

1/2

7/8

Full

Full
Slide courtesy of Elfar Adalsteinsson
Slide courtesy of Elfar Adalsteinsson
Image reconstruction
Image reconstruction
Image reconstruction
**k-space and spatial resolution**

\[
\Delta x = \frac{1}{2k_{x_{\text{max}}}} \quad \Delta y = \frac{1}{2k_{y_{\text{max}}}}
\]

- \(2k_{x_{\text{max}}} = N_x \gamma G_x T\)
- \(2k_{y_{\text{max}}} \)

\(T\): temps d’application de \(G_x\). Vaut l’inverse de la fréquence d’échantillonnage.
k-space and FOV

• FOV (Field of View)

\[ \text{FOV}_x = \frac{1}{\Delta k_x} \quad \text{FOV}_y = \frac{1}{\Delta k_y} \]

\[ \Delta k_x = \gamma G_x T \]

\[ \Delta k_y \]

Slide courtesy of Julien Cohen-Adad
k-space and FOV

- What if we subsample in k-space?
Sub-Nyquist $\rightarrow$ aliasing !!!
From Sparse Signals to Sparse Sampling
From Sparse Signals to Sparse Sampling

uniform subsampling

cohherent folding
From Sparse Signals to Sparse Sampling

uniform subsampling

coherent folding
From Sparse Signals to Sparse Sampling

random subsampling

non-coherent artifact
From Sparse Signals to Sparse Sampling

variable density random

noise-like artifact
From Sparse Signals to Sparse Sampling

undersampled radial


streaking artifact
**Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging**

Michael Lustig,* David Donoho,² and John M. Pauly¹

\[
\begin{align*}
\text{minimize} & \quad \| \Psi m \|_1 \\
\text{s.t.} & \quad \| \mathcal{F}_u m - y \|_2 < \epsilon 
\end{align*}
\]

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MRI IS SLOW...

click click

ARE WE DONE?

WHY?

NOPE...

NYQUIST SAYS WE NEED MORE DATA
Compressed Sensing - Basic Idea

Ingredients:

- Take **compressible** signals
  Have sparse transform representations

- Make **incoherent measurements**
  Make artifacts look like noise (after transform)

- Reconstruct using sparsity-enforcing **non-linear** reconstruction
  Iteratively separate sparse signals from artifacts
Intuitive Example of Compressed Sensing
Intuitive Example of Compressed Sensing

sampling → \[ \text{Nyquist} \]
Intuitive Example of Compressed Sensing

equispaced  \rightarrow \quad \text{sub-Nyquist}

M. Lustig, EECS UC Berkeley
Intuitive Example of Compressed Sensing
Intuitive Example of Compressed Sensing

random → [signal representation] → iFFT → sub-Nyquist
Intuitive Example of Compressed Sensing

Looks like "random noise"

sub-Nyquist
Intuitive Example of Compressed Sensing

But it’s not noise!

sub-Nyquist
Intuitive Example of Compressed Sensing

Example inspired by Donoho et. Al, 2007
• Signal is sparse, and we sample it directly. Would CS still work?

A. Yes, most times
B. Only if it is very sparse
C. Unlikely to work
D. Only with sophisticated recon.
Domains in Compressed Sensing

Signal

Sparse Domain

Not Sparse!

Sampling Domain

Sparse!

incoherent

Sparse Domain
Domains in Compressed Sensing

Signal domain → k-space

Wavelet domain → incoherent

\[ \text{minimize} \quad \| \Psi m \|_1 \]
\[ \text{s.t.} \quad \| \mathcal{F}_u m - y \|_2 < \epsilon \]
EVERYTHING TO DO WITH COMPRESSION IS VERY CONTENTIOUS IN MEDICAL - IMAGING. EVERY TIME YOU THROW BITS AWAY SOMEBODY GETS VERY NERVOUS ABOUT IT.

THE FORTUNATE THING ABOUT... [COMRESSED SENSING]... IS THAT YOU DON'T COLLECT THEM IN THE FIRST PLACE, AND THAT'S A MUCH BETTER SITUATION!

John Pauly, 2008