solution of Midterm exam

ele 8401, hiver 2018

Problème 1)

$$\begin{cases} N_1 i_1 &= \mathcal{R}_g\phi_1 + \mathcal{R}_g(\phi_1 - \phi_2) \\ N_2 i_2 &= \mathcal{R}_g(\phi_2 - \phi_1) + \mathcal{R}_g\phi_2 \end{cases} \implies \begin{cases} \phi_1 &= (2N_1 i_1 + N_2 i_2)/3\mathcal{R}_g \\ \phi_2 &= (N_1 i_1 + 2N_2 i_2)/3\mathcal{R}_g \end{cases}$$

$$\lambda_1 = N_1\phi_1 = \left(\frac{2N_1^2}{3\mathcal{R}_g}\right) i_1 + \left(\frac{N_1 N_2}{3\mathcal{R}_g}\right) i_2 = L_{11} i_1 + L_{12} i_2$$

$$\lambda_2 = N_2\phi_2 = \left(\frac{N_2 N_1}{3\mathcal{R}_g}\right) i_1 + \left(\frac{2N_2^2}{3\mathcal{R}_g}\right) i_2 = L_{21} i_1 + L_{22} i_2$$

(a) $$L_{11} = \frac{2N_1^2}{3\mathcal{R}_g} = \frac{2\mu_o A N_1^2}{3g} = \frac{2\times 4\pi\times 10^{-7}\times 200\times 10^{-6}\times 100^2}{3\times 0.001} \cong 1.68 \text{ mH}$$ (0.75 Pt

$$L_{22} = \frac{2N_2^2}{3\mathcal{R}_g} = \frac{2\mu_o A N_2^2}{3g} = \frac{2\times 4\pi\times 10^{-7}\times 200\times 10^{-6}\times 200^2}{3\times 0.001} \cong 6.70 \text{ mH}$$ (0.75 Pt

$$L_{21} = L_{21} = \frac{N_2 N_1}{3\mathcal{R}_g} = \frac{\mu_o A N_2 N_1}{3g} = \frac{4\pi\times 10^{-7}\times 200\times 10^{-6}\times 200\times 100}{3\times 0.001} \cong 1.68 \text{ mH}$$ (0.75 Pt

(b) $$W_m = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{12}i_1 i_2 + \frac{1}{2}L_{21}i_2 i_1 + \frac{1}{2}L_{22}i_2^2 = \frac{1}{2}(1.68 + 1.68 + 1.68 + 6.70) \cong 5.87 \text{ mJ}$$ (1 Pt)

(c) $$L_{12} = L_{21} = 0 \text{ H}$$ (0.75 Pt)

Problème 2

$$T_e = -i_1 i_2 M \sin\theta_r = -I_{s1} I_{s2} \cos\omega_1 t \cos(\omega_2 t + \varphi_2) M \sin(\theta_r(0) + \omega_r t)$$

(1) Pt $$= -\tfrac{1}{2} I_{s1} I_{s2} M \left\{ \cos((\omega_1+\omega_2)t + \varphi_2) + \cos((\omega_1-\omega_2)t + \varphi_2) \right\} \times \sin(\omega_r t + \theta_r(0))$$

$$\omega_1 = \omega_2 \neq 0 \Rightarrow T_e = -\tfrac{1}{2} I_{s1} I_{s2} M \left\{ \cos(2\omega_1 t + \varphi_2) + \cos\varphi_2 \right\} \sin(\omega_r t + \theta_r(0))$$

$$= -\tfrac{1}{4} I_{s1} I_{s2} M \left\{ \sin(\omega_r t + \theta_r(0) + 2\omega_1 t + \varphi_2) + \sin(\omega_r t + \theta_r(0) - 2\omega_1 t - \varphi_2) + 2\cos\varphi_2 \sin(\omega_r t + \theta_r(0)) \right\}$$

$T_e \neq 0$ if
- a) $\omega_r = 0$, $\theta_r(0) \neq k\pi$, $\varphi_2 = k\pi/2$ (1) Pt
- b) $\omega_r = 2\omega_1$, $\varphi_2 \neq \theta_r(0)$ (1) Pt
- c) $\omega_r = -2\omega_1$, $\varphi_2 \neq -\theta_r(0)$ (1) Pt

Problem 3 : See the Matlab Code on Moodle

$\tilde{I}'_{ar} = \tilde{I}'_{qr} = -j\,\tilde{I}'_{dr} \Rightarrow \tilde{I}'_{ar} = 100\angle 60^\circ$ (1 Pt)

(1.75 Pt) $\tilde{I}_{as} = 110\angle -140.8^\circ \Rightarrow T_e = 706.2513$ N.m (1.75 Pt)

système référentiel synchron

$\sqrt{2}\,\tilde{f}_{as} = f^e_{qs} - j f^e_{ds} \Rightarrow \sqrt{2}\,\tilde{I}_{as} = I^e_{qs} - j I^e_{ds}$

$\Rightarrow I^e_{qs} = \text{Real}\{\sqrt{2}\,\tilde{I}_{as}\} = \text{Real}\{\sqrt{2}\times 110\angle -140.8\}$

$= -121.1220$ (A) (1.5 Pt)

Probleme 4: See the Matlab Code on Moodle

$\underbrace{G = \pm 1.6571}_{(2\,Pt)} \Rightarrow$ if V is (L-L) Voltage $\Rightarrow V_{ph} = 460/\sqrt{3}$

$\Rightarrow \begin{cases} T_{e,max}\text{(motor)} = 780.40 \text{ N.m.} & (1.5\text{ Pt}) \\ T_{e,max}\text{(gen)} = -1030 \text{ N.m.} & (1.5\text{ Pt}) \end{cases}$

if $V = 460 \Rightarrow$

$\begin{cases} T_{e,max}\text{(motor)} = 2343 \text{ N.m} \\ T_{e,max}\text{(gen)} = -3041 \text{ N.m.} \end{cases}$

$T_e \propto |V_{as}|^2 \Rightarrow V_{as2} = \frac{1}{2} V_{as1} \Rightarrow T_{e2} = \frac{1}{4} T_{e2}$

(1 Pt)