# The analytic nodal method

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# Content (week 8)



- The linear transformation technique
- The flat leakage approximation
- The Matlab script `anm_coupling_2D`

# Introduction 1

- The two-dimensional (2D) Cartesian heterogeneous reactor configurations correspond to the case where the neutron flux is function of two spatial variables.
- These cases cannot be solved analytically and the analytic nodal method (ANM) is an attempt to find a solution with the smallest possible approximation.
- We are limiting our investigations to a 2D Cartesian domain made from the assembly of many $x$–$y$ rectangular nodes which are infinite in the $z$ direction.
- Each node is assumed to be homogeneous, so that the corresponding nuclear properties $D_g(x,y)$, $\Sigma_{\mathrm{r}g}(x,y)$, $\Sigma_{g\leftarrow h}(x,y)$, $\chi_g(x,y)$ and $\nu\Sigma_{\mathrm{f}h}(x,y)$ are piecewise continuous.
- The reactor domain is divided into $I \times J$ regions of indices $1 \leq i \leq I$ and $1 \leq j \leq J$, in such a way that the nuclear properties in node $(i,j)$ are constant and equal to $D_{g,i,j}$, $\Sigma_{\mathrm{r}g,i,j}$, $\Sigma_{g\leftarrow h,i,j}$, $\chi_{g,i,j}$ and $\nu\Sigma_{\mathrm{f}h,i,j}$.

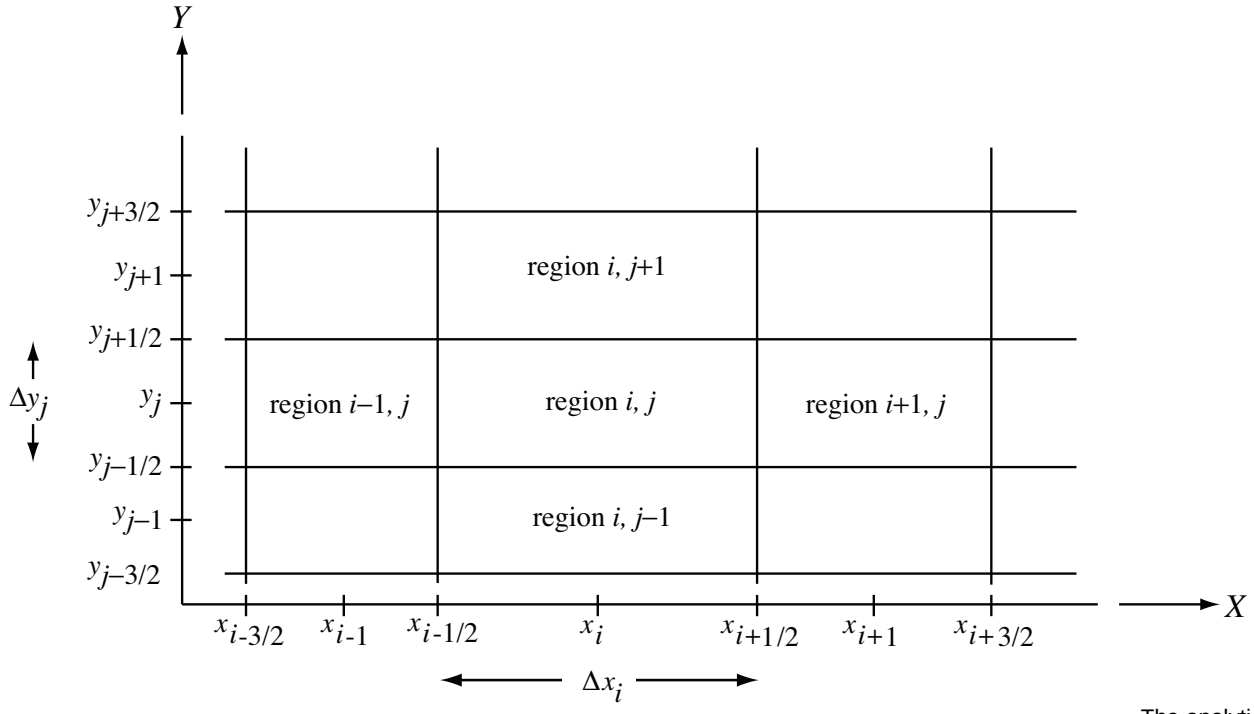
# Introduction



The nuclear properties of the reactor are only function of the independent variables $x$ and $y$. The multigroup diffusion equation in 2D Cartesian geometry simplifies to

$$-\frac{\partial}{\partial x}D_g(x,y)\frac{\partial \phi_g}{\partial x}-\frac{\partial}{\partial y}D_g(x,y)\frac{\partial \phi_g}{\partial y}+\Sigma_{\mathrm{r}g}(x,y)\,\phi_g(x,y)=Q_g^{\diamond}(x,y)$$

$$=\sum_{\substack{h=1\\h\neq g}}^{G}\Sigma_{g\leftarrow h}(x,y)\,\phi_h(x,y)+\frac{\chi_g(x,y)}{K_{\mathrm{eff}}}\sum_{h=1}^{G}\nu\Sigma_{\mathrm{f}h}(x,y)\,\phi_h(x,y)\quad . \tag{1}$$



# The linear transformation technique 1

The linear transformation technique is applied on each node, leading to the linear transformation $G \times G$ matrix $\mathbb{T}_{i,j}$ and to a set of $G$ eigenvalues $\lambda_{\ell,i,j}$. The transformation process is repeated for each node, leading to $I \times J$ matrix equations written as

(2) $$\frac{\partial^2}{\partial x^2}\mathbf{\Psi}(x,y) + \frac{\partial^2}{\partial y^2}\mathbf{\Psi}(x,y) + \mathrm{diag}(\lambda_{\ell,i,j})\,\mathbf{\Psi}(x,y) = \mathbf{0}$$

if $x_{i-1/2} < x < x_{i+1/2}$ and $y_{j-1/2} < y < y_{j+1/2}$. Each equation is uncoupled in energy, and can be written as $G$ differential equations of the form

(3) $$\frac{\partial^2}{\partial x^2}\psi_g(x,y) + \frac{\partial^2}{\partial y^2}\psi_g(x,y) + \lambda_{g,i,j}\,\psi_g(x,y) = 0\ ;\quad g = 1, G\ .$$

Unfortunately, it is impossible to find the analytical solution of Eq. (3) because its dependent variable $\psi_g(x,y)$ is generally not separable.

# The linear transformation technique 1

The ANM is based on transverse integration of Eq. (3), leading to

$$\int_{y_{j-1/2}}^{y_{j+1/2}} dy\, \frac{\partial^2}{\partial x^2}\psi_g(x,y) \;+\; \int_{y_{j-1/2}}^{y_{j+1/2}} dy\, \frac{\partial^2}{\partial y^2}\psi_g(x,y) + \lambda_{g,i,j} \int_{y_{j-1/2}}^{y_{j+1/2}} dy\, \psi_g(x,y) = 0 \tag{4}$$

which can be rewritten as

$$\frac{\partial^2}{\partial x^2}\psi^y_{g,j}(x) + \lambda_{g,i,j}\, \psi^y_{g,j}(x) = \frac{1}{\Delta y_j}\, \mathcal{F}^y_{g,j}(x) \tag{5}$$

where $\Delta y_j = y_{j+1/2} - y_{j-1/2}$,

$$\psi^y_{g,j}(x) = \frac{1}{\Delta y_j} \int_{y_{j-1/2}}^{y_{j+1/2}} dy\, \psi_g(x,y) \tag{6}$$

and where we introduced the $X$–directed transverse leakage term as

$$\mathcal{F}^y_{g,j}(x) = -\int_{y_{j-1/2}}^{y_{j+1/2}} dy\, \frac{\partial^2}{\partial y^2}\psi_g(x,y) = -\left.\frac{\partial}{\partial y}\psi_g(x,y)\right|_{y_{j-1/2}}^{y_{j+1/2}} . \tag{7}$$

# The linear transformation technique 2

Similarly, the transverse integration along the $X$ axis leads to

(8) $$\frac{\partial^2}{\partial y^2}\psi^x_{g,i}(y) + \lambda_{g,i,j}\,\psi^x_{g,i}(y) = \frac{1}{\Delta x_i}\,\mathcal{F}^x_{g,i}(y)$$

where $\Delta x_i = x_{i+1/2} - x_{i-1/2}$,

(9) $$\psi^x_{g,i}(y) = \frac{1}{\Delta x_i}\int_{x_{i-1/2}}^{x_{i+1/2}} dx\,\psi_g(x,y)$$

and where we introduced the $Y$–directed transverse leakage term as

(10) $$\mathcal{F}^x_{g,i}(y) = -\int_{x_{i-1/2}}^{x_{i+1/2}} dx\,\frac{\partial^2}{\partial x^2}\psi_g(x,y) = -\left.\frac{\partial}{\partial x}\psi_g(x,y)\right|_{x_{i-1/2}}^{x_{i+1/2}} .$$

Equations (5) and (8) can be solved analytically, provided that the $x$ and $y$ variation of the transverse leakage terms $\mathcal{F}^y_{g,j}(x)$ and $\mathcal{F}^x_{g,i}(y)$ are known.

# The linear transformation technique 3

This is where we introduce the unique approximation of the ANM:

- Shober initially assumed that the transverse leakages and the one-dimensional fluxes had the same shape. He wrote

$$\begin{aligned} \mathcal{F}^y_{g,j}(x) &= B^y_{g,i,j}\,\psi^y_{g,j}(x) \\ \mathcal{F}^x_{g,i}(y) &= B^x_{g,i,j}\,\psi^x_{g,i}(y) \quad . \end{aligned} \tag{11}$$

Shober found that the use of the buckling approximation led to large errors in highly nonseparable cases.

- As an alternative to the buckling approximation, Shober proposed to use a flat leakage approximation in which the transverse leakage shape is spatially flat over each node:

$$\begin{aligned} \mathcal{F}^y_{g,j}(x) &= \bar{\mathcal{F}}^y_{g,i,j} = \mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2}) \\ \mathcal{F}^x_{g,i}(y) &= \bar{\mathcal{F}}^x_{g,i,j} = \mathcal{J}^y_{g,j}(x_{i+1/2}) - \mathcal{J}^y_{g,j}(x_{i-1/2}) \end{aligned} \tag{12}$$

where the transformed currents are defined as

$$\mathcal{J}^y_{g,j}(x) \equiv -\frac{\partial}{\partial x}\psi^y_{g,j}(x) \quad \text{and} \quad \mathcal{J}^x_{g,i}(y) \equiv -\frac{\partial}{\partial y}\psi^x_{g,i}(y) \quad . \tag{13}$$

# The linear transformation technique 4

- Later, Smith introduced a quadratic leakage approximation in the ANM, leading to the version that is now currently used in legacy codes.
  - The expansion coefficients of the leakage fit are calculated by assuming that the quadratic polynomial extends over the two neighboring nodes and satisfies the average leakages in the central and two neighboring nodes.
  - The quadratic leakage fit does not rely on the diffusion equation itself and can only be justified if the transverse leakages vary smoothly across the three nodes.
  - Such an approximation can be constructed for node $(i,j)$, in the $X$–direction, using $\bar{F}^y_{g,i-1,j}$, $\bar{F}^y_{g,i,j}$ and $\bar{F}^y_{g,i+1,j}$, the transverse leakage terms without linear transformation.
  - In his thesis, Smith developed the ANM with a quadratic leakage approximation in two–group, 3D Cartesian geometry.

# The flat leakage approximation 1

Consider the flat leakage approximation in $G$–group, 2D Cartesian geometry. Eq. (5) leads to

$$\frac{\partial^2}{\partial x^2}\psi^y_{g,j}(x) + \lambda_{g,i,j}\,\psi^y_{g,j}(x) = \frac{1}{\Delta y_j}\left[\mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2})\right] \quad . \tag{14}$$

Integration over node $(i,j)$ leads to the transformed nodal balance equation, written as

$$\begin{aligned} \bar{\psi}_{g,i,j} &= \frac{1}{\Delta x_i\,\lambda_{g,i,j}}\left[\mathcal{J}^y_{g,j}(x_{i+1/2}) - \mathcal{J}^y_{g,j}(x_{i-1/2})\right] \\ &+ \frac{1}{\Delta y_j\,\lambda_{g,i,j}}\left[\mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2})\right] \quad . \end{aligned} \tag{15}$$

Let us first consider the case where $\lambda_{g,i,j} \geq 0$. In energy group $g$ and in node $(i,j)$, Eq. (14) has an analytical solution of the form

$$\begin{aligned} \psi^y_{g,j}(x) &= \frac{1}{\Delta y_j\,\lambda_{g,i,j}}\left[\mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2})\right] + A_{g,i,j}\cos(\sqrt{\lambda_{g,i,j}}\,x) \\ &+ B_{g,i,j}\sin(\sqrt{\lambda_{g,i,j}}\,x) \end{aligned} \tag{16}$$

if $x_{i-1/2} < x < x_{i+1/2}$.

# The flat leakage approximation 2

Integrating Eq. (16) over the node leads to

$$\bar{\psi}_{g,i,j} = \frac{1}{\Delta y_j \, \lambda_{g,i,j}} \left[ \mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2}) \right] + \frac{A_{g,i,j}}{\Delta x_i \sqrt{\lambda_{g,i,j}}} \sin(\sqrt{\lambda_{g,i,j}}\, x) \Big|_{x_{i-i/2}}^{x_{i+i/2}}$$

$$- \frac{B_{g,i,j}}{\Delta x_i \sqrt{\lambda_{g,i,j}}} \cos(\sqrt{\lambda_{g,i,j}}\, x) \Big|_{x_{i-i/2}}^{x_{i+i/2}} \quad . \tag{17}$$

Differentiating Eq. (16) over the node leads to

$$\mathcal{J}^y_{g,j}(x) = A_{g,i,j} \sqrt{\lambda_{g,i,j}} \sin(\sqrt{\lambda_{g,i,j}}\, x) - B_{g,i,j} \sqrt{\lambda_{g,i,j}} \cos(\sqrt{\lambda_{g,i,j}}\, x) \quad . \tag{18}$$

# The flat leakage approximation 3

Equations (16) to (18) can be rewritten, if $\lambda_{g,i,j} \leq 0$, as

$$\begin{aligned} \psi^y_{g,j}(x) &= \frac{1}{\Delta y_j \, \lambda_{g,i,j}} \left[ \mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2}) \right] \\ &+ C_{g,i,j} \cosh(\sqrt{-\lambda_{g,i,j}}\, x) + E_{g,i,j} \sinh(\sqrt{-\lambda_{g,i,j}}\, x) \quad , \end{aligned} \tag{19}$$

$$\begin{aligned} \bar{\psi}_{g,i,j} &= \frac{1}{\Delta y_j \, \lambda_{g,i,j}} \left[ \mathcal{J}^x_{g,i}(y_{j+1/2}) - \mathcal{J}^x_{g,i}(y_{j-1/2}) \right] \\ &+ \frac{C_{g,i,j}}{\Delta x_i \sqrt{-\lambda_{g,i,j}}} \sinh(\sqrt{-\lambda_{g,i,j}}\, x) \Big|_{x_{i-i/2}}^{x_{i+i/2}} \\ &+ \frac{E_{g,i,j}}{\Delta x_i \sqrt{-\lambda_{g,i,j}}} \cosh(\sqrt{-\lambda_{g,i,j}}\, x) \Big|_{x_{i-i/2}}^{x_{i+i/2}} \end{aligned} \tag{20}$$

and

$$\begin{aligned} \mathcal{J}^y_{g,j}(x) &= -C_{g,i,j} \sqrt{-\lambda_{g,i,j}} \sinh(\sqrt{-\lambda_{g,i,j}}\, x) \\ &- E_{g,i,j} \sqrt{-\lambda_{g,i,j}} \cosh(\sqrt{-\lambda_{g,i,j}}\, x) \quad . \end{aligned} \tag{21}$$

# The flat leakage approximation



To proceed further, we need to rewrite Eqs. (16) to (21) in matrix algebra. We define

$$\begin{aligned}
\boldsymbol{\Psi}_j^y(x) &= \{\psi_{g,j}^y(x)\,;\ \ g=1,G\} \qquad \bar{\boldsymbol{\Psi}}_{i,j} = \{\bar{\psi}_{g,i,j}\,;\ \ g=1,G\} \\
\boldsymbol{\mathcal{J}}_j^y(x) &= \{\mathcal{J}_{g,j}^y(x)\,;\ \ g=1,G\} \qquad \boldsymbol{\mathcal{J}}_i^x(y) = \{\mathcal{J}_{g,i}^x(y)\,;\ \ g=1,G\} \\
\boldsymbol{A}_{i,j} &= \{A_{g,i,j}\,;\ \ g=1,G\} \qquad \boldsymbol{B}_{i,j} = \{B_{g,i,j}\,;\ \ g=1,G\}
\end{aligned}$$

so that the above equations can be cast into

(22)
$$\begin{bmatrix} \bar{\boldsymbol{\Psi}}_{i,j} \\ \boldsymbol{\mathcal{J}}_j^y(x_{i-1/2}) \end{bmatrix} = \mathbb{M}_{i,j}^- \begin{bmatrix} \boldsymbol{A}_{i,j} \\ \boldsymbol{B}_{i,j} \end{bmatrix} + \begin{bmatrix} -\mathbb{Y}_{i,j} & \mathbb{Y}_{i,j} \\ \mathbb{O} & \mathbb{O} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{J}}_i^x(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}_i^x(y_{j+1/2}) \end{bmatrix} ,$$

(23)
$$\begin{bmatrix} \bar{\boldsymbol{\Psi}}_{i,j} \\ \boldsymbol{\mathcal{J}}_j^y(x_{i+1/2}) \end{bmatrix} = \mathbb{M}_{i,j}^+ \begin{bmatrix} \boldsymbol{A}_{i,j} \\ \boldsymbol{B}_{i,j} \end{bmatrix} + \begin{bmatrix} -\mathbb{Y}_{i,j} & \mathbb{Y}_{i,j} \\ \mathbb{O} & \mathbb{O} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{J}}_i^x(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}_i^x(y_{j+1/2}) \end{bmatrix} ,$$

(24)
$$\boldsymbol{\Psi}_j^y(x_{i-1/2}) = \mathbb{N}_{i,j}^- \begin{bmatrix} \boldsymbol{A}_{i,j} \\ \boldsymbol{B}_{i,j} \end{bmatrix} + \begin{bmatrix} -\mathbb{Y}_{i,j} & \mathbb{Y}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{J}}_i^x(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}_i^x(y_{j+1/2}) \end{bmatrix}$$

and

(25)
$$\boldsymbol{\Psi}_j^y(x_{i+1/2}) = \mathbb{N}_{i,j}^+ \begin{bmatrix} \boldsymbol{A}_{i,j} \\ \boldsymbol{B}_{i,j} \end{bmatrix} + \begin{bmatrix} -\mathbb{Y}_{i,j} & \mathbb{Y}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{J}}_i^x(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}_i^x(y_{j+1/2}) \end{bmatrix}$$

# The flat leakage approximation



where $\mathbb{M}^{\mp}_{i,j}$ are two $2G \times 2G$ matrices, $\mathbb{N}^{\mp}_{i,j}$ are two $G \times 2G$ matrices and $\mathbb{Y}_{i,j}$ is a $G \times G$ matrix whose coefficients are recovered from Eqs. (16) to (21).

- Coefficients $\boldsymbol{A}_{i,j}$ and $\boldsymbol{B}_{i,j}$ from Eq. (24) can be eliminated using Eq. (22)
- Coefficients $\boldsymbol{A}_{i,j}$ and $\boldsymbol{B}_{i,j}$ from Eq. (25) can be eliminated using Eq. (23).

The resulting equations can be cast into

$$\text{(26)} \quad \boldsymbol{\Psi}^y_j(x_{i-1/2}) = \mathbb{P}_{i,j} \begin{bmatrix} \bar{\boldsymbol{\Psi}}_{i,j} \\ \boldsymbol{\mathcal{J}}^y_j(x_{i-1/2}) \\ \boldsymbol{\mathcal{J}}^x_i(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}^x_i(y_{j+1/2}) \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Psi}^y_j(x_{i+1/2}) = \mathbb{Q}_{i,j} \begin{bmatrix} \bar{\boldsymbol{\Psi}}_{i,j} \\ \boldsymbol{\mathcal{J}}^y_j(x_{i+1/2}) \\ \boldsymbol{\mathcal{J}}^x_i(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}^x_i(y_{j+1/2}) \end{bmatrix}$$

where $\mathbb{P}_{i,j}$ and $\mathbb{Q}_{i,j}$ are two $G \times 4G$ matrices.

The transformed nodal balance equation (15) can be written in matrix form as

$$\text{(27)} \quad \bar{\boldsymbol{\Psi}}_{i,j} = \begin{bmatrix} -\mathbb{X}_{i,j} & \mathbb{X}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{J}}^y_j(x_{i-1/2}) \\ \boldsymbol{\mathcal{J}}^y_j(x_{i+1/2}) \end{bmatrix} + \begin{bmatrix} -\mathbb{Y}_{i,j} & \mathbb{Y}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mathcal{J}}^x_i(y_{j-1/2}) \\ \boldsymbol{\mathcal{J}}^x_i(y_{j+1/2}) \end{bmatrix} .$$

# The flat leakage approximation



Finally, the linear transformation can be inverted, with the help of the following definitions:

$$\begin{aligned}
\boldsymbol{\Phi}_j^y(x) &= \{\phi_{g,j}^y(x)\ ;\ \ g=1,G\} = \mathbb{T}_{i,j}\,\boldsymbol{\Psi}_j^y(x) \\
\bar{\boldsymbol{\Phi}}_{i,j} &= \{\bar{\phi}_{g,i,j}\ ;\ \ g=1,G\} = \mathbb{T}_{i,j}\,\bar{\boldsymbol{\Psi}}_{i,j} \qquad \mathbb{S}_{i,j} = \mathbb{T}_{i,j}^{-1}\mathrm{diag}(D_{g,i,j})^{-1} \\
\boldsymbol{J}_j^y(x) &= \left\{ -D_{g,i,j}\frac{d}{dx}\phi_{g,j}^y(x)\ ;\ \ g=1,G \right\} = \mathrm{diag}(D_{g,i,j})\,\mathbb{T}_{i,j}\,\boldsymbol{\mathcal{J}}_j^y(x)
\end{aligned}$$

We can show that

$$\text{(28)} \qquad \boldsymbol{\Phi}_j^y(x_{i-1/2}) = \mathbb{L}_{i,j}\begin{bmatrix} \bar{\boldsymbol{\Phi}}_{i,j} \\ \boldsymbol{J}_j^y(x_{i-1/2}) \\ \boldsymbol{J}_i^x(y_{j-1/2}) \\ \boldsymbol{J}_i^x(y_{j+1/2}) \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Phi}_j^y(x_{i+1/2}) = \mathbb{R}_{i,j}\begin{bmatrix} \bar{\boldsymbol{\Phi}}_{i,j} \\ \boldsymbol{J}_j^y(x_{i+1/2}) \\ \boldsymbol{J}_i^x(y_{j-1/2}) \\ \boldsymbol{J}_i^x(y_{j+1/2}) \end{bmatrix}$$

where

$$\text{(29)} \qquad \mathbb{L}_{i,j} = \mathbb{T}_{i,j}\mathbb{P}_{i,j}\begin{bmatrix} \mathbb{T}_{i,j}^{-1} & \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{S}_{i,j} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{S}_{i,j} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \mathbb{S}_{i,j} \end{bmatrix} \quad \text{and} \quad \mathbb{R}_{i,j} = \mathbb{T}_{i,j}\mathbb{Q}_{i,j}\begin{bmatrix} \mathbb{T}_{i,j}^{-1} & \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{S}_{i,j} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{S}_{i,j} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \mathbb{S}_{i,j} \end{bmatrix}.$$

# The flat leakage approximation



The $X$–directed ANM coupling relations are therefore written by imposing the flux continuity on $x_{i-1/2}$, as

(30)
$$\mathbb{R}_{i-1,j}\begin{bmatrix}\bar{\boldsymbol{\Phi}}_{i-1,j}\\ \boldsymbol{J}^y_j(x_{i-1/2})\\ \boldsymbol{J}^x_{i-1}(y_{j-1/2})\\ \boldsymbol{J}^x_{i-1}(y_{j+1/2})\end{bmatrix}=\mathbb{L}_{i,j}\begin{bmatrix}\bar{\boldsymbol{\Phi}}_{i,j}\\ \boldsymbol{J}^y_j(x_{i-1/2})\\ \boldsymbol{J}^x_i(y_{j-1/2})\\ \boldsymbol{J}^x_i(y_{j+1/2})\end{bmatrix}.$$

The nodal balance equation can be written in term of non-transformed variables as

(31)
$$\bar{\boldsymbol{\Phi}}_{i,j}=\mathbb{B}_{i,j}\begin{bmatrix}\boldsymbol{J}^y_j(x_{i-1/2})\\ \boldsymbol{J}^y_j(x_{i+1/2})\end{bmatrix}+\mathbb{C}_{i,j}\begin{bmatrix}\boldsymbol{J}^x_i(y_{j-1/2})\\ \boldsymbol{J}^x_i(y_{j+1/2})\end{bmatrix}$$

where

(32)
$$\mathbb{B}_{i,j}=\mathbb{T}_{i,j}\begin{bmatrix}-\mathbb{X}_{i,j} & \mathbb{X}_{i,j}\end{bmatrix}\begin{bmatrix}\mathbb{S}_{i,j} & \mathbb{O}\\ \mathbb{O} & \mathbb{S}_{i,j}\end{bmatrix}$$

and

(33)
$$\mathbb{C}_{i,j}=\mathbb{T}_{i,j}\begin{bmatrix}-\mathbb{Y}_{i,j} & \mathbb{Y}_{i,j}\end{bmatrix}\begin{bmatrix}\mathbb{S}_{i,j} & \mathbb{O}\\ \mathbb{O} & \mathbb{S}_{i,j}\end{bmatrix}.$$

Relations (30) are used together with the $Y$–directed ANM coupling relations and with the nodal balance equation (31) to build the global matrix system.

# The Matlab script anm_coupling_2D 1

Calculation of matrices $\mathbb{L}_{i,j}$, $\mathbb{R}_{i,j}$, $\mathbb{B}_{i,j}$ and $\mathbb{C}_{i,j}$ for each node represents the core of the ANM. These matrices are function of the $G$–group cross sections and diffusion coefficients, of the node size and of the effective multiplication factor $K_{\text{eff}}$. They must be updated during the power iteration, as $K_{\text{eff}}$ change. The linear transformation and the calculation of the nodal coupling matrices for a single node is implemented in Matlab script `anm_coupling_2D`.

The script is called as

```
[L,R,B]=anm_coupling_2D(keff, xm, xp, dely, diff, sigr, chi, nusigf)
```

`keff`= effective multiplication factor

`xm`= value of $x_{i-1/2}$ (cm)

`xp`= value of $x_{i+1/2}$ (cm)

`dely`= node size $\Delta y_j$ along the $Y$–axis (cm)

`diff`= diffusion coefficient array of size $G$ (cm)

`sigr`= macroscopic removal cross section array of size $G$ (cm$^{-1}$)

`chi`= fission spectrum array of size $G$

`nusigf`= $\nu$ times the macroscopic fission cross section array of size $G$ (cm$^{-1}$)

`[L, R, B]`= nodal coupling matrices

# Closing remarks



- Equations (28) are written in a convenient form to introduce a zero-flux or albedo boundary condition on the left or right boundary.
- The relation between the surfacic flux and current, required to implement an albedo boundary condition, is obtained directly from

$$\mathbb{D}_g(\boldsymbol{r})\boldsymbol{\nabla}\phi_g(\boldsymbol{r})\cdot\boldsymbol{N}(\boldsymbol{r})+\frac{1}{2}\frac{1-\beta(\boldsymbol{r})}{1+\beta(\boldsymbol{r})}\,\phi_g(\boldsymbol{r})=0\quad\text{if }\boldsymbol{r}\in\partial W_i$$

- The above Matlab script does not deal with situations where $\lambda_{g,i,j}$ approaches zero. A production implementation should take this possibility into consideration.
- The ANM unknowns are the average fluxes on the node volumes and the average net currents on the node surfaces. This choice of unknowns is a characteristics of dual discretization approaches such as the mesh-centered finite differences method or the Raviart-Thomas finite element technique. The ANM belongs to the family of dual discretization approaches.
- The ANM in 2D leads to a matrix system of order approximatively equal to $3GN$ where $G$ is the number of energy groups and $N$ is the number of nodes.