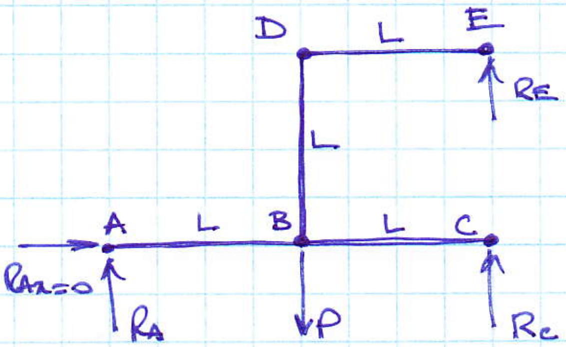
SOLUTION TD3 – MEC2405 A2012

– Question 1

a) . DCL + Equilibre :



. $\sum F_y = 0$: $R_A - P + R_C + R_E = 0$ (a)

. $\sum M_C = 0$ ↺+ : $-2R_A L + PL = 0$

→ $R_A = P/2$ (b)

3 inc. : R_A, R_C et R_E

2 eq. d'équilibre : $\sum F_y = 0$; $\sum M = 0$

HYPERSTATIQUE !

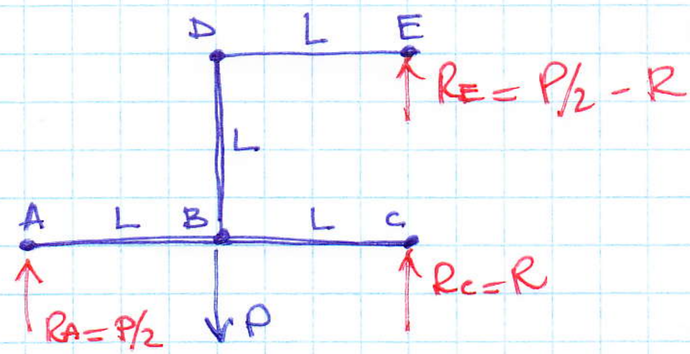
3 possibilités de surabondantes : R_A, R_C ou R_E

→ Choisissons $R_C = R$

→ (b) dans (a) :

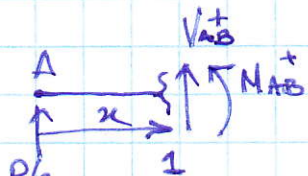
$R_E = P/2 - R$ (c)

→ Le DCL devient :



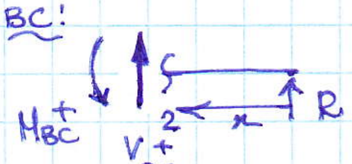
. Efforts internes :

AB :



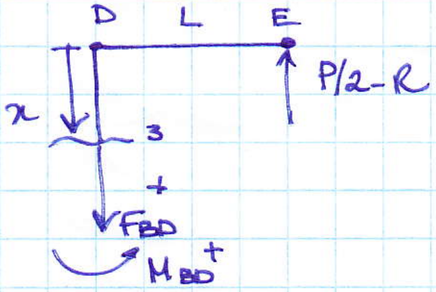
$\sum M_1 = 0$ ↺+ : $M_{AB} = \frac{Px}{2}$ $0 \le x \le L$

BC :



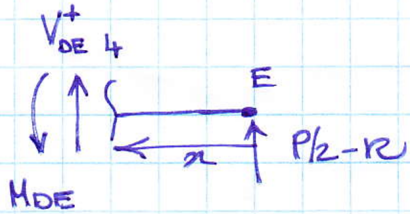
$\sum M_2 = 0$ ↺+ : $M_{BC} = -Rx$ $0 \le x \le L$

BD:



$\Sigma M_3 = 0 \circlearrowleft^{+}$: $M_{BD} = -(P/2 - R)L$ $\quad 0 \le x \le L$

DE:



$\Sigma M_4 = 0 \circlearrowleft^{+}$: $M_{DE} = -(P/2 - R)x$ $\quad 0 \le x \le L$

• Castigliano :

$$\delta_C = 0 = \frac{\partial U}{\partial R} = \Sigma \int_0^L \frac{M(\partial M/\partial R)\,dx}{EI} = \int_0^L \frac{M_{AB}\cancel{(\partial M_{AB}/\partial R)}^{\,0}\,dx}{EI} + \int_0^L \frac{M_{BC}(\partial M_{BC}/\partial R)\,dx}{EI} + \int_0^L \frac{M_{BD}(\partial M_{BD}/\partial R)\,dx}{EI} + \int_0^L \frac{M_{DE}(\partial M_{DE}/\partial R)\,dx}{EI}$$

AB: $\partial M_{AB}/\partial R = 0$

BC: $\partial M_{BC}/\partial R = -x$

BD: $\partial M_{BD}/\partial R = L$

DE: $\partial M_{DE}/\partial R = x$

$$= \int_0^L \frac{-Rx \cdot -x\,dx}{EI} + \int_0^L \frac{-(P/2-R)L \cdot L\,dx}{EI} + \int_0^L \frac{-(P/2-R)x \cdot x\,dx}{EI}$$

$$\Rightarrow \frac{Rx^3}{3EI}\Big|_0^L + -\frac{(P/2-R)L^2 x}{EI}\Big|_0^L + -\frac{(P/2-R)x^3}{3EI}\Big|_0^L = 0$$

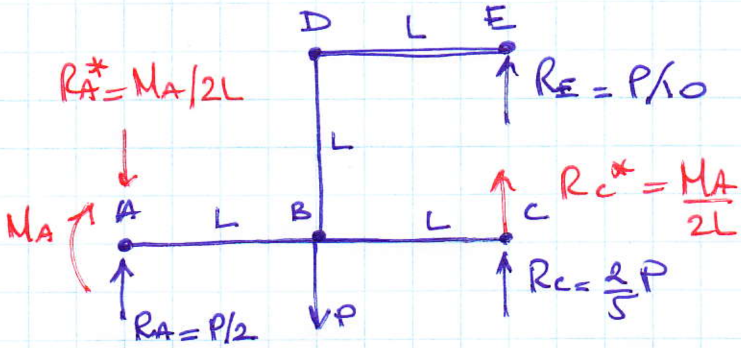
$$\Rightarrow \frac{RL^3}{3EI} - \frac{PL^3}{2EI} + \frac{RL^3}{EI} - \frac{PL^3}{6EI} + \frac{RL^3}{3EI} = 0$$

$$R = R_C = \frac{2}{5}P \quad \text{(d)}$$

(d) dans (c):

$$R_E = P/2 - 2/5\,P = P/10$$

b) $\theta_A = ?$



- Si on ajoute un moment fictif M_A en "A" pour déterminer la rotation θ_A en "A", il faudra refaire l'équilibre. Pour $\Sigma M = 0$, on peut ajouter par exemple une réaction supplémentaire R_A^* en "A":

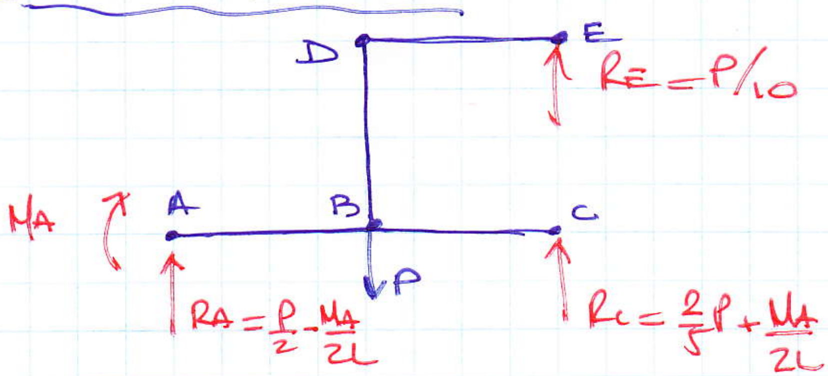
$\Sigma M_C = 0$ ↺+ :

$$-M_A - \frac{P}{2}\cdot 2L + 2L R_A^* + PL = 0$$

$$\Rightarrow R_A^* = \frac{M_A}{2L}$$

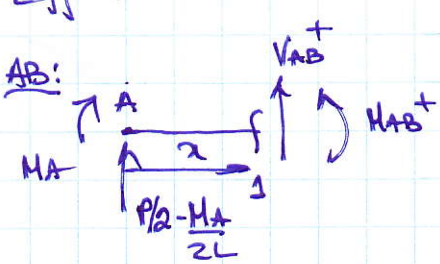
- Pour $\Sigma F_y = 0$, on peut ajouter une réaction supplémentaire R_C^* en C $= \frac{M_A}{2L}$

→ Le DCL devient alors:



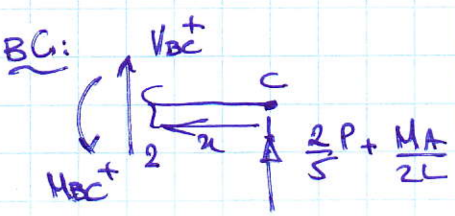
• Efforts internes

AB:



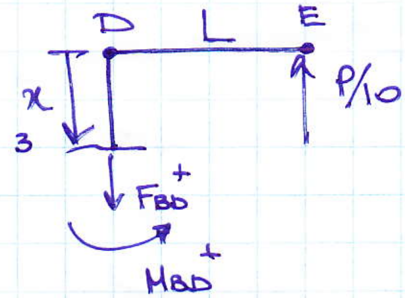
$\Sigma M_1 = 0$ ↺+ : $M_{AB} = \left(\frac{P}{2} - \frac{M_A}{2L}\right)x + M_A$

BC:



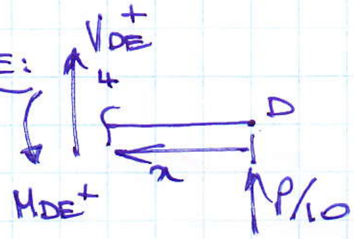
$\Sigma M_2 = 0$ ↺+ : $M_{BC} = -\left(\frac{2}{5}P + \frac{M_A}{2L}\right)x$

BD:



$\Sigma M_3 = 0 \circlearrowleft^+$: $M_{BD} = -\frac{P}{10} L$

DE:



$\Sigma M_4 = 0 \circlearrowleft^+$: $M_{DE} = -\frac{P}{10} x$

- Castigliano:

$$\theta_A = \frac{\partial U}{\partial M_A} = \Sigma \int_0^L \frac{M(\partial M/\partial M_A)\,dx}{EI} = \int_0^L \frac{M_{AB}(\partial M_{AB}/\partial M_A)\,dx}{EI} + \int_0^L \frac{M_{BC}\,\partial M_{BC}/\partial M_A\,dx}{EI}$$

$$+ \int_0^L \frac{M_{BD}\cancelto{0}{(\partial M_{BD}/\partial M_A)}\,dx}{EI} + \int_0^L \frac{M_{DE}\cancelto{0}{(\partial M_{DE}/\partial M_A)}\,dx}{EI}$$

AB: $\partial M_{AB}/\partial M_A = -x/2L + 1$

BC: $\partial M_{BC}/\partial M_A = -x/2L$

BD: $\partial M_{BD}/\partial M_A = 0$

DE: $\partial M_{DE}/\partial M_A = 0$

$$\Rightarrow \theta_A = \int_0^L \frac{((P/2 - \cancelto{0}{M_A/2L})x + \cancelto{0}{M_A})(-x/2L+1)}{EI}\,dx +$$

$$\int_0^L \frac{-(2/5\,P + \cancelto{0}{M_A/2L})x \cdot -x/2L}{EI}\,dx$$

$$= \int_0^L \left(-\frac{Px^2}{4EIL} + \frac{Px}{2EI}\right)dx + \int_0^L \frac{Px^2}{5EIL}\,dx$$

$$\Rightarrow \theta_A = -\frac{PL^2}{12EI} + \frac{PL^2}{4EI} + \frac{PL^2}{15EI} = \frac{-5PL^2 + 15PL^2 + 4PL^2}{60EI} = \frac{14PL^2}{60EI}$$

$$= \frac{7PL^2}{30EI}$$

- QUESTION 2

- Plan xy: Moment autour de l'axe "z" + Charge en compression → Poutre-Colonne
- Plan xz: Charge en compression seulement → Colonne

- Plan xz : (Rotule - Rotule ; K = 1) → ($C/Cr \leq 1,0$)

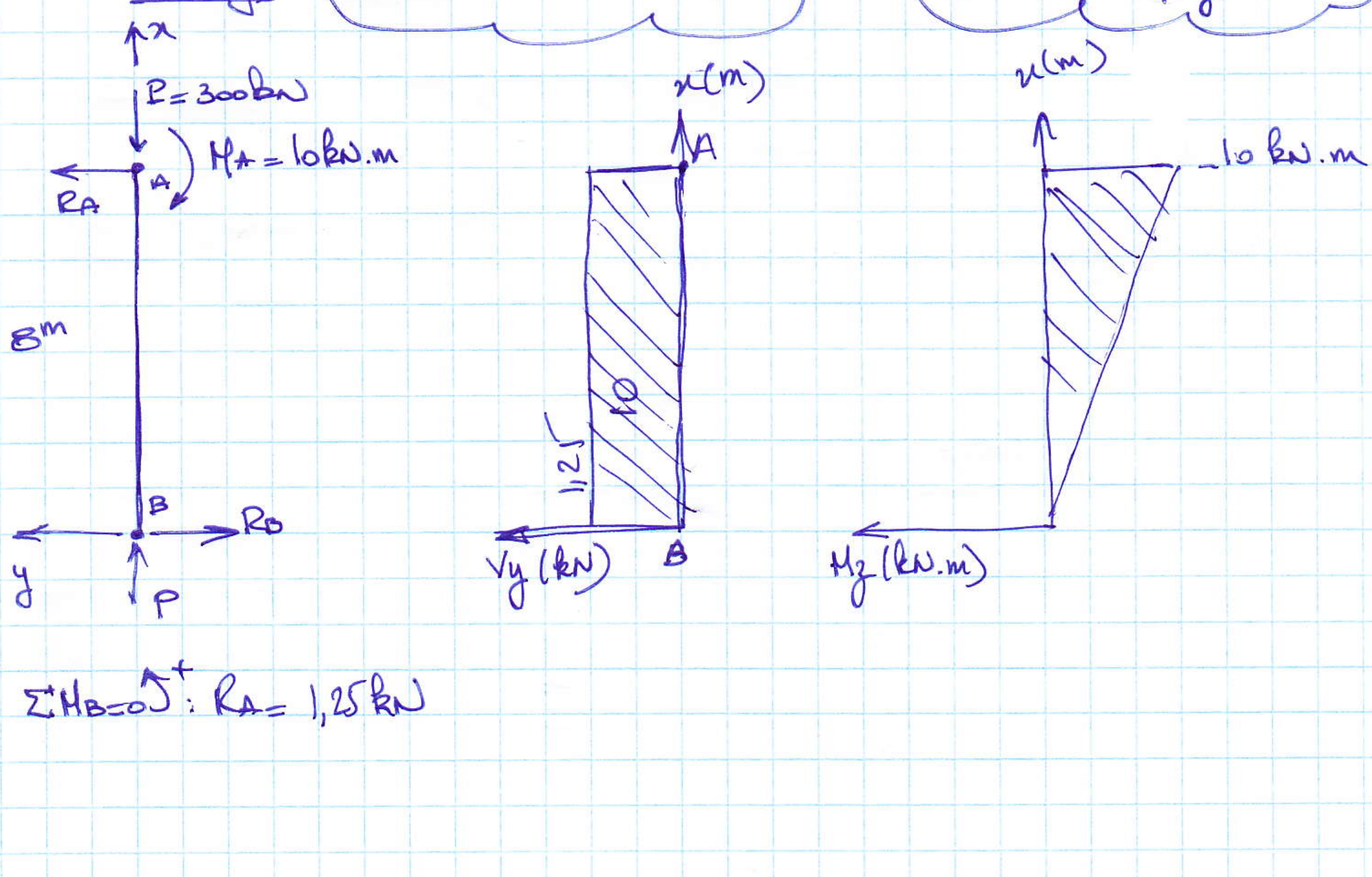
- $C = \alpha P = (1.5)(300) = \underline{450 \text{ kN}}$

- $Cr = \phi A S_y (1 + \lambda^{2n})^{-1/n} = (0,9)(6620)(500)(1 + 2,453^{2\times1,34})^{-1/1,34} = 464,06 \times 10^3 N = \underline{464,03 \text{ kN}}$

$$\lambda = \frac{KL}{r_y}\sqrt{\frac{S_y}{\pi^2 E}} = \frac{(1)(8\times10^3)}{51,9}\sqrt{\frac{500}{\pi^2(200\times10^3)}} = 2,453$$

- $\frac{C}{Cr} = \frac{450}{464,03} = 0,97 \leq 1$ (Limite)

- Plan xy: (Rotule - Rotule : K = 1) → ($C/Cr + \frac{Famp/z \cdot M_{fz}}{M_{rz}} \leq 1,0$)



$\Sigma M_B = 0 \circlearrowleft^{+}$: $R_A = 1,25 \text{ kN}$

$\cdot\ C = \alpha P = (1{,}5)(300) = 450\,kN$

$\cdot\ C_r = \phi A S_y \left(1 + \lambda^{2n}\right)^{-1/n} = (0{,}9)(6620)(500)\left(1 + 1{,}4306^{2\times 1{,}34}\right)^{-1/1{,}34} = 1{,}143\times10^{6}\ N = \underline{1143\ kN}$

$$\lambda = \frac{KL}{r_z}\sqrt{\frac{S_y}{\pi^2 E}} = \frac{(1)(8\times10^3)}{89}\sqrt{\frac{500}{\pi^2(200\times10^3)}} = 1{,}4306$$

$$\cdot\ F_{amp}/z = \frac{1}{1 - \frac{C}{P_{cr}}} = \frac{1}{1 - \frac{450}{1619}} = 1{,}385$$

$$P_{cr} = \frac{\pi^2 E I_z}{(KL)^2} = \frac{\pi^2(200\times10^3)(52{,}5\times10^6)}{(1\times8000)^2} = 1{,}619\times10^6\,N = \underline{1619\ kN}$$

$\cdot\ M_{rz} = \phi S_y S_z = (0{,}9)(500)(509\times10^3) = \underline{229{,}05\times10^6\ N.mm}$

$\cdot\ M_z = (1{,}5)(10) = \underline{15\ kN.m} = \underline{15\times10^6\,N.mm}$

$$\longrightarrow\quad \frac{450}{1143} + \frac{(1{,}385)(15\times10^6)}{229{,}05\times10^6} = 0{,}484 \leq 1{,}0 \qquad (O.K)$$