The neutron flux – part 2

Alain Hébert

alain.hebert@polymtl.ca

Institut de génie nucléaire

École Polytechnique de Montréal

Content (week 7)

- The steady-state source density
- Eigensolutions
- The transport correction
- Multigroup discretization
- Multigroup steady-state transport equation

The steady-state source density 1

- The neutral particles are neutrons and the source density is a multiplicative source of secondary fission, scattering and (n,xn) secondary neutrons.
- The steady-state source density is used in both lattice and static full-core calculations.
- In the following developments, the energy variable $E = mV_{\mathrm{n}}^2/2$ is used in replacement of the neutron velocity V_{n} as independent variable.

In the case of a multiplying medium with neutrons, the differential form of the steady-state transport equation is given by

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla} \phi(\boldsymbol{r}, E, \boldsymbol{\Omega}) \; + \; \Sigma(\boldsymbol{r}, E) \, \phi(\boldsymbol{r}, E, \boldsymbol{\Omega}) \; = \; Q(\boldsymbol{r}, E, \boldsymbol{\Omega}) \quad . \tag{1}$$

The steady-state source density 2

Assuming that the fission reactions are isotropic in the LAB, the steady-state source density is written

$$Q(\boldsymbol{r}, E, \boldsymbol{\Omega}) = \int_{4\pi} d^2\Omega' \int_0^\infty dE'\, \Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \boldsymbol{\Omega} \leftarrow \boldsymbol{\Omega}')\, \phi(\boldsymbol{r}, E', \boldsymbol{\Omega}') + \frac{1}{4\pi K_{\mathrm{eff}}} Q^{\mathrm{fiss}}(\boldsymbol{r}, E) \quad (2)$$

where

$\Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \boldsymbol{\Omega} \leftarrow \boldsymbol{\Omega}') =$ macroscopic differential scattering cross section taking into account diffusion and (n,xn) reactions.

$K_{\mathrm{eff}} =$ effective multiplication factor. K_{eff} is the factor by which the fission sources must be divided in order to maintain the steady-state condition.

$Q^{\mathrm{fiss}}(\boldsymbol{r}, E) =$ isotropic fission sources.

In isotropic media, the scattering cross section is only a function of the scattering angle, and

$$Q(\boldsymbol{r}, E, \boldsymbol{\Omega}) = \frac{1}{2\pi} \int_{4\pi} d^2\Omega' \int_0^\infty dE'\, \Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \boldsymbol{\Omega}\cdot\boldsymbol{\Omega}')\, \phi(\boldsymbol{r}, E', \boldsymbol{\Omega}') + \frac{1}{4\pi K_{\mathrm{eff}}} Q^{\mathrm{fiss}}(\boldsymbol{r}, E) \quad (3)$$

The steady-state source density 3

It is convenient to expand the scattering cross section in term of Legendre polynomials, as

$$\Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \boldsymbol{\Omega} \cdot \boldsymbol{\Omega}') = \sum_{\ell=0}^{L} \frac{2\ell+1}{2} \Sigma_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E')\, P_\ell(\boldsymbol{\Omega} \cdot \boldsymbol{\Omega}') \tag{4}$$

where L is the scattering order of the medium where the neutron is moving. $L = 0$ and $L = 1$ correspond to isotropic scattering and to linearly anisotropic scattering in the LAB, respectively. The Legendre coefficients $\Sigma_{\mathrm{s},\ell}(E \leftarrow E')$ are defined as

$$\Sigma_{\mathrm{s},\ell}(E \leftarrow E') = \int_{-1}^{1} d\mu\, \Sigma_{\mathrm{s}}(E \leftarrow E', \mu)\, P_\ell(\mu) \quad . \tag{5}$$

The steady-state source density

Using the addition theorem of spherical harmonics, it is possible to rewrite the scattering source of Eq. (1) in term of the spherical harmonics components of the flux:

$$\begin{aligned} Q(\boldsymbol{r}, E, \boldsymbol{\Omega}) &= \int_0^\infty dE' \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi} \Sigma_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E') \sum_{m=-\ell}^{\ell} R_\ell^m(\boldsymbol{\Omega})\, \phi_\ell^m(\boldsymbol{r}, E') \\ &+ \frac{1}{4\pi K_{\mathrm{eff}}} Q^{\mathrm{fiss}}(\boldsymbol{r}, E) \end{aligned} \tag{6}$$

where

$$\phi_\ell^m(\boldsymbol{r}, E) = \int_{4\pi} d^2\Omega\; R_\ell^m(\boldsymbol{\Omega})\, \phi(\boldsymbol{r}, E, \boldsymbol{\Omega}) \quad . \tag{7}$$

The steady-state source density

We assume that the isotropic fission source is independent of the energy of the incident neutron. For each fissile nuclide i, the energy of emitted neutrons is distributed according to a probability density known as fission spectrum $\chi_i(E)$:

> The quantity $\chi_i(E)\,dE$ is the probability for an emitted neutron to have an energy equal to E (within a dE interval) in the LAB.

(8) $$\text{with} \quad \int_0^\infty dE\,\chi_i(E) = 1 \quad .$$

The isotropic fission source is written

(9) $$Q^{\text{fiss}}(\boldsymbol{r}, E) = \sum_{j=1}^{J^{\text{fiss}}} \chi_j(E) \int_0^\infty dE'\,\nu\Sigma_{\text{f},j}(\boldsymbol{r}, E')\,\phi(\boldsymbol{r}, E')$$

where

$J^{\text{fiss}} =$ total number of fissile isotopes

$\nu\Sigma_{\text{f},j}(\boldsymbol{r}, E) =$ number of emitted neutrons per fission times the macroscopic fission cross section of the j^{th} fissile isotope.

The steady-state source density

Substitution of Eq. (9) in Eq. (6) gives the complete expression of a multiplicative source in reactor physics:

$$
\begin{aligned}
Q(\boldsymbol{r}, E, \boldsymbol{\Omega}) &= \int_0^\infty dE' \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi} \Sigma_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E') \sum_{m=-\ell}^{\ell} R_\ell^m(\boldsymbol{\Omega})\, \phi_\ell^m(\boldsymbol{r}, E') \\
&+ \frac{1}{4\pi K_{\mathrm{eff}}} \sum_{j=1}^{J^{\mathrm{fiss}}} \chi_j(E) \int_0^\infty dE'\, \nu\Sigma_{\mathrm{f},j}(\boldsymbol{r}, E')\, \phi(\boldsymbol{r}, E')
\end{aligned}
\tag{10}
$$

where $\phi(\boldsymbol{r}, E) \equiv \phi_0^0(\boldsymbol{r}, E)$.

Eigensolutions

The steady-state transport equation is an eigenproblem featuring a number of properties:

- $\phi = 0$ is a trivial solution. A set of non trivial eigensolutions exists for some discrete values of K_{eff}. We clearly obtain an eigenvalue problem, with K_{eff} as the eigenvalue and ϕ as the eigenvector. The fundamental solution corresponds to the maximum possible value of K_{eff} and is the only eigensolution with a physical meaning.
- Only the flux distribution of the fundamental solution is almost positive everywhere in the domain. The other eigensolutions are the harmonics of the flux and are partly positive and partly negative.
- Each eigensolution can be arbitrarily normalized. If $\phi(\boldsymbol{r}, E)$ is a non-trivial solution, then $C\,\phi(\boldsymbol{r}, E)$ is also a non-trivial solution $\forall$ non-zero value of the constant C. The value of C can be computed from the thermal power P of the reactor using

$$\int_0^\infty dE \int_V d^3r\; H(\boldsymbol{r}, E)\,\phi(\boldsymbol{r}, E) = P \tag{11}$$

  where V is the volume of the reactor and $H(\boldsymbol{r}, E)$ is the power factor giving the recoverable energy in term of the flux.
- It is possible to define a mathematical adjoint problem with the same eigenvalues as the original problem.

Eigensolutions

A multiplicative domain with $K_{\text{eff}} > 1$ is said to be over-critical because its flux level will increase with time if we remove the eigenvalue.

Each term of the steady-state transport equation is an operator for which an adjoint operator can be found. The general rules for creating the adjoint of an operator are:

1. Transpose the matrix operators.
2. Change the sign of odd-parity differential operators.
3. Interchange the arguments of the kernels of integral operators.

Using these rules, the adjoint transport equation is written

$$-\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\phi^*(\boldsymbol{r},E,\boldsymbol{\Omega}) + \Sigma(\boldsymbol{r},E)\,\phi^*(\boldsymbol{r},E,\boldsymbol{\Omega}) = Q^*(\boldsymbol{r},E,\boldsymbol{\Omega}) \tag{12}$$

where the adjoint source density is

$$\begin{aligned} Q^*(\boldsymbol{r},E,\boldsymbol{\Omega}) &= \int_0^\infty dE' \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi}\,\Sigma_{\text{s},\ell}(\boldsymbol{r},E'\leftarrow E) \sum_{m=-\ell}^{\ell} R_\ell^m(\boldsymbol{\Omega})\,\phi_\ell^{*m}(\boldsymbol{r},E') \\ &+ \frac{1}{4\pi K_{\text{eff}}} \sum_{j=1}^{J^{\text{fiss}}} \nu\Sigma_{\text{f},j}(\boldsymbol{r},E) \int_0^\infty dE'\,\chi_j(E')\,\phi^*(\boldsymbol{r},E') \quad . \end{aligned} \tag{13}$$

Eigensolutions

The adjoint flux is a function of E. The adjoint flux cannot be a distribution of E because the term $\chi_j(E')\,\phi^*(\boldsymbol{r},E')$ in Eq. (13) cannot involve the product of two distributions of E'. The adjoint flux solution is generally normalized to an arbitrary value:

$$\int_0^\infty dE \int_V d^3r\ \phi^*(\boldsymbol{r},E) = 1 \quad . \tag{14}$$

The adjoint transport equation is also an eigenvalue problem. Its eigenvalues are the same as those of the original transport equation. Each eigenvalue can be expressed in term of the corresponding eigenvectors $\phi(\boldsymbol{r},E)$ and $\phi^*(\boldsymbol{r},E)$ using the Rayleigh ratio:

$$K_{\text{eff}} = \frac{\displaystyle\sum_{j=1}^{J^{\text{fiss}}} \int_0^\infty dE'\,\chi_j(E')\,\phi^*(\boldsymbol{r},E') \int_0^\infty dE\,\nu\Sigma_{\text{f},j}(\boldsymbol{r},E)\,\phi(\boldsymbol{r},E)}{\displaystyle\int_0^\infty dE'\,\phi^*(\boldsymbol{r},E')\left[\Sigma(\boldsymbol{r},E')\,\phi(\boldsymbol{r},E') - \int_0^\infty dE\,\Sigma_{\text{s},0}(\boldsymbol{r},E'\leftarrow E)\,\phi(\boldsymbol{r},E)\right]} \quad .$$

The Rayleigh ratio is stationary with respect to a small variation $\delta\Sigma$. The first order variation δK_{eff} corresponding to such a variation in cross sections can be written in term of ϕ, ϕ^*, Σ and $\delta\Sigma$, without using $\delta\phi$ or $\delta\phi^*$. This is the origin of the classical perturbation theory.

The transport correction 1

- The transport equation is frequently solved by assuming the isotropy of the scattering sources in the LAB (i. e., $L = 0$).
- This approximation is generally not valid but can be mitigated by performing a transport correction on the cross sections appearing in the transport equation.

The basic principle is to add a forward-peaked component in the Legendre expansion of the differential scattering cross section. This additional component takes the form of a Dirac delta term in Eq. (4).

$$\begin{aligned} \Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \mu) &= \sum_{\ell=0}^{L} \frac{2\ell+1}{2} \overline{\Sigma}_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E')\, P_\ell(\mu) \\ &+ \Delta\Sigma_{\mathrm{tr}}(\boldsymbol{r}, E')\, \delta(E - E')\, \delta(\mu - 1) \end{aligned} \tag{15}$$

where $\overline{\Sigma}_{\mathrm{s},\ell}(\boldsymbol{r}, E' \leftarrow E)$ is a modified Legendre coefficient and $\Delta\Sigma_{\mathrm{tr}}(\boldsymbol{r}, E')$ is the additional coefficient multiplying the Dirac delta term.

The transport correction

Coefficients $\overline{\Sigma}_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E')$ and $\Delta\Sigma_{\mathrm{tr}}(\boldsymbol{r}, E')$ are computed so as to preserve the Legendre moments of Eq. (5). We write

$$\int_{-1}^{1} d\mu\, \Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \mu)\, P_\ell(\mu) = \Sigma_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E') \tag{16}$$

for $0 \leq \ell \leq L+1$. Substituting Eq. (15) in Eq. (16) and using the relation $P_\ell(1) = 1$, we obtain

$$\overline{\Sigma}_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E') + \Delta\Sigma_{\mathrm{tr}}(\boldsymbol{r}, E')\, \delta(E - E') = \Sigma_{\mathrm{s},\ell}(\boldsymbol{r}, E \leftarrow E') \quad \text{if } 0 \leq \ell \leq L \tag{17}$$

and

$$\Delta\Sigma_{\mathrm{tr}}(\boldsymbol{r}, E') = \Sigma_{\mathrm{s},L+1}(\boldsymbol{r}, E'). \tag{18}$$

Writing $L = 0$, Eqs. (15) and (18) reduce to

$$\begin{aligned} \Sigma_{\mathrm{s}}(\boldsymbol{r}, E \leftarrow E', \mu) &= \frac{1}{2}\left[\Sigma_{\mathrm{s},0}(\boldsymbol{r}, E \leftarrow E') - \Sigma_{\mathrm{s},1}(\boldsymbol{r}, E')\, \delta(E - E')\right] \\ &+ \Sigma_{\mathrm{s},1}(\boldsymbol{r}, E')\, \delta(E - E')\, \delta(\mu - 1) \end{aligned} \tag{19}$$

The transport correction

so that the substitution of Eq. (19) in the steady-state transport equation leads to

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla} \phi(\boldsymbol{r}, E, \boldsymbol{\Omega}) + \bar{\Sigma}(\boldsymbol{r}, E)\, \phi(\boldsymbol{r}, E, \boldsymbol{\Omega}) = \overline{Q}(\boldsymbol{r}, E, \boldsymbol{\Omega}) \tag{20}$$

where the transport-corrected macroscopic total cross section is written

$$\bar{\Sigma}(\boldsymbol{r}, E) = \Sigma(\boldsymbol{r}, E) - \Sigma_{\mathrm{s},1}(\boldsymbol{r}, E) \tag{21}$$

and where the transport-corrected steady-state source density is now given by

$$\begin{aligned} \overline{Q}(\boldsymbol{r}, E, \boldsymbol{\Omega}) &= \frac{1}{4\pi} \int_0^\infty dE' \left[\Sigma_{\mathrm{s},0}(\boldsymbol{r}, E \leftarrow E') - \Sigma_{\mathrm{s},1}(\boldsymbol{r}, E')\, \delta(E - E') \right] \phi(\boldsymbol{r}, E') \\ &+ \frac{1}{4\pi K_{\mathrm{eff}}} \sum_{j=1}^{J^{\mathrm{fiss}}} \chi_j(E) \int_0^\infty dE'\, \nu\Sigma_{\mathrm{f},j}(\boldsymbol{r}, E')\, \phi(\boldsymbol{r}, E'). \end{aligned} \tag{22}$$

The transport correction

- The transport equation (20) with the transport-corrected source density (22) is as easy to solve as the transport equation for an isotropic collision in the LAB, but does include a correction for anisotropic scattering effects.
- This technique of transport correction is very useful with the integral form of the transport equation, as these equations are otherwise limited to isotropic scattering in the LAB.
- In this case, the optical path is computed using the transport corrected total cross section defined in Eq. (21). Lattice calculations are generally performed this way.
- This formulation of the transport correction may lead to negative values of the within-group scattering cross section components after multigroup discretization.
- Equation (19) is known as the outscatter form of the transport correction. It should not be used to compute diffusion coefficients.

Multigroup discretization 1

- Consists to divide the energy domain in a set of G energy groups inside which the neutrons are assumed to behave as one-speed particles and to take averages of all energy-dependent quantities over these groups.
- The groups are defined in term of the lethargy variable. The average gain in lethargy per collision is almost constant over the complete energy range.
- We assume constant cross sections in each group. The definition of the lethargy limits is made after close inspection of the position of the resolved resonances. The cross sections of low-energy resonances must be precisely discretized and other resolved resonances must be enclosed in an energy group.
- Multigroup discretization may be performed in many instances of the global computational scheme.
  - The first energy condensation occurs in the groupr module of NJOY, over the energy structure of the cross section library, typically with $50 \leq G \leq 400$.
  - The next energy condensation occurs in the lattice code, with $2 \leq G \leq 8$.
  - In some case, the lattice code uses a multilevel energy representation and performs two energy condensations; the first toward an intermediate energy structure with $20 \leq G \leq 35$, and the second toward the energy structure of the reactor calculation.

Multigroup discretization 2

The reference energy E_0 used to define the lethargy $u = \ln(E_0/E)$ is generally taken above 10 MeV in order to correspond to the maximum energy of neutrons in a nuclear reactor. The lethargy is zero for the neutrons of energy E_0 and increases as neutrons slow-down. We divide the domain $0 \leq E \leq E_0$ into G groups W_g, so that

$$W_g = \{u\,;\; u_{g-1} \leq u < u_g\} = \{E\,;\; E_g < E \leq E_{g-1}\}\,;\quad g = 1, G$$

where $u_g = \ln(E_0/E_g)$ and $u_0 = 0$.
Before proceding with energy condensation, we define the average $\langle X \rangle_g$ of a function or of a distribution $X(E)$ over group g. The following definition is used:

(23) $$\langle X \rangle_g = \int_{u_{g-1}}^{u_g} du\, X(u) = \int_{E_g}^{E_{g-1}} dE\, X(E) \quad \text{if } X(E) \text{ is a distribution}$$

and

(24) $$\langle X \rangle_g = \frac{1}{u_g - u_{g-1}} \int_{u_{g-1}}^{u_g} du\, X(u) = \frac{1}{\ln(E_{g-1}/E_g)} \int_{E_g}^{E_{g-1}} \frac{dE}{E}\, X(E)$$

if $X(E)$ is a function

Multigroup discretization

The group-averaged values of the flux, cross section, differential cross section and source density are therefore defined as

$$\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) \equiv \langle \phi(\boldsymbol{r},\boldsymbol{\Omega}) \rangle_g = \int_{u_{g-1}}^{u_g} du\, \phi(\boldsymbol{r},u,\boldsymbol{\Omega}) \quad , \tag{25}$$

$$\phi_g(\boldsymbol{r}) \equiv \langle \phi(\boldsymbol{r}) \rangle_g = \int_{u_{g-1}}^{u_g} du\, \phi(\boldsymbol{r},u) \quad , \tag{26}$$

$$\langle \Sigma(\boldsymbol{r})\, \phi(\boldsymbol{r}) \rangle_g = \int_{u_{g-1}}^{u_g} du\, \Sigma(\boldsymbol{r},u)\, \phi(\boldsymbol{r},u) \quad , \tag{27}$$

$$\left\langle \Sigma_{\mathrm{s},\ell}(\boldsymbol{r})\, \phi(\boldsymbol{r}) \right\rangle_{g\leftarrow h} = \int_{u_{g-1}}^{u_g} du \int_{u_{h-1}}^{u_h} du'\, \Sigma_{\mathrm{s},\ell}(\boldsymbol{r}, u \leftarrow u')\, \phi(\boldsymbol{r},u') \tag{28}$$

and

$$\langle Q(\boldsymbol{r},\boldsymbol{\Omega}) \rangle_g = \int_{u_{g-1}}^{u_g} du\, Q(\boldsymbol{r},u,\boldsymbol{\Omega}) \quad . \tag{29}$$

Multigroup discretization

The angular and integrated multigroup flux components are defined as

$$\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) \equiv \langle \phi(\boldsymbol{r},\boldsymbol{\Omega}) \rangle_g \quad \text{and} \quad \phi_g(\boldsymbol{r}) \equiv \langle \phi(\boldsymbol{r}) \rangle_g \quad .$$

The multigroup cross section components preserve the values of the reaction rates:

(30) $$\Sigma_g(\boldsymbol{r}) = \frac{1}{\phi_g(\boldsymbol{r})} \langle \Sigma(\boldsymbol{r})\,\phi(\boldsymbol{r}) \rangle_g \quad , \quad \nu\Sigma_{\mathrm{f},j,g}(\boldsymbol{r}) = \frac{1}{\phi_g(\boldsymbol{r})} \left\langle \nu\Sigma_{\mathrm{f},j}(\boldsymbol{r})\,\phi(\boldsymbol{r}) \right\rangle_g \quad ,$$

(31) $$\Sigma_{\mathrm{s},\ell,g\leftarrow h}(\boldsymbol{r}) = \frac{1}{\phi_h(\boldsymbol{r})} \left\langle \Sigma_{\mathrm{s},\ell}(\boldsymbol{r})\,\phi(\boldsymbol{r}) \right\rangle_{g\leftarrow h}$$

The multigroup transport correction obtained by energy condensation of Eq. (18), leading to

(32) $$\Delta\Sigma_{\mathrm{tr},g}(\boldsymbol{r}) = \Sigma_{\mathrm{s},1,g}(\boldsymbol{r}).$$

Multigroup discretization

The multigroup transport corrected macroscopic cross section is written

$$\overline{\Sigma}_g(\boldsymbol{r}) = \Sigma_g(\boldsymbol{r}) - \Delta\Sigma_{\mathrm{tr},g}(\boldsymbol{r}) \tag{33}$$

and the multigroup P_0 transport-corrected component of the differential scattering cross section is written

$$\overline{\Sigma}_{\mathrm{s},0,g\leftarrow h}(\boldsymbol{r}) = \Sigma_{\mathrm{s},0,g\leftarrow h}(\boldsymbol{r}) - \delta_{gh}\,\Delta\Sigma_{\mathrm{tr},g}(\boldsymbol{r}) \tag{34}$$

where δ_{gh} is the Kronecker delta function. Equation (34) is likely to produce negative $\overline{\Sigma}_{\mathrm{s},0,g\leftarrow g}(\boldsymbol{r})$ components.

Multigroup steady-state equation 1

The transport equation for neutrons can be written in multigroup form, leading to a set of G coupled integro-differential equations. We will now present the steady-state transport equations. The multigroup of the steady-state transport equation in group g is written

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla} \phi_g(\boldsymbol{r}, \boldsymbol{\Omega}) + \Sigma_g(\boldsymbol{r}) \, \phi_g(\boldsymbol{r}, \boldsymbol{\Omega}) = Q_g(\boldsymbol{r}, \boldsymbol{\Omega}) \tag{35}$$

where $1 \leq g \leq G$. The multigroup source density is

$$\begin{aligned} Q_g(\boldsymbol{r}, \boldsymbol{\Omega}) &= \sum_{h=1}^{G} \sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi} \Sigma_{\mathrm{s},\ell,g \leftarrow h}(\boldsymbol{r}) \sum_{m=-\ell}^{\ell} R_\ell^m(\boldsymbol{\Omega}) \, \phi_{\ell,h}^m(\boldsymbol{r}) \\ &+ \frac{1}{4\pi K_{\mathrm{eff}}} \sum_{j=1}^{J^{\mathrm{fiss}}} \chi_{j,g} \sum_{h=1}^{G} \nu \Sigma_{\mathrm{f},j,h}(\boldsymbol{r}) \, \phi_h(\boldsymbol{r}) \end{aligned} \tag{36}$$

and the average fission spectrum in group g is

$$\chi_{j,g} \equiv \langle \chi_j \rangle_g = \int_{u_{g-1}}^{u_g} du \, \chi_j(u) \ . \tag{37}$$

Multigroup steady-state equation 2

The differential transport equation, with a first-order transport correction, is written:

$$\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) + \overline{\Sigma}_g(\boldsymbol{r})\,\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) + \frac{1}{4\pi}\Delta\Sigma_{\mathrm{tr},g}(\boldsymbol{r})\,\phi_g(\boldsymbol{r}) = Q_g(\boldsymbol{r},\boldsymbol{\Omega}) \quad . \tag{38}$$

The differential transport equation, without transport correction, is written:

$$\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) + \Sigma_g(\boldsymbol{r})\,\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) = Q_g(\boldsymbol{r},\boldsymbol{\Omega}) \quad . \tag{39}$$

The integral infinite-domain form, without transport correction, is

$$\phi_g(\boldsymbol{r},\boldsymbol{\Omega}) = \int_0^\infty ds\; \mathrm{e}^{-\tau_g(s)}\; Q_g(\boldsymbol{r}-s\,\boldsymbol{\Omega},\boldsymbol{\Omega}) \tag{40}$$

where the optical path in group g is

$$\tau_g(s) = \int_0^s ds'\; \Sigma_g(\boldsymbol{r}-s'\,\boldsymbol{\Omega}) \quad . \tag{41}$$

Adjoint multigroup equation 1

We have obtain an eigenproblem taking the form if a set of coupled differential equations.

- This problem is not self-adjoint, due to slowing-down effects.
- The corresponding adjoint equation is written in term of the multigroup adjoint flux.
- The adjoint flux is a function of energy; not a distribution. They are therefore defined as

$$\phi_g^*(\boldsymbol{r},\boldsymbol{\Omega}) \equiv \langle \phi^*(\boldsymbol{r},\boldsymbol{\Omega}) \rangle_g = \frac{1}{u_g - u_{g-1}} \int_{u_{g-1}}^{u_g} du\, \phi^*(\boldsymbol{r},u,\boldsymbol{\Omega}) \quad , \tag{42}$$

The adjoint multigroup form of the steady-state transport equation in group g is written

$$-\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\phi_g^*(\boldsymbol{r},\boldsymbol{\Omega}) + \Sigma_g(\boldsymbol{r})\,\phi_g^*(\boldsymbol{r},\boldsymbol{\Omega}) = Q_g^*(\boldsymbol{r},\boldsymbol{\Omega}) \tag{43}$$

where $1 \leq g \leq G$. The adjoint multigroup source density is

$$\begin{aligned} Q_g^*(\boldsymbol{r},\boldsymbol{\Omega}) &= \sum_{h=1}^{G}\sum_{\ell=0}^{L} \frac{2\ell+1}{4\pi}\,\Sigma_{\mathrm{s},\ell,h\leftarrow g}(\boldsymbol{r}) \sum_{m=-\ell}^{\ell} R_\ell^m(\boldsymbol{\Omega})\,\phi_{\ell,h}^{*m}(\boldsymbol{r}) \\ &+ \frac{1}{4\pi K_{\mathrm{eff}}} \sum_{j=1}^{J^{\mathrm{fiss}}} \nu\Sigma_{\mathrm{f},j,g}(\boldsymbol{r}) \sum_{h=1}^{G} \chi_{j,h}\,\phi_h^*(\boldsymbol{r}) \quad . \end{aligned} \tag{44}$$

Solution approaches

- The most accurate, and most expensive, solution technique is the Monte Carlo method. Many million of particles histories are simulated, based on a sequence of random numbers. The simulation of each particle is taking into account its interactions with an accurate representation of the geometric domain and using a continuous-energy or multigroup representation of the cross sections.
- The second class of approaches are the deterministic solution techniques. These approaches do not use any random number generator. They are based on the application of numerical analysis techniques to the transport equation.

  The deterministic approaches are based on many approximations related to their energetic and spatial discretization and to the limitation of the angular representation:
  - integro-differential form $\Rightarrow$ P_n or S_N methods,
  - characteristics form $\Rightarrow$ method of characteristics or
  - integral form $\Rightarrow$ collision probability method.

  The particle flux is used as dependent variable and the transport equation is solved with its boundary conditions. A multigroup representation of the cross sections is generally imposed.