Problème 05-04

Définition de variables pour les calculs

In[1]:= $J = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \; K = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - J;$

Défintion du chargement

In[10]:=
```
H[t_] = HeavisideTheta[t];
```

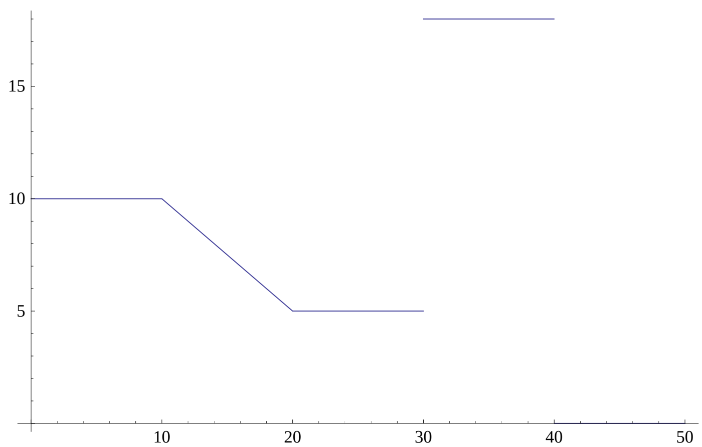
In[11]:=
```
σ1 = 10; σ2 = 5; σ3 = 18; t1 = 10; t2 = 20; t3 = 30; t4 = 40;
```

In[12]:= $\text{histoire}[t\_] = \sigma1\, H[t] + \frac{\sigma2 - \sigma1}{t2 - t1}\,(t - t1)\, H[t - t1] + \left(-\sigma1 - \frac{\sigma2 - \sigma1}{t2 - t1}\,(t - t1) + \sigma2\right) H[t - t2] + (\sigma3 - \sigma2)\, H[t - t3] - \sigma3\, H[t - t4];$

(Ce graphe montre l'histoire de chargement qui est imposée)

In[13]:=
```
Plot[histoire[t], {t, 0, 50}]
```

Out[13]=



Définiton des matrices internes

In[29]:= $S0 = \frac{3.}{3} J + \frac{7.}{2} K; \; S1 = \frac{2.}{3} J + \frac{2.}{2} K; \; S2 = \frac{1.}{3} J + \frac{3.}{2} K; \; \omega1 = \frac{1.}{5}; \; \omega2 = \frac{1.}{70};$

In[39]:=
```
A2 = Array[f, {6, 12}]; A21 = Transpose[CholeskyDecomposition[ω1 S1]];
A22 = Transpose[CholeskyDecomposition[ω2 S2]];
Do[A2[[i, j]] = A21[[i, j]];
  A2[[i, j + 6]] = A22[[i, j]];
  , {i, 1, 6}, {j, 1, 6}];
Id = Array[f, {12, 12}]; Do[Id[[i, j]] = 0, {i, 1, 12}, {j, 1, 12}];
Do[Id[[i, i]] = 1, {i, 1, 12}];
B = Id;
A3 = Array[f, {12, 12}]; Do[A3[[i, j]] = 0, {i, 1, 12}, {j, 1, 12}];
Do[A3[[i, i]] = ω1, {i, 1, 6}]; Do[A3[[i, i]] = ω2, {i, 7, 12}];
A1 = S0;
```

Matrice A2 associée au premier temps de retard

In[45]:= **MatrixForm[A21]**

Out[45]//MatrixForm=

0.421637	0.	0.	0.	0.	0.
−0.0527046	0.41833	0.	0.	0.	0.
−0.0527046	−0.0597614	0.414039	0.	0.	0.
0.	0.	0.	0.447214	0.	0.
0.	0.	0.	0.	0.447214	0.
0.	0.	0.	0.	0.	0.447214

Matrice A2 associée au deuxième temps de retard

In[46]:= **MatrixForm[A22]**

Out[46]//MatrixForm=

0.125988	0.	0.	0.	0.	0.
−0.0440959	0.118019	0.	0.	0.	0.
−0.0440959	−0.0635489	0.099449	0.	0.	0.
0.	0.	0.	0.146385	0.	0.
0.	0.	0.	0.	0.146385	0.
0.	0.	0.	0.	0.	0.146385

Matrice A2 totale

In[47]:= **MatrixForm[A2]**

Out[47]//MatrixForm=

0.421637	0.	0.	0.	0.	0.	0.125988	0.	0.
−0.0527046	0.41833	0.	0.	0.	0.	−0.0440959	0.118019	0.
−0.0527046	−0.0597614	0.414039	0.	0.	0.	−0.0440959	−0.0635489	0.0994
0.	0.	0.	0.447214	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.447214	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.447214	0.	0.	0.

Matrice A3

In[48]:= **MatrixForm[A3]**

Out[48]//MatrixForm=

0.2	0	0	0	0	0	0	0	0	0	0	0
0	0.2	0	0	0	0	0	0	0	0	0	0
0	0	0.2	0	0	0	0	0	0	0	0	0
0	0	0	0.2	0	0	0	0	0	0	0	0
0	0	0	0	0.2	0	0	0	0	0	0	0
0	0	0	0	0	0.2	0	0	0	0	0	0
0	0	0	0	0	0	0.0142857	0	0	0	0	0
0	0	0	0	0	0	0	0.0142857	0	0	0	0
0	0	0	0	0	0	0	0	0.0142857	0	0	0
0	0	0	0	0	0	0	0	0	0.0142857	0	0
0	0	0	0	0	0	0	0	0	0	0.0142857	0
0	0	0	0	0	0	0	0	0	0	0	0.0142857

Matrice A1

In[49]:= **MatrixForm[A1]**

Out[49]//MatrixForm=

2.66667	−0.833333	−0.833333	0.	0.	0.
−0.833333	2.66667	−0.833333	0.	0.	0.
−0.833333	−0.833333	2.66667	0.	0.	0.
0.	0.	0.	3.5	0.	0.
0.	0.	0.	0.	3.5	0.
0.	0.	0.	0.	0.	3.5

Définition du schéma de Euler

In[50]:=
```
W1[z_] = Inverse[Id + z Inverse[B].A3];
W2[z_] = -z Inverse[Id + z Inverse[B].A3].Inverse[B].Transpose[A2];
```

Définition du Schéma de Crank-Nicholson

In[52]:=
```
W3[z_] = Inverse[Id + z/2 Inverse[B].A3].(Id - z/2 Inverse[B].A3);

W4[z_] = -z/2 Inverse[Id + z/2 Inverse[B].A3].Inverse[B].Transpose[A2];
```

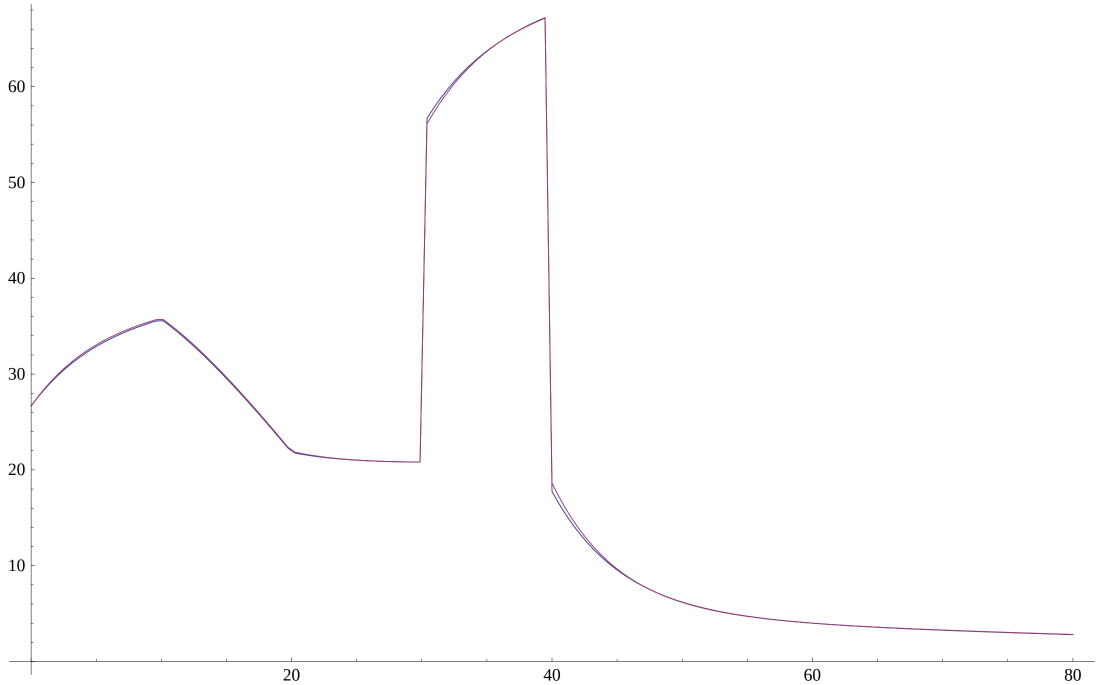
Programmation de l’intégration

In[54]:=
```
programmecomplexe[tf_, n_] := (
   h = tf / n;
   t = Table[tf/n (i - 1) + 1 * 10^-4, {i, 1, n + 1}];
   σ = Array[f, {n + 1, 6}];
   Do[σ[[i, 1]] = histoire[t[[i]]]; σ[[i, j]] = 0, {i, 1, n + 1}, {j, 2, 6}];
   ξEuler = Array[f, {n + 1, 12}];
   ϵEuler = Array[f, {n + 1, 6}];
   ξCN = Array[f, {n + 1, 12}];
   ϵCN = Array[f, {n + 1, 6}];
   (* Pour t = 0 *) ϵEuler[[1]] = A1.σ[[1]];
   ϵCN[[1]] = A1.σ[[1]]; ξEuler[[1]] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
   ξCN[[1]] = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0};
   Do[
    ξEuler[[i]] = W1[h].ξEuler[[i - 1]] + W2[h].(σ[[i]]);
    ϵEuler[[i]] = A1.σ[[i]] - A2.ξEuler[[i]];
    ξCN[[i]] = W3[h].ξCN[[i - 1]] + W4[h].(σ[[i]] + σ[[i - 1]]);
    ϵCN[[i]] = A1.σ[[i]] - A2.ξCN[[i]];
    , {i, 2, n + 1}];
   resultEuler = Array[f, {n + 1, 2}];
   resultCN = Array[f, {n + 1, 2}];
   Do[resultEuler[[i, 1]] = t[[i]]; resultEuler[[i, 2]] = ϵEuler[[i, 1]];
    resultCN[[i, 1]] = t[[i]]; resultCN[[i, 2]] = ϵCN[[i, 1]], {i, 1, n + 1}];
  )
```

In[55]:=
```
programmecomplexe[80, 150]
```

In[56]:=
```
ListLinePlot[{resultEuler, resultCN}]
```

Out[56]=



In[69]:=
```
ϵCNsol = Array[f, {150, 7}]; ϵCNsol = Transpose[ϵCNsol]; ϵCNsol[[1]] = N[t];
Do[ϵCNsol[[i + 1]] = Transpose[ϵCN][[i]], {i, 1, 6}]; exp = Transpose[ϵCNsol];

exportexcel = Array[f, {150, 7}]; exportexcel = Transpose[exportexcel];
exportexcel[[1]] = t; Do[exportexcel[[i + 1]] = Transpose[σ][[i]], {i, 1, 6}];
exportexcel = Transpose[exportexcel];
```

In[71]:=
```
Export["05-04-solution-CN.xls", exp]
```

Out[71]= 05-04-solution-CN.xls

```
Export["05-06-histoire-contrainte.xls", exportexcel]
```

05-06-histoire-contrainte.xls