Solution problème 03-04

$$\frac{d\sigma}{d\tau} = \overset{①}{\frac{\sigma_1}{t_1} t \; \delta(t)} + \overset{②}{2\left(\sigma_1 - \frac{\sigma_1}{t_1} t\right) \delta(t - t_1)} - \overset{③}{\left(\sigma_1 - \frac{\sigma_1}{t_1}(t - t_1)\right) \delta(t - 2t_1)}$$

$$+ \overset{④}{\frac{\sigma_1}{t} H(t)} - \overset{⑤}{\frac{2\sigma_1}{t_1} H(t - t_1)} + \overset{⑥}{\frac{\sigma_1}{t_1} H(t - 2t_1)}$$

$$\varepsilon(t) = \int_0^t \left(s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right) \frac{d\sigma}{d\tau} \, d\tau.$$

On va calculer les intégrales ① à ⑥ individuellement.

① $$\int_0^t \left(s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right) \frac{\sigma_1}{t_1} \tau \; \delta(\tau) \, d\tau$$

$$= \left(s_0 + s_1\left(1 - \exp(-(t-\tau)\lambda)\right)\right) \frac{\sigma_1}{t_1} \tau \Big|_{\tau=0} = 0$$

② $$\int_0^t 2\left(s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right) \left(\sigma_1 - \frac{\sigma_1}{t_1} \tau\right) \delta(\tau - t_1) \, d\tau$$

$$= \dots \left(\sigma_1 - \frac{\sigma_1}{t_1} \tau\right) \Big|_{\tau = t_1} = 0$$

③ $$\int_0^t \dots \left(\sigma_1 - \frac{\sigma_1}{t_1}(\tau - t_1)\right) \delta(\tau - 2t_1) \, d\tau =$$

$$\dots \; \sigma_1 - \frac{\sigma_1}{t_1}(\tau - t_1) \Big|_{\tau = 2t_1} = 0$$

④ $$\int_0^t \left(s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right) \frac{\sigma_1}{t_1} H(\tau)\, d\tau$$

$$= \frac{\sigma_1}{t_1}\left[(s_0 + s_1)\tau - s_1 \frac{\exp[(t-\tau)\lambda]}{\lambda}\right]\Bigg|_{\tau=0}^{\tau=t}$$

$$= \left[(s_0 + s_1)t - \frac{s_1}{\lambda} + \frac{s_1}{\lambda}\exp(-t\lambda)\right]\frac{\sigma_1}{t_1} H(t)$$

$$= \left[s_0 t + s_1\left(t - \frac{(1 - \exp(-t\lambda))}{\lambda}\right)\right]\frac{\sigma_1}{t_1} H(t)$$

⑤ $$\int_0^t -\left[s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right] \frac{2\sigma_1}{t_1} H(\tau - t_1)\, d\tau$$

$$= -\int_{t_1}^t \left[s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right] \frac{2\sigma_1}{t_1}\, d\tau\; H(t-t_1)$$

$$= -\frac{2\sigma_1}{t_1}\left[(s_0 + s_1)\tau - \frac{s_1 \exp[-(t-\tau)\lambda]}{\lambda}\right]_{\tau=t_1}^{\tau=t} H(t-t_1)$$

$$= -\frac{2\sigma_1}{t_1}\left[(s_0 + s_1)(t - t_1) - \frac{s_1}{\lambda} + \frac{s_1}{\lambda}\exp[-(t-t_1)\lambda]\right] H(t-t_1)$$

$$= -2\left[s_0(t-t_1) + s_1\left((t-t_1) - \frac{1 - \exp[-(t-t_1)\lambda]}{\lambda}\right)\right]\frac{\sigma_1}{t_1} H(t-t_1)$$

(6) $$\int_0^t \left(s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right) \frac{\sigma_1}{t_1} H(\tau - 2t_1)\, d\tau$$

$$= \int_{2t_1}^t \left(s_0 + s_1\left(1 - \exp[-(t-\tau)\lambda]\right)\right) \frac{\sigma_1}{t_1}\, d\tau \; H(t - 2t_1)$$

$$= \left[s_0(t - 2t_1) + s_1\left((t - 2t_1) - \frac{1 - \exp[-(t-2t_1)\lambda]}{\lambda}\right)\right] \frac{\sigma_1}{t_1} H(t - 2t_1)$$

Alors, $\varepsilon(t) =$ (4) + (5) + (6).