COURS

THÉORIE DE LA CIRCULATION

LES MODÈLES DE POURSUITE

CAR FOLLOWING

(Recueil des acétates utilisées)

par:

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Attention: Il n'est pas suffisant de consulter ces acétates. Elles ne remplacent pas les cours et les références bibliographiques choisies pour le cours.

Janvier 2003

INTRODUCTION

- L'ESPACEMENT LONGITUDINAL, L'INVERSE DE LA DENSITÉ EST UNE VARIABLE DE BASE DE LA CIRCULATION.
- ELLE EST RELIÉE A LA CAPACITE ET A LA SECURITE DE LA ROUTE.
- ELLE REPRESENTE LE NIVEAU DE SERVICE.
- LES MODELES MICROSCOPIQUES DE POURSUITE ESSAIENT DE REPRESENTER LE COMPORTEMENT DU CONDUCTEUR LORSQU'IL SUIT DE PRES UN AUTRE VEHICULE.
- SUIVRE DE PRES ÇA VEUT DIRE 15 à 30m OU A UN ECART D'ENVIRON 6 à 8 SEC.
- DANS LE CAS DE FAIBLES DENSITES, OÙ IL N'Y A PAS D'INTERACTION, IL FAUT APPLIQUER UNE AUTRE THEORIE.
- LES MODELES S'APPLIQUENT DE MANIERE RIGOUREUSE DANS LE CAS D'UNE SEULE VOIE SANS POSSIBILITES DE DEPASSEMENT.

- HYPOTHESE :

  - LE CONDUCTEUR BASE SON COMPORTEMENT SUR CELUI DU VEHICULE PRECEDENT.
  - LES REACTIONS POSSIBLES SONT L'ACCELERATION ET LA DECELERATION
  - ON IGNORE GENERALEMENT L'INTER-ACTION ENTRE LE CONDUCTEUR ET LE DEUXIEME VEHICULE DEVANT LUI.

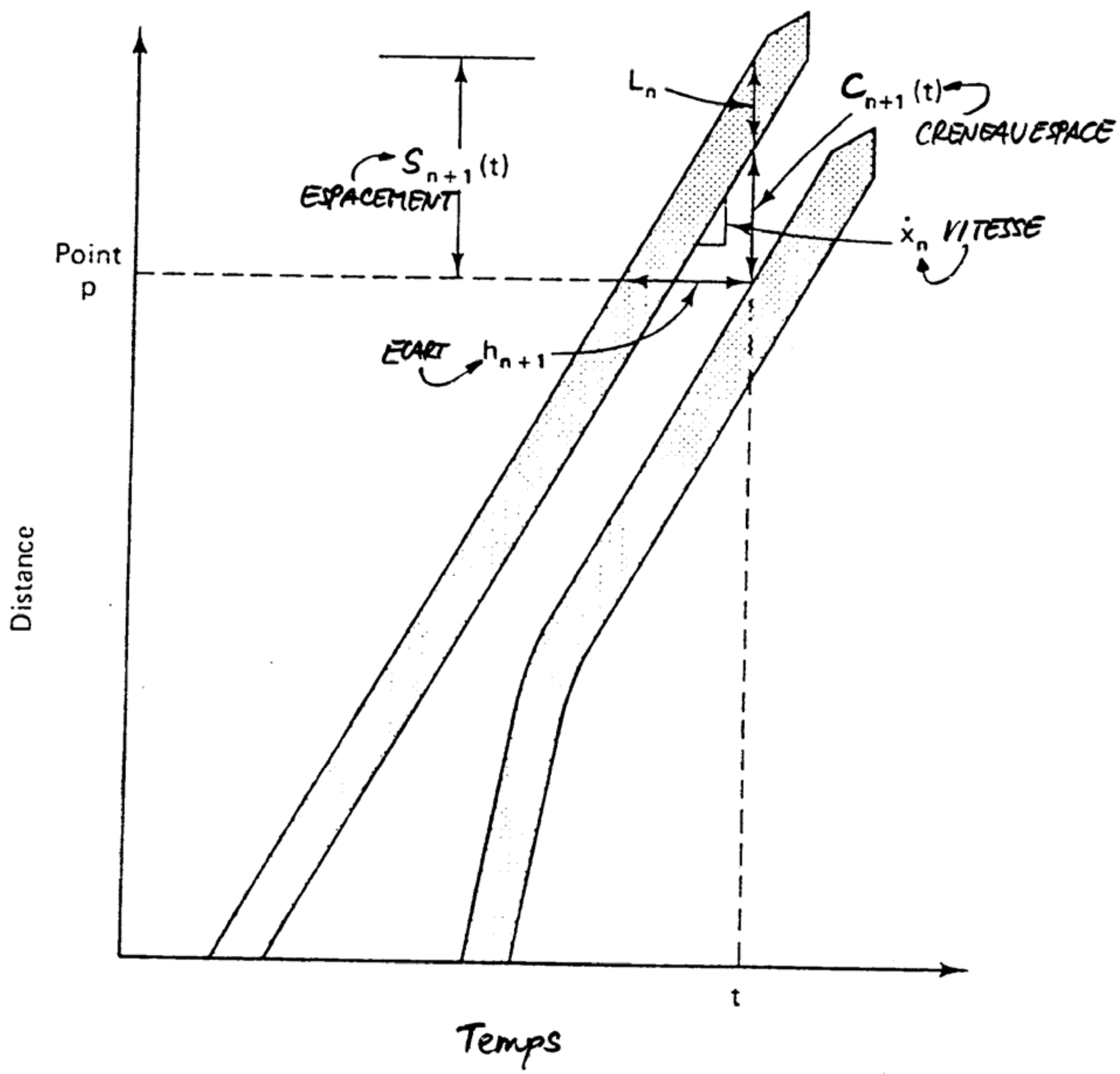
UTILISATION

- COMPREHENSION DE LA DYNAMIQUE DE LA CIRCULATION
- ANALYSE DE SECURITE
- ETUDE DU COMPORTEMENT DU CONDUCTEUR
- REPERCUSSION D'UNE PERTURBATION PONCTUELLE SUR LA CIRCULATION
- ANALYSE DE LA CAPACITE ET DU NIVEAU DE SERVICE

DEFINITIONS

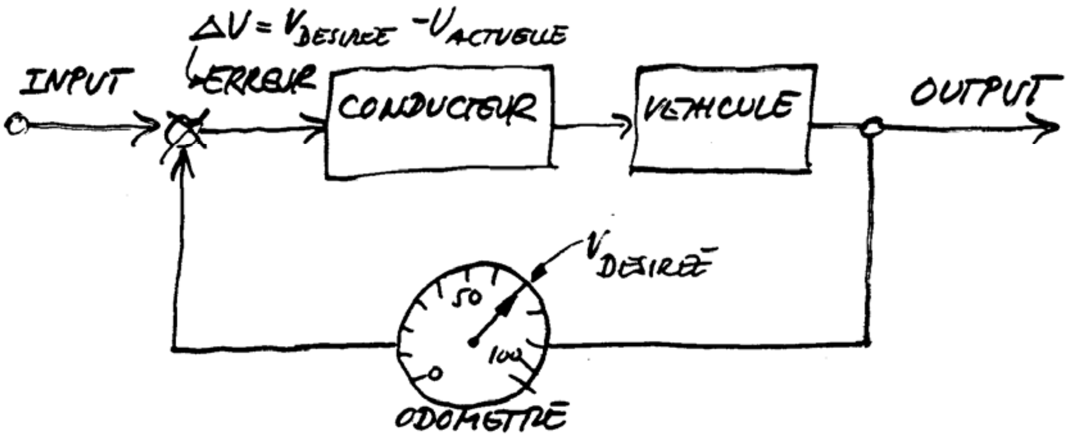
$$K = \frac{1000}{\bar{s}} \quad (\text{véh/Km}) \quad \text{DENSITE}$$

$$\bar{s} = \frac{\sum_{n=1}^{N} d_n}{N} \quad \text{ESPACEMENT MOYEN}$$



LE SYSTEME DE CONDUITE D'UN VEHICULE

- ON PEUT VOIR LA CONDUITE D'UNE AUTOMOBILE COMME UN SYSTEME A BOUCLE FERMEE OU COMME UN SYSTEME A CONTROLE PAR RETRO-ACTION.

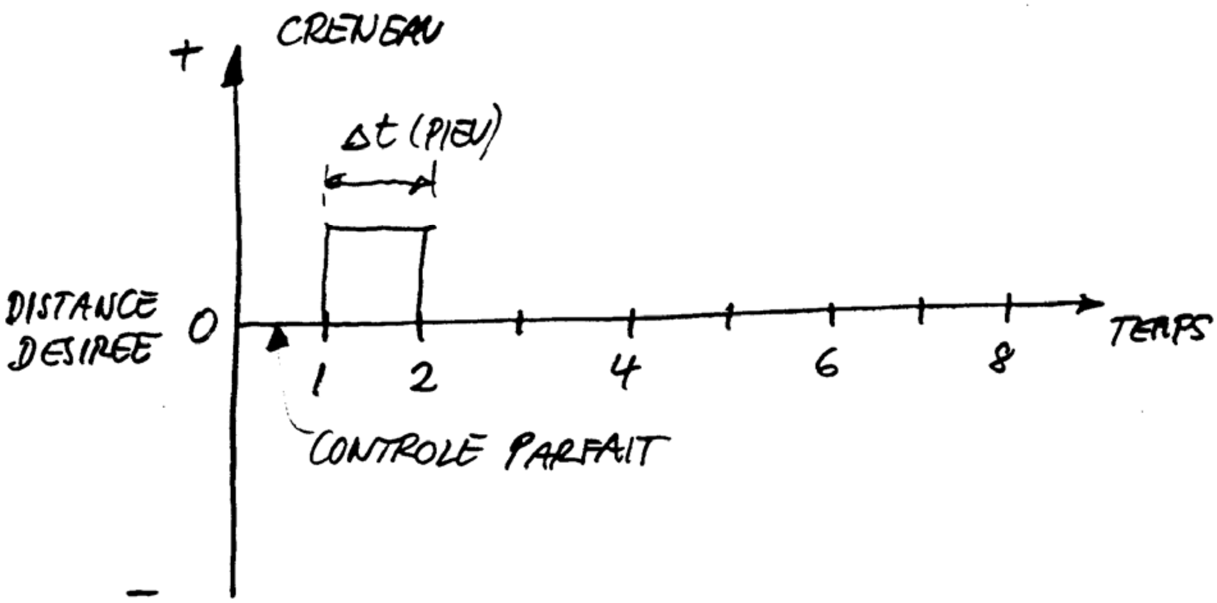


- LE CONDUCTEUR DESIRE CONSERVER UNE VITESSE CONSTANTE PAR. EX.

- INPUT : ROUTE, VEHICULE, ENVIRONNEMENT
  CONTROLES : VOLANT, ACCELERATEUR, FREINS
  OUTPUT : MOUVEMENT DANS LE TEMPS ET DANS L'ESPACE
  FEEDBACK : LECTURE DE LA VITESSE ACTUELLE

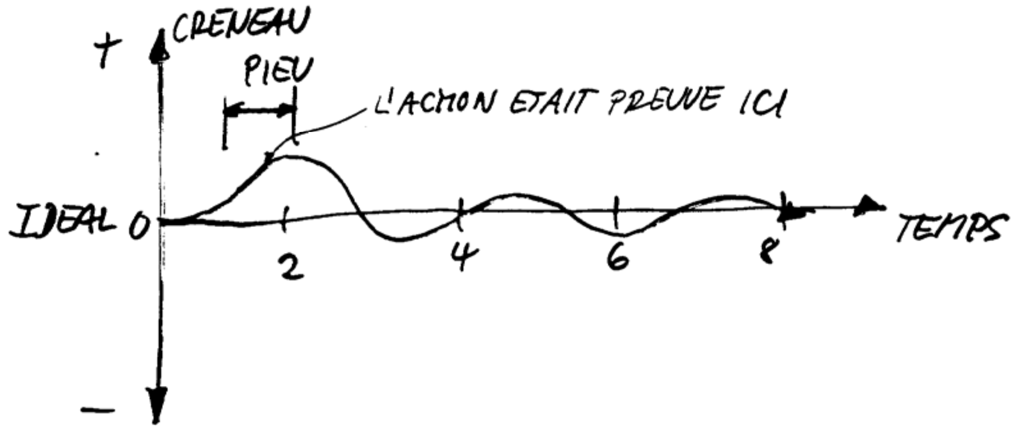
CE SYSTEME EST DIFFICILE A CONTROLER A CAUSE DU TEMPS DE REACTION Δt (PIEV)

- SOUVENT LA CORRECTION EST APPLIQUEE A UNE SITUATION QUI A DEJA CHANGÉ.
- LE CONDUCTEUR VEUT GARDER UNE DISTANCE DE SECURITE (CRENEAU) QU'IL CONSIDERE SECURITAIRE:

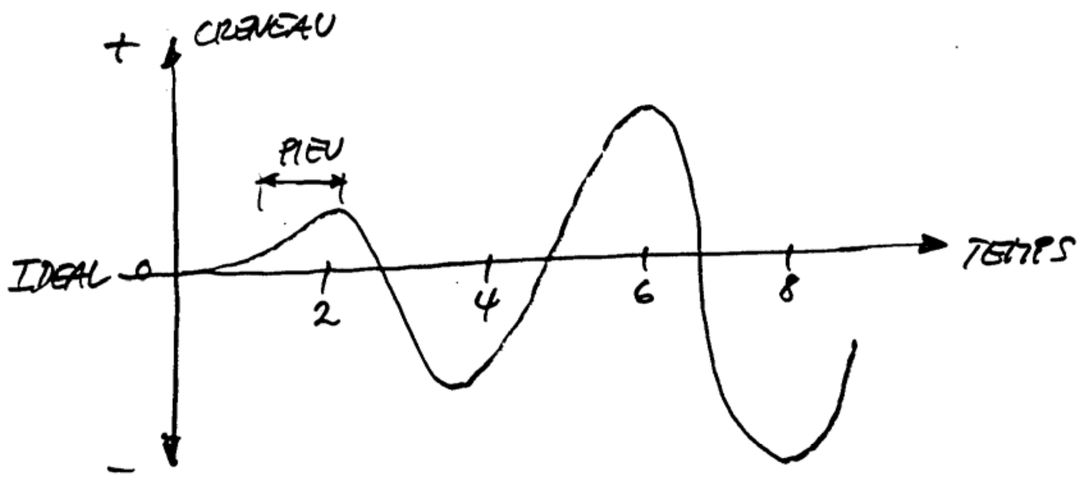


AU MOMENT 0 LE CRENEAU EST EXACT, AU MOMENT 1 LE CRENEAU AUGMENTÉ. LE CONDUCTEUR LE REMARQUE (1s) ET CORRIGE LA SITUATION.
CE CONTROLE EST IDEAL ET SIMPLIFIE. EN PRATIQUE LES CRENEAUX AUGMENTENT GRADUELLEMENT COMME AUSSI LES ACTIONS ET REACTIONS DE CONTROLE.

- LA DISTANCE AUGMENTE GRADUELLEMENT ET LA CORRECTION EST EGALEMENT GRADUELLE.



CE CONTROLE EST STABLE, CAR ON TROUVE LE CRENEAU IDEAL APRES UN CERTAIN TEMPS.

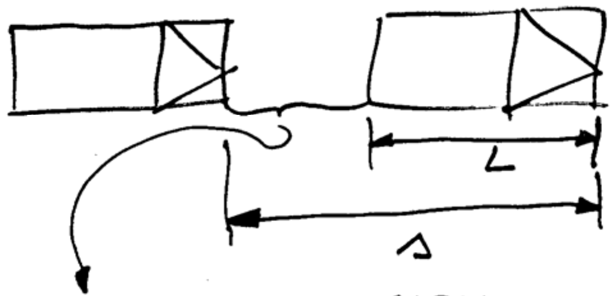


SI LA REACTION ETAIT INAPPROPRIEE AU MOMENT 2, ON PEUT ABOUTIR A UNE SITUATION INSTABLE. LA CORRECTION QUI AURAIT ETE LA BONNE AU MOMENT t PEUT ETRE MAUVAISE AU MOMENT t + Δt LORSQU'ELLE EST EFFECTUEE.

IL FAUT FAIRE ATTENTION A CE PROBLEME D'INSTABILITE, SI ON CONSTRUIT DES MODELES MATHEMATIQUES.

DONC EVITER DES PARAMETRES ET VALEURS DE VARIABLES QUI ENGENDRENT CETTE INSTABILITE!!

MODELES DETERMINISTES



DISTANCE DE FRENAGE = $\frac{PIEV \cdot V}{3.6} + \frac{V^2}{254(f \pm p)}$ (m)

ESPACEMENT $\boxed{s = L + \frac{PIEV \cdot V}{3.6} + \frac{V^2}{254(f \pm p)} \quad (m)}$

- **SECURITE**
  - TEMPS DE REACTION PIEV
    - LE CONDUCTEUR 2 REAGIT TROP LENT.
    - LE CONDUCTEUR 2 REAGIT TROP RAP.
  - FROTTEMENT $f = ?$
  - DECLIVITES

- LE PROBLEME DE SECURITE PEUT ETRE RESOLU EN CHOISISSANT DES ESPACEMENTS PLUS GRANDS ! MAIS CELA REDUIT LA CAPACITE.

- LE CONDUCTEUR A UN PIEV LONG ET PEUT DIFFICILEMENT APPRECIER LES DISTANCES ET VITESSES. UN SYSTEME AUTOMATIQUE POURRAIT L'AIDER.

• **CAPACITE**

$$Q = K \cdot V = \frac{V}{S}$$

$$\boxed{Q = \frac{1000 \cdot V}{L + \frac{PIEV \cdot V}{3.6} + \frac{V^2}{254(f \pm p)}}} \quad (\text{véh}/h)$$

$$\frac{dQ}{dV} = 0$$

$$V_{opt} = 254\, L\,(f \pm p)$$

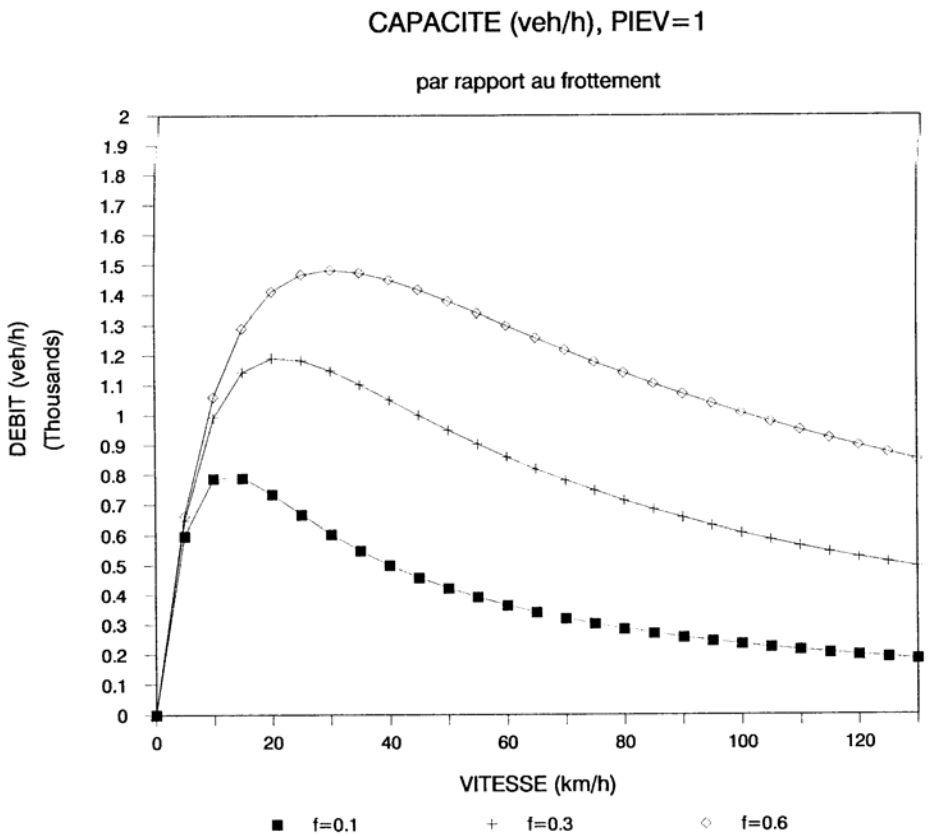
$$Q_{max} = \frac{1000}{2\sqrt{\frac{L}{254(f \pm p)}} + \frac{PIEV}{3.6}} \quad (v/h)$$

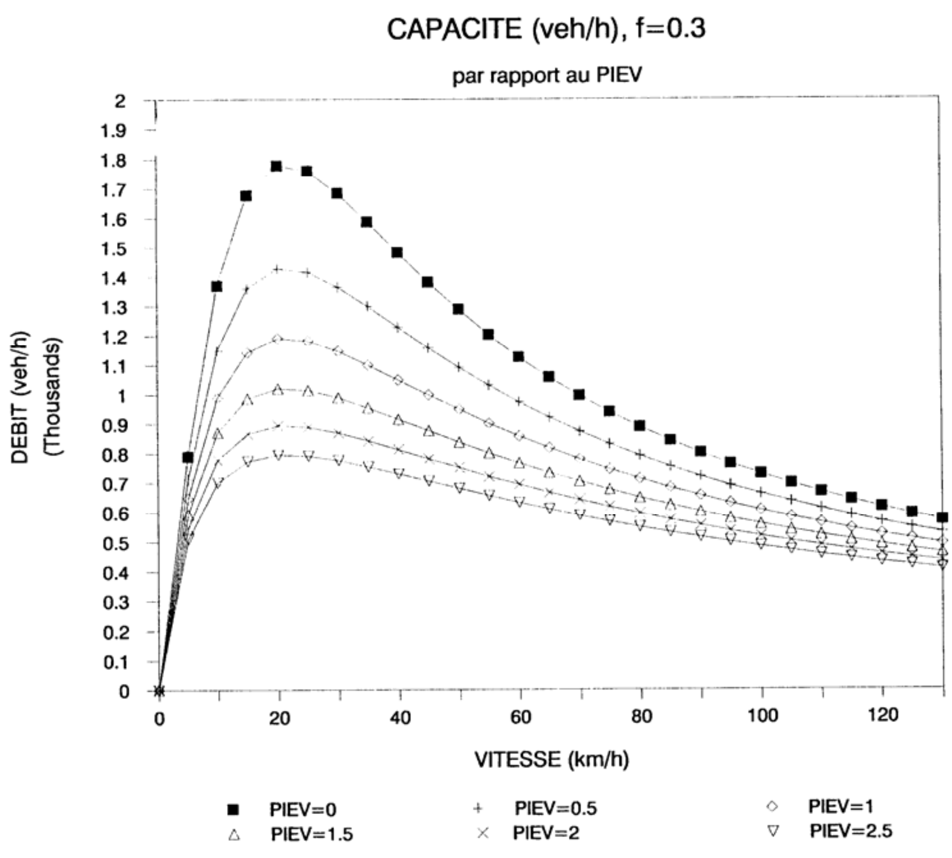
ON A OBSERVE :

$L = 9m$, $f = 1.0$, $PIEV = 0.5$

$Q_{max} = 1925$ véh/h

$V_{opt} = 47.5$ Km/h


CAPACITE (veh/h), PIEV=1
par rapport au frottement
DEBIT (veh/h)
(Thousands)
VITESSE (km/h)
f=0.1
f=0.3
f=0.6


CAPACITE (veh/h), f=0.3
par rapport au PIEV
DEBIT (veh/h)
(Thousands)
VITESSE (km/h)
PIEV=0
PIEV=0.5
PIEV=1
PIEV=1.5
PIEV=2
PIEV=2.5

DRIVER EDUCATION FOR SAFER TRAVEL ON HIGHWAYS BY PAVEMENT MARKINGS.

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ABSTRACT:

When one analyzes traffic accidents on high speed roads, one realizes that rear end collisions happen frequently even in good weather conditions. Very often these accidents are due to too short gaps between the cars under prevailing conditions. Drivers misjudge the available and the required distances.
Several rules are published as to how to choose the safe headways (for example the 2 seconds rule), but these are fairly impracticable and difficult to use and, furthermore, they do not allow to fix in a simple way in the driver's memory a safe distance to adopt, depending on weather and speed conditions.
The aim of the present article is to critically review the existing rules for safe headways and to propose a new one which is adapted from a french experience. Special marks are painted on the highway at regular intervals and the driver has to perceive a certain number of these markings (depending on speed and weather, as indicated on special traffic signs) between himself and the preceding car, in order to drive safely.
These pavement markings are only applied in special test sections of several kilometers of length as a means to permit the calibration of safe distances in the drivers memory. The learned safe behavior is then, hopefully, also applied to other road sections.
This new and inexpensive means could significantly contribute to safer highways, since drivers would realize in a very visual way, how dangerous too close driving could be.

1988 Annual Conference of the Canadian Transportation Association in Halifax.

Introduction

The aim of the article is to discuss the problem of inter-vehicular distances and the related safety issues. In the first part of the article, the author discusses the probable reasons for these accidents and how driver performance can be improved to increase safety. While technological solutions to control and to provide safe car following distances become possible due to developments in microcomputers, it appears that real world applications are still not yet fully operational and require further research and development.

Driver education remains an important way of reducing this type of accident and the author discusses, in the second part of this article, the different car following rules available for evaluating and keeping safe distances. In a subsequent part, the available rules are critically reviewed and a new method based on pavement markings is proposed. This method was developed and used in France and tested on several sections of french freeways.

The accidents related to too short inter-vehicular distances.

In the range of higher traffic volumes rear end collisions represent the most frequent type of accident on highways. There are also many accidents with several vehicles involved, where obviously too short distances between cars in a platoon generated a chain reaction. These accidents often occur in optimal driving conditions and even more so when bad weather conditions such as fog, rain, snow or icy conditions prevail.

Colbourn (1978) found that 12 % of the total vehicle involvement in road traffic accidents is due to the vehicle following situation. This is very similar in France, where Carré (1985) observed that 15% of all accidents on rural highways were rear end accidents and up to 41% on expressways.

It becomes obvious that the car following problem is a fairly important problem which merits further investigation and solutions should be found to improve safety.

How and why these accidents happen:

It is apparent that drivers don't leave enough space between vehicles, so that accidents can happen even in good driving conditions. There may be different reasons for this, for example:

- The underestimation of necessary braking distances;
- A misjudgment of the actual distance between cars and of their relative speeds;
- An underestimation of necessary perception reaction times;
- An active risk taking behavior;
- A misjudgment of the danger involved.

Some of these points are reviewed in more detail to contribute to a better understanding of the problem.

Recent observations in Germany have shown that drivers underestimate necessary braking distances.
13% of the drivers interviewed estimated that a distance of 28m would be sufficient given a speed of 100 km/h and 23% thought it would take 36 meters to brake to a stop from this speed in good driving conditions (while the distance necessary is at least 75 m).
This underestimation of necessary braking distances may be one of the main reasons why car following distances are frequently too short.

The same survey also showed that, at a speed of 100 km/h 10% of the drivers keep only a gap of 10 m (measured from the rear of one vehicle to the front of the following), 18% keep a gap of 30 m and 20% keep a distance of 40 m.

When studying headways, (defined as the time elapsed between the passage of the front bumpers of two cars) similar remarks can be made. The Highway Capacity Manual (1965) indicates that on a 4 lane highway the percentage of headways shorter than one second increases with volume as given in table 1.

Volume (v/h)	200	400	600	800	1000	1200	1400	1600	1800
% < 1 sec	4	8	11	15	19	21	24	27	28
% < 2 sec	8	17	26	34	42	48	53	57	60

Table 1. Percentage of drivers leaving headways of less than 1 and less than 2 seconds.

Brilon (1976) defines the degree of dangerously short headways in the following manner:

$$P_r = \frac{\text{\# of vehicles with } t < 1 \text{ sec}}{\text{volume}}$$

Figure 1 illustrates this percentage.

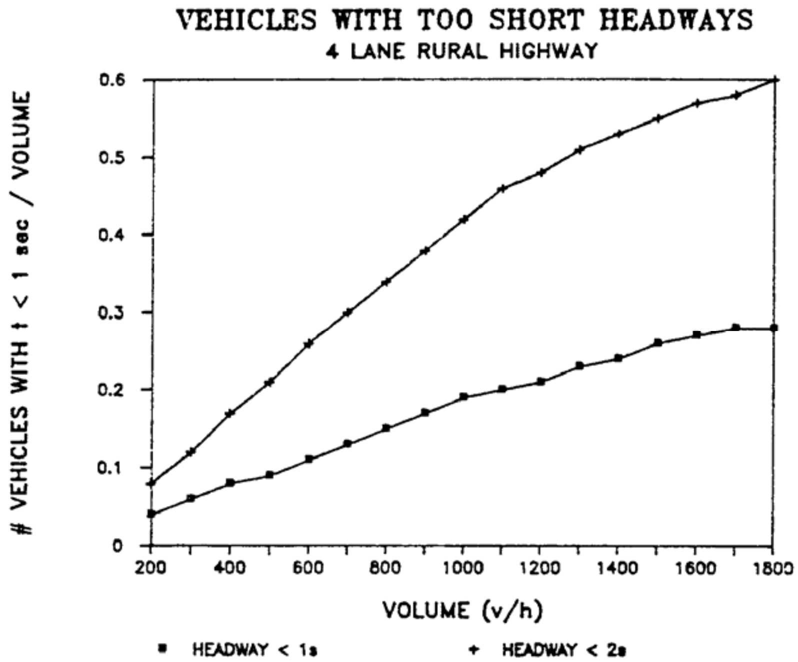

Figure 1. Percentage of dangerously short headways.

This clearly indicates that a certain percentage of drivers are "tailgating".

Harris reports comparable values for Britain. He found that the percentage of vehicles following too closely (< 1 sec) increases linearly with increasing flow from 5% at 150 v/h to 25% at 1000 veh/h. Colbourn (1978) reports mean values of headways and spacings at different speeds (table 2).

speed (km/h)	48.3	66	80.5
spacing (m)	24	33	45
headway (s)	1.77	1.81	2.02

Table 2. spacings and headways at different speeds.

Drivers seem, on the average, to adopt a 2 second headway but it is apparent that slight variations about the mean may be of practical importance in the accident experience.

Another factor which might explain the occurrence of rear end accidents is the difficulty drivers have in evaluating <u>actual</u>

distances and actual speed differences between vehicles. This information is necessary in order to estimate the rate of change of spacing with time. Koehler (1979) distinguishes between three situations in car following, the uninfluenced, the following range and the approaching and removing range. The distances oscillate in fact around the desired safety distance, up to the moment where a misjudgment causes an accident. He shows the approaching and following procedure on a highway in figure 2.

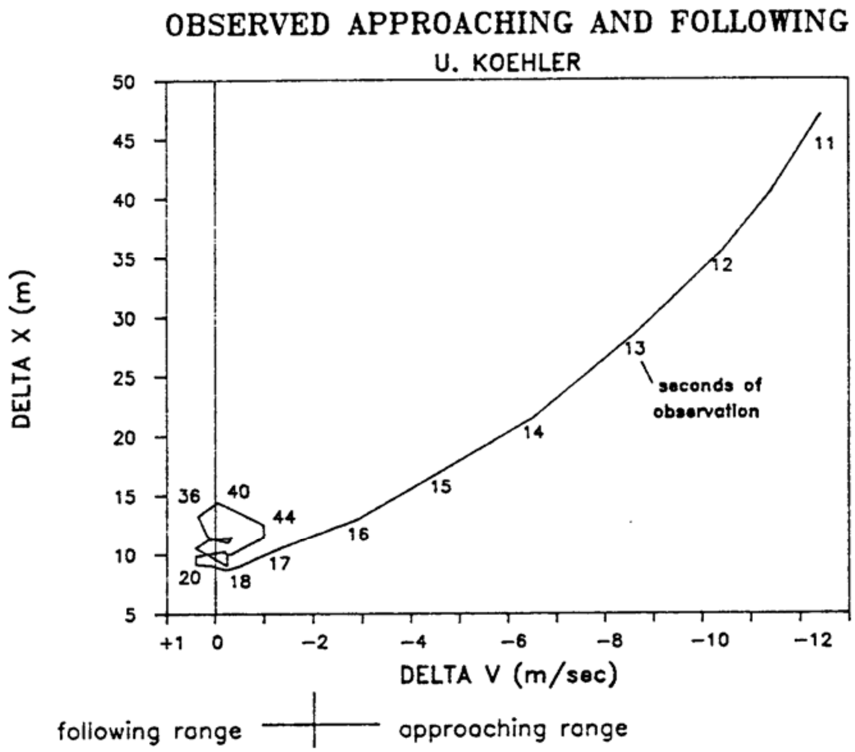

Figure 2. Vehicle approaching and following another.

Koehler shows that the proportion of vehicles maintaining too small a distance is rather high and that in the approaching range more vehicles had too short distances than in the following range. Harte (1976) conducted an experiment where car following distances were to be estimated and found that at 65 km/h the average underestimation error was 33%, while Rockwell and Snyder (1967) found that car following distances were underestimated by 41% at a speed of 80 km/h. Drivers are inaccurate in their judgments of car following distances. While an underestimation of distances may increase safety in braking situations it may lead to an underestimation of the time necessary to overtake and to pass a vehicle.

Colbourn et al. (1978) report that absolute judgment of distances have been observed to be in error by between 20% and 100% among experienced drivers and up to 300% in error among novice drivers.

The least one can say is that the driver's ability to estimate speeds and distances is limited.

Barwell (1982) finds that the change in size of the image of an object on the retina of the eye is the physiological correlate of distance perception and Michaels (1965) suggests that the driver estimates the angle subtended by the lead vehicle. The contours of the lead vehicle expand or contract at a rate which is proportional to both relative velocity and the distance apart.

The car following situation is indeed very complex since many variables intervene. The fundamental problem for the driver is to keep safe distances, but what are these and how do we estimate them in a given situation? We have to find a simple and efficient way to explain this to the driver. The message contained in the Highway Code: " The driver of a vehicle following another shall keep at a safe distance from it, taking into account the speed and density of the traffic as well as the conditions of the road" is well meant but too vague to be useful.

Improved driver performance with respect to inter-vehicular sensing will improve the accident situation; providing the driver with accurate relative velocity and headway information could indeed improve his car following performance.

What are presently the means for improving driver performance in car following?
One can achieve an improvement in essentially three ways, by acting on the three components of the driving system; which are: the driver, the control system and the car.

<u>Driver education:</u>

This may be done along two lines: Learning about necessary braking distances and distance estimation, and learning rules which allow to keep a safe distance. There are several rules in use and four of these will be presented and analyzed subsequently with respect to their efficiency. These are:

- <u>Rule 1</u>: The rule of the 15 km/h. This rule stipulates to leave a space gap equal to one car length for each 15 km/h of speed.

- <u>Rule 2</u>: A rule used frequently in Germany asks for a space gap (in meter) of equal to half the speed in km/h indicated on the tachometer of the car.

- <u>Rule 3.1</u>: The rule of the two seconds. The driver should leave a time gap of 2 seconds between the two vehicles and this independent of his speed. The driver is warned that this rule applies only to optimal driving situations.

- Rule 3.2: This rule is an extension of rule 3.1. If the driver is tired or if he drives in unfavorable weather conditions the time gap should be increased to 4 or 5 seconds. Trucks should keep a distance corresponding to a time gap of 4 seconds or more.

- Rule 4: Pavement markings at fixed distances as developed in France. Yellow pavement markings having the shape of chevrons were painted in the center of the lane of certain freeways in France. These markings are painted at intervals of 40 meters. Traffic signs explain the principle of the method to the driver: If you see only one mark between your car and the preceding one, this means "danger", two marks mean "safety". This suggests a safety distance of 60 to 80 meters depending on the interpretation of "seeing two marks". These pavement markings were applied on certain test sections of a length of 5 km and should allow drivers to calibrate safe distances in their visual memory in real driving situations.

  These test zones are designed to contribute to driver education in car following and it is hoped that the acquired knowledge of safe distances would be transferred to other types of highways.

The Control System:

Traffic engineers try to induce safer car following behavior by means of variable message signs. This concept is known as "following too closely" (FTC).
Traffic signs "following too closely" are installed at high accident spots on freeways. Time gaps between vehicles are measured and a variable message indicates "danger" when gaps are between 0.7 and 1.25 seconds and "violation" when gaps are smaller than 0.7 seconds.

These systems allowed to reduce the percentage of short gaps and reduced the number of accidents, but the effects decreased and vanished after a time and police surveillance became necessary to improve efficiency. The problem with this system is that it does not teach the driver to appreciate safe distances and is only used at high accident spots.

In-car information on gaps.

Given the insufficient abilities of the human driver in car-following, it became apparent that instrument aids are required to make more accurate judgments. In addition, since the capacity of the highway is determined by the characteristics of the average driver, an automatic device could increase capacity and safety by improving one or all of the three functions, perception, decision and response.

There are three types of headway control systems:

- the automatic highway concept;
- the radar distance warning and control system;
- the head-up display (HUD).

The automatic highway concept is based on detector loops incorporated into the pavement at certain intervals. The leading vehicle passing over the loop creates a signal which is transmitted to the following vehicle that receives a warning signal indicating the presence of the vehicle ahead when he follows too closely. The driver can adjust his speed accordingly or a computer on board does it in an automatic way. This is similar to the block system in railway engineering. Even if this system is technologically feasible there are many unsolved problems related to financial, insurance and legal responsibility.

The second type of system does not need any investment into a system incorporated into the pavement. Distances between vehicles and speeds are measured by radar and gap and relative speed information is displayed inside the car in acoustic and optical way in order to assist and improve the estimation capabilities of the driver.

Research in Germany (Jahnke (1982)) shows interesting results but the technology needs to be improved since there are problems related to false alarms as well as to misinterpretations of distances in horizontal and vertical curves. One major disadvantage is that the optical display acts as a distraction to the driver.

The inconvenience of the driver having to lower his head to look at the instruments or displays inside the vehicle is eliminated in the case of the so- called "head-up display".

Strobel (1985) describes this system: "Two vertical lines are projected at the car's windshield by means of a specially designed optical device. An electronic control unit changes the horizontal distance between these two lines as a function of the speed and the driving conditions. If the windshield projection of the leading car remains between the two vertical lines, then the distance is sufficiently large." The display tells the driver that he was off the desired headway and shows also how fast he is getting further off.

Other devices were designed to put more information into stop lights in order to improve driver performance. A few examples are:
The third braking light, the variation of light intensity or number of braking lights to indicate the rate of deceleration or simply the recommendation to use the emergency flashing lights when approaching a critical situation in order to inform the driver behind.

Developments in microcomputer technology will certainly contribute to the improvement of these on-board systems within a few years. The idea of the "intelligent car" is studied in Europe (Project Prometheus) and in Japan but these cars will not be available before 10 or 15 years. Up to then we should concentrate our efforts on the diver and his education in order to improve car-following behavior.

The aim of the following paragraphs is to analyze the efficiency of the 4 rules cited earlier. This will be done by comparing space gaps provided by the rules with the space gaps necessary for safe braking.

Formulae for safe space and time gaps.

The formula for the space gap has three parts, these are the distance driven during the perception and reaction time (PIEV), the braking distance of the following vehicle (number 2) and the distance necessary to decelerate the leading vehicle (number 1).

$$s = PIEV\ v^2 + \frac{v_2^{\,2}}{2g(f_2+p)} - \frac{v_1^{\,2}}{2g(f_1+p)} \qquad (1)$$

where:
- s = space gap (m)
- PIEV = perception reaction time (s)
- v = speed (m/sec)
- f_1, f_2 = friction between tire and pavement for vehicles 1 and 2.
- p = grade of highway

if one supposes level terrain the time gap h becomes:

$$h = \frac{s}{v} = PIEV + \frac{v_2}{2gf_2} - \frac{v_1^{\,2}}{2gf_1 v_2}$$

Clearly, the gap size is related to highway capacity and as gaps become larger capacity decreases. Koehler (1979) finds, that if drivers would behave in a safe way, that is if they would adopt formula (1), capacity would decrease by approximately 17%.

The question of safety versus fluidity or capacity is a very interesting one and was treated by Carré et al. (1984) but this question will not be considered here.

Analysis of the variables.

In the gap formula there are five explicit variables, the perception-reaction time, the speeds v_1 and v_2 and the decelerations of the two vehicles given by the formula $d_1 = gf_1$ and $d_2 = gf_2$. These variables themselves depend on the geometry of the highway, on visibility conditions, on the pavement surface and on the driver's characteristics.

In the following we will consider, for more simplicity, the speed as a constant and we distinguish between two driving conditions.

Favorable driving conditions are given when weather conditions are good, providing a sufficient sight distance and the driver is able to anticipate the leading driver's manoeuvres by looking through and around the leading vehicle.

In the second case the weather conditions are unfavorable (fog, rain, or snow) with insufficient sight distance, so that the driver cannot anticipate the manoeuvres of the leading car. This situation is similar when one follows a truck.

Regarding these two driving conditions one can analyze the three basic variables visibility, pavement conditions (which determine the maximum deceleration available) and the state of attention of the driver.

a) Visibility

In the case of a good visibility the driver normally follows the leading car observing the highway ahead of the leading car and the braking lights in front of him. The driver is thus well informed of the presence of an obstacle on the roadway and he supposes that he will be able to brake at the same rate of deceleration as the driver of the leading car.

Supposing that the speeds of the two cars are nearly the same (which is the case in the following range) the necessary space gap reduces to:

$$s = PIEV\ v_2 \quad (m)$$

and the time gap becomes:

$$h = PIEV \quad (sec)$$

Jahnke calls this the minimal safety criterion.

In the case of insufficient visibility (following a truck or in fog...) the following driver can only count on his ability to detect rapidly enough the braking lights of the leading car. In this case there would be a certain difference in deceleration between the two cars. This leads to the criterion of relative safety:

$$s = PIEV\ v_2 + \frac{v_2^2\ (d_1 - d_2)}{2\ d_1\ d_2}$$

And, finally, there is the "brick-wall criterion" where there is insufficient visibility and the leading car is forced to stop suddenly as if hitting a parked truck in the fog or a brick-wall. The following driver should have chosen a space gap according to the criterion of absolute safety:

$$s = PIEV\ v_2 + \frac{v_2^2}{2gf_2}$$

b) State of the pavement surface

One can distinguish between three cases, dry pavement with a friction coefficient of f=0.6, wet pavement with f=0.3 and icy pavement with f=0.05. The last case, however, must be considered as exceptional and the highway user should be aware of the dangers involved. It must also be kept in mind, that 0.6 is a fairly high value for the friction coefficient. Colbourn (1978) states that in practice "Only 0.1% of decelerations used in braking exceed 0.5g.

Therefore rapid deceleration is a low probability event for many drivers. So in an emergency they are likely to initiate a braking response which produces inappropriately low levels of deceleration."

c) State of attention of the driver.

Johansson et al. (1971) show that braking reaction time measured in real driving situations are not a constant and not even a constant for a given individual.

A person watchful and prepared for danger or emergency may have a

braking reaction time between 0.3 and 1.6 seconds but this time may be more than 2 seconds for a tired and inattentive driver.
For the purpose of the following analysis two cases will be considered, an attentive driver having a PIEV of 0.7 seconds and an inattentive and tired driver having a PIEV of 2 seconds.
The two driving conditions together with the different values of the parameters give rise to 6 cases shown in figure 3.

DRIVING CONDITION	FORMULA	VISIBILITY	PAVEMENT	DRIVER ATTENTION	CASE Nb.
FAVORABLE	$s = PIEV\ v$	GOOD	DRY	NORMAL	1
				TIRED	2
UNFAVORABLE	$s = PIEVv + v^2/2gf$	POOR	DRY	NORMAL	3
				TIRED	4
			WET	NORMAL	5
				TIRED	6

Figure 3. The different cases to be analyzed.

The space gaps necessary for car following in these 6 cases are calculated and shown in table 3 and depicted in figure 4.

GAPS NECESSARY FOR THE SIX CASES ANALYZED

Friction dry pavement: 0.6 , wet pavement: 0.3

SPEED	time gaps (sec)						space gap (m)					
km/h	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
20	0.7	2	1.2	2.5	1.6	2.9	4	11	7	14	9	16
30	0.7	2	1.4	2.7	2.1	3.4	6	17	12	23	18	28
40	0.7	2	1.6	2.9	2.6	3.9	8	22	18	33	29	43
50	0.7	2	1.9	3.2	3.1	4.4	10	28	26	44	42	61
60	0.7	2	2.1	3.4	3.5	4.8	12	33	35	57	59	81
70	0.7	2	2.4	3.7	4.0	5.3	14	39	46	71	78	103
80	0.7	2	2.6	3.9	4.5	5.8	16	44	58	86	99	128
90	0.7	2	2.8	4.1	4.9	6.2	18	50	71	103	124	156
100	0.7	2	3.1	4.4	5.4	6.7	19	56	85	121	151	187
110	0.7	2	3.3	4.6	5.9	7.2	21	61	101	140	180	220
120	0.7	2	3.5	4.8	6.4	7.7	23	67	118	161	212	255
130	0.7	2	3.8	5.1	6.8	8.1	25	72	136	183	247	294

Table 3. Necessary space gaps for driving in different levels of safety (6 cases).

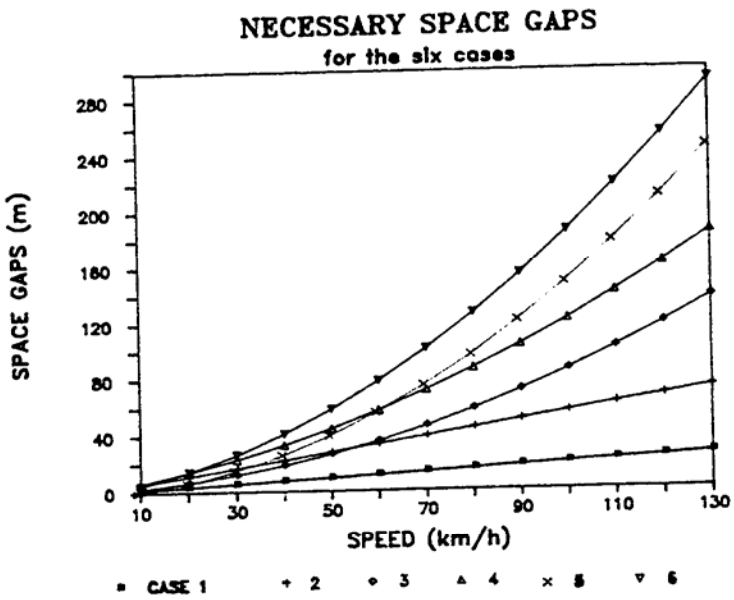

Figure 4. Necessary space gaps for driving in different levels of safety (6 cases).

The evaluation of necessary space gaps, as shown in figure 4, was done for a pair of closely following cars. Car-following theory as described for example by Fox and Lehman (1967) shows that when cars are driving in platoons, variations in space gaps between two cars can be such that no accident occurs between them. This situation is termed locally stable.

But these variations can be amplified by the time lags of the following drivers and become too important further upstream and cause an accident. In this case the spacings are called locally stable but unstable for the platoon. Jahnke developed a formula for platoon stable spacings, but in the following analysis the problem of stability in car-following is neglected, it is however, important to note that stability has to be considered when an automatic distance control system has to be designed.

<u>The available gaps if the driver follows the highway rules.</u>

4 different rules are to be analyzed and these are the rule of the 15 km/h (rule 1), the rule of half of the speed indicated on the tachometer (rule 2), the rule of the 2 seconds (rule 3.1) and its extension (rule 3.2) and the pavement marking method (rule 4).

The available spacings are calculated in table 4 and are illustrated in figure 5.

COMPUTATION OF AVAILABLE GAPS GIVEN BY THE DRIVING RULES											
Length of vehicle: 5.8 m											
SPEED	Rule 1		Rule 2		Rule 3				Rule 4		
km/h	s	h	s	h	h	s	h	s	s	h	
20	6	1.0	10	1.8	2.0	11	4.0	22	80	14.4	
30	12	1.4	15	1.8	2.0	17	4.0	33	80	9.6	
40	17	1.6	20	1.8	2.0	22	4.0	44	80	7.2	
50	17	1.3	25	1.8	2.0	28	4.0	56	80	5.8	
60	23	1.4	30	1.8	2.0	33	4.0	67	80	4.8	
70	29	1.5	35	1.8	2.0	39	4.0	78	80	4.1	
80	29	1.3	40	1.8	2.0	44	4.0	89	80	3.6	
90	35	1.4	45	1.8	2.0	50	4.0	100	80	3.2	
100	41	1.5	50	1.8	2.0	56	4.0	111	80	2.9	
110	41	1.3	55	1.8	2.0	61	4.0	122	80	2.6	
120	46	1.4	60	1.8	2.0	67	4.0	133	80	2.4	
130	52	1.4	65	1.8	2.0	72	4.0	144	80	2.2	

Table 4. Available space gaps if the driver follows the rules.

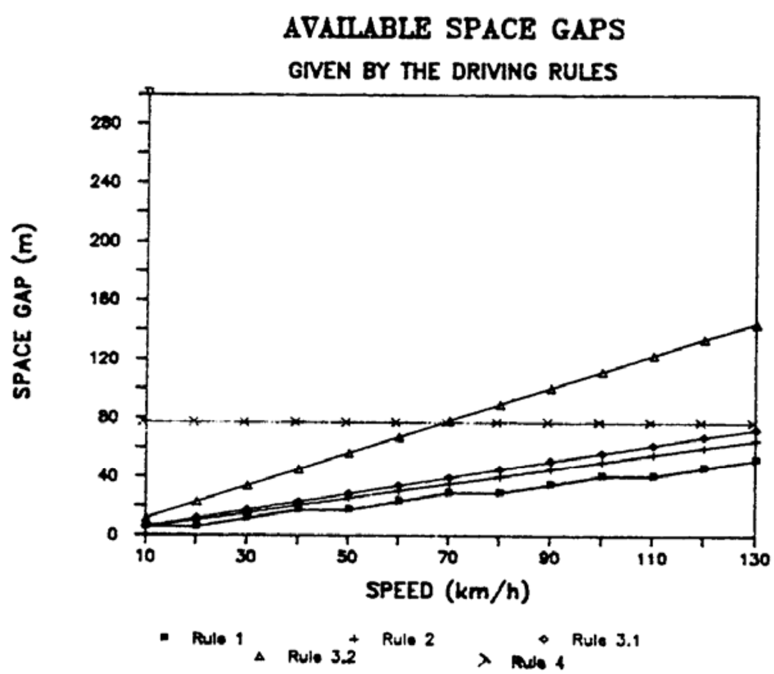

Figure 5. Available space gaps if the driver follows the rules.

Analysis of the rules.

Jahnke defined a safety factor d = available gaps/necessary gaps These safety factors are given for the five rules and for the two cases of minimal and absolute safety. The resulting values are indicated in figures 6 and 7.

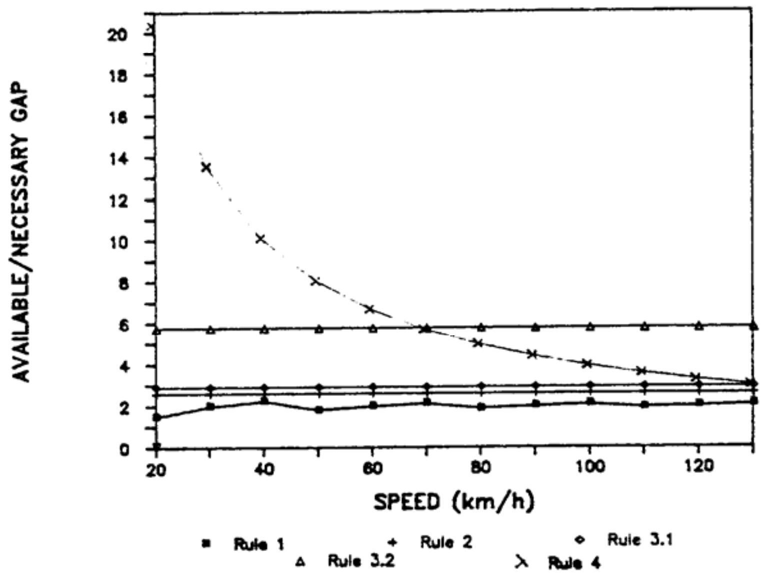

Figure 6. Minimal safety factors, if the driver follows the rules.

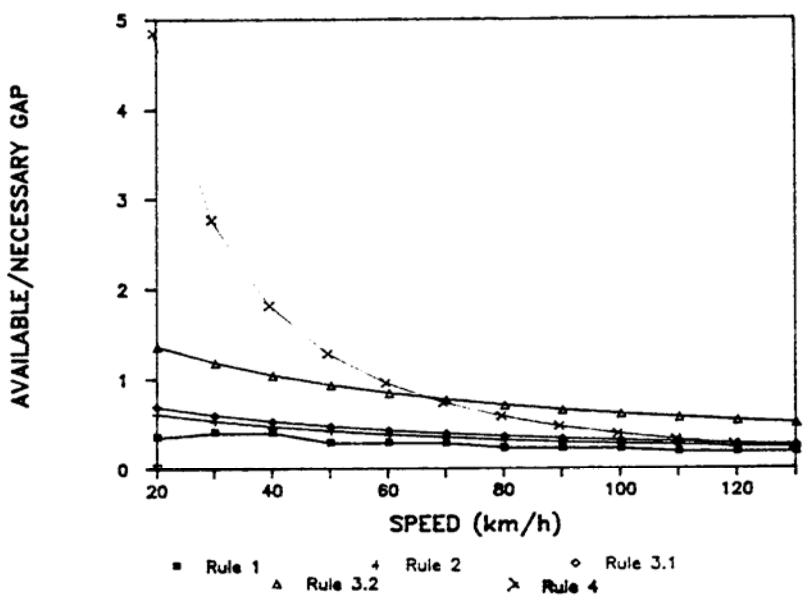

Figure 7. Absolute safety factors if the driver follows the rules.

Analysis of the rules.

Jahnke defined a safety factor d = available gaps/necessary gaps These safety factors are given for the five rules and for the two cases of minimal and absolute safety. The resulting values are indicated in figures 6 and 7.

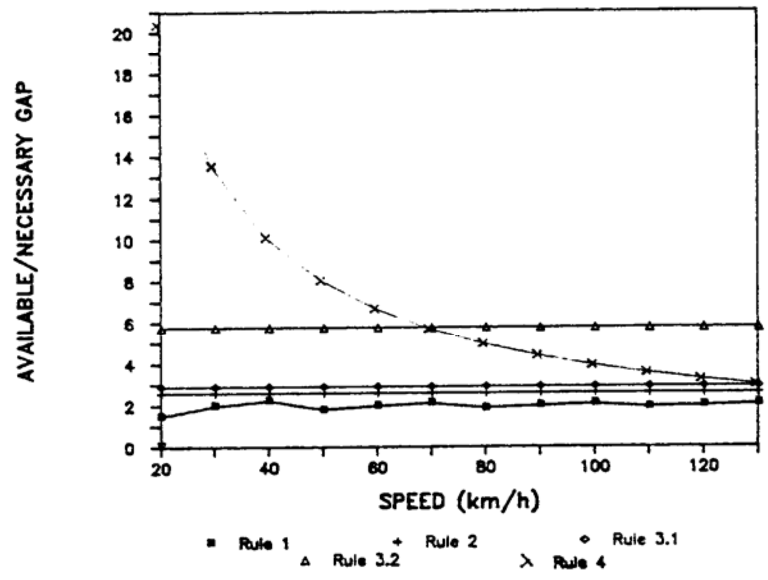

Figure 6. Minimal safety factors, if the driver follows the rules.

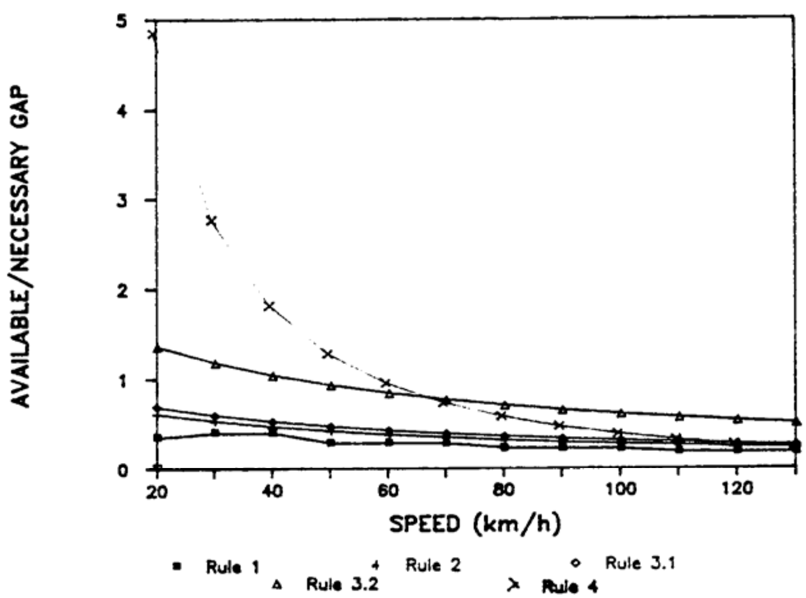

Figure 7. Absolute safety factors if the driver follows the rules.

Adaptation of the pavement markings method.

The idea to use delineators or pavement markings to allow drivers to better estimate distances when driving is not new. Harte for example studied the effect of existing pavement lines on distance estimation.

The french method, however, is much more explicit and one can choose an adequate interval between chevrons and apply different rules for different driving conditions in order to further improve the method. As an example one could use chevrons at intervals of 30 m and use the rules shown in table 5.

Speed (km/h)	SAFETY DISTANCES	
	favorable	unfavorable
0 - 50	1 mark	2 marks
50-100	2 marks	4 marks

Table 5. Proposed method.

The message to display on traffic signs should be easy to understand in order to help the driver to discriminate easily between situations. This evidently increases the complexity of the traffic signs, but pictograms could be used to simplify the message as shown in figure 8.

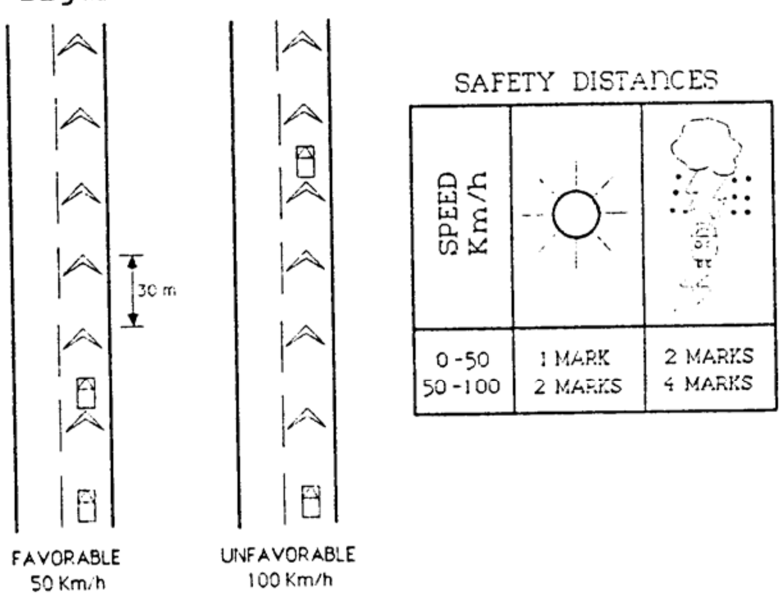

Figure 8. Illustration of the proposed method.

Figure 9 illustrates the space gaps given by the proposed method drawn over the necessary gaps for three levels of safety, minimal, relative and absolute safety.

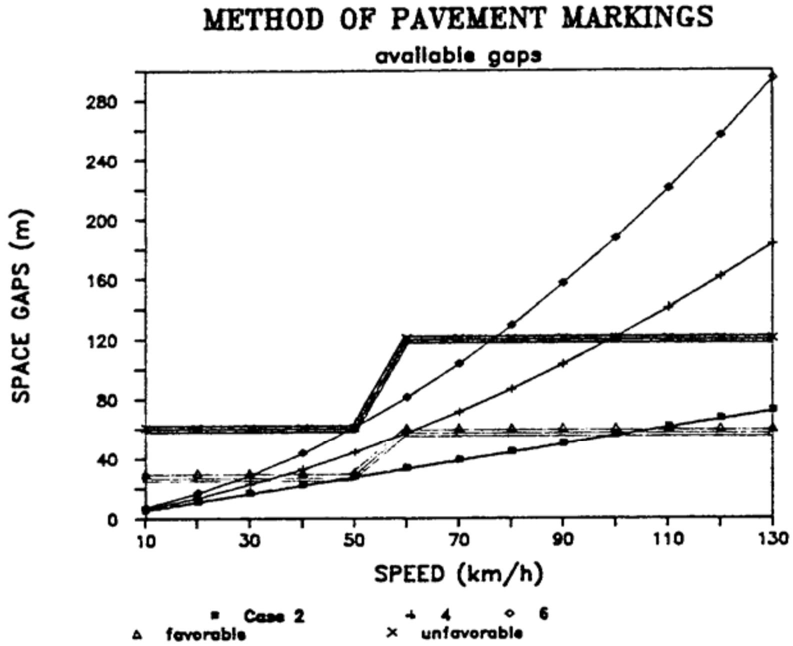

Figure 9. Space gaps given by the proposed method.

Figure 10 shows the domain of safe space gaps covered by this adaptation of the pavement markings rule.

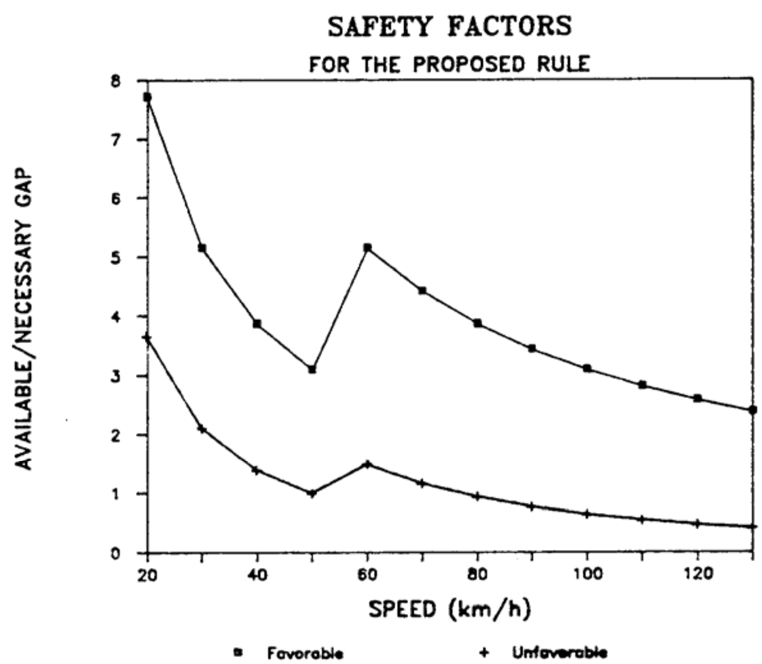

Figure 10. Safety factors for the proposed method.

Test zones on expressways and highways could be implemented and drivers would be able to calibrate safe space gaps. The acquired knowledge would be useful also outside test zones and drivers would be able to better understand the real danger involved in driving too close.

French experience with this kind of pavement markings are encouraging. Limited surveys carried out in France in 1985 have shown that 98% to 99% of the drivers have seen the markings and 61% to 70% have understood the rule, while 52% to 56% of the drivers interviewed had tried the rule and 36% to 45% found the rule easy to understand. But even those who did not actually apply the rule were at least sensibilized to the problem of inter-vehicular distances. It is not foreseen in the near future to use it on all the length of the highway system, but rather to continue the experience in test sections of 5 to 10 km of length.

Conclusion

The method of pavement markings should be tested on certain freeways in Canada in order to sensibilize the driver to the problem of car-following distances. The markings could contribute to imprinting in the driver's mind safer inter-vehicular distances for different driving conditions and this would help reduce the percentage of misjudgments of actual gaps by a process of mental distance calibration. The benefits would be felt at medium or long term even outside the test sections since it would educate drivers on safe distancing. Highway traffic safety at higher speeds would thus be considerably improved and this, at a very low cost of investment.

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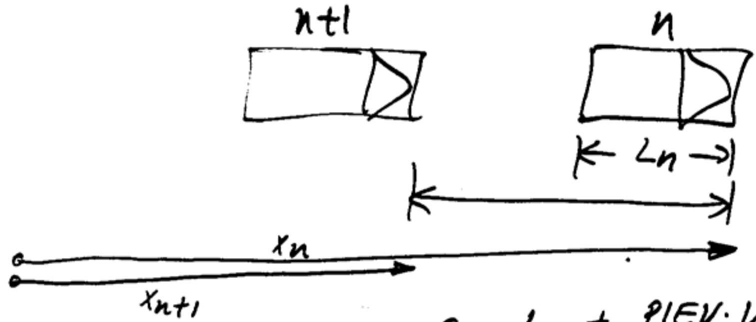
LE MODELE LINEAIRE

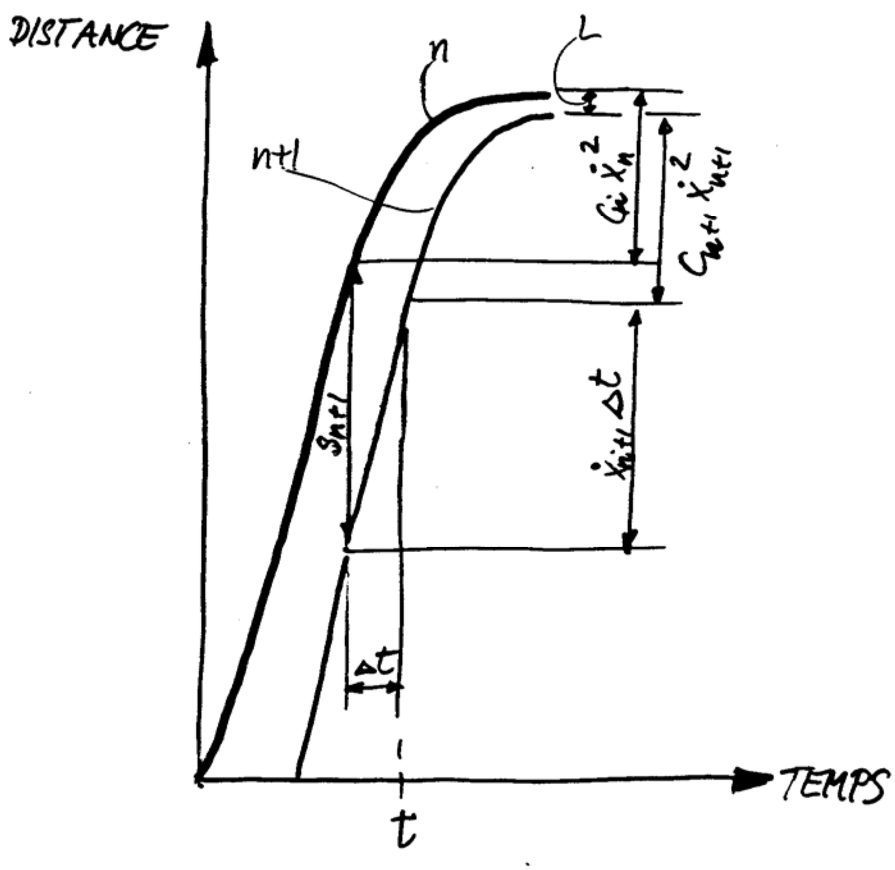
- LE MODELE EST BASE SUR L'OBSERVATION

REPONSE $(t+\Delta t)$ = STIMULUS(t) $*$ SENSIBILITE DU COND.

- LA REPONSE SE MANIFESTE APRES UN TEMPS Δt DE REACTION.
- LE CONDUCTEUR SUIT A UNE DISTANCE DE SECURITE
- LA REPONSE NE PEUT ETRE QU'UNE ACCELERATION OU UNE DECELERATION.
- LE STIMULUS EST L'OBSERVATION D'UNE DIFFERENCE DE VITESSE ENTRE n ET $n+1$.
- LA SENSIBILITE DU CONDUCTEUR DEPEND :
  - SON TEMPS DE REACTION
  - DE SA VITESSE
  - DU CRENEAU ACTUEL



$$S = L_n + \frac{PIEV \cdot V_{n+1}}{36} + \frac{V_{n+1}^2}{254 f_{n+1}} - \frac{V_n^2}{254 f_n}$$



S_{n+1} = ESPACEMENT DE SECURITE

$$S_{n+1}(t-\Delta t) = C_{n+1} \cdot \dot{x}_{n+1}^2(t) + L_n + \Delta t\, \dot{x}_{n+1}(t) - C_n\, \dot{x}_n^2(t-\Delta t)$$

$$C_n = \frac{1}{2 d_n} = \frac{1}{2 f_n g}$$

- ON FAIT UNE HYPOTHESE IMPORTANTE :

LE COMPORTEMENT (TAUX) DE CHANGEMENT DE VITESSE EST IDENTIQUE POUR LES CONDUCTEURS n ET $n+1$

DONC: $$C_{n+1}\,\dot{x}_{n+1}^{2}(t) = C_n\,\dot{x}_n^{2}(t-\Delta t)$$

LA DISTANCE ENTRE LE VEHICULE n ET $n+1$, OU LE CRENEAU EST :

$$s_{n+1}(t-\Delta t) = \Delta t\,\dot{x}_{n+1}(t)$$

ET : $$x_n(t-\Delta t) - x_{n+1}(t-\Delta t) = \Delta t\,\dot{x}_{n+1}(t)$$

LA REACTION DU CONDUCTEUR EST UNE ACCELERATION

$$\ddot{x}_{n+1}(t) = \frac{\dot{x}_n(t-\Delta t) - \dot{x}_{n+1}(t-\Delta t)}{\Delta t}$$

OU ENCORE

$$\underbrace{\ddot{x}_{n+1}(t+\Delta t)}_{\text{REPONSE}} = \frac{\overbrace{\dot{x}_n(t) - \dot{x}_{n+1}(t)}^{\text{STIMULUS}}}{\underbrace{\Delta t}_{\text{SENSIBILITE}}}$$

$$\boxed{\ddot{x}_{n+1}(t+\Delta t) = \alpha\left\{\dot{x}_n(t) - \dot{x}_{n+1}(t)\right\}}$$

SENSIBILITE $\alpha = \frac{1}{\Delta t}$ $\left\{\begin{array}{l}\Delta t_{\text{médiane}} = 1,5\,s\\ \text{VARIE ENTRE 1 ET } 2.2\,s\end{array}\right\}$

- CE MODELE EST UN MODELE LINEAIRE, CAR α EST UNE CONSTANTE DANS CE CAS.

LES MODELES NON-LINEAIRES

- DANS LE MODELE LINEAIRE ON A SUPPOSE QUE LA SENSIBILITE α RESTAIT CONSTANTE.
- POUR UNE DIFFERENCE DE VITESSE DONNEE, LA REPONSE DU DEUXIEME CONDUCTEUR SERAIT ALORS INDEPENDANTE DE L'ESPACEMENT ENTRE LES VEHICULES.
- ON OBSERVE CEPENDANT QUE LA REPONSE N'EST PAS LA MEME SI LE CRENEAU EST + GRAND OU SI LA VITESSE EST + GRANDE
- LA SENSIBILITE DEVRAIT TENIR COMPTE DE CES OBSERVATIONS.

α = fonction (créneau, vitesse)

- LA SENSIBILITE PEUT PRENDRE PLUSIEURS FORMES:
  - $\alpha = \text{const.}$
  - $\alpha = \begin{cases} a & \text{si } S \leq S_{crit} \\ b & \text{si } S > S_{crit} \end{cases}$
  - $\alpha = \dfrac{c}{S}$
  - $\alpha = \dfrac{\dot{x}_{n+1}}{S^2}$
  - $\alpha = \dfrac{c}{S^2}$

OU EN GENERAL

$$\alpha = \frac{\alpha' \, \dot{x}_{n+1}(t+\Delta t)^m}{[x_n(t) - x_{n+1}(t)]^l}$$

où m ET l SONT DES CONSTANTES

$$\ddot{x}_{n+1}(t+\Delta t) = \frac{\alpha' [\dot{x}_{n+1}(t+\Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [\dot{x}_n(t) - \dot{x}_{n+1}(t)]$$

- AVEC $m = 0$ NOUS OBTENONS UNE EQUATION DE MOUVEMENT QUI RESSEMBLE A CELLE UTILISEE DANS L'ANALOGIE HYDRODYNAMIQUE :

$$\ddot{x}_n = -c^2 k_n^l k_n'(x)$$

- LE CHOIX D'UNE FORME OU D'UNE AUTRE DEPEND DES CONDITIONS DE LA CIRCULATION.

- ON PEUT, A PARTIR CETTE EQUATION DEVELOPPER LES MODELES MACROSCOPIQUES.

LOI D'INTERACTION ENTRE 3 VEHICULES

- HYPOTHESE : IL Y A UNE INTERACTION ENTRE LE CONDUCTEUR n+2 ET LES VEHICULES PRECEDENTS n+1 ET n.

$$\ddot{x}_{n+2}(t+T) = C_1\left[\dot{x}_{n+1}(t) - \dot{x}_{n+2}(t)\right] + C_2\left[\dot{x}_n(t) - \dot{x}_{n+2}(t)\right]$$

OÙ :

$C_1 C_2$ LES PARAMETRES DE SENSIBILITE CONCERNANT LE CONDUCTEUR n+2 PAR RAPPORT AUX VITESSES DU VEHICULE n ET n+1.

- LES EXPERIENCES SEMBLENT INDIQUER QUE L'INFLUENCE DU 2° VEHICULE PRECEDANT SOIT NEGLIGEABLE, AU MOINS DANS CE MODELE.

ASYMETRIE ENTRE ACCELERATION ET DECELERATION

- ON A OBSERVE QUE LES CONDUCTEURS NE REPONDENT PAS AVEC LE MEME TAUX ABSOLU DE CHANGEMENT DE VITESSE, LORSQUE LE STIMULUS Δv EST POSITIF OU NEGATIF.
- ON A GENERALEMENT UNE REPONSE DE DECELERATION PLUS FORTE A UN Δv

$$\ddot{x}_{n+1}(t+T) = \alpha_{-}\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

- ET SI LA VITESSE RELATIVE EST POSITIVE

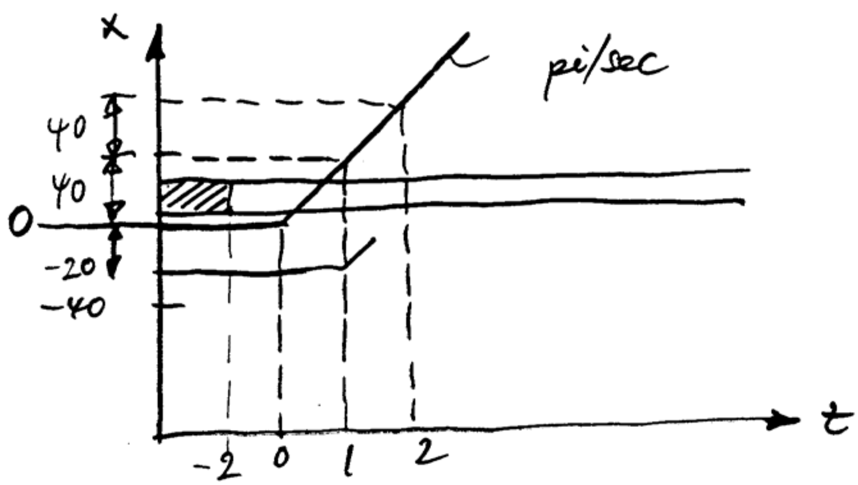
$$\ddot{x}_{n+1}(t+T) = \alpha_{+}\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

- L'UTILISATION DE α_{-} ET α_{+} AMELIORE LE MODELE.

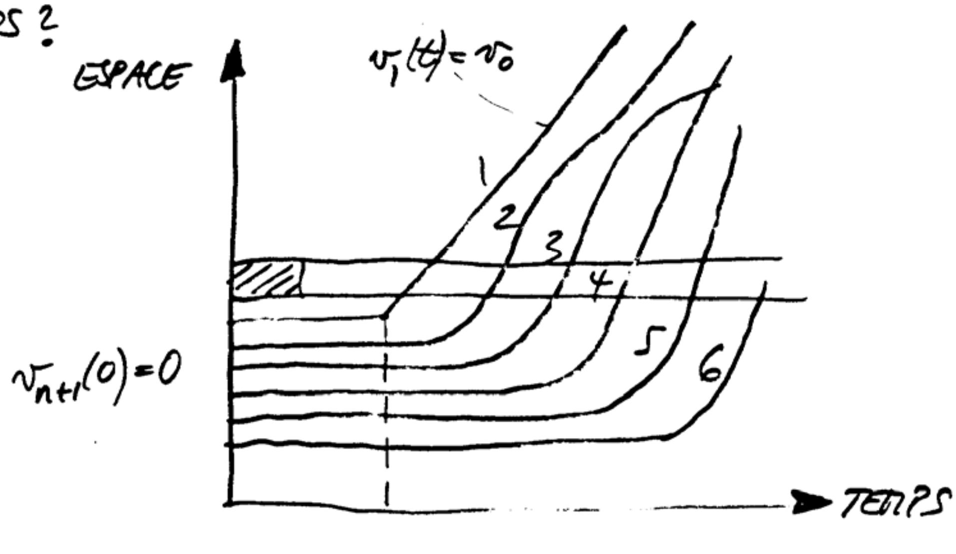
CALCULS DES POSITIONS DE VEHICULES EN INTERACTION

EXEMPLE 1

DEMARRAGE DE 6 VEHICULES AU FEU VERT



- LA VITESSE DU PREMIER VEHICULE EST CONSTANTE = 40 pi/s
- LES AUTRES VEHICULES DEMARRENT APRES $T = \Delta t = 1s$
- QUELLES SONT LEURS POSITIONS DANS LE DIAGRAMME ESPACE TEMPS ?



SI ON VEUT CONNAITRE LES VITESSES DES VEHICULES, IL FAUT TROUVER LA SOLUTION GENERALE POUR L'EQUATION DIFFERENTIELLE

$$T\dot{v}_{n+1}(t) = v_n(t-T) - v_{n+1}(t-T)$$

CETTE SOLUTION EST LONGUE ET FASTIDIEUSE!

RAPPEL:

- ON APPLIQUE LA TRANSFORMATION DE LAPLACE A L'EQUATION EN t. $\mathcal{L}\{\text{équ.}\}$
- L'EQUATION EN s QUE L'ON OBTIENT PEUT ETRE MANIPULEE ALGEBRIQUEMENT
- L'EXPRESSION AINSI OBTENUE PEUT ETRE RETRANSFORMEE DANS UNE EXPRESSION EN t QUI EST LA SOLUTION DE L'EQUATION. $\mathcal{L}^{-1}\{\ \}$
- DES TABLEAUX DONNENT LES TRANSFORMATIONS DE LAPLACE.

$$\mathcal{L}\{v_n(t-T)\} - \mathcal{L}\{v_{n+1}(t-T)\} = T\mathcal{L}\{\dot{v}_{n+1}(t)\}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$V_n(s)e^{-Ts} - V_{n+1}(s)e^{-Ts} = T\left(sV_{n+1}(s) - v_{n+1}(0)\right)$$

$$\boxed{V_{n+1}(s) = \frac{e^{-Ts}}{Ts+e^{-Ts}}V_n(s) + \frac{T}{Ts+e^{-Ts}}v_{i+1}(0)}$$

CETTE EQUATION NE PEUT ETRE RESOLUE QUE SI ON CONNAIT CERTAINES CONDITIONS DU PROBLEME, COMME PAR EX. $v_{n+1}(0)=0$ ET $v_1(t)=v_0$

AVEC $\mathcal{L}\{v_1(t)\} = \mathcal{L}\{v_0\}$

ON TROUVE $\boxed{V_1(s) = \frac{v_0}{s}}$

ET $$\boxed{V_{n+1}(s) = \frac{e^{-Ts}}{Ts + e^{-Ts}} V_n(s)}$$

PAR RECURSION :

$$V_2(s) = \left(\frac{e^{-Ts}}{Ts+e^{-Ts}}\right)\frac{v_0}{s}$$

$$V_3(s) = \left(\frac{e^{-Ts}}{Ts+e^{-Ts}}\right)^2\frac{v_0}{s}$$

$$\vdots$$

$$V_{n+1}(s) = \left(\frac{e^{-Ts}}{Ts+e^{-Ts}}\right)^n\frac{v_0}{s}$$

IL FAUT RETRANSFORMER CETTE EQUATION. ON PEUT DEVELOPPER EN SERIE :

$$\frac{V_2(s)}{v_0} = \frac{1}{s}\left(\frac{e^{-Ts}}{Ts+e^{-Ts}}\right) = \frac{1}{s}\left\{\frac{e^{-Ts}}{Ts} - \underbrace{\frac{e^{-2Ts}}{T^2s^2}} + \frac{e^{-3Ts}}{T^3s^3} - \dots\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-2Ts}}{T^2s^2}\right\} = \frac{(t-2T)^2}{2T^2}$$

$$\mathcal{L}^{-1}\left(\frac{e^{-kTs}}{s^n}\right) \longrightarrow \frac{(t-kT)^{n-1}}{n\,\Gamma(n)}(t-kT)$$

$\{\Gamma(1)=1 \ ; \ \Gamma(2)=1 \ ; \ \Gamma(3)=2 \ ; \ \Gamma(n+1)=n!\}$

DONC :

$$\frac{v_2(t)}{v_0} = \frac{t-T}{T} - \frac{(t-2T)^2}{2T^2} + \frac{(t-3T)^3}{6T^3} - + \cdots$$

$$\frac{v_3(t)}{v_0} = \frac{(t-2T)^2}{2T^2} - \frac{(t-3T)^3}{3T^3} + \frac{(t-4T)^4}{8T^4} - + \cdots$$

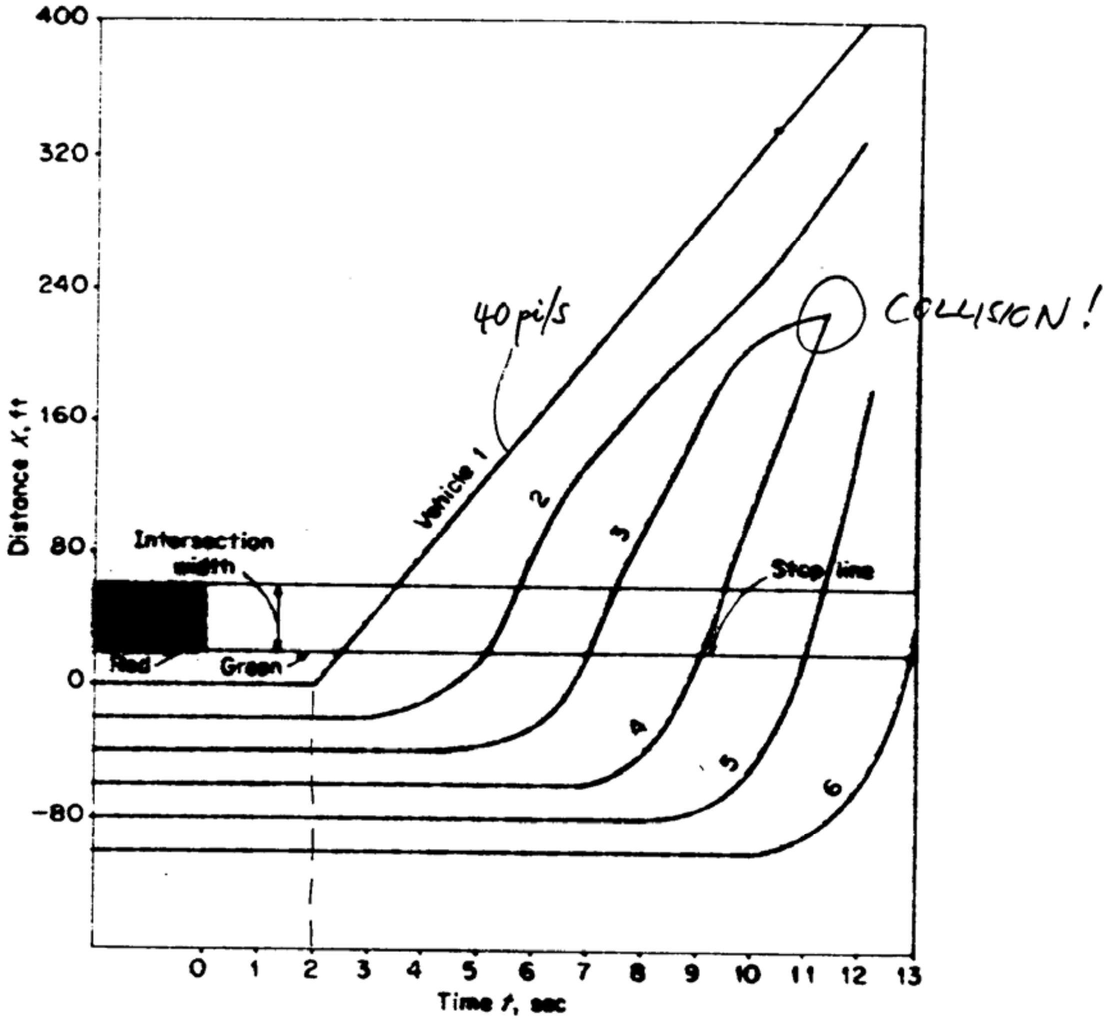
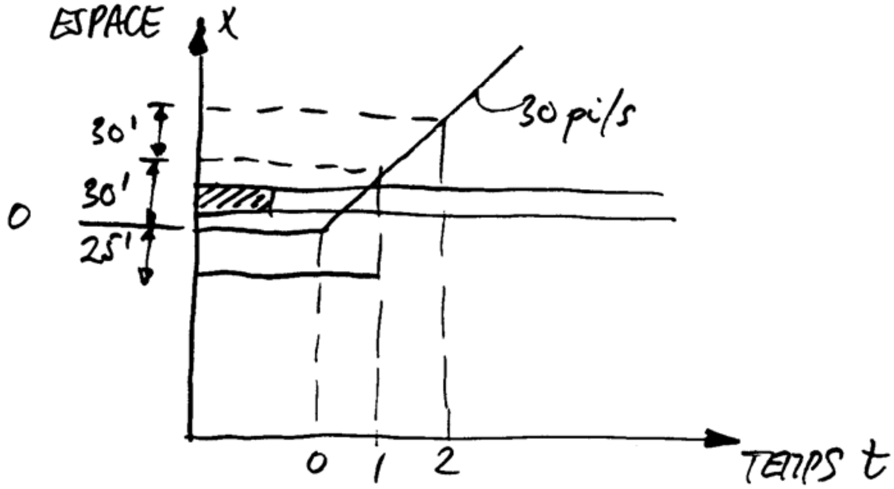


Fig. 13.7 Time-space relationship for vehicles obeying the car-following (simple law) (DREW)

ON PEUT EGALEMENT RESOUDRE CE PROBLEME A L'AIDE DE L'ANALYSE NUMERIQUE :



$$\ddot{x}_{n+1}(t+1) = 1{,}0\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$$

- ON CHOISIT DES INTERVALLES D'ANALYSE (POUR PLUS DE PRECISION 0,1 S)
- ON CALCULE L'ACCELERATION DU 2° VEHICULE ET ON CONSIDERE QUE $\ddot{x}_{n+1}$ RESTE CONST. PENDANT L'INTERVALLE i.

$\{v = v_0 + at\}$

$$\dot{x}_2(t) = \dot{x}_2(t-i) + \frac{\ddot{x}_2(t-i) + \ddot{x}_2(t)}{2} \cdot i$$

$$x_2(t) = x_2(t-i) + \frac{\dot{x}_2(t-i) + \dot{x}_2(t)}{2} \cdot i$$

$$\ddot{x}_2(t+1) = 1{,}0\left[\dot{x}_1(t) - \dot{x}_2(t)\right]$$

$\dot{x}_1 = 30\ p/s \qquad x_2(0) = -25$

$$x_2(t) - x_{n+1}(t) = T\,\dot{x}_{n+1}(t+T) + L \qquad L = 25$$

TABLE 6.1 Car-Following Calculations

Time (sec)	$\dot{x}_1$ (ft/sec)	$\ddot{x}_2$ (ft/sec²)	$\dot{x}_2$ (ft/sec)	$\dot{x}_1 - \dot{x}_2$ (ft/sec)	x_1 (ft)	x_2 (ft)	$x_1 - x_2$ (ft)
0.0	30.0	0.0	0.0	30.0	0	−25.0	25.0
1.0	30.0	30.0	0.0	30.0	30	−25.0	55.0
2.0	30.0	30.0	30.0	0	60	−10.0	70.0
3.0	30.0	0.0	45.0	−15.0	90	27.5	62.5
4.0	30.0	−15.0	37.50	−7.5	120	68.8	51.2
5.0	30.0	−7.50	26.25	3.75	150	100.6	49.4
6.0	30.0	3.75	24.375	5.625	180	125.9	54.1
7.0	30.0	5.625	29.062	0.938	210	152.7	57.3
8.0	30.0	0.938	32.345	−2.345	240	183.4	56.6
9.0	30.0	−2.345	31.641	−1.641	270	215.4	54.6
10.0	30.0	−1.641	29.648	0.352	300	246.0	54.0
11.0	30.0	0.352	29.004	0.996	330	275.3	54.7
12.0	30.0	0.996	29.678	0.332	360	304.7	55.3
13.0	30.0	0.322	30.342	−0.342	390	334.7	55.3
14.0	30.0	−0.342	30.332	−0.332	420	365.0	55.0
15.0	30.0	−0.332	29.995	0.005	450	395.2	54.8
16.0	30.0	0.005	29.831	0.169	480	425.1	54.9
17.0	30.0	0.169	29.918	0.082	510	455.0	55.0
18.0	30.0	0.082	30.046	−0.046	540	484.9	55.1
19.0	30.0	−0.046	30.064	−0.064	570	515.0	55.0
20.0	30.0	−0.064	30.009	−0.009	600	545.0	55.0

$$\ddot{x}_2(t+1) = 1.0[\dot{x}_1(t) - \dot{x}_2(t)]$$

$$\dot{x}_2(t) = x_2(t-1) + \frac{1}{2}[\ddot{x}_2(t-1) + \ddot{x}_2(t)]$$

$$x_2(t) = x_2(t-1) + \frac{1}{2}[\dot{x}_2(t-1) + \dot{x}_2(t)]$$

INTERVALLE DE CALCUL = 1 S

ICI L'INTERVALLE DE CALCUL A ETE 0.1 sec

TABLE 6.2 Solution of Car-Following Model

Time (sec)	$\dot{x}_1$ (ft/sec)	x_1 (ft)	$\dot{x}_2$ (ft/sec)	x_2 (ft)	$\dot{x}_3$ (ft/sec)	x_3 (ft)	$\dot{x}_4$ (ft/sec)	x_4 (ft)	$\dot{x}_5$ (ft/sec)	x_5 (ft)
0.0	30	0.0	0.00	−25	0.00	−50.0	0.00	−75.0	0.00	−100.0
1.0	30	30.0	0.00	−25	0.00	−50.0	0.00	−75.0	0.00	−100.0
2.0	30	60.0	30.00	−10	0.00	−50.0	0.00	−75.0	0.00	−100.0
3.0	30	90.0	45.00	27.5	15.00	−42.5	0.00	−75.0	0.00	−100.0
4.0	30	120.0	35.00	67.5	50.00	−10.0	5.00	−72.5	0.00	−100.0
5.0	30	150.0	23.75	96.9	58.75	44.4	36.25	−51.9	1.25	−99.4
6.0	30	180.0	25.25	121.4	29.00	88.2	78.00	−4.8	19.00	−89.2
7.0	30	210.0	31.71	149.8	6.96	106.23 [a]	109.75	89.1 [a]	70.08	−44.7

[a] Front ends of vehicles are separated by less than 18.0 ft (collision imminent).

ON REMARQUE UNE AUGMENTATION DE LA PRECISION PAR RAPPORT AU CALCUL PRECEDENT

- LE MODELE UTILISE EST LE MODELE LINEAIRE, MOINS REALISTE

AUTRE EXEMPLE (MAY, 1990)

- 2 VEHICULES SE SUIVENT.
- AU DEBUT LES DEUX SONT ARRETES AVEC UN ESPACEMENT DE 25'.
- VEHICULE 1 ACCELERE AU TAUX CONSTANT 3.3 pi/s² JUSQU'A 44 pi/s DE VITESSE.
- CETTE VITESSE EST GARDEE PENDANT 10S.
- APRES DECELERATION AU TAUX CONST. DE 4.6 pi/s².

$$\dot{x}_1(t+T) = \dot{x}_1(t) + \left[\frac{\ddot{x}_1(t) + \ddot{x}_1(t+T)}{2}\right] T$$

AVEC $T = 1\,s$

$$\dot{x}_1(t+1) = \dot{x}_1(t) + \ddot{x}_1$$

$$x_1(t+T) = x_1(t) + \dot{x}_1(t)\,T + \left[\frac{\ddot{x}_1(t) + \ddot{x}_1(t+T)}{2}\right]\frac{T^2}{2}$$

$$x_1(t+1) = x_1(t) + \dot{x}_1(t) + \frac{\ddot{x}_1}{2}$$

MODELE SIMPLE: $\ddot{x}_{n+1}(t+\Delta t) = \alpha\left[\dot{x}_n(t) - \dot{x}_{n+1}(t)\right]$

TEMPS DE REACTION: $\Delta t = 1\,s$

$$\alpha = 0,5$$

$$\ddot{x}_2(t+1) = 0,5\left\{\dot{x}_1(t) - \dot{x}_2(t)\right\}$$

TABLE 6.3 Car-Following Trajectories

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Lead Vehicle			Following Vehicle			Relative	
Time (sec)	Accel., $\ddot{x}_1$	Speed, $\dot{x}_1$	Distance, x_1	Accel., $\ddot{x}_2$	Speed, $\dot{x}_2$	Distance, x_2	Speed, $\dot{x}_1 - \dot{x}_2$	Distance, $x_1 - x_2$
0		0	0	0.0000	0.0000	−25.00	0.0000	25.00
	3.3			0.0000[a]				
1		3.3	1.6	0.0000	0.0000	−25.00	3.3000	26.60
	3.3			0.8250				
2		6.6	6.6	1.6500	0.8250	−24.59	5.7750	31.19
	3.3			2.2688				
3		9.9	14.8	2.8875	3.0938	−22.63	6.8062	37.43
	3.3			3.1453				
4		13.2	26.4	3.4031	6.2391	−17.96	6.9609	44.36
	3.3			3.4418				
5		16.5	41.2	3.4804	9.6809	−10.00	6.8191	51.20
	3.3			3.4450				
6		19.8	59.4	3.4096	13.1259	1.40	6.6741	58.00
	3.3			3.3733				
7		23.1	80.8	3.3370	16.4992	16.21	6.6008	64.59
	3.3			3.3187				
8		26.4	105.6	3.3004	19.8179	34.37	6.5821	71.23
	3.3			3.2957				
9		29.7	133.6	3.2910	23.1136	55.83	6.5864	77.76
	3.3			3.2921				
10		33.0	165.0	3.2932	26.4057	80.59	6.5943	84.41
	3.3			3.2952				
11		36.3	199.6	3.2971	29.7009	108.64	6.5912	90.96
	3.3			3.2983				
12		39.6	237.6	3.2996	33.0000	139.99	6.6008	97.61
	3.3			3.3000				
13		42.9	278.8	3.3004	36.3000	174.64	6.6000	104.16
	3.3							
13.33		44.0	293.3	—	—	—	—	—
	0.0			3.3002				
14		44.0	322.6	3.3000	39.6002	212.59	4.3998	110.01
	0.0			2.7500				
15		44.0	366.6	2.1999	42.3502	253.56	1.6498	113.03
	0.0			1.5124				
16		44.0	410.6	0.8249	43.8626	296.67	0.1374	113.93
	0.0			0.4468				
17		44.0	454.6	0.0687	44.3094	340.76	−0.3094	113.84
	0.0			−0.0430				
18		44.0	498.6	−0.1547	44.2664	385.05	−0.2664	113.55
	0.0			−0.1440				
19		44.0	542.6	−0.1332	44.1224	429.24	−0.1224	113.36
	0.0			−0.0972				

TABLE 6.3 Car-Following Trajectories (continued)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Lead Vehicle			Following Vehicle			Relative	
Time (sec)	Accel., $\ddot{x}_1$	Speed, $\dot{x}_1$	Distance, x_1	Accel., $\ddot{x}_2$	Speed, $\dot{x}_2$	Distance, x_2	Speed, $\dot{x}_1 - \dot{x}_2$	Distance, $x_1 - x_2$
20		44.0	586.6	−0.0612	44.0252	473.31	−0.0252	113.29
	0.0			−0.0369				
21		44.0	630.6	−0.0126	43.9883	517.32	0.0117	113.28
	0.0			−0.0034				
22		44.0	674.6	+0.0058	43.9849	561.31	0.0151	113.29
	0.0			+0.0067				
23		44.0	718.6	+0.0075	43.9916	605.30	0.0084	113.30
	0.0							
23.33		44.0	733.3	—	—		—	
	−4.6			0.0058				
24		40.9	761.6	0.0042	43.9974	649.29	−3.0974	112.31
	−4.6			−0.7722				
25		36.3	800.2	−1.5487	43.2252	692.90	−6.9252	107.30
	−4.6			−2.5056				
26		31.7	834.2	−3.4626	40.7196	734.87	−9.0296	99.33
	−4.6			−3.9862				
27		27.1	863.6	−4.5098	36.7334	773.60	−9.6334	90.00
	−4.6			−4.6632				
28		22.5	888.4	−4.8167	32.0702	808.00	−0.5702	80.40
	−4.6			−4.8009				
29		17.9	908.6	−4.7851	27.2693	837.63	−9.3693	70.93
	−4.6			−4.7349				
30		13.3	924.2	−4.6847	22.5344	862.57	−9.2344	61.63
	−4.6			−4.6510				
31		8.7	935.2	−4.6172	17.8834	882.78	−9.1834	52.42
	−4.6			−4.6045				
32		4.1	941.6	−4.5917	13.2789	893.36	−9.1789	43.24
	−4.6			−4.5906				
32.89		0.0	943.4	−4.5895	8.6883	909.34	−8.6883	34.06
	0.0			−4.4668				
33		0.0	943.4	−4.3442	4.2215	915.79	−4.2215	27.61
	0.0			−3.2275				
34		0.0	943.4	−2.1107	0.9940	918.40	−0.9940	25.00

[a]Average acceleration (deceleration) rate during sampling time interval.

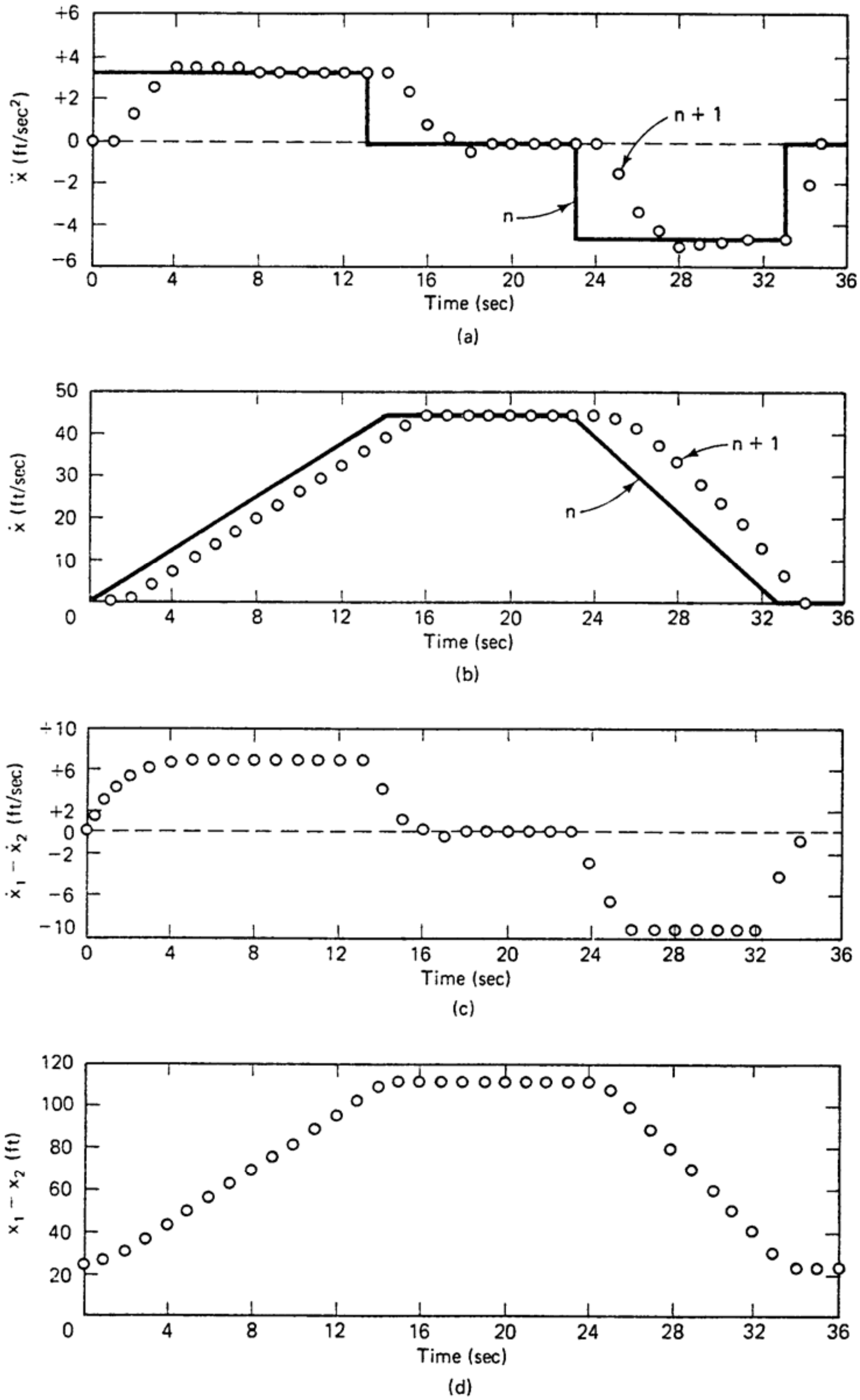


Figure 6.7 Car-Following Trajectories

LA STABILITE

- COMPORTEMENT DU CONDUCTEUR
  - INSTABLE

    UN CONDUCTEUR QUI, EN SUIVANT UN VEHICULE, REAGIT LENTEMENT ($\Delta t >$) ET TRES FORTEMENT ($\alpha >$) A UN COMPORTEMENT INSTABLE, QUI PEUT ENTRAINER DES COLLISIONS ENTRE DES VEHICULES EN FILE EN AMONT DE LUI.
  - STABLE

    UN CONDUCTEUR ATTENTIF, NE PRODUISANT PAS D'ACCELERATIONS ET DE DECELERATIONS BRUSQUES. FAIBLE TEMPS DE REACTION ET PLUS FAIBLE SENSIBILITE

- LE PRODUIT $C = \alpha \cdot \Delta t$ CARACTERISE LA STABILITE DU PROCESSUS. DES VALEURS C ELEVEES PRODUISENT DES OSCILLATIONS DANS LES ESPACEMENTS PAR RAPPORT AU TEMPS.

- ON PARLE DE 2 TYPES DE STABILITE
  - STABILITE LOCALE QUI SE LIMITE A 2 VEHICULES SE SUIVANT
  - STABILITE ASYMPTOTIQUE QUI ETUDIE LE COMPORTEMENT D'UNE FILE DE VEHICULES. LE COMPORTEMENT INSTABLE 2 DEUXIEME CONDUCTEUR PEUT ENTRAINER DES ACCIDENTS EN AMONT.

- LES CRITERES DE STABILITE SONT DIFFICILES A ETABLIR.

- ON PEUT OBTENIR UNE IDEE PLUS PRECISE SUR LA STABILITE EN TROUVANT UNE SOLUTION POUR L'EQUATION DIFFERENTIELLE.

- POUR PLUS DE SIMPLICITE ON SE BASE SUR LE MODELE LINEAIRE.

$$\dot{v}_{n+1}(t) = \alpha\left[v_n(t-T) - v_{n+1}(t-T)\right]$$

POUR RESOUDRE CETTE EQUATION ON PEUT FAIRE UNE <u>APPROXIMATION</u>:

$$\frac{d\,v_{n+1}(t)}{dt} = \alpha \underbrace{\frac{ds_{n+1}(t-T)}{dt}}_{\text{DEVELOPPER EN SERIE DE TAYLOR}}$$

$$\frac{dv_{n+1}(t)}{dt} \approx \alpha \left\{ \frac{d}{dt} s_{n+1}(t) - T \frac{d^2}{dt^2} s_{n+1}(t) + \frac{T^2}{2} \frac{d^3}{dt^3} s_{n+1}(t) \right\}$$

AVEC
$$\frac{ds_{n+1}}{dt} = v_n - v_{n+1}$$
$$\frac{d^2 s_{n+1}}{dt^2} = v_n' - v_{n+1}'$$
$$\frac{d^3 s_{n+1}}{dt^3} = v_n'' - v_{n+1}''$$

$$v_{n+1}' = \alpha \left\{ v_n - v_{n+1} - T v_n' + T v_{n+1}' + \frac{T^2}{2} v_n'' - \frac{T^2}{2} v_{n+1}'' \right\}$$

$$\frac{\alpha T^2}{2} v_{n+1}'' + (1-\alpha T) v_{n+1}' + \alpha v_{n+1} = \frac{\alpha T^2}{2} v_n'' - \alpha T v_n' + \alpha v_n$$

v_n, v_n', v_n'' SONT CONNUES.

EN REMPLAÇANT $v_{n+1} = e^{rt}$ AVEC r COMME CONST.

$$v_{n+1}' = re^{rt} \qquad v_{n+1}'' = r^2 e^{rt}$$

$$\left(\frac{\alpha T^2}{2}\right) r^2 e^{rt} + (1-\alpha T) r e^{rt} + \alpha e^{rt}$$

$$\underbrace{\left\{\left(\frac{\alpha T^2}{2}\right)r^2+(1-\alpha T)r+\alpha\right\}}_{\text{EQUATION CARACTERISTIQUE}} e^{rt}=0$$

$$r^2+\frac{2(1-\alpha T)}{\alpha T^2}r+\frac{2\alpha}{\alpha T^2}=0$$

RACINES $\qquad r_{1,2}=-\frac{(1-\alpha T)}{\alpha T^2}\pm\sqrt{\frac{(1-\alpha T)^2}{\alpha^2T^4}-\frac{2}{T^2}}$

__RAPPEL__

SI $\frac{(1-\alpha T)^2}{\alpha^2 T^4}>\frac{2}{T^2}$ ALORS IL Y A 2 SOLUTIONS REELLES

$v_{n+1,1}=e^{r_1 t}$ ET $v_{n+1,2}=e^{r_2 t}$

SI $\frac{(1-\alpha T)^2}{\alpha^2 T^4}=\frac{2}{T^2}$ ALORS $r=-\frac{(1-\alpha T)}{\alpha T^2}$

$v_{n+1}=e^{rt}$

SI $\frac{(1-\alpha T)^2}{\alpha^2 T^4}<\frac{2}{T^2}$ ALORS SOLUTIONS COMPLEXES

AVEC $\tau=-\frac{(1-\alpha T)}{\alpha T^2}$ ET $\omega=\sqrt{}$

$r_1=\tau+i\omega \qquad r_2=\tau-i\omega$

$$v_{n+1,1}=e^{r_1 t}=e^{(\tau+i\omega)t}=e^{\tau t}(\cos(\omega t)+i\sin(\omega t))$$

$$v_{n+1,2}=e^{r_2 t}=e^{(\tau-i\omega)t}=e^{\tau t}(\cos(\omega t)-i\sin(\omega t))$$

IL N'Y A PAS D'OSCILLATION (LE PROCESSUS EST STABLE) SI

$$\frac{(1-\alpha T)^2}{\alpha^2 T^4} \geq \frac{2}{T^2}$$

DONC

$0 \leq \alpha T < 0{,}414$	NON OSCILLATOIRE ET AMORTI
$0{,}414 \leq \alpha T < 1{,}0$	OSCILLATOIRE MAIS AMORTI
$\alpha T \geq 1{,}0$	OSCILLATION CROISSANTE

CES RESULTATS NE SONT QUE APPROXIMATIFS LA SOLUTION PRECISE EST LONGUE ET COMPLIQUEE. ON OBTIENT POUR L'ESPACEMENT:

$0 < \alpha T < \frac{1}{e} = 0{,}368$	NON OSCILLATOIRE, AMORTI
$\frac{1}{e} < \alpha T < \frac{\pi}{2} = 1.57$	OSCILLATOIRE, AMPLITUDE AMORTIE
$\alpha T = \frac{\pi}{2}$	OSCILLATOIRE, AMPLITUDE CONSTANTE
$\alpha T > \frac{\pi}{2}$	ESPACEMENT OSCILLATOIRE AVEC AMPLITUDE CROISSANTE

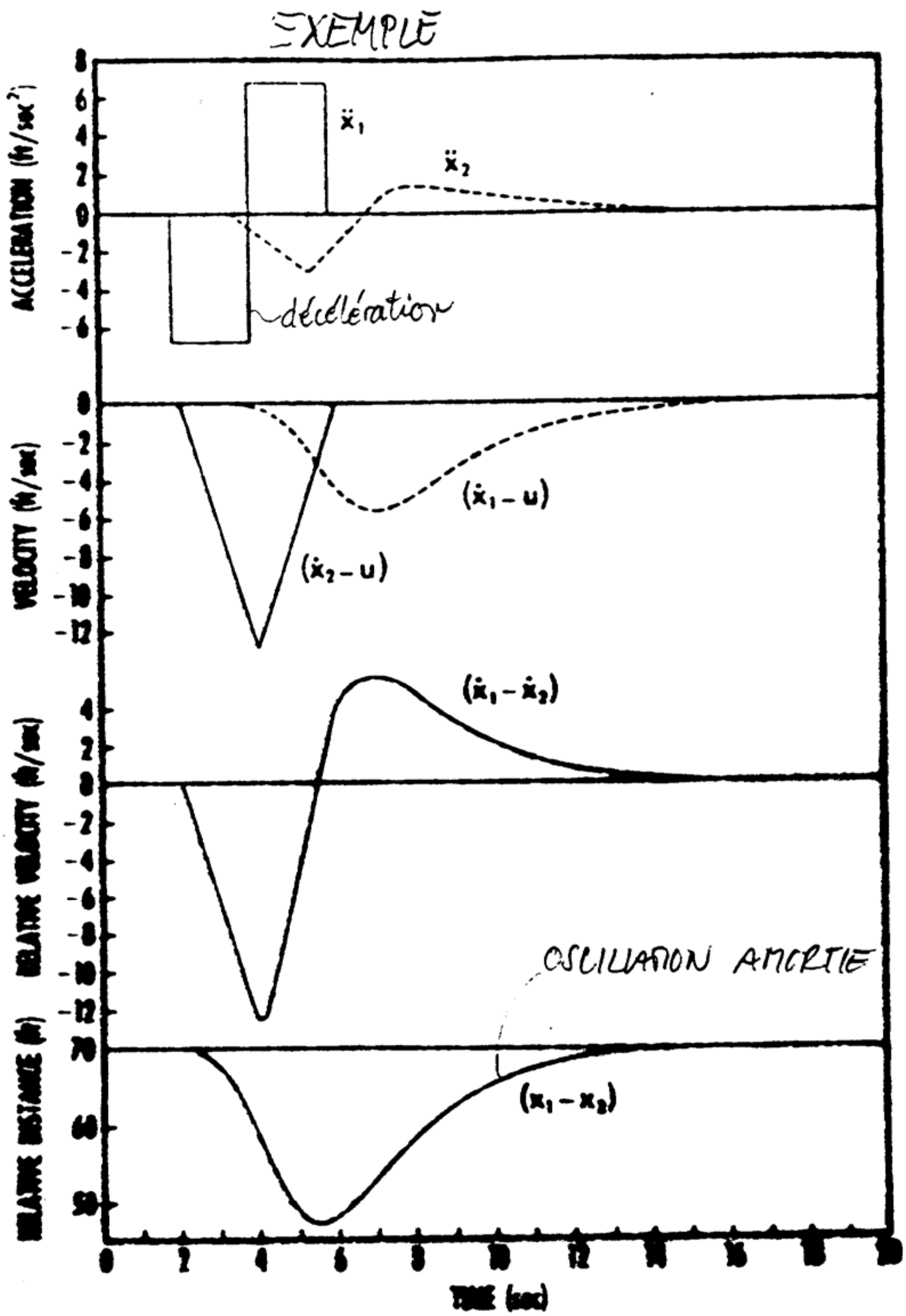


FIGURE 32. Detailed motion of two cars showing the effect of a fluctuation in the acceleration of the lead car. The second car follows the first with relative speed control and a time lag $T = 1.5$ sec and $C = aT = e^{-1} = 0.368$.

EXEMPLES : $\alpha T > e^{-1} = 0.37$

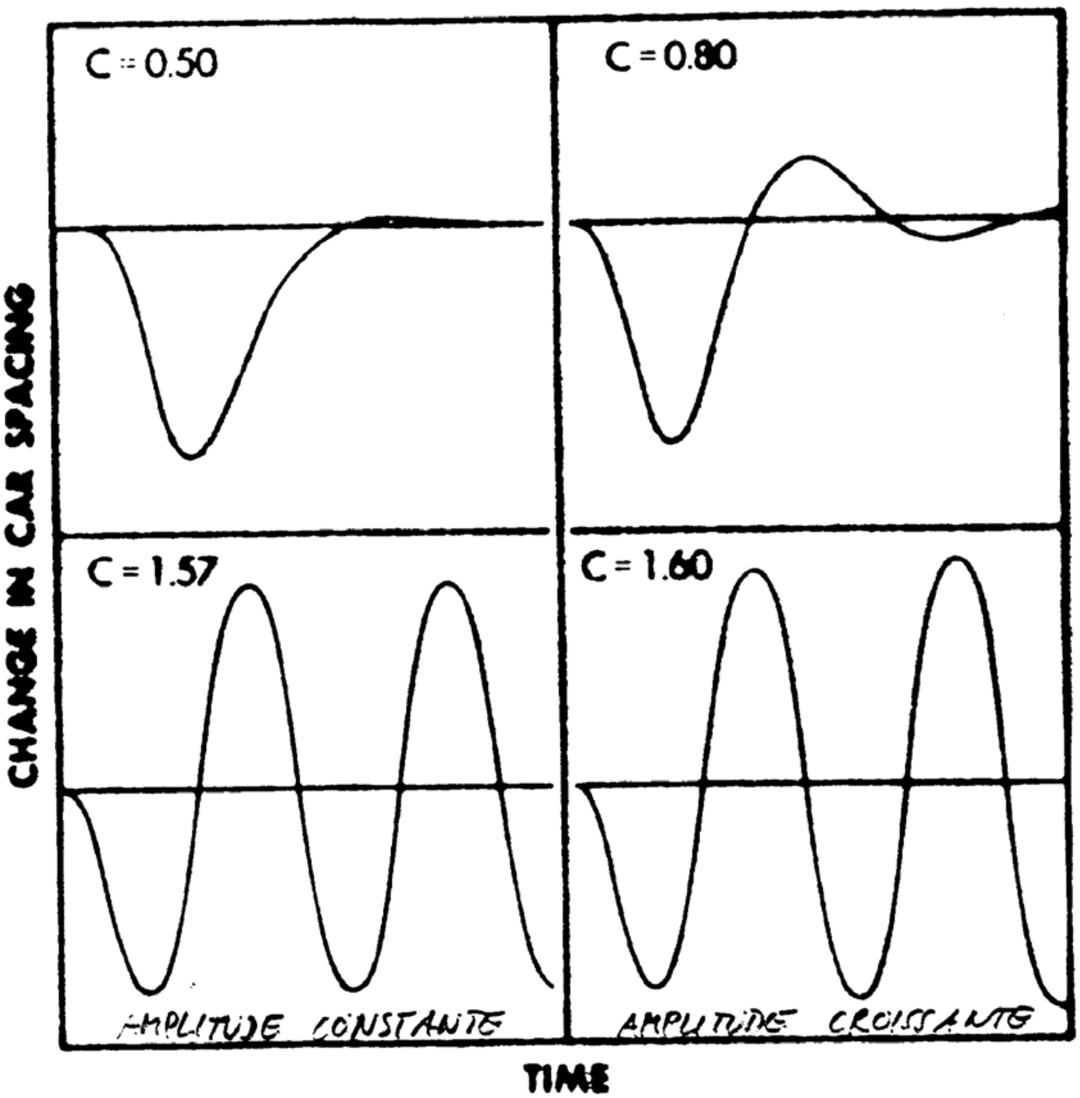


FIGURE 33. Change in car spacing of two cars with different values of $C = aT$ for the follower.

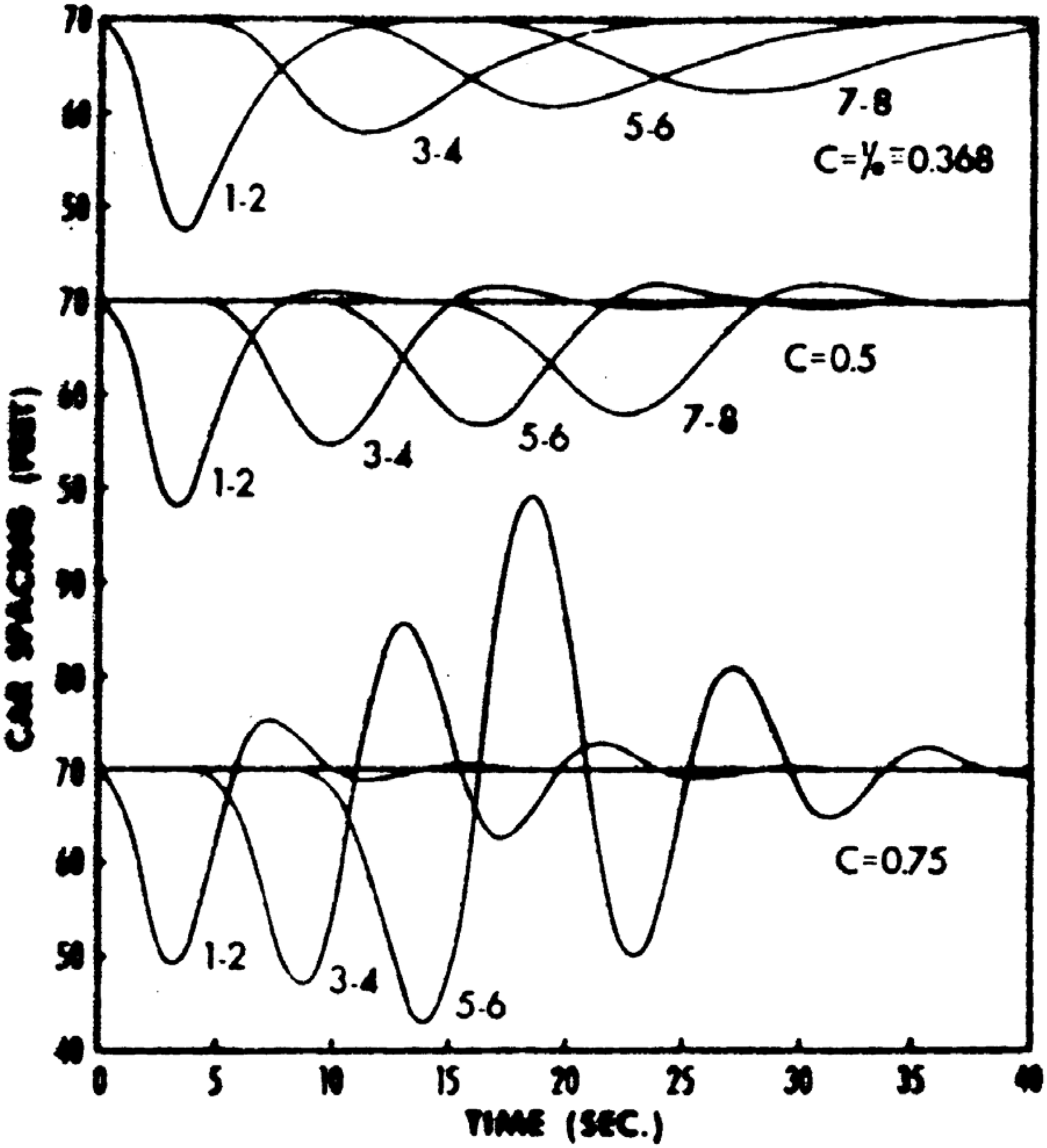
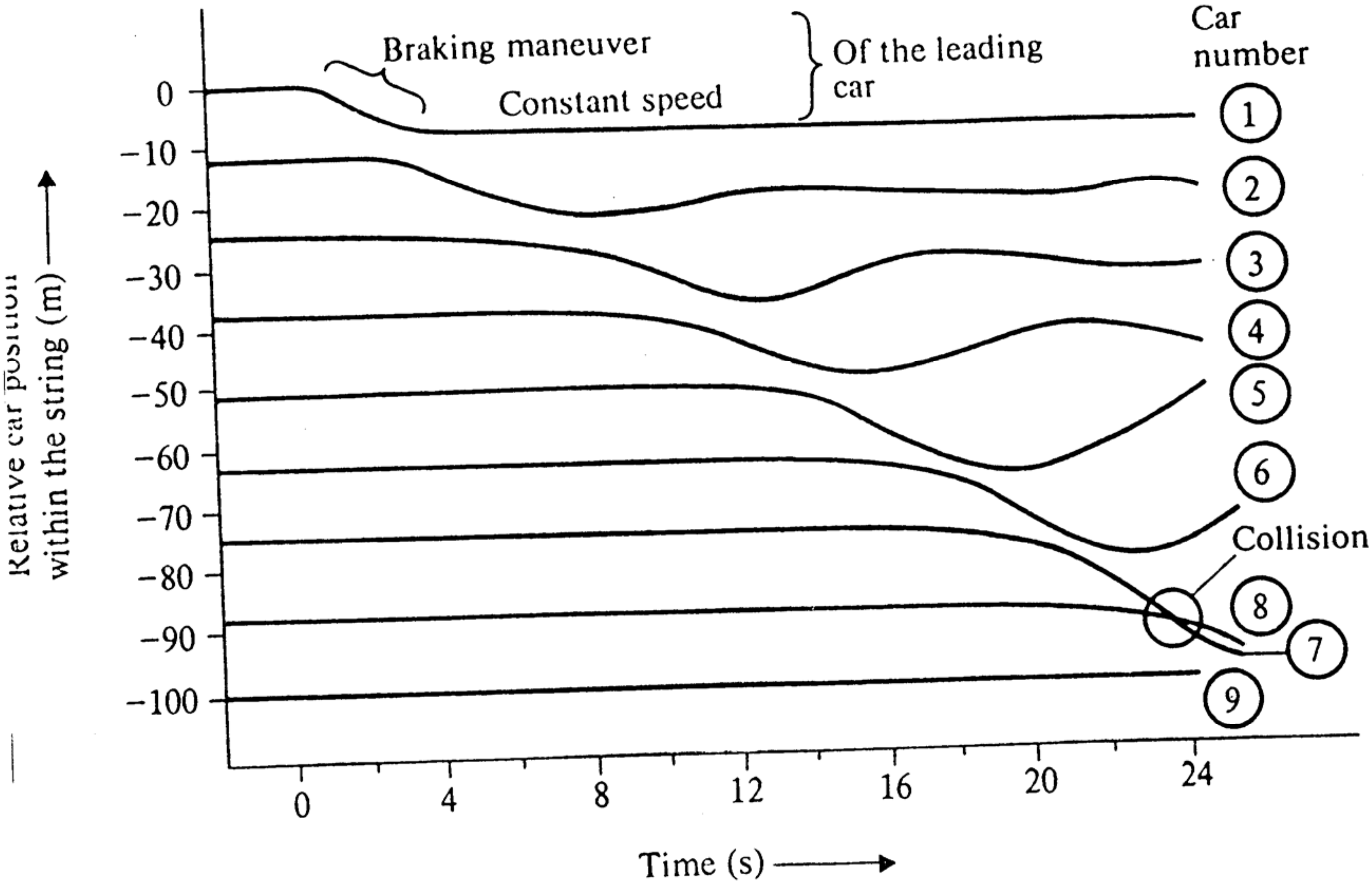


FIGURE 34. Car spacing of a line of cars with constant sensitivity, (a) for various values of T, and $C = aT$.



Simulation example illustrating the phenomenon of queue instability

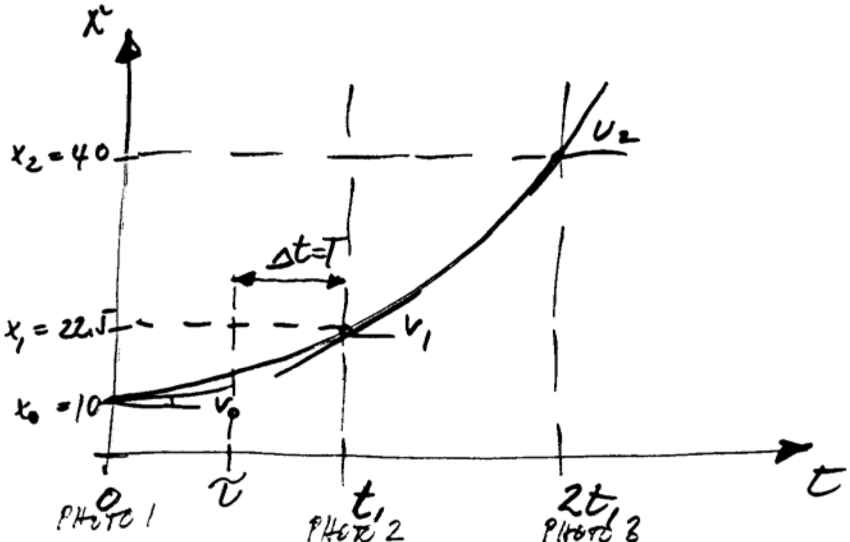
. Gazis 1972, : LA PERTURBATION EST AMPLIFIEE, JUSQU'A CE QU'IL Y AIT COLLISION ENTRE VEHICULE 7 ET 8.

STABILITÉ

$\alpha \cdot \Delta t$	STABILITÉ LOCALE	STABILITÉ ASYMPTOTIQUE
0	AUCUNE OSCILLATION	OSCILLATION AMORTIE
0,37	OSCILLATION AMORTIE	OSCILLATION AMORTIE
0,5	OSCILLATION AMORTIE	OSCILLATION CROISSANTE
1.0	OSCILLATION AMORTIE	OSCILLATION CROISSANTE
1.57	OSCILLATION CROISSANTE	OSCILLATION CROISSANTE
2.0	OSCILLATION CROISSANTE	OSCILLATION CROISSANTE

MAY, 1990

CALCUL DES VARIABLES A PARTIR D'OBSERVATIONS



EN SUPPOSANT UNE ACCELERATION CONSTANTE ON CALCULE:

$$x(t) = \frac{1}{2} a t^2 + v_0 t + x_0$$
$$v(t) = a t + v_0$$

AVEC DES INTERVALLES DE PHOTOGRAPHIE CONSTANTS ON AURA:

$$x_1 = \frac{1}{2} a t_1^2 + v_0 t_1 + x_0$$
$$x_2 = 2 a t_1^2 + 2 v_0 t_1 + x_0$$

$$v_0 = \frac{4x_1 - x_2 - 3x_0}{2t_1} \qquad a = \frac{x_2 + x_0 - 2x_1}{t_1^2}$$

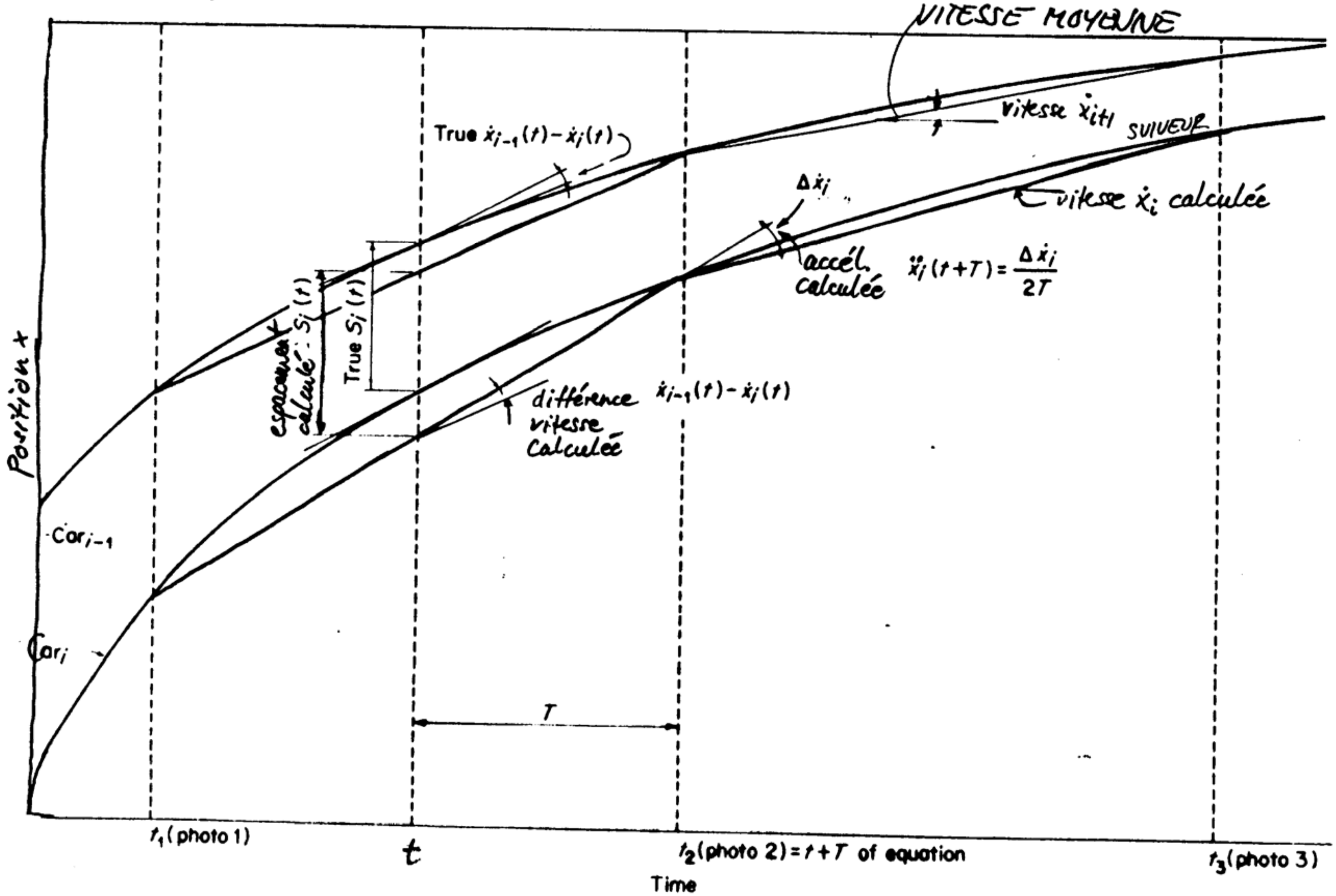
$$v_1 = \frac{x_2 - x_0}{2t_1}$$

$$\boxed{\ddot{x}_n(t_1) = \frac{\alpha\left[\dot{x}_{n-1}(t_1-T) - \dot{x}_n(t_1-T)\right]}{\left[x_{n-1}(t_1-T) - x_n(t_1-T)\right]^{\ell}}}$$

$$v_2 = \frac{x_0 - 4x_1 + 3x_2}{2t_1}$$

$$v(t_1 - T) = a(t_1 - T) + v_0$$
$$x(t_1 - T) = \frac{1}{2} a (t_1 - T)^2 + v_0 (t_1 - T) + x_0$$

ON SUPPOSE UN CHANGEMENT DE VITESSE CONSTANT, QUI A LIEU A t_2.

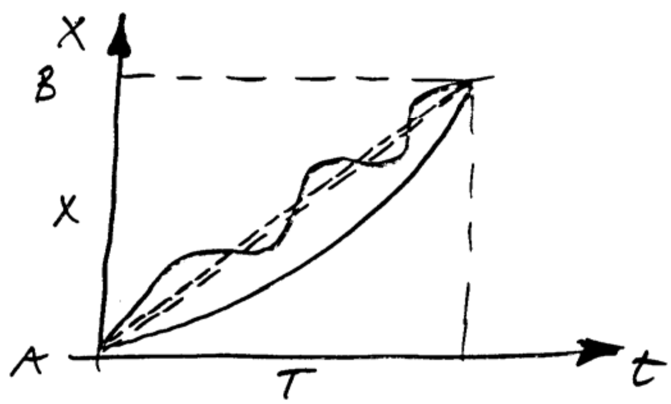


LA VARIATION DE L'ACCELERATION

BRUIT D'ACCELERATION

- CETTE VARIABLE PEUT ETRE UTILISEE POUR INDIQUER LE NIVEAU DE SERVICE, CAR ELLE INDIQUE LA FREQUENCE ET LA TAILLE DES VARIATIONS DE LA VITESSE.

- SOUVENT ON A UTILISE LA VITESSE DE PARCOURS COMME INDICATEUR DU NIVEAU DE SERVICE.



MAIS LES 3 PROFILS DE VITESSE D'UN PARCOURS ENTRE A ET B ONT LE <u>MEME</u> TEMPS (ET VITESSE) DE PARCOURS

$$v = \frac{X}{T}$$

- L'INCONFORT N'EST DONC PAS INCLU DANS CETTE VARIABLE
- HYPOTHESE: LE CONDUCTEUR EVALUE LE NIVEAU D'UNE ROUTE EN FONCTION DE LA VITESSE QU'IL PEUT ATTEINDRE, MAIS EGALEMENT EN FONCTION DE L'UNIFORMITE DE CETTE VITESSE.
- LA VARIABLE "BRUIT D'ACCELERATION" PEUT DECRIRE LA QUALITE DE SERVICE.
- LES FLUCTUATIONS DEPENDENT
  - DU CONDUCTEUR
  - DE LA ROUTE
  - DE LA CIRCULATION AMBIANTE
- CONDUCTEUR

CONDUCTEUR "SPORTIF" POTENTIELLEMENT DANGEREUX

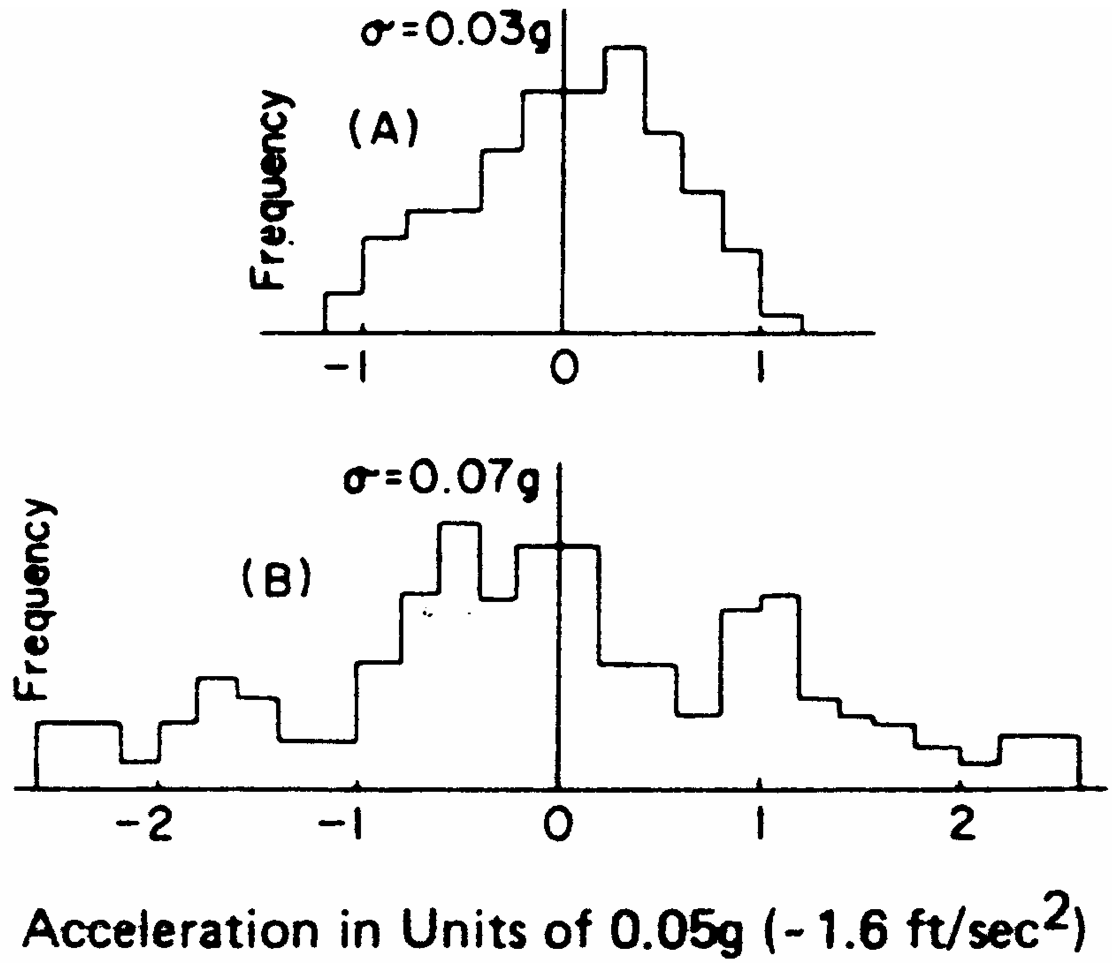


Figure 6.10 Acceleration distribution functions for a driver (A) moving with a traffic stream at approximately 35 mph and (B) attempting to drive 5 to 10 mph faster than the stream average.[10]

- ROUTE
  - CONDITIONS IDEALES ~ 0.32 pi/s²

  (VARIATIONS CAUSEES PAR IMPERFECTION DE LA RETRO ACTIONS)

  QUAND LA GEOMETRIE VARIE LE BRUIT AUGMENTE

  - TUNNEL N.Y. ~ 0.78 pi/s²
  - ROUTE RURALE 1.45 pi/s²

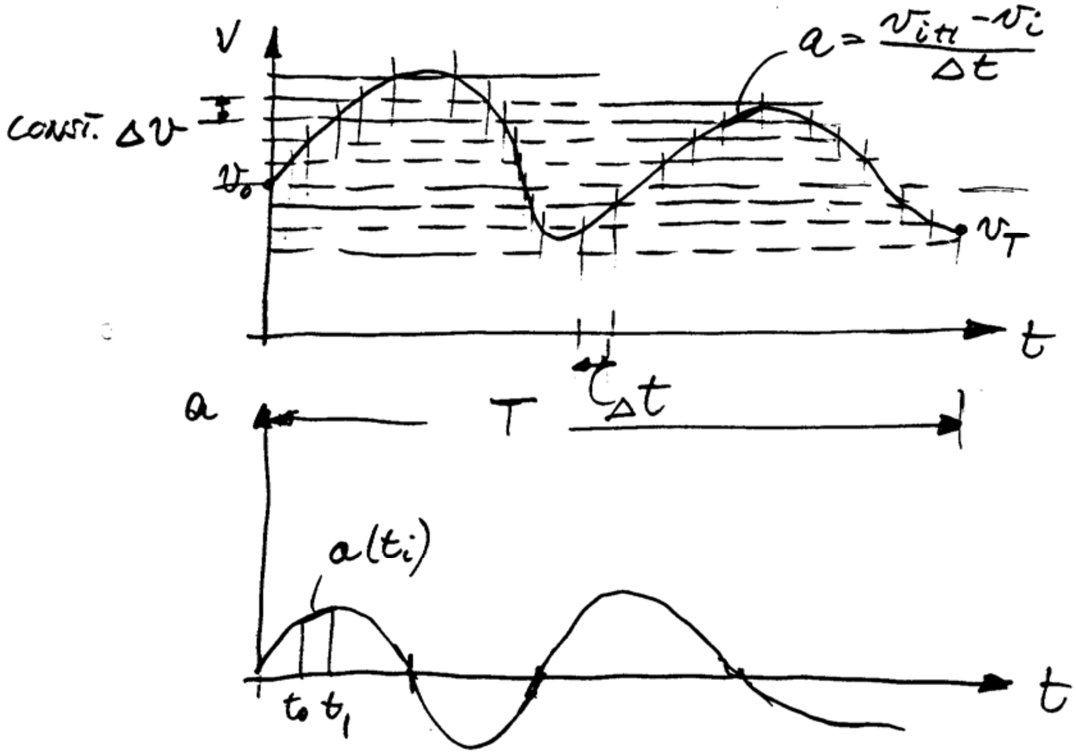
- CIRCULATION

  LE BRUIT AUGMENTE LORSQUE LA DENSITE AUGMENTE.

- ON APPELLE "BRUIT NATUREL" LE BRUIT QUI SE PRESENTE EN ABSCENCE DE CIRCULATION.

- ON POURRAIT UTILISER LA DIFFERENCE BRUIT NATUREL - BRUIT IDEAL COMME INDICATEUR DE LA GEOMETRIE, DES CONDITIONS DE ROULEMENT QUI DÉVIENT DE L'IDEAL.

RELATIONS MATHEMATIQUES



L'ACCELERATION MOYENNE POUR UN DEPLACEMENT DE DUREE T

$$a_{MOYENNE} = \frac{1}{T}\int_0^T a(t_i)\,dt = \frac{1}{T}\left[v(T) - v(0)\right]$$

L'ECART TYPE SE CALCULE

$$\sigma = \sqrt{\frac{1}{T}\int_0^T \left(a(t_i) - a_{moy}\right)^2 dt}$$

$$\sigma^2 = \frac{1}{T}\int_0^T \left[a(t_i) - a_m\right]^2 dt$$

EN FAISANT L'EXPANSION STANDARD DE CETTE FORMULE :

$$\sigma^2 = \frac{1}{T}\left\{ \int_0^T a^2(t_i)\,dt - \int_0^T \underbrace{2a(t_i)}_{a_m} a_m\,dt + \int_0^T a_m^2\,dt \right.$$

$$\vdots$$

$$\sigma^2 = \frac{1}{T}\int_0^T a^2(t_i)\,dt - a_m^2$$

ON PEUT ESTIMER σ^2 EN APPROXIMANT :

$$\sigma^2 \approx \frac{1}{T}\sum_{i=0}^{T}\left(\frac{\Delta v}{\Delta t}\right)^2 \Delta t - \left(\frac{v_T - v_0}{T}\right)^2$$

MAIS SI LA VITESSE INITIALE = VITESSE FINALE, OU SUR UNE LONGUE ROUTE $v_T \approx v_0$.

DE PLUS SI ON ADOPTE UN INTERVALLE FIXE Δv ($\approx$ 2 à 3 Km/h) ALORS ON A :

$$\sigma^2 = \frac{\Delta v^2}{T}\sum_{i=0}^{T}\frac{n^2}{\Delta t_i}$$

OU n EST LE NOMBRE DE CHANGEMENTS DE VITESSE DE TRANCHES DE Δv km/h PENDANT LE TEMPS Δt.
SI Δt EST MESURE POUR CHAQUE CHANGEMENT SUCCESSIF DE VITESSE DE Δv ALORS $n=1$ ET :

$$\sigma = \left[\frac{(\Delta v)^2}{T}\sum_{i=0}^{T}\frac{1}{\Delta t_i}\right]^{1/2}$$

LES DEVIATIONS DU VEHICULE ARRETE CONTRIBUENT TROP FORTEMENT A σ. ON UTILISE DONC UNIQUEMENT LE TEMPS DE MARCHE POUR L'ANALYSE.

ON RELEVE Δt_i ET V_i POUR CHAQUE CHANGEMENT SIGNIFICATIF DE LA VITESSE.

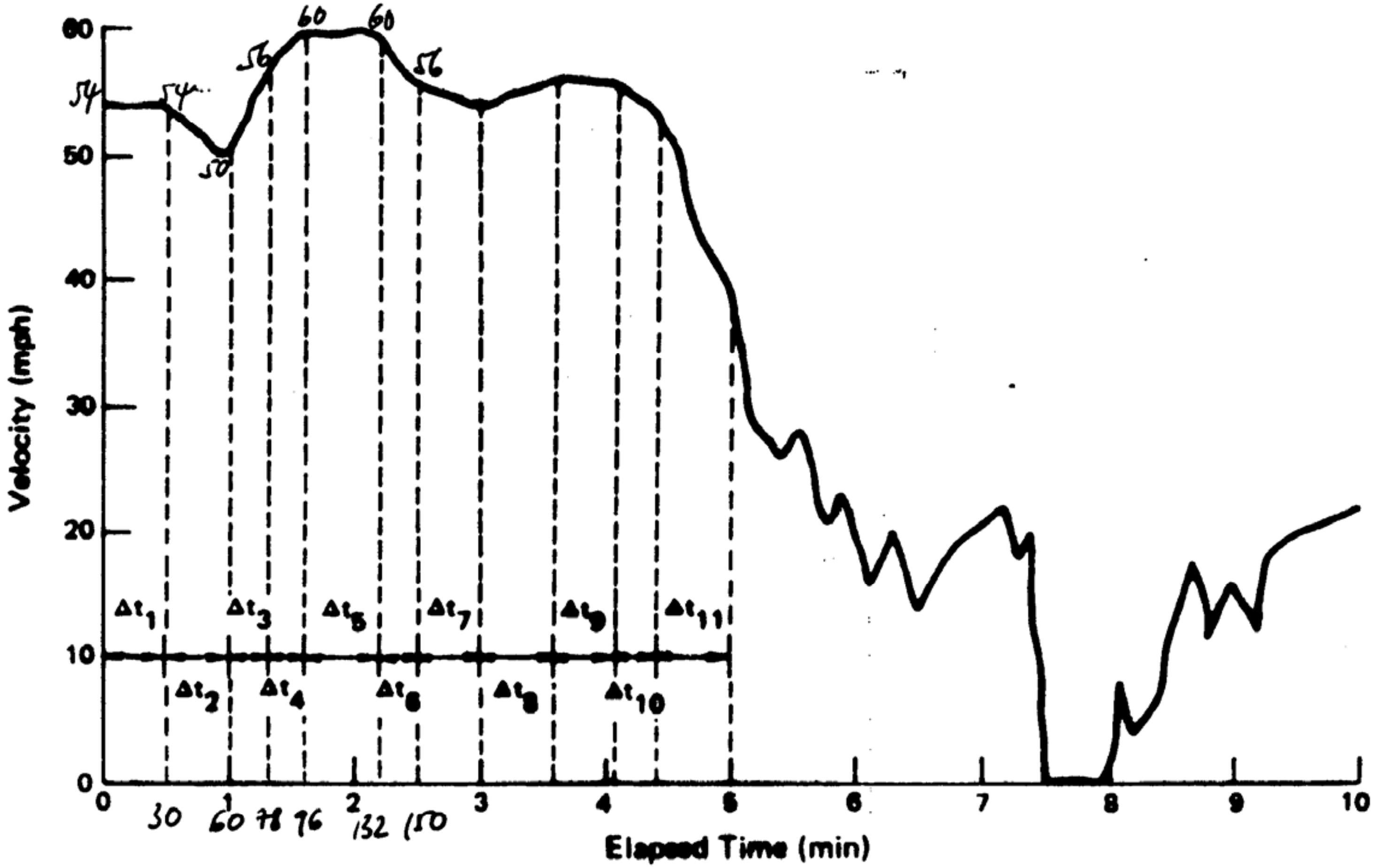


Figure 6.8 Velocity trace over 10-min time interval.

EXEMPLE

TABLE 6.7 Example Showing Calculation of Acceleration Noise

Interval	Elapsed Time at End of Interval (sec)	Velocity u at End of Interval (mph)	nombre de TRANCHES de 2 mi/h n	Δt_i (sec)	$n_i^2/\Delta t_i$
0	0	54	—	—	—
1	30.0	54	0	30.0	0.00
2	60.0	50	54-50 = 2	60-30 = 30.0	0.13
3	78.0	56	56-50 = 3	78-60 = 18.0	0.50
4	96.0	60	60-56 2	18.0	0.22
5	132.0	60	0	36.0	0.00
6	150.0	56	2	18.0	0.22
7	180.0	54	1	30.0	0.03
8	216.0	56	1	36.0	0.03
9	246.0	56	0	30.0	0.00
10	264.0	54	1	18.0	0.06
11	300.0	40	7	36.0	1.36
TOTAL				300.0	2.55

If Δt is in seconds, the running time T in seconds, and $\Delta u = 2.0$ mph, $(\Delta u)^2 = (2.08 \times 88/66)^2 \approx 8.60$ ft²/sec.*

$$\sigma = \left[\frac{1{,}465^2 \Delta U^2}{T} \sum_{i=0}^{I} \frac{n_i^2}{\Delta t_i} - 1{,}465^2 \left(\frac{U_T - U_0}{T} \right)^2 \right]^{1/2}$$

POUR LES 5 PREMIERES MINUTES $T = 5 \cdot 60 = 300s$

$$\sigma = \left[\frac{1{,}47^2 \cdot 4}{300} \cdot 2{,}55 - 1{,}47^2 \left(\frac{40-54}{300} \right)^2 \right]^{1/2} = 0{,}26 \ pi/s^2$$

BRUIT D'ACCELERATION DANS UNE FILE DE VOITURES

LE BRUIT AUGMENTE POUR LES 5 PREMIERS CONDUCTEURS ET RESTE ENSUITE CONSTANT.

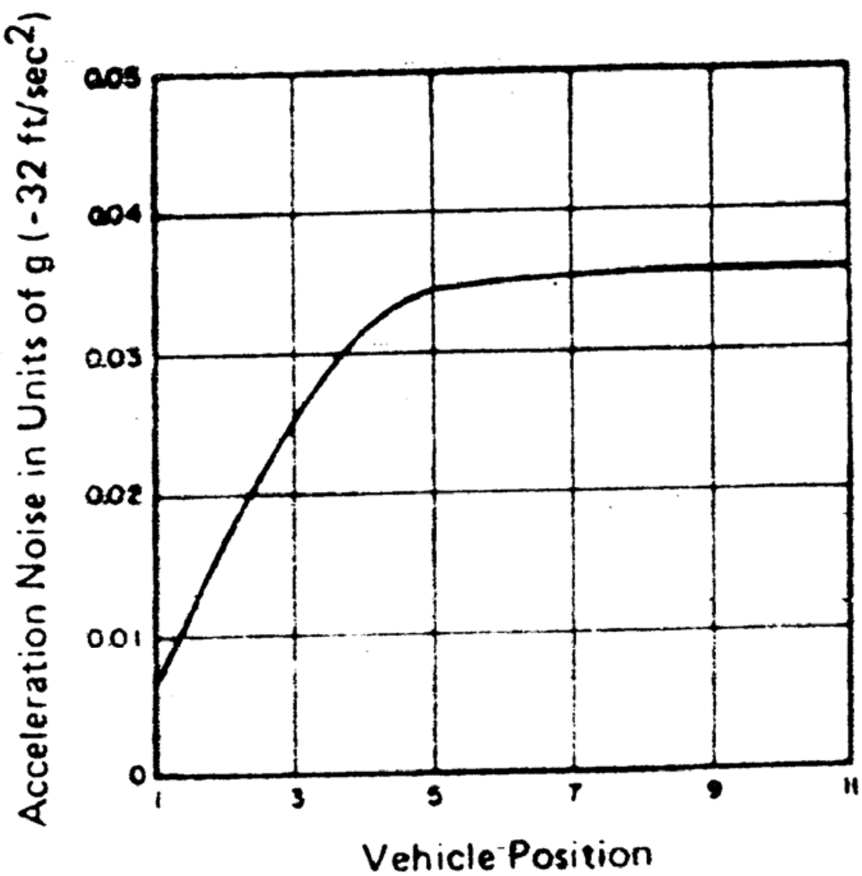


Figure 6.9 Acceleration noise of vehicles at different locations in a platoon.[41]

$$\ddot{x}_{n+1}(t+T) = \frac{\alpha}{[x_n(t)-x_{n+1}(t)]}[\dot{x}_n(t)-\dot{x}_{n+1}(t)] + \beta(t)$$

$$c = \frac{\alpha_0}{\Delta}$$

- COMMENT SONT RELIES LES VARIANCES σ DE $\ddot{x}_n(t)$ ET σ_0 DE $\beta(t)$, DU BRUIT ?

ON PEUT MONTRER QUE POUR DES VEHICULES LOINS DU DEBUT DU PELOTON

$$\sigma = \frac{\sigma_0}{(1-2CT)^{1/2}}$$

- SI $CT < \frac{1}{2}$ ALORS STABILITE ASYMPTOTIQUE
- SI $2CT \longrightarrow 1$ ON ATTEINT LA LIMITE DE LA STABILITE CAR σ AUGMENTE DEMESUREMENT.
- L'ESPACEMENT MOYEN AVEC $C = \frac{\alpha_0}{s}$

$$1-2\frac{\alpha_0}{s}T = \frac{\sigma_0^2}{\sigma^2}$$

$$s = \frac{2\alpha_0 T}{\left(1-\frac{\sigma_0^2}{\sigma^2}\right)}$$

ON A OBSERVE DANS LE TUNNEL HOLLAND $\frac{\sigma}{\sigma_0} = 1,5$ à $1,75$ DEPENDANT DE k. $T \approx 1,5\,s$

SI ON SUPPOSE DES VALEURS RAISONNABLES POUR α_0, T ET $\frac{\sigma_0}{\sigma}$ ON PEUT CALCULER $s = \frac{1}{k}$. LES VALEURS AINSI OBTENUES CORRESPONDENT AUX VALEURS OBSERVEES A 10-15% PRES.

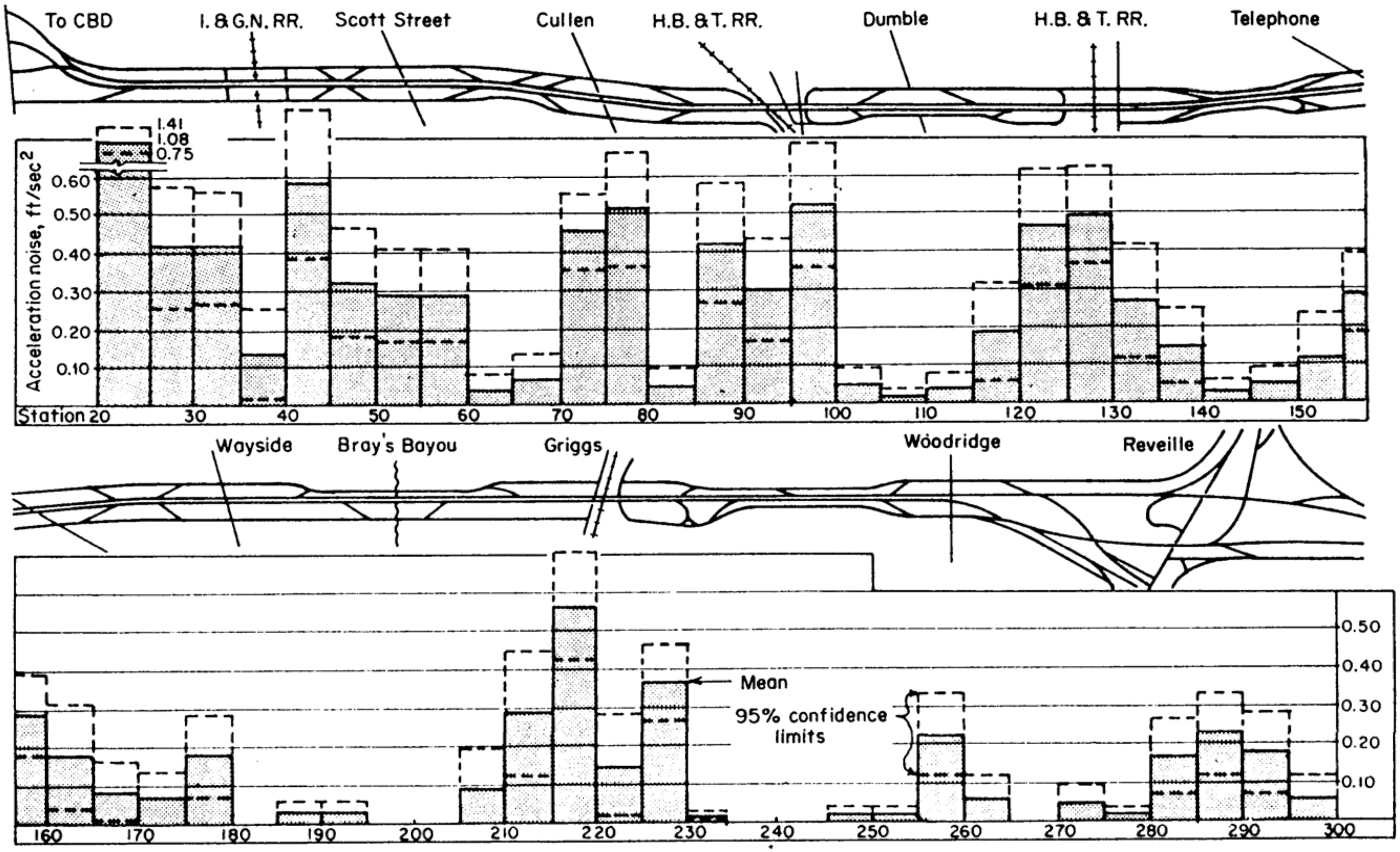


Fig. 14.8 Natural acceleration noise, σ_n.

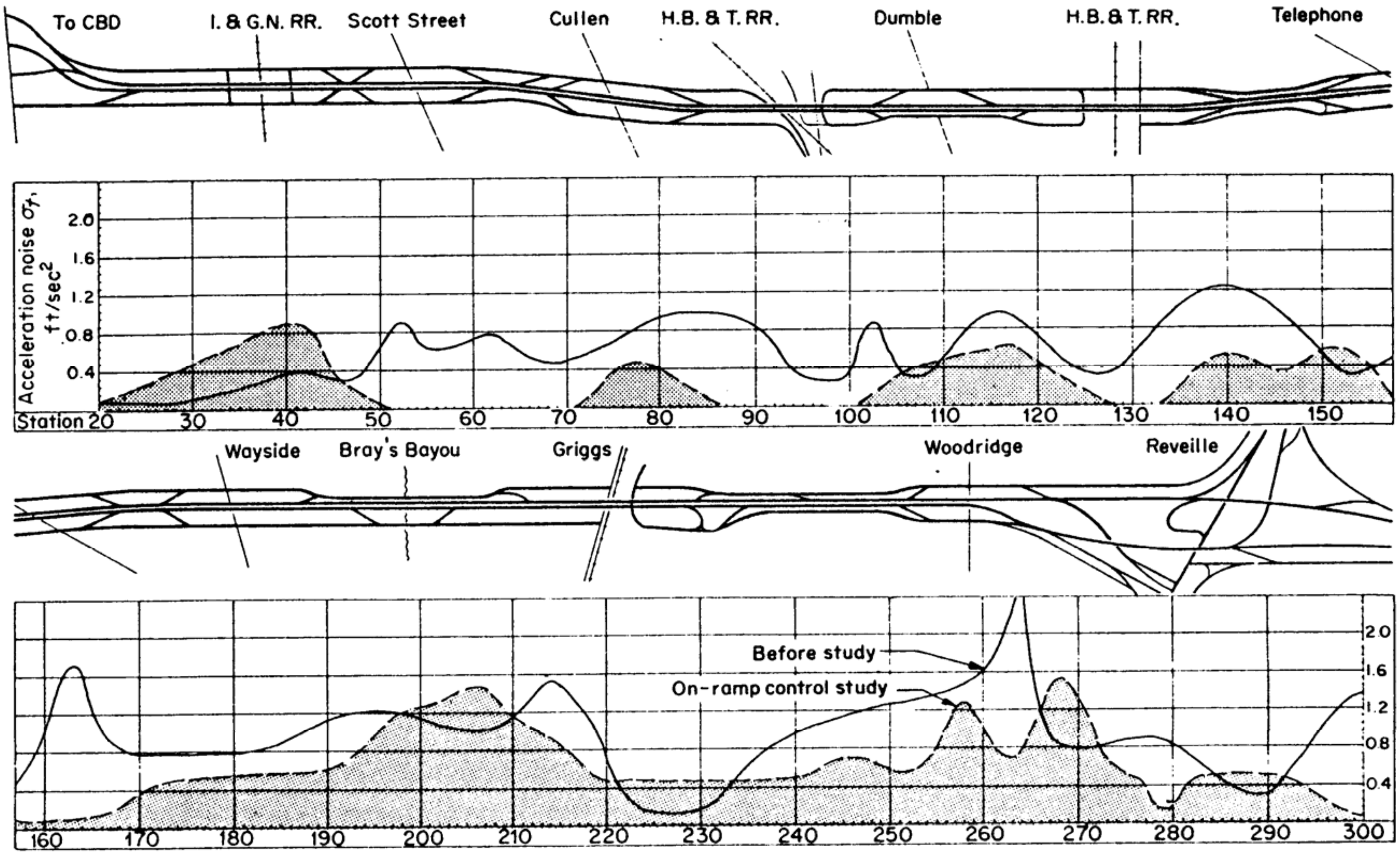


Fig. 14.9 Acceleration noise profiles, 7:30 A.M.

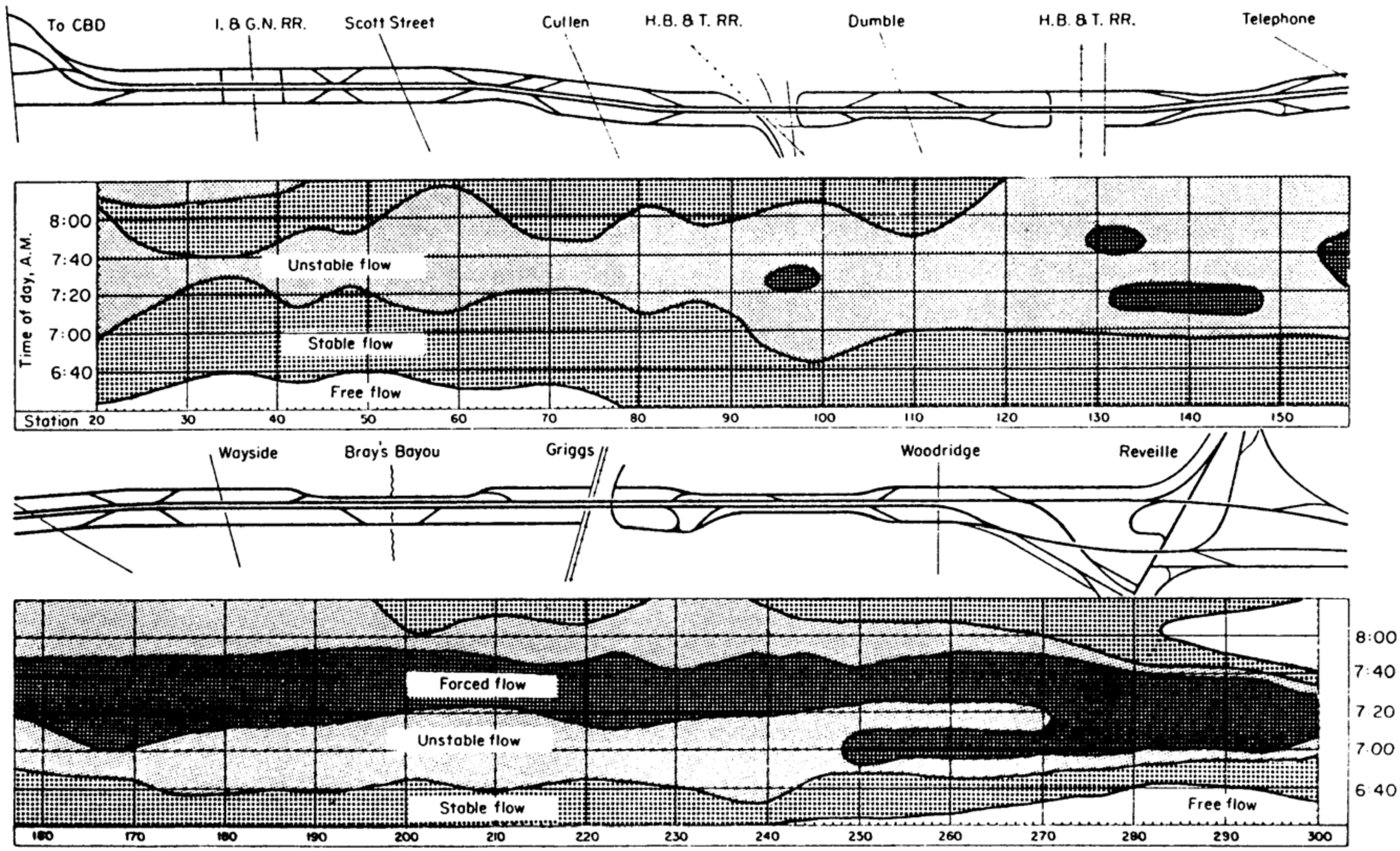


Fig. 14.12 Level-of-service contours, Tuesday, inbound.