Définition des propriétés mécaniques

In[57]:=
```
k0 = 10.; μ0 = 2.; k1 = 2.; μ1 = 1. / 2; ω1 = 1.; k2 = 4.; μ2 = 1.; ω2 = 10.;
```

$$Id = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \; J = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \; K = Id - J;$$

```
C0 = 3 k0 J + 2 μ0 K; C1 = 3 k1 J + 2 μ1 K;  C2 = 3 k2 J + 2 μ2 K;
```

Définition des matrices internes

In[59]:=
```
L21 = Transpose[CholeskyDecomposition[ω1 C1]];
L22 = Transpose[CholeskyDecomposition[ω2 C2]];
L2 = Array[f, {6, 12}];
Do[L2[[i, j]] = L21[[i, j]], {i, 1, 6}, {j, 1, 6}];
Do[L2[[i, j + 6]] = L22[[i, j]], {i, 1, 6}, {j, 1, 6}];
L1 = C0 + C1 + C2;
L3 = Array[f, {12, 12}];
Do[L3[[i, j]] = 0, {i, 1, 12}, {j, 1, 12}];
Do[L3[[i, i]] = ω1, {i, 1, 6}];
Do[L3[[i, i]] = ω2, {i, 7, 12}];
B = Array[f, {12, 12}];
Do[B[[i, j]] = 0, {i, 1, 12}, {j, 1, 12}];
Do[B[[i, i]] = 1, {i, 1, 12}];
```

Affichage des matrices

In[72]:=
```
MatrixForm[B]
```

Out[72]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[73]:=
```
MatrixForm[L1]
```

Out[73]//MatrixForm=

$$\begin{pmatrix} 20.6667 & 13.6667 & 13.6667 & 0. & 0. & 0. \\ 13.6667 & 20.6667 & 13.6667 & 0. & 0. & 0. \\ 13.6667 & 13.6667 & 20.6667 & 0. & 0. & 0. \\ 0. & 0. & 0. & 7. & 0. & 0. \\ 0. & 0. & 0. & 0. & 7. & 0. \\ 0. & 0. & 0. & 0. & 0. & 7. \end{pmatrix}$$

In[74]:=
```
MatrixForm[L2]
```

Out[74]//MatrixForm=

$$\begin{pmatrix} 1.63299 & 0. & 0. & 0. & 0. & 0. & 7.30297 & 0. & 0. & 0. & 0. & 0. \\ 1.02062 & 1.27475 & 0. & 0. & 0. & 0. & 4.56435 & 5.70088 & 0. & 0. & 0. & 0. \\ 1.02062 & 0.49029 & 1.1767 & 0. & 0. & 0. & 4.56435 & 2.19265 & 5.26235 & 0. & 0. & 0. \\ 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 4.47214 & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 4.47214 & 0. \\ 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 4.4721 \end{pmatrix}$$

In[75]:=
```
MatrixForm[L3]
```

Out[75]//MatrixForm=

$$\begin{pmatrix} 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1. & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10. & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10. & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10. & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10. & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10. & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10. \end{pmatrix}$$

Définition des matrices pour la méthode d’Euler

In[76]:=
```
W1[h_] = Inverse[B + h B.L3];
W2[h_] = -h Inverse[B + h B.L3].Inverse[B].Transpose[L2];
```

In[78]:=
```
MatrixForm[W1[h]]
```

Out[78]//MatrixForm=

$$\begin{pmatrix} \frac{1.}{1.+1.\,h} & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & \frac{1.}{1.+1.\,h} & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & \frac{1.}{1.+1.\,h} & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & \frac{1.}{1.+1.\,h} & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & \frac{1.}{1.+1.\,h} & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+1.\,h} & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+10.\,h} & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+10.\,h} & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+10.\,h} & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+10.\,h} & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+10.\,h} & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & \frac{1.}{1.+10.\,h} \end{pmatrix}$$

In[80]:=
```
MatrixForm[Simplify[W2[h]]]
```

Out[80]//MatrixForm=

$$\begin{pmatrix} 0.-\frac{1.63299\,h}{1.+1.\,h} & 0.-\frac{1.02062\,h}{1.+1.\,h} & 0.-\frac{1.02062\,h}{1.+1.\,h} & 0. & 0. & 0. \\ 0. & 0.-\frac{1.27475\,h}{1.+1.\,h} & 0.-\frac{0.49029\,h}{1.+1.\,h} & 0. & 0. & 0. \\ 0. & 0. & 0.-\frac{1.1767\,h}{1.+1.\,h} & 0. & 0. & 0. \\ 0. & 0. & 0. & 0.-\frac{1.\,h}{1.+1.\,h} & 0. & 0. \\ 0. & 0. & 0. & 0. & 0.-\frac{1.\,h}{1.+1.\,h} & 0. \\ 0. & 0. & 0. & 0. & 0. & 0.-\frac{1.\,h}{1.+1.\,h} \\ 0.-\frac{7.30297\,h}{1.+10.\,h} & 0.-\frac{4.56435\,h}{1.+10.\,h} & 0.-\frac{4.56435\,h}{1.+10.\,h} & 0. & 0. & 0. \\ 0. & 0.-\frac{5.70088\,h}{1.+10.\,h} & 0.-\frac{2.19265\,h}{1.+10.\,h} & 0. & 0. & 0. \\ 0. & 0. & 0.-\frac{5.26235\,h}{1.+10.\,h} & 0. & 0. & 0. \\ 0. & 0. & 0. & 0.-\frac{4.47214\,h}{1.+10.\,h} & 0. & 0. \\ 0. & 0. & 0. & 0. & 0.-\frac{4.47214\,h}{1.+10.\,h} & 0. \\ 0. & 0. & 0. & 0. & 0. & 0.-\frac{4.47214\,h}{1.+10.\,h} \end{pmatrix}$$

Essai numérique de l'implémentation avec N=1000 pas de temps, pour 0 à 5 secondes (h = 0.005), pour une relaxation à 0.01 de déformation

In[81]:=
```
n = 1000;
```

In[82]:=
```
ξ = Array[f, {n, 12}]; Do[ξ[[i, j]] = 0, {i, 1, n}, {j, 1, 12}];
σ = Array[f, {n, 6}]; Do[σ[[i, j]] = 0, {i, 1, n}, {j, 1, 6}];
ϵ = Array[f, {n, 6}]; Do[ϵ[[i, j]] = 0, {i, 1, n}, {j, 1, 6}];
Do[ϵ[[i, 1]] = 0.01 , {i, 1, n}];
```

In[85]:=
```
ξtmp = Array[f, 12]; Do[ξtmp[[i]] = 0, {i, 1, 12}]; ϵtmp = Array[f, 6];
Do[ϵtmp[[i]] = 0, {i, 1, 6}]; σtmp = Array[f, 6]; Do[σtmp[[i]] = 0, {i, 1, 6}];
```

Premier pas de temps

In[86]:=
```
h = 5 / n;
```

In[87]:=
```
Do[ϵtmp[[i]] = ϵ[[1, i]], {i, 1, 6}]
ξtmp = W2[h].ϵtmp;
σtmp = L1.ϵtmp + L2.ξtmp;
Do[ξ[[1, i]] = ξtmp[[i]], {i, 1, 12}];
Do[σ[[1, i]] = σtmp[[i]], {i, 1, 6}];
```

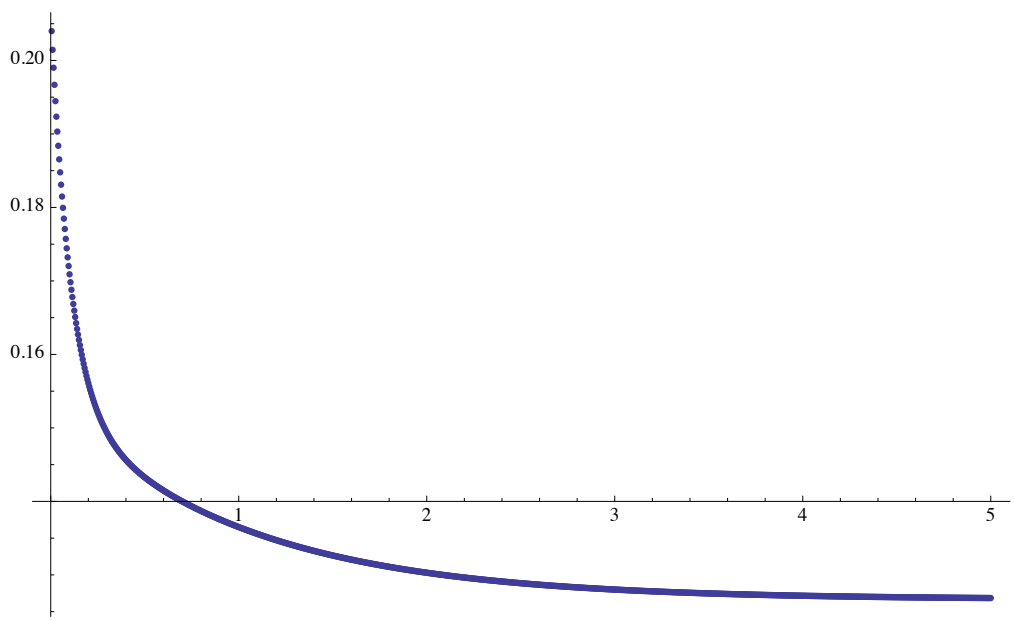
Tous les autres pas de temps

In[92]:=
```
Do[
 Do[ϵtmp[[z]] = ϵ[[i, z]], {z, 1, 6}];
 Do[ξtmp[[z]] = ξ[[i - 1, z]], {z, 1, 12}];
 ξtmp = W1[h].ξtmp + W2[h].ϵtmp;
 σtmp = L1.ϵtmp + L2.ξtmp;
 Do[ξ[[i, z]] = ξtmp[[z]], {z, 1, 12}];
 Do[σ[[i, z]] = σtmp[[z]], {z, 1, 6}];
 , {i, 2, n}]
```

Vérification graphique

In[93]:=
```
σtemps = Array[f, {n, 2}];
Do[σtemps[[i, 1]] = h i; σtemps[[i, 2]] = σ[[i, 1]], {i, 1, n}];
a1 = ListPlot[σtemps, PlotRange → All]
```

Out[95]=



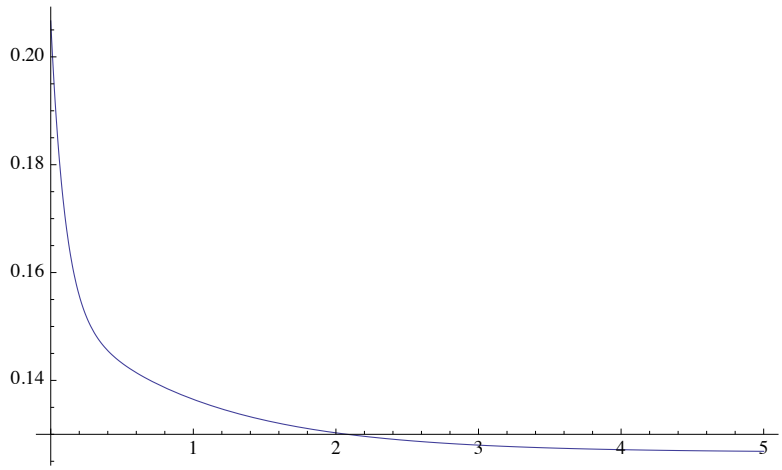
In[96]:=
```
Cve[t_] = C0 + C1 Exp[-ω1 t] + C2 Exp[-ω2 t]; ϵverif = {0.01, 0, 0, 0, 0, 0};
```

In[97]:=
```
σverif[t_] = Expand[Cve[t].εverif]
```

Out[97]=
$$\{0.126667 + 0.0533333\, e^{-10.\, t} + 0.0266667\, e^{-1.\, t},\ 0.0866667 + 0.0333333\, e^{-10.\, t} + 0.0166667\, e^{-1.\, t},\ 0.0866667 + 0.0333333\, e^{-10.\, t} + 0.0166667\, e^{-1.\, t},\ 0.,\ 0.,\ 0.\}$$

In[98]:=
```
a2 = Plot[σverif[t][[1]], {t, 0, 5}, PlotRange → All]
```

Out[98]=



In[99]:=
```
Show[a1, a2]
```

Out[99]=