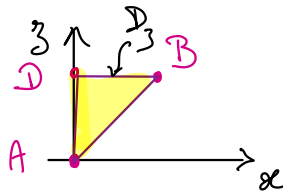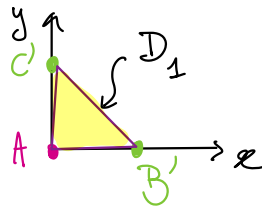en x puis sur O_{yz} (type II) I_2
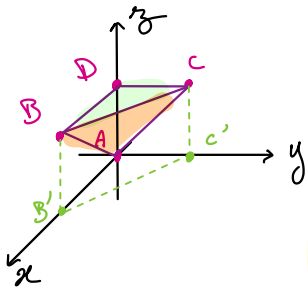
en y puis sur O_{xz} (type III) I_3

en z puis sur O_{xy} (type I) I_1

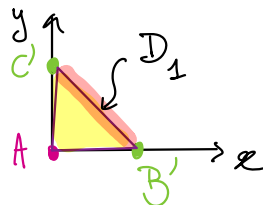# ① type I

$u_2(x,y) = 1$

$u_1(x,y) = x+y$ (plan ABC)

$$I_1 = \iint_{D_1} \int_{x+y}^{1} xz\, dz\; dA = \iint_{D_1} \frac{x}{2}(1-x^2-2xy-y^2)\, dA$$

$y = 1-x$

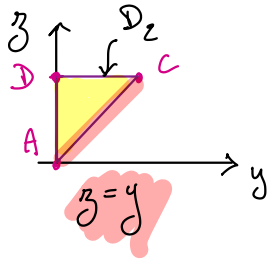$$= \int_0^1 \int_0^{1-x} \frac{x}{2} - \frac{x^3}{2} - x^2 y - \frac{xy^2}{2}\; dy\, dx$$

$$= \int_0^1 \frac{x}{2}(1-x) - \frac{x^3}{2}(1-x) - \frac{x^2}{2}(1-x)^2 - \frac{x}{6}(1-x)^3 dx$$

$$= \int_0^1 \frac{x}{2} - \frac{x^2}{2} - \frac{x^3}{2} + \frac{x^4}{2} - \frac{x^2}{2} + x^3 - \frac{x^4}{2} + \frac{x^4}{6} - \frac{1}{2}x^3 + \frac{1}{2}x^2 - \frac{1}{6}x\, dx$$

$$= \int_0^1 \frac{1}{3}x - \frac{1}{2}x^2 + \frac{1}{6}x^4\, dx$$

$$= \frac{1}{6} - \frac{1}{6} + \frac{1}{6\times 5} = \boxed{\frac{1}{30}}$$

② type II
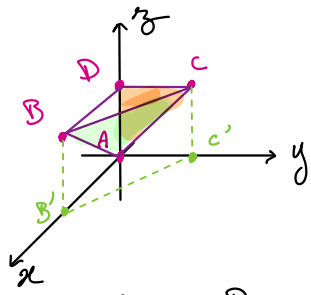
$w_2(y, z) = z - y$ (plan ABC)

$w_1(y, z) = 0$ (plan ACD)

$$I_2 = \iint_{D_2} \int_0^{z-y} xz \, dx \, dA = \iint_{D_2} \frac{1}{2} z \left(z^2 - 2zy + y^2 \right) \, dA$$

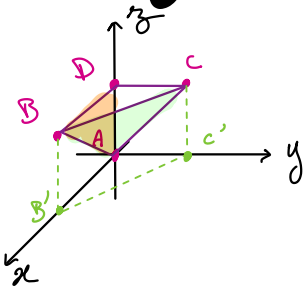$$= \int_0^1 \frac{1}{2} \int_y^1 z^3 - 2yz^2 + y^2 z \, dz \, dy$$

$z = y$

$$= \frac{1}{2} \int_0^1 \frac{1 - y^4}{4} - \frac{2}{3} y (1 - y^3) + \frac{1}{2} y^2 (1 - y^2) \, dy$$

$$= \frac{1}{2} \int_0^1 \frac{1}{4} - \frac{1}{4} y^4 - \frac{2}{3} y + \frac{2}{3} y^4 + \frac{1}{2} y^2 - \frac{1}{2} y^4 \, dy$$

$$= \frac{1}{2} \int_0^1 \frac{1}{4} - \frac{2}{3} y + \frac{1}{2} y^2 - \frac{1}{12} y^4 \, dy$$

$$= \frac{1}{2} \left(\frac{1}{4} - \frac{1}{3} + \frac{1}{6} - \frac{1}{12 \times 5} \right) = \boxed{\frac{1}{30}}$$
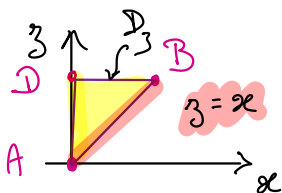
③ type III

$v_2(x, z) = z - x$

$v_1(x, z) = 0$

$$I_3 = \iint_{D_3} \int_0^{z-x} xz \, dy \, dA$$

$$= \iint_{D_3} xz^2 - x^2 z \, dA$$

$z = x$

$$= \int_0^1 \int_x^1 xz^2 - x^2 z \, dz \, dx$$

$$I_3 = \int_0^1 \int_x^1 xz^2 - x^2z \, dz \, dx$$

$$= \int_0^1 \frac{x}{3}(1-x^3) - \frac{x^2}{2}(1-x^2) \, dx$$

$$= \int_0^1 \frac{x}{3} - \frac{x^4}{3} - \frac{x^2}{2} + \frac{x^4}{2} \, dx$$

$$= \int_0^1 \frac{x}{3} - \frac{x^2}{2} + \frac{1}{6}x^4 \, dx$$

$$= \frac{1}{6} - \frac{1}{6} + \frac{1}{6 \times 5}$$

$$= \frac{1}{30}$$