WORKSPACE-BASED DESIGN OF PARALLEL MANIPULATORS OF STAR TOPOLOGY WITH A GENETIC ALGORITHM

Luc Baron
Department of Mechanical Engineering
Ecole Polytechnique de Montréal
C.P. 6079, succ. CV, Montréal, QC, Canada H3C 3A7
Email: baron@meca.polymtl.ca
http://www.meca.polymtl.ca/profs/baron

ABSTRACT
This paper presents the results of the implementation of a genetic algorithm for the design of parallel manipulators of Star topology based on the characteristics of its workspace (W). The algorithm allows to propose new geometries of manipulators that maximize the weighted sum of the volume of W, the percentage of W having a dexterity index greater than a minimum threshold, and a shape ration of W. The algorithm has proven to be effective by proposing a new design that overcomes, by a factor of 3.636, the performances of the original Y Star design, after evaluating the performances of only 2000 alternative designs, which correspond to a traveling of only $2\times10^{-16}\%$ through the search space of this design problem.

NOMENCLATURE

- $A, B$: Frames attached to body $A$ and $B$.
- $A_i, A'_i$: Displacement limits of leg $i$ along the screw axis $i$.
- $B_i, B'_i$: Attachment points of leg $i$ on body $B$, i.e., the EE.
- $E_i, E'_i$: Attachment points of leg $i$ along the screw axis $i$.
- $a_i$: Position vector of $A_i$ in $A$.
- $b_i$: Position vector of $B_i$ in $B$.
- $e_i$: Unit vector along the screw axis $i$.
- $f_i$: Unit vector from $E_i$ toward $B'_i$ in $A$.
- $m_i$: Position vector of $B'_i$ relative to $E_i$ in $A$.
- $q_i$: Controlled and measured displacement of $E_i$ along $e_i$.
- $l_i$: Leg length between points $E_i$ and $B'_i$.

INTRODUCTION
In general, parallel manipulators are particularly interesting because they possess complementary characteristics to serial ones. In fact, the formers can be used in situations where characteristics of the latter cannot satisfy the application requirements. It is noteworthy that parallel manipulators possess a higher load carrying capacity, a better positioning accuracy or speed, but also a lower workspace volume, more singularity problems and an increased complexity when solving the direct kinematic problem compared to serial manipulators of equivalent size (Baron and Angeles, 2000). The design of parallel manipulators is a complex activity involving many technical and creative behaviors. The designer must evaluate the capacity of each potential design to meet the application requirements. Among others, the manipulator mobility, stiffness, singularity and dexterity are posture dependent ratings, and hence, require to be evaluated a set of postures within the manipulator workspace. Unfortunately, these ratings are usually available only in an algorithmic form or as a non-derivable equation, and thus, standard optimization methods based on the gradient cannot be used to reach the specifications. The designer must use evaluation programs in order to measure the performances of each potential design and then determined the geometrical modifications that would improve the performances of these designs. This iterative process is extremely long and tedious since it is based only on the designer’s intuition and experiences. Genetic algorithms (GAs) are powerful stochastic optimization techniques (Goldberg, 1989) and are considered here for that purpose.
an actuated \( R \)-joint connecting link 2 to 3, two passive \( R \) joints connecting link 3 to links 4 and 5, and a passive \( R \) joint connecting link 6 to link 7.

In order to maintain the translational three-degree-of-freedom mobility of the EE, the four \( R \) joint axes of the closed loops formed by links 3 to 6 must always be parallel, while the axis of the \( R \) joint connecting each leg to the EE must be parallel to its corresponding \( H \) worm screw axis. All the other relative positions and orientations of the kinematic pairs on the different links of the parallel manipulators of this topology can be general, and hence, be described with geometrical parameters. Although being restrictive, the following assumption is made in order to limit the dimension of the search space.

**Assumption:** The three worm screw axes intersect at a point.

This simplification assumption allows to remove the few geometrical parameters that would be required to describe three none intersecting axes. This limit to the scope of possible designs appears reasonable to the author’s knowledge.

**GEOMETRIC PARAMETERS**

As shown in Fig.2, frames \( A \) and \( B \) are attached to the base and the EE, which are denoted \( A \) and \( B \), respectively. Without lost of generality, the origin of frame \( A \) is located at the intersection point of three worm screw axes, the \( x \)-axis aligned along the worm screw 1. The orientation of the three axes is described with three geometric parameters, i.e., \( \gamma_2, \omega_2, \gamma_1 \) and \( \omega_1 \), and thus, the unit vector \( e_i \) along each worm screw axis \( i \) is given as

\[
e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} \cos \gamma_2 \\ \sin \gamma_2 \sin \omega_2 \end{bmatrix}, \quad e_3 = \begin{bmatrix} \cos \gamma_1 \\ \sin \gamma_1 \sin \omega_1 \end{bmatrix}
\]

The controlled joint variable, namely \( q_i \), allows the displacement of points \( E_i \) and \( E'_i \) of Link 3 along the worm screw axis \( e_i \) between the limits defined by points \( A_i \) and \( A'_i \). These last two points are located along \( e_i \) at a distance \( v_i \) and \( u_i \) from the origin, i.e.,

\[
a_i = v_i e_i, \quad a'_i = u_i e_i, \quad i = 1, 2, 3.
\]

while the distance between \( E_i \) and \( E'_i \) is defined as \( e_i \). For the sake of removing designs that differs only from a scale factor, \( v_1 \) is arbitrarily set to 150 units of length, namely \( u \), and consequently, any further lengths will be expressed with this unit \( u \).

The geometry of the EE is defined at the home position of the controlled joints, i.e., \( q_1 = q_2 = q_3 = 0 \), as the position in \( A \) of three points of general coordinates, i.e.,

\[
[b_1]_A = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, \quad [b_2]_A = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, \quad [b_3]_A = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}
\]

Again here, for the sake of removing designs that differs only from a scale factor of the EE relative to the base, we arbitrarily set \( z_1 \) to 50 \( u \). Finally, the total number of geometric parameters of this design problem is 20, i.e., \( \gamma_2, \omega_2, v_2, x_1, y_1, x_2, y_2, \omega_3, v_3, e_1, z_2, x_3, y_3, z_3, e_2, e_3, u_1, u_2, u_3 \) and \( \gamma_3 \).

**KINEMATIC MODEL**

Each leg of the manipulator forms a kinematic loop passing through the origin of frames \( A \) and \( B \) and points \( A_i, E_i, E'_i \) and \( B_i \). The closure equation of each loop is written as

\[
m_i = z_i + q_i e_i, \quad i = 1, 2, 3
\]

where vector \( z_i \) is defined as

\[
z_i \equiv p + b_i + e_i e_i - a_i
\]
manipulators can be written as

\[ m_i^{-\frac{1}{2}} m_i A_i' [b_i] A_i = B_i, \]

where the two solutions of \( q_i \) are, respectively, the parallel and serial Jacobian matrices of the manipulator at hand, i.e.,

\[ A = \begin{bmatrix} m_1^{-\frac{1}{2}} & m_2^{-\frac{1}{2}} & m_3^{-\frac{1}{2}} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}, \]

where \( b_i \equiv -m_i^{-\frac{1}{2}} e_i \).

\section*{GENETIC ALGORITHM}

GAs are powerful stochastic optimization techniques based on the analogy of the mechanics of natural genetics, and imitates the Darwinian survival-of-the-fittest approach. The method uses bits of string as the \textit{genotype} to represent the numerical values of the geometric parameters, called in this context \textit{phenotype}, of a potential design. The coding of the phenotype, denoted \( p_i \) into the genotype, denoted \( g_i \), requires the discretization of \( p_i \) between the limits \( p_{\text{imin}} \) and \( p_{\text{imax}} \) with a chosen resolution, denoted \( r_i \), in order to obtain the corresponding genotype \( g_i \in [g_{\text{imin}}, g_{\text{imin}} + r_i, \ldots, g_{\text{imax}}] \) as

\[ g_i = f(p_i), \quad p_{\text{imin}} \leq p_i \leq p_{\text{imax}}, \]

where \( f(p_i) \) produces an integer computed as

\[ f(p_i) \equiv \text{round}(\frac{p_i - p_{\text{imin}}}{r_i}) + g_{\text{imin}} \]

with \( r_i \) defined as

\[ r_i \equiv \frac{(p_{\text{imax}} - p_{\text{imin}})}{(g_{\text{imax}} - g_{\text{imin}})} \]

Alternatively, the decoding of \( g_i \) into \( p_i \) is performed with

\[ p_i = f^{-1}(g_i), \quad g_i \in [g_{\text{imin}}, g_{\text{imin}} + r_i, \ldots, g_{\text{imax}}], \]

where \( f^{-1}(g_i) \) is defined as

\[ f^{-1}(g_i) \equiv r_i(g_i - g_{\text{imin}}) + p_{\text{imin}} \]

As shown in Table 1, the phenotypes of this design problem require a total of 88 bits, and for the sake of computer handling facility three 32-bits variables (96 bits) is used as follows

\[ G_p = \begin{bmatrix} 1001 & 1100 & 0000 & 0100 & \ldots & 1001 \\
 1111 & 0010 & 1101 & 1101 & \ldots & 1110 \\
 0111 & 0101 & 0101 & 0101 & \ldots & 1001 & 1111 \end{bmatrix} \]

\[ s_1, s_2, s_3, s_4, \ldots, s_{20} \]
The GA uses iterative improvement of a population of potential designs by mean of three basic operations: reproduction, evaluation and natural selection.

Reproduction: Based on an elitist strategy, the characteristics of the most promising designs are recombined by crossover and mutation in order to create new designs. As shown in Fig.3, the crossover operation is formed by inverting the end part of the genotype of two fellow designs at a randomly selected crossover site. The mutation is the inversion of a bit during the crossover operation.

Evaluation: The performances of each new design are measured along three indexes, while a weighted linear combination of the three former is the fourth index.

Natural selection: The fellow and new designs are classified along the four indexes, and only the best designs of each index are kept for the next generation.

In this work, the first generation start with 100 designs, 100 new designs are obtained by reproduction, and the best 25 non-identical designs of the four indexes are kept for the next generation. The probability of mutation must be lower than 5% in order to let the population mainly improve itself by crossover. The mutation produces movement in any direction, and thus, allows the possibility of jumping out of a local optimum and potentially found a more promising design.

WORKSPACE-BASED DESIGN

All our performance indexes are based directly or indirectly on the manipulator’s workspace, denoted $W$, and hence, requires its determination. For this purpose, the box method (Merlet, 1997) is used, and for which a short summary appear below.

An initial box sufficiently large to completely contain $W$ is chosen. This box is divided in 8 sub-boxes The corners of each box are verify with eq.(8) in order to produce solutions of $q_i$ within the joint limits, and for which 3 cases may arise: completely included (black), partially included (grey), or totally outside of $W$ (white). The first box (always grey if properly chosen) and all other grey boxes are repeatedly subdivided and analyzed up to $d$ times, a predefined depth. As a result, $W$ is efficiently represented in the computer as an octree. For example, the workspace $W$ (the black boxes) of the $4 \times 4$ array shown in Fig.4 can be represented with a quadtree (a concept equivalent to octree in 2D).

Unfortunately, this method can not distinguish between boxes coming from different assembly modes, and hence can pro-
roduce completely wrong results as it is the case in Tremblay and Baron (1999). It is recalled that a parallel manipulator can not move its end-effector from an assembly mode to another without disassembling the manipulator. Therefore, the workspace $W$ must contain only boxes of the same assembly mode. This distinction can intuitively be done for Stewart-Gough platforms or the Y Star parallel manipulator (see Fig5), since their geometries are known. However in this work, the geometry is generalized and unknown, and consequently, the assembly mode becomes intuitively unpredictable. For this purpose, the mathematical concept of aspect (Chablat, 1998) is used to distinguish between portions of assembly modes that correspond to different operational modes. The limits between two aspects are defined as singularities of the serial Jacobian matrix. For example, the 8 aspects (or operational modes) of the first assembly mode of the Y Star parallel manipulator are shown in Fig.6.

Our three performance indexes are, respectively, the volume of $W$, denoted $v_W$, the shape ratio of $W$, denoted $\rho_s$, and the dexterity ratio, denoted $\rho_d$. The first index $v_W$ is easily computed as the total number of smallest boxes totally included in $W$. The second index $\rho_s$ is defined as the ratio of the volume of the largest cube completely inscribed in $W$ over the volume of the smallest cube completely containing $W$. Finally, the third index is defined as the percentage of smallest boxes of $W$ having a dexterity index greater than a minimum threshold of 0.25, where the dexterity index (Salisbury, 1982) of is computed as the ratio the minimum over the maximum singular values of the combined Jacobian matrix, i.e., $\mathbf{J} = \mathbf{B}^{-1}\mathbf{A}$. Thus, boxes with a dexterity index of zero and one mean singular and isotropic boxes, respectively. Moreover, a dexterity ratio of 0.5, for example, means 50% of $W$ have a dexterity index greater than 0.25, which is a very interesting property, because manipulators exhibit their best kinematic performances at high dexterity indexes. The fourth performance index, denoted $\rho$, is defined as a weighted sum of the first three indexes so that the performance index of the Y Star produces an equal weight between the three indexes, and hence, we have

$$\rho = k_1 v_W + k_2 \rho_s + k_3 \rho_d,$$

where $k_1$, $k_2$ and $k_3$ are defined as $k_1 = 1.179 \times 10^{-6} \text{ m}^{-3}$, $k_2 = 1.996$ and $k_3 = 0.555$, so that $\rho = 0.333 + 0.333 + 0.333 = 1.000$ for the Y Star parallel manipulator.

### DESIGN RESULTS

The number of discrete values of each genotype $g_i$ is given as $(g_{i_{\text{max}}} - g_{i_{\text{min}}} + 1)$, and hence, the total number of possible solutions of this design problem is computed as

$$\prod_{i=1}^{20} (g_{i_{\text{max}}} - g_{i_{\text{min}}} + 1) = 9.9 \times 10^{22}$$

Considering an average computing time for each design of 21 sec. on a Pentium-II at 300 MHz, it would require $3.1 \times 10^{15}$ years to compute the performances of all possible designs of this search space. Obviously, such a computation is unfeasible, and only approximative solutions to this design problem are seek. Table 2 and 3 show, respectively, the geometric parameters and the performance indexes of three expert designs found after only 20 generations relative to those of the Y Star design. These solutions have required the evaluation 2000 designs, which correspond to a traveling of only $2.02 \times 10^{-16}$% through the search space.

The three expert designs, shown in Fig.7, has been found with three different runs of our GA-based synthesis program from which the design having the best weighted sum $\rho$ is selected. The experts #1 and #2 have been found from the same
initial and randomly generated population, but the crossover and mutation operations are randomly different, and converge differently toward slightly different designs. Alternatively, expert #3 has been found from a randomly different initial population, and has converged toward different designs. It is noteworthy that for all three runs, the population of designs after only 20 generations includes designs having performance indexes three times greater than those of the Y Star design. Moreover, our synthesis program includes designs having performance indexes three times greater than the Y Star, but also a slightly smaller $\rho_s$, where

\[
\rho_s = \frac{v_w}{v_d}
\]

has a higher $\rho_s$ than expert #1, but also a slightly smaller $v_w$. However, expert #3 has a higher $v_w$ than any others, a better $\rho_s$ than the Y Star, but also a lower $\rho_d$. The latter being not really a disadvantage, since the dextrous volume, denoted $v_d$, can be computed as $v_d = \rho_d v_w$, where $v_d = 830286 \text{ u}^3$ for expert #3 and $v_d = 173140 \text{ u}^3$ for the Y Star. Although $\rho_d$ of expert #3 is lower than the one of the Y Star, its dextrous volume $v_d$ is almost 5 times larger than the corresponding volume of the Y Star. Clearly, expert #3 overcomes all the three indexes of the Y Star, and is globally 3.636 times better than the Y Star.

Table 2. GEOMETRY OF THE THREE EXPERTS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>Y Star</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>15</td>
<td>15</td>
<td>75</td>
<td>120</td>
<td>deg</td>
</tr>
<tr>
<td>$p_2$</td>
<td>-60</td>
<td>-60</td>
<td>0</td>
<td>0</td>
<td>deg</td>
</tr>
<tr>
<td>$p_3$</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>u</td>
</tr>
<tr>
<td>$p_4$</td>
<td>-20</td>
<td>-50</td>
<td>-10</td>
<td>40</td>
<td>u</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-50</td>
<td>-70</td>
<td>-20</td>
<td>0</td>
<td>u</td>
</tr>
<tr>
<td>$p_6$</td>
<td>20</td>
<td>0</td>
<td>-20</td>
<td>-20</td>
<td>u</td>
</tr>
<tr>
<td>$p_7$</td>
<td>-80</td>
<td>-70</td>
<td>-30</td>
<td>40</td>
<td>u</td>
</tr>
<tr>
<td>$p_8$</td>
<td>0</td>
<td>0</td>
<td>-75</td>
<td>-75</td>
<td>u</td>
</tr>
<tr>
<td>$p_9$</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>u</td>
</tr>
<tr>
<td>$p_{10}$</td>
<td>30</td>
<td>75</td>
<td>0</td>
<td>30</td>
<td>u</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>50</td>
<td>-10</td>
<td>80</td>
<td>50</td>
<td>u</td>
</tr>
<tr>
<td>$p_{12}$</td>
<td>70</td>
<td>-30</td>
<td>70</td>
<td>-20</td>
<td>u</td>
</tr>
<tr>
<td>$p_{13}$</td>
<td>-40</td>
<td>-20</td>
<td>80</td>
<td>-40</td>
<td>u</td>
</tr>
<tr>
<td>$p_{14}$</td>
<td>100</td>
<td>50</td>
<td>-60</td>
<td>50</td>
<td>u</td>
</tr>
<tr>
<td>$p_{15}$</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>u</td>
</tr>
<tr>
<td>$p_{16}$</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>u</td>
</tr>
<tr>
<td>$p_{17}$</td>
<td>15</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>u</td>
</tr>
<tr>
<td>$p_{18}$</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>u</td>
</tr>
<tr>
<td>$p_{19}$</td>
<td>40</td>
<td>10</td>
<td>45</td>
<td>10</td>
<td>u</td>
</tr>
<tr>
<td>$p_{20}$</td>
<td>255</td>
<td>315</td>
<td>240</td>
<td>240</td>
<td>deg</td>
</tr>
</tbody>
</table>

CONCLUSION

Our genetic algorithm has proven to be a very effective design tool for proposing new geometries of parallel manipulators of Star topology that maximize some characteristics of its workspace. The concepts of assembly and operational modes allowed the algorithm to compute performance indexes on only portions of $W$ that are either free of parallel singularities or free of both parallel and serial singularities, respectively. The main result of the paper lies in the proposition of a new design that overcomes the performance indexes of the original Y Star design. This design, called for instance the Expert Star is further shown in Fig. 8 under a translational motion of its triangular end-effector. This design is globally 3.636 times better than the Y Star design. In the future, we intend to remove the assumption of three worm screw axes intersecting at a point and takes into account links obstruction in the computation of the workspace. Obviously, this will result in a significant increase in the size of the search space.

ACKNOWLEDGMENT

The author acknowledge the financial support of NSERC (National Sciences and Engineering Research Council of Canada) under grant OGPIN-203618 and FCAR (Fond Concerté d’aide à la Recherche of Quebec) under grant NC-66861.

REFERENCES


Figure 6. THE 8 OPERATIONAL MODES OF THE 1st ASSEMBLY MODE OF THE Y STAR PARALLEL MANIPULATOR

Figure 7. THE THREE EXPERT MANIPULATORS
Table 3. PERFORMANCES OF THE THREE EXPERTS

<table>
<thead>
<tr>
<th>Design</th>
<th>( v_W )</th>
<th>( \rho_d )</th>
<th>( \rho_s )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>expert #1</td>
<td>1890625</td>
<td>0.674</td>
<td>0.414</td>
<td>3.422</td>
</tr>
<tr>
<td>expert #2</td>
<td>1857178</td>
<td>0.902</td>
<td>0.414</td>
<td>3.504</td>
</tr>
<tr>
<td>expert #3</td>
<td>2281006</td>
<td>0.364</td>
<td>0.375</td>
<td>3.636</td>
</tr>
<tr>
<td>Y Star</td>
<td>288086</td>
<td>0.601</td>
<td>0.167</td>
<td>1.000</td>
</tr>
<tr>
<td>units</td>
<td>( u^3 )</td>
<td>none</td>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

Figure 8. MOTION OF THE EXPERT STAR

(a) \( \mathbf{p} = (-87.5, 23.2, -81.1) \)  
(b) \( \mathbf{p} = (-38.3, 10, -65.5) \)  
(c) \( \mathbf{p} = (-61.1, 10, -0.7) \)  
(d) \( \mathbf{p} = (-68.3, 32.8, 23.3) \)