Proceedings of DETC'01
2001 ASME Design Engineering Technical Conferences
September 9-12, 2001, Pittsburgh, Pennsylvania, USA

DETC2001/DAC-21025

THE DESIGN OF PARALLEL MANIPULATORS OF STAR TOPOLOGY UNDER ISOTROPIC CONSTRAINT

Luc Baron* **and Ghislain Bernier**
Department of Mechanical Engineering
Ecole Polytechnique de Montréal
C.P. 6079, succ. CV, Montréal, QC, Canada H3C 3A7
Email: baron@meca.polymtl.ca, URL: www.meca.polymtl.ca/profs/baron

ABSTRACT

This paper proposes a necessary and sufficient number of 18 geometrical parameters allowing to describe the design manifold of the Star topological class, i.e., all geometries of manipulators having the same topology and mobility as the Y-Star parallel manipulator. The isotropic constraints are then applied on this manifold in order to define the constraint manifold of isotropic designs, i.e., those having isotropic Jacobian matrices at their home position. This constraint manifold is of dimension 11 and greatly facilitates the design of isotropic manipulators in this topological class.

NOMENCLATURE

$\mathcal{A},\mathcal{B}$ Frames attached to body A and B.
A_i, B_i Attachment points of leg i on body A and B.
M_i Attachment point of leg i along the screw axis i.
$\mathbf{p}$ Position vector of the origin of $\mathcal{B}$ in $\mathcal{A}$.
$\mathbf{R}$ 3×3 rotation matrix from $\mathcal{A}$ to $\mathcal{B}$.
$\mathbf{a}_i$ Position vector of A_i in $\mathcal{A}$.
$\mathbf{b}_i$ Position vector of B_i in $\mathcal{B}$.
$\mathbf{e}_i$ Unit vector along the screw axis i.
$\mathbf{f}_i$ Unit vector from M_i toward B_i in $\mathcal{A}$.
$\mathbf{m}_i$ Position vector of B_i relative to M_i in $\mathcal{A}$ when $\{q_i\}_1^3 = 0$.
q_i Controlled and measured displacement of M_i along $\mathbf{e}_i$.
l_i Leg length, i.e., the distance between points M_i and B_i.

*Address all correspondence to this author.

INTRODUCTION

As shown in Fig.1, the Y-Star parallel manipulator (Hervé, 1991) allows 3D translations of the end-effector (EE) using three motorized worm screws acting in parallel. In general, parallel manipulators are particularly interesting because they possess complementary characteristics to serial manipulators. The former can be used in situations where the characteristics of latter does not satisfy the application requirements. The design of parallel manipulators is a complex activity involving many technical and creative behaviors. The designer must evaluate the capacity of each potential design to meet the application requirements. Among others, the *dexterity* (Salisbury, 1982), i.e., the capacity of the manipulator to produce an equal motion in all directions from an equal motion of its actuators, is a very interesting property, because the manipulator exhibits, at this state, its best kinematic performances. Unfortunately, this property depends on both the manipulator posture and design, and hence, only a few designs can reached isotropy at only one, or a few, posture(s), because this constraint has been included within the design process. A genetic algorithm has succeeded to produce designs of Star-like parallel manipulators using three geometric parameters allowing to generalize the relative orientation of the three worm screws (Tremblay, 1999). However, this generalization does not describe all the geometries of this topology and mobility of the EE. In fact, the three worm screws do not need to intersect at a point in order to maintain the translational mobility of the EE, but can have three distinct common perpendiculars. In this paper, we propose such a set of geometric parameters together with the constraints allowing to obtain isotropic designs.

 Copyright © 2001 by ASME

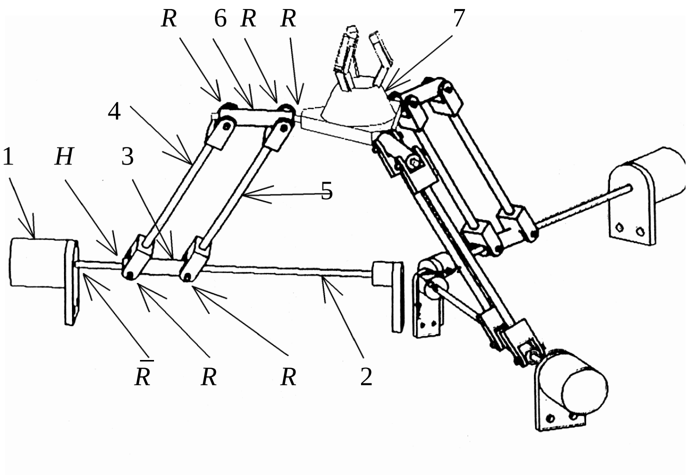
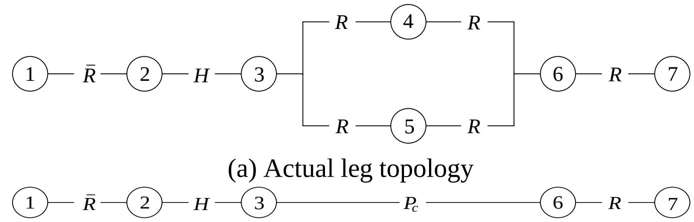


Figure 1. THE Y-STAR PARALLEL MANIPULATOR



(b) Simplified leg topology

Figure 2. TOPOLOGY OF THE Y-STAR PARALLEL MANIPULATOR

PROBLEM FORMULATION

The kinematics of robotic manipulators can be described with the concept of *kinematic chain*, which carries both topological and geometrical informations. A kinematic chain is defined as a mechanical system in which rigid bodies, called *links*, are coupled by *lower kinematic pairs*. There are six of such pairs, namely, revolute (R), prismatic (P), cylindrical (C), helicoidal (H), planar (E), and spherical (S). The *topology* refers to the layout of these pairs along the chain, while the *geometry* refers to the relative location of the pairs on each link. As shown in Fig.2(a), the kinematic chain of the Y-Star parallel manipulator is composed of three serial chains in parallel of identical topology, i.e., an actuated R joint—denoted $\bar{R}$—connecting link 1 to link 2, a passive H joint connecting link 2 to 3, two passive R joints connecting link 3 to links 4 and 5, two passive R joints connecting links 4 and 5 to link 6, and a passive R joint connecting link 6 to link 7. The topology of the closed loop formed by links 3 to 6 can be obtained with several geometries. However, only translations of the EE are required. The motion of link 6 relative to link 3 can be regarded as the motion obtained through a *circular prismatic* (P_c) pair, i.e., a circularpath at constant orientation, as shown in Fig.2(b). Hence, if the circular prismatic pair is obtained from the four R joints of the loop, the joints must have parallel axis and form a parallelogram in order to maintain the desired mobility of the EE.

The *design problem* is to find the numerical values of a set of geometric parameters of a parallel manipulator that belong to the Star *topological class*. In this context, the following two definitions are necessary.

Definition 1: Topological Class
A topological class is the set of all mechanisms having the same topology, disregarding the link geometry.

Definition 2: Geometric parameters
A necessary and sufficient number of geometric parameters must describe uniquely all geometries of the topological class.

Different values of geometrical parameters must produce different mechanisms, and not only a displacement of frames. Consequently, a location of each frame must be chosen without loss of generality. Moreover, the geometrical parameters must only describe the kinematic architecture of the links. The next section proposes such a set of geometric parameters.

GEOMETRIC PARAMETERS

Each screw of the manipulator is modeled as a unit vector $\mathbf{e}_i$ pointing in the increasing direction of the joint axis, while the home position of link 3 along the screw is described by point A_i.

Definition 3: Location of frame $\mathcal{A}$
The origin of frame $\mathcal{A}$ is located at the intersection point of the first screw axis and the common perpendicular with the second screw axis, its x-axis aligned along the first screw axis and its y-axis aligned along the common perpendicular.

As shown in Fig.3, the orientation of three screw axes are given in frame $\mathcal{A}$ as

$$\mathbf{e}_1 = \mathbf{i}, \quad \mathbf{e}_2 = \mathbf{R}_y(\alpha_2)\,\mathbf{i}, \quad \mathbf{e}_3 = \mathbf{R}_x(\theta_3)\,\mathbf{R}_y(\alpha_3)\,\mathbf{i}, \tag{1}$$

and the position vector of points A_i in frame $\mathcal{A}$ as

$$\mathbf{a}_1 = v_1\mathbf{i}, \qquad \mathbf{a}_2 = a_2\mathbf{j} + \mathbf{R}_y(\alpha_2)\,v_2\mathbf{i}, \quad \mathbf{a}_3 = d_3\mathbf{i} + \mathbf{R}_x(\theta_3)(a_3\mathbf{j} + \mathbf{R}_y(\alpha_3)\,v_3\mathbf{i}), \tag{2}$$

where v_1, a_2, α_2, v_2, d_3, θ_3, a_3, α_3, and v_3 are the geometric parameters of the base, $\mathbf{R}_a(\theta)$ denotes the 3×3 rotation matrix of an angle θ around axis a, while $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the unit vectors along x-, y- and z-axis, respectively. As shown in Fig.4, the geometry of the EE is described by the location of the attachment points B_i in $\mathcal{A}$, when the joints are located at their home positions, i.e.,

$$[\mathbf{b}_i]_{\mathcal{A}} = \mathbf{a}_i + \mathbf{m}_i, \quad i = 1,2,3, \tag{3}$$

Copyright © 2001 by ASME

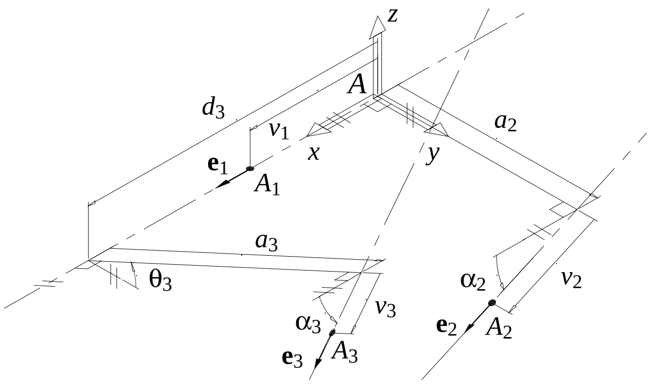


Figure 3. GEOMETRIC PARAMETERS OF THE BASE

with the leg vector $\mathbf{m}_i$ defined at the home position as

$$\begin{aligned}
\mathbf{m}_1 &= \mathbf{R}_x(\gamma_1)\ \mathbf{R}_z(\beta_1)\ l_1\mathbf{j}\\
\mathbf{m}_2 &= \mathbf{R}_y(\alpha_2)\ \mathbf{R}_x(\gamma_2)\ \mathbf{R}_z(\beta_2)\ l_2\mathbf{j}\\
\mathbf{m}_3 &= \mathbf{R}_x(\theta_3)\ \mathbf{R}_y(\alpha_3)\ \mathbf{R}_x(\gamma_3)\ \mathbf{R}_z(\beta_3)\ l_3\mathbf{j}
\end{aligned} \tag{4}$$

where γ_1, β_1, l_1, γ_2, β_2, l_2, γ_3, β_3, and l_3 are the 9 geometric parameters describing the legs and the EE.

Definition 4: Location of frame $\mathcal{B}$

The origin of frame $\mathcal{B}$ is located on the EE at point B_1 its x-, y- and z-axis parallel to those of frame $\mathcal{A}$.

The constant location of point B_i in frame $\mathcal{B}$ is given as

$$\mathbf{b}_i = [\mathbf{b}_i]_A - [\mathbf{p}_0]_A^B, \quad i = 1,2,3 \tag{5}$$

where $\mathbf{p}_0$ is the position of the origin of $\mathcal{B}$ in $\mathcal{A}$, when the joints are at their home positions, i.e., $[\mathbf{p}_0]_A^B = [\mathbf{b}_1]_A$. In summary, both the base and the EE require 9 geometric parameters, for a total of 18 geometric parameters.

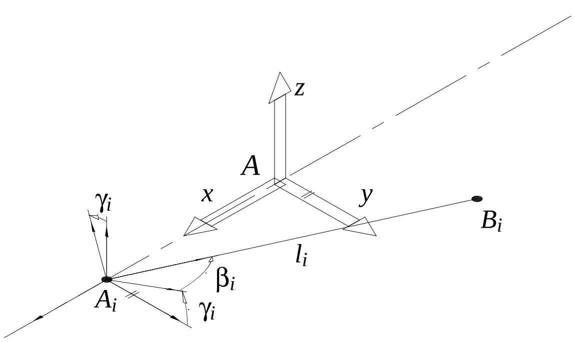


Figure 4. GEOMETRIC PARAMETERS OF THE END-EFFECTOR

KINEMATIC MODEL

Each leg of the parallel manipulator defines a kinematic loop passing through the origin of frames $\mathcal{A}$ and $\mathcal{B}$, and points A_i, M_i and B_i. The closure of each kinematic loop can be expressed in the form

$$\mathbf{a}_i + q_i\mathbf{e}_i + l_i\mathbf{f}_i = \mathbf{p} + \mathbf{b}_i, \quad i = 1,2,3 \tag{6}$$

where q_i is the controlled joint variable, $\mathbf{p}$ the position vector of the origin of $\mathcal{B}$ in $\mathcal{A}$, while $\mathbf{e}_i$ and $\mathbf{f}_i$ are the unit vectors along the screw axis and along the line starting from point M_i to B_i, respectively. At the home position, i.e., when $\{q_i\}_1^3 = 0$, eq.(6) reduces to

$$\mathbf{a}_i + \mathbf{m}_i = \mathbf{p}_0 + \mathbf{b}_i, \tag{7}$$

where $\mathbf{p}_0$ is the home position of $\mathcal{B}$ in $\mathcal{A}$, and $\mathbf{m}_i$ is the home position of $l_i\mathbf{f}_i$, i.e., $\mathbf{m}_i = l_i\mathbf{f}_i$ for $q_i = 0$. In general, the position of $\mathcal{B}$ in $\mathcal{A}$ can alternatively be written as

$$\mathbf{p} \equiv \mathbf{p}_0 + \Delta \tag{8}$$

where Δ is a displacement from $\mathbf{p}_0$. Substituting eqs.(8) and (7) into (6) yields

$$l_i\mathbf{f}_i = \Delta + \mathbf{m}_i - q_i\mathbf{e}_i, \tag{9}$$

where $\mathbf{f}_i$ and q_i are the only time-varying quantities, while the others are constant for a specific manipulator geometry.

For trajectory planning, it is necessary to solve the inverse kinematic problem, i.e., the computation the joint variables $\{q_i\}_1^3$ from a desired EE position Δ. Upon taking the norm of both sides of eq.(9) to eliminate the unknown vector $\mathbf{f}_i$, we obtain

$$l_i^2 = \Delta^T\Delta + 2\Delta^T\mathbf{m}_i + \mathbf{m}_i^T\mathbf{m}_i - 2(\Delta + \mathbf{m}_i)^T\mathbf{e}_i q_i + q_i^2 \tag{10}$$

where the two solutions for each leg are given as

$$q_i = u_i \pm \sqrt{u_i^2 - v_i}, \tag{11a}$$

where u_i and v_i are defined as

$$\begin{aligned}
u_i &\equiv (\Delta + \mathbf{m}_i)^T\mathbf{e}_i,\\
v_i &\equiv \Delta^T\Delta + 2\Delta^T\mathbf{m}_i + \mathbf{m}_i^T\mathbf{m}_i - l_i^2.
\end{aligned} \tag{11b}$$

Copyright © 2001 by ASME

JACOBIAN MATRICES

The time derivative of both sides of eqs.(8) and (10) yields

$$\dot{\mathbf{p}} = \dot{\Delta}, \qquad (12)$$
$$0 = \Delta^T\dot{\Delta} + \mathbf{m}_i^T\dot{\Delta} - \Delta^T\mathbf{e}_i\dot{q}_i - q_i\mathbf{e}_i^T\dot{\Delta} - \mathbf{m}_i^T\mathbf{e}_i\dot{q}_i + q_i\dot{q}_i,$$

which can be rewritten in a matrix form as

$$\mathbf{A}\dot{\mathbf{p}} = \mathbf{B}\dot{\mathbf{q}}, \qquad (13a)$$

where $\mathbf{A}$ and $\mathbf{B}$ are, respectively, the parallel and serial Jacobian matrices of the manipulator defined such as

$$\mathbf{A} \equiv \begin{bmatrix} \Delta^T + \mathbf{m}_1^T - q_1\mathbf{e}_1^T \\ \Delta^T + \mathbf{m}_2^T - q_2\mathbf{e}_2^T \\ \Delta^T + \mathbf{m}_3^T - q_3\mathbf{e}_3^T \end{bmatrix}, \quad \mathbf{B} \equiv \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}, \qquad (13b)$$
$$b_i \equiv -\Delta^T\mathbf{e}_i - \mathbf{m}_i^T\mathbf{e}_i + q_i, \qquad \dot{\mathbf{q}} \equiv [\dot{q}_1\ \dot{q}_2\ \dot{q}_3]^T. \qquad (13c)$$

with $\dot{q}_i$ being the controlled-joint velocity and $\dot{\mathbf{p}}$ the velocity of the origin of B in A. ¿From eq.(13), it is clear that the numerical conditioning of the two Jacobian matrices of the parallel manipulators of the Star topological class depend on the manipulator geometry and its posture.

ISOTROPIC CONSTRAINT

In order to design isotropic parallel manipulators, the Jacobian matrix, namely $\mathbf{J}$, must be isotropy at one or more posture(s) within the manipulator workspace, i.e.,

$$\mathbf{J}\dot{\mathbf{p}} = \dot{\mathbf{q}}, \quad \mathbf{J} \equiv \mathbf{B}^{-1}\mathbf{A} = \sigma^2\mathbf{1}_{3\times 3} \qquad (14)$$

The home position is arbitrary chosen as the manipulator posture, where the isotropic property is required. A sufficient, but not necessary, condition for isotropy at this posture is that both parallel and serial Jacobian matrices, $\mathbf{A}$ and $\mathbf{B}$, being individually isotropic. Upon substituting $q_i = 0$ in eq.(13), one can write the isotropic constraints of eq.(14) as

$$\mathbf{B}^T\mathbf{B} = \begin{bmatrix} (\mathbf{m}_1^T\mathbf{e}_1)^2 & 0 & 0 \\ 0 & (\mathbf{m}_2^T\mathbf{e}_2)^2 & 0 \\ 0 & 0 & (\mathbf{m}_3^T\mathbf{e}_3)^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_1^2 & 0 \\ 0 & 0 & \sigma_1^2 \end{bmatrix} \qquad (15a)$$
$$\mathbf{A}^T\mathbf{A} = \begin{bmatrix} \mathbf{m}_1^T\mathbf{m}_1 & \mathbf{m}_1^T\mathbf{m}_2 & \mathbf{m}_1^T\mathbf{m}_3 \\ \mathbf{m}_2^T\mathbf{m}_1 & \mathbf{m}_2^T\mathbf{m}_2 & \mathbf{m}_2^T\mathbf{m}_3 \\ \mathbf{m}_3^T\mathbf{m}_1 & \mathbf{m}_3^T\mathbf{m}_2 & \mathbf{m}_3^T\mathbf{m}_3 \end{bmatrix} = \begin{bmatrix} \sigma_2^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_2^2 \end{bmatrix}, \qquad (15b)$$

where $\mathbf{m}_i$ is recalled to be the leg vector $l_i\mathbf{f}_i$ with the three joints at their home positions, i.e., $q_1 = q_2 = q_3 = 0$. Apparently, three important design rules can be drawn from eq.(15).

Constraint 1: Projection of the leg along the screw axis
The projection of the legs along their corresponding screw axis must be of equal length.
Proof:
¿From the diagonal elements of eq.(15a), one can readily write

$$(\mathbf{m}_1^T\mathbf{e}_1)^2 = (\mathbf{m}_2^T\mathbf{e}_2)^2 = (\mathbf{m}_3^T\mathbf{e}_3)^2 = \sigma_1^2, \qquad (16)$$

where σ_1 is clearly the length of the projection of $\mathbf{m}_i$ along $\mathbf{e}_i$.◇
Constraint 2: Leg length
The three legs must be of equal length.
Proof:
¿From the diagonal elements of eq.(15b), one can readily write

$$\mathbf{m}_1^T\mathbf{m}_1 = \mathbf{m}_2^T\mathbf{m}_2 = \mathbf{m}_3^T\mathbf{m}_3 = \sigma_2^2 \qquad (17)$$

where σ_2 is clearly the length of the three legs.◇
Constraint 3: Relative leg orientation
The three legs must be orthogonal.
Proof:
¿From the off-diagonal elements of eq.(15b), one can readily write

$$\mathbf{m}_1^T\mathbf{m}_2 = \mathbf{m}_1^T\mathbf{m}_3 = \mathbf{m}_2^T\mathbf{m}_3 = 0, \qquad (18)$$

which is satisfied when $\mathbf{m}_1$, $\mathbf{m}_2$, and $\mathbf{m}_3$ form an orthogonal base.◇

CONSTRAINT MANIFOLD

As previously defined, a total of 18 parameters is required to describe all possible geometries of parallel manipulators of the Star topological class, i.e., v_1, β_1, γ_1, l_1, a_2, α_2, v_2, β_2, γ_2, l_2, d_3, θ_3, a_3, α_3, v_3, β_3, γ_3, l_3. Therefore, designs of this class can be described by a *manifold* of dimension 18. Below, the three isotropic constraints are used to describe the *constrained manifold* of isotropic designs, i.e., only those having isotropic Jacobian matrices at their home position.

By virtue of constraint 2, the length $l_i = \|\mathbf{m}_i\|$ of the three legs must be identical to a user chosen length, e.g. l, that defines the scale of the manipulator, i.e.,

$$l_1 = l_2 = l_3 = l \qquad (19)$$

This constraint allows to replace the parameters l_1, l_2 and l_3 by l, and hence, reduce by two the dimension of the design manifold.

By virtue of constraint 1, the projection of each $\mathbf{m}_i$ along its screw axis $\mathbf{e}_i$ must be of equal length., i.e.,

$$\mathbf{m}_i^T\mathbf{e}_i = l_i\cos(\frac{\pi}{2} + \beta_i) = \sigma_1 \qquad (20)$$

Copyright © 2001 by ASME

Upon substitution of eqs.(20) and (19) into (16) yields

$$\beta_1 = \beta_2 = \beta_3 = \beta, \tag{21}$$

where the angle β can be chosen by the user. Again here, this constraint allows to replace the parameters β_1, β_2 and β_3 by β, and hence, reduces by two the dimension of the design manifold.

By virtue of constraint 3, the three leg vectors $\mathbf{m}_i$ must be orthogonals. The relative orientation of leg 1 and 2 depends on parameters β, γ_1, α_2 and γ_2. Because there is not a solution for all of the combinations of α_2 and β, the value of angle β must be limited between $-\beta_{max}$ and β_{max}.

Because e_2 and m_1 are unitary vectors, the angle θ between screw 2 and leg 1 can be found with :

$$\mathbf{e}_2 \cdot \mathbf{m}_1 = \cos\theta \tag{22}$$

θ can be replace by $(\pi/2+\psi)$ where $\pi/2$ represent the orthogonality condition between legs 1 and 2, and ψ is the minimal angle between screw 2 and leg 2 to be able to obtain the orthogonality condition. The angle β is the orthogonal complement to the angle between screw 2 and leg 2, thus $\psi = \beta - \pi/2$. This lead to $\theta = \beta$. So Eq. (22) can be written :

$$-\sin\beta\cos\alpha_2 + \cos\beta\sin\gamma_1\sin\alpha_2 = \cos\beta \tag{23}$$

Thus :

$$\tan\beta_{max} = (1+\sin\gamma_1\sin\alpha_2)/\cos\alpha_2. \tag{24}$$

The latter defines an interval on β, i.e., $\beta_{max} > \beta > -\beta_{max}$, which ensure that leg 1 and 2 can be perpendicular for a chosen value of α_2. Once a valid set of β, γ_1 and α_2 chosen, it is possible to compute γ_2 to satisfy to following orthogonality condition, i.e.,

$$\mathbf{m}_1^T\mathbf{m}_2 = 0, \tag{25}$$

which can be rewritten as

$$A\cos\gamma_2 + B\sin\gamma_2 + C = 0, \tag{26}$$

whose coefficients A, B and C are all computable as

$$\begin{aligned}
A &\equiv \cos\gamma_1\cos^2\beta \\
B &\equiv -\sin\beta\cos\beta\sin\alpha_2 + \sin\gamma_1\cos\beta\cos\beta\cos\alpha_2 \\
C &\equiv \sin^2\beta\cos\alpha_2 + \sin\gamma_1\cos\beta\sin\beta\sin\alpha_2
\end{aligned} \tag{27}$$

The solution of eq.(26) for γ_2 is found using the following set well-known trigonometric identities, namely,

$$\sin\theta \equiv \frac{2\tau}{(1+\tau^2)}, \quad \cos\theta \equiv \frac{(1-\tau^2)}{(1+\tau^2)}, \quad \tau \equiv \tan(\theta/2) \tag{28}$$

Upon substitution of eq.(28) into eq.(26) and multiplying both sides of eq.(26) by $(1-\tau_1^2)$, yield the following quadratic monovariate polynomial in $\tau_1 \equiv \tan(\gamma_2/2)$, i.e.,

$$(C-A)\tau_1^2 + 2B\tau_1 + C + A = 0, \tag{29}$$

The constraint of eq.(29) allows to compute γ_2, and hence, reduce by one the dimension of the design manifold.

Finally, the orthogonality of leg 3 relative to leg 1 and 2 can be constrained using the following vector product, i.e.,

$$\mathbf{m}_3 = \mathbf{m}_1 \times \mathbf{m}_2, \tag{30}$$

where the leg lengths are already known to be equal to l. With β, γ_1, α_2, γ_1 known, γ_3 can arbitrarily be chosen, and thus eq.(30) is used to compute α_3 and θ_3 as:

$$D\cos\alpha_3 + E\sin\alpha_3 + F = 0, \tag{31}$$

whose coefficients D, E and F are all computable as

$$\begin{aligned}
D &\equiv -\sin\beta \\
E &\equiv \sin\gamma_3\cos\beta \\
F &\equiv -\cos\gamma_1\cos\beta(\sin\gamma_2\cos\beta\cos\alpha_2 \\
&\qquad +\sin\beta\sin\alpha_2) + \sin\gamma_1\cos^2\beta\cos\gamma_2
\end{aligned} \tag{32}$$

and

$$G\cos\theta_3 + H\sin\theta_3 + I = 0, \tag{33}$$

whose coefficients G, H and I are also all computable as

$$\begin{aligned}
G &= \sin\gamma_3\cos\beta\cos\alpha_3 + \sin\beta\sin\alpha_3 \\
H &= \cos\gamma_3\cos\beta \\
I &= \sin\beta\cos\gamma_2\cos\beta + \cos\gamma_1\cos\beta(\sin\gamma_2\cos\beta\sin\alpha_2 \\
&\qquad -\sin\beta\cos\alpha_2)
\end{aligned} \tag{34}$$

Again here, eq.(28) is used to transform eqs.(31) and (33) into two quadratic monovariate polynomials in $\tau_2 \equiv \tan(\alpha_3/2)$ and $\tau_3 \equiv \tan(\theta_3/2)$, respectively, i.e.,

$$(F-D)\tau_2^2 + 2E\tau_2 + F + D = 0, \tag{35}$$

Copyright © 2001 by ASME

and

$$(I-G)\tau_3^2+2H\tau_3+I+G=0 \tag{36}$$

These two polynomials are used to compute α_3 and θ_3, and hence, reduce by two the dimension of the design manifold.

In summary, the isotropic constraints determine 7 of the 18 geometric parameters describing the design manifold, and thus, produces a constrained manifold of isotropic designs of dimension 11. Among the remaining parameters, only β, γ_1, α_2 and γ_3 influence the conditioning of the Jacobian matrices, while l, v_1, a_2, v_2, d_3, a_3, v_3 determine the manipulator scale and workspace.

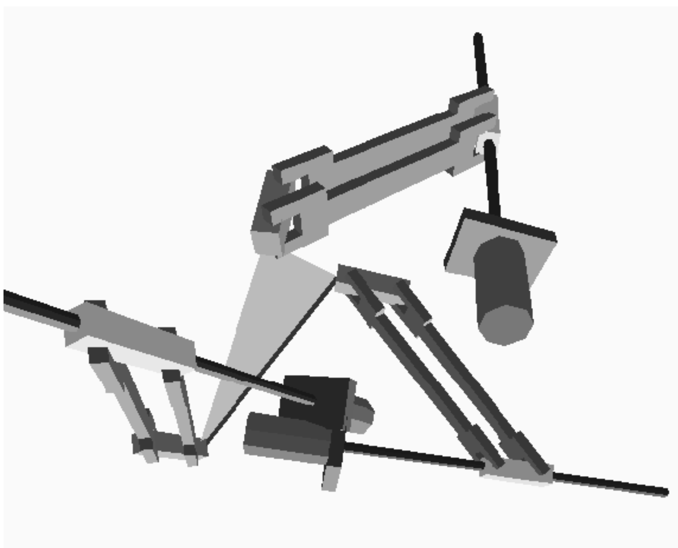


Figure 5. AN ISOTROPIC MANIPULATOR OF STAR TOPOLOGY

NUMERICAL EXAMPLE

This constraint manifold is now used to propose a new design of isotropic parallel manipulator of the Star topological class. First, the four angles β, γ_1, α_2 and γ_3 are chosen so that to satisfy eq.(24), e.g.,

$$\beta = 0.7854, \quad \gamma_1 = 58\pi/36, \quad \alpha_2 = \pi/2, \quad \gamma_3 = 58\pi/36.$$

Second, the angles γ_2, α_3 and θ_3 are computed with eqs.(29), (35) and (36) as

$$\gamma_2 = -1.7166\,rad, \quad \alpha_3 = 2.2190\,rad, \quad \theta_3 = 2.0620\,rad.$$

Third, the size of manipulator is chosen as $l = 1$ m, while the other lengths v_1, a_2, v_2, d_3, a_3 and v_3 are chosen such that points A_1, A_2, A_3 on base and points B_1, B_2 and B_3 on the EE are approximately two equilateral triangles of sides 1.5 m and 0.3 m, respectively, i.e.,

$$v_1 = 0.3751m, \quad v_2 = 1.1185m, \quad v_3 = 0.7594m,$$
$$a_2 = 0.7750m, \;\; a_3 = -0.1589m, \;\; d_3 = -0.5022m.$$

As shown in Fig.5, the orientation of the screw axis and the leg vectors at the home position are given as

$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} -0.6038 \\ 0.7029 \\ -0.3760 \end{bmatrix},$$
$$\mathbf{m}_1 = \begin{bmatrix} -0.7071 \\ 0.2418 \\ -0.6645 \end{bmatrix}, \; \mathbf{m}_2 = \begin{bmatrix} -0.6996 \\ -0.1027 \\ 0.7071 \end{bmatrix}, \; \mathbf{m}_3 = \begin{bmatrix} -0.1027 \\ -0.9649 \\ -0.2418 \end{bmatrix},$$

from which the two Jacobian matrices are readily computed as

$$\mathbf{A} = \begin{bmatrix} -0.7071 & 0.2418 & -0.6645 \\ -0.6996 & -0.1027 & 0.7071 \\ -0.1027 & -0.9649 & -0.2418 \end{bmatrix}, \tag{37}$$

$$\mathbf{B} = \begin{bmatrix} 0.7071 & 0.0000 & 0.0000 \\ 0.0000 & 0.7071 & 0.0000 \\ 0.0000 & 0.0000 & 0.7071 \end{bmatrix}, \tag{38}$$

where it is clear that $\mathbf{A}$ and $\mathbf{B}$ are both isotropic at the home position, since $\mathbf{A}^T\mathbf{A} = \mathbf{1}$ and $\mathbf{B}^T\mathbf{B} = \mathbf{1}$.

CONCLUSION

The necessary and sufficient number of geometric parameters to describe the design manifold of the Star topological class is 18, while the constraint manifold of isotropic designs requires only 11. Moreover, there are only 4 angular parameters among the 11 that influence the conditioning the Jacobian matrices, while the 7 remaining parameters influence the manipulator size and workspace. This constraint manifold greatly facilitates the design of isotropic manipulators.

ACKNOWLEDGMENT

The first author acknowledge the financial support of NSERC (National Sciences and Engineering Research Council of Canada) under grant OGPIN-203618 and FCAR (Fond Concerté d'aide à la Recherche of Quebec) under grant NC-66861.

Copyright © 2001 by ASME

REFERENCES

Hervé, J.M., “Dispositif pour le déplacement en translation spaciale d’un élément dans l’espace, en particulier pour robot mécanique”, *French Patent No. 9100286*, January 1991, *European Patent No. 91403521.7*, December 1991.

Salisbury, J.-K. and Craig, J.-J., “Articulated Hands: Force Control and Kinematic Issues”, *The Int. J. of Robotics Research*, Vol. 1, No. 1, pp. 4–17, 1982.

Tremblay, A. and Baron, L., “Geometrical Synthesis of Parallel Manipulators of Star-Like Topology with a Genetic Algorithm”, *Proc. IEEE Int. Conf. on Robotics and Automation*, pp. 2446-2451, Detroit, USA, Mai 1999.

Copyright © 2001 by ASME