A Raviart-Thomas implementation of the Baff-Refl nodal equivalence technique

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Introduction

The Baff-Refl reflector model

- The legacy reflector model in the SCIENCE platform, known as Baff-Refl, is based on a 1D nodal equivalence technique. We will adapt this technique for use in the ODYSSEE project. The following issues will be investigated:
  - Adaptation of Baff-Refl to the Raviart-Thomas finite element method
  - Support of both diffusion and SPN solutions
  - Calculation of diffusion coefficients for diffusion theory and albedos
  - Production of APEX-formatted reflector databases.

Traditionnally, nodal equivalence techniques are related to nodal codes. In the 1D particular case, nodal equivalence techniques can be applied to other types of solutions. This is no longer true with 2D nodal equivalence techniques.

Baff-Refl is a rigorous equivalence technique, leading to a conservative verification macro calculation.

Introduction

Nodal codes

- Industry-standard methods for production calculations.
- Two variants derived from the work of Kord Smith (MIT):
  - Analytical Nodal Method (ANM): based on a linear transformation over each node. 1D analytic solutions are used.
  - Nodal expansion method (NEM): based on weighted residuals method.
- The orthodox formulation is based on three specific numerical techniques:
  - Transverse quadratic leakage approximation
    - Integration of the 2D/3D diffusion equation in the transverse directions to produce a 1D equation.
  - Use of discontinuity factors
  - Iterative strategy based on the nodal adjustment procedure.
- Several orthodox implementations:
  - QPANDA solver from the SIMULATE platform (Studsvik of America)
  - PARCS system (Perdue University and University of Michigan)
  - NESTLE system (North Carolina State University and University of Tennessee)
  - NEMO solver (G. H. Hobson, Babcock & Wilcox)
  - SMART solver from the SCIENCE platform (Framatome)
  - ARTEMIS solver from the ARCADIA platform (Framatome)
  - implementations exist at Westinghouse, CGN (code COCO) and SNPTC
  - BRISINGR solver (Framatome) used for R&D work
  - NSS Analytic nodal solver in DONJON5.

Introduction

Raviart-Thomas finite element codes

- Industry-standard method in France
- based on a rigorous numerical formulation
- adapted to the diffusion or SPN solution of the neutron flux
- Several implementations:
  - MINOS solver in CRONOS2 and APOLLO3
  - COCAGNE solver in ANDROMÈDE and ODYSSEE
  - TRIVAC dual solver in DONJON5.

	Studsvik	Areva		EDF		EDF/Framatome	Framatome
Framework	Simulate	Science V2	ARCADIA	Cassiopée	Andromède	Odyssee	NEMESI
Lattice code	CASMO5	APOLLO2-F	APOLLO2-A	APOLLO2.5	APOLLO2.8	APOLLO2.8	APOLLO3
Computational scheme	PIJ/MOC (2 levels)	depleting S_n (2 levels)	PIJ/S_n/MOC (3 levels)	PIJ-99 groups (1 level)	REL-2005 (2 levels)	REL-2005/amélioré (2 levels)	REL-2005/amélioré (2 levels)
Full-core simulation code	QPANDA (nodal/NEM)	SMART (nodal/NEM)	ARTEMIS (nodal/NEM)	Coccinelle (nodal)	Cocagne (Raviart-Thomas)	Cocagne (Raviart-Thomas)	MINOS3 (Raviart-Thomas)
Reactor types	PWR / BWR / VVER	PWR	PWR	PWR	PWR	PWR	PWR / VVER

Nodal equivalence techniques

Introducing SPH or discontinuity factors

1. **The SPH technique**
   - The equivalence technique of superhomogenization (SPH) is a correction procedure based on equivalence factors.
   - The SPH factors are obtained as the solution of a non-linear system (using a fixed point or a Newtonian approach).
   - Diffusion coefficients and P_0 cross sections (including total cross section) are multiplied by the SPH factor $\mu_{i,g}$.
2. **The nodal equivalence technique**
   - Cross sections are not modified
   - Flux discontinuity factors (DFs) are imposed at the mesh interfaces:

$$f_{i-1}^{+}\phi(x_{i-1/2}^{-}) = f_{i}^{-}\phi(x_{i-1/2}^{+}) \tag{1}$$

and

$$f_{i}^{+}\phi(x_{i+1/2}^{-}) = f_{i+1}^{-}\phi(x_{i+1/2}^{+}) \tag{2}$$

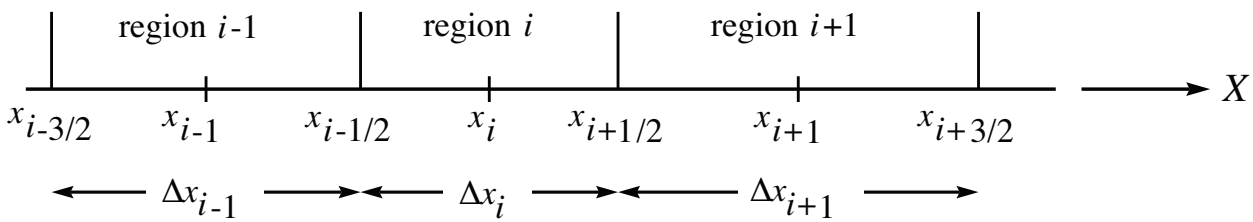


Figure 1: 1D Cartesian macro-geometry.

The nodal equivalence technique

Introducing discontinuity factors

It is possible to obtain the DFs over Fig. 1 using a non-iterative approach.

- A reference solution is first obtained over the geometry with fine mesh and fine-group discretization of the Boltzmann equation.
- Let's consider f_i^-, the DF on the right of region i. This factor is equal to

$$f_i^- = \frac{\phi^*(x_{i-1/2})}{\phi(x_{i-1/2}^+)}. \tag{3}$$

where

$\phi^*(x_{i-1/2}) =$ reference surfacic flux obtained after condensation of the reference fine-group flux. This flux is continuous.

$\phi(x_{i-1/2}^+) =$ few-group surfacic flux of the macro-calculation. This value is the result of a Raviart-Thomas (RT) finite element solution. Analytical expressions of $\phi(x_{i\mp1/2}^{\pm})$ are available.

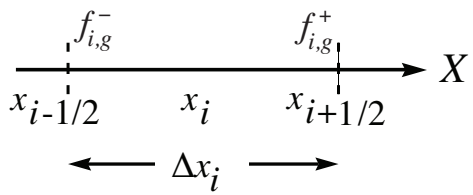


Figure 2: Single Cartesian mesh with DFs.

Equation (3) is the basis of the baff-refl equivalence procedure.

The nodal equivalence technique

Introducing discontinuity factors

There is a distinction between discontinuity factors and assembly discontinuity factors:

- Discontinuity factors (DF) are computed in non-fundamental mode condition using the technique presented in this presentation. They are used in reflector models and in full-core equivalence of small cores.
- Assembly discontinuity factors (ADF) are computed in fundamental mode condition. They are written in single-assembly multiparameter (Multicompo, Saphyb, APEX or MPO) databases.
- If the single-assembly is completely homogenized (1×1 or 2×2 nodal calculations), the surfacic flux of the macro-calculation in the denominator of Eq. (3) is flat and is equal to the flux $\bar{\phi}$ averaged over the assembly and condensed over macrogroups. The ADF is obtained from a modified version of Eq. (3) as

$$f^{-} = \frac{\phi^{*}_{\text{blade}}}{\bar{\phi}} \tag{4}$$

where ϕ^{*}_{blade} is the average flux in the water blade (typically $\simeq 0.042$ cm) surrounding the assembly (depicted in Fig. 3) and condensed over macrogroups.

The nodal equivalence technique

Introducing discontinuity factors

A typical PWR assembly is represented in eight-of-assembly symmetry and features a water blade where ADF information is recovered.

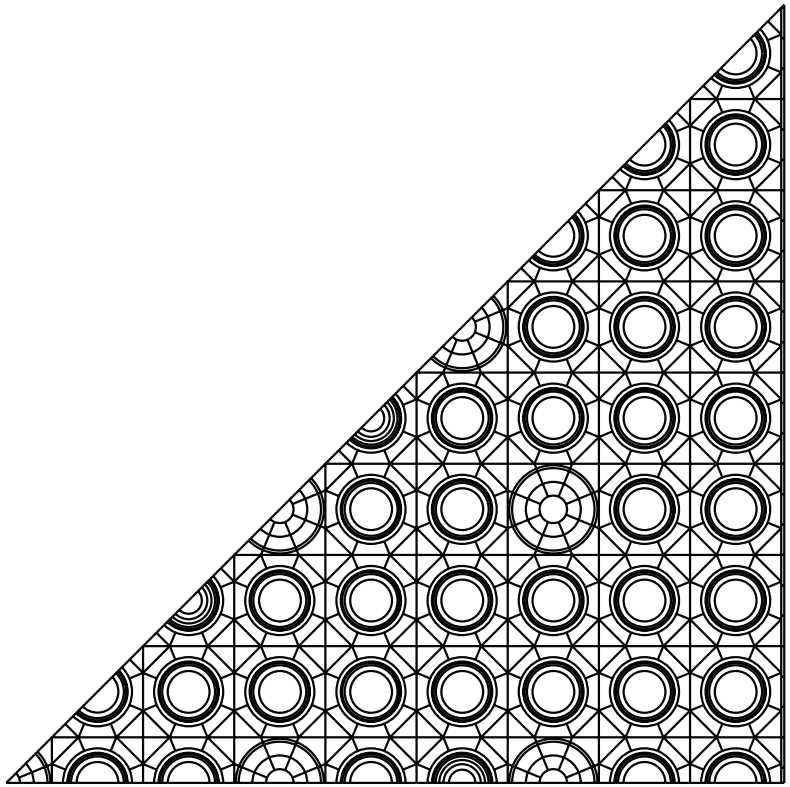


Figure 3: Geometry of an eight-symmetry PWR assembly with a water blade.

The Raviart-Thomas finite element method

Theory

The Raviart-Thomas finite element method is a mixed-dual formulation with the following characteristics:

- The diffusion (or SPN) equation is discretized over a 1D/2D/3D Cartesian or hexagonal 2D/3D finite elements.
- A weak formulation is used where net current continuity is forced (essential condition) and flux continuity is relaxed (natural condition).
- in Cartesian 1D (slab) geometry, unknowns are the net currents $J_{i\mp 1/2,g}$ between finite elements and Legendre coefficients of the flux inside them:

$$\phi_{i,g}(u) = \sum_{k=0}^{K} a_{k,i,g}\, \tilde{P}_k(u) \tag{5}$$

where $\tilde{P}_\ell(u)$ is a normalized Legendre polynomial defined as

$$\begin{aligned} \tilde{P}_0(u) &= 1 \\ \tilde{P}_1(u) &= 2\sqrt{3}u \end{aligned} \tag{6}$$

and

$$\tilde{P}_{k+1}(u) = 2\sqrt{\frac{2k+3}{2k+1}}\,\frac{2k+1}{k+1}u\tilde{P}_k(u) - \sqrt{\frac{2k+3}{2k-1}}\,\frac{k}{k+1}\tilde{P}_{k-1}(u) \quad \text{if } k \geq 1. \tag{7}$$

The Raviart-Thomas finite element method

Theory

The matrix system resulting from the Raviart-Thomas discretization of the diffusion equation has been presented by Hébert:

Application of a dual variational formulation to finite element reactor calculations, ANE (1993).

Mixed-dual implementations of the simplified Pn method, ANE (2010)

The one-group matrix system for a diffusion approximation in Cartesian 1D geometry in group g is written (group indices g are omitted)

$$\mathbb{M} \begin{bmatrix} \boldsymbol{\phi} \\ \boldsymbol{J}_x \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_\phi \\ \boldsymbol{S}_x \end{bmatrix} \tag{8}$$

where $\boldsymbol{\phi}$ contains Legendre coefficients of the flux, $\boldsymbol{J}_x$ contains net currents between finite elements, and $\mathbb{M}$ is a symmetric matrix defined as

$$\mathbb{M} = \begin{bmatrix} \mathbb{T} & \mathbb{R}_x^\top \\ \mathbb{R}_x & \mathbb{A}_x \end{bmatrix} \tag{9}$$

where matrix $\mathbb{T}$ is diagonal and matrix $\mathbb{A}_x$ is diagonally profiled. Matrix $\mathbb{R}$ is stored in Perdue sparse storage mode. The source vector contains fission and out-of-group contributions.

The Raviart-Thomas finite element method

Theory

Preconditionning is based on inversion of the coefficient matrix $\mathbb{M}$. The linear system (8) can be rewritten as

$$\tilde{\mathbb{A}}_x \boldsymbol{J}_x = -\boldsymbol{S}_x + \mathbb{R}_\phi \, \mathbb{T}^{-1} \, \boldsymbol{S}_\phi \tag{10}$$

where matrix $\tilde{\mathbb{A}}_x$ is factorized as $\mathbb{L}\mathbb{U}$ factors:

$$\tilde{\mathbb{A}}_x = -\mathbb{A}_x + \mathbb{R}_\phi \, \mathbb{T}^{-1} \, \mathbb{R}_\phi^\top \tag{11}$$

so that

$$\boldsymbol{J}_x = \mathbb{A}_x^{-1} \left[-\boldsymbol{S}_x + \mathbb{R}_\phi \, \mathbb{T}^{-1} \, \boldsymbol{S}_\phi \right] \tag{12}$$

The fluxes are obtained afterwards using the first line of Eq. (8):

$$\boldsymbol{\phi} = \mathbb{T}^{-1} \left[\boldsymbol{S}_\phi - \mathbb{R}_\phi^\top \boldsymbol{J}_x \right]. \tag{13}$$

- Equation (13) is used to compute Legendre coefficients of the flux in Baff-Refl, as a function of reference net currents $\boldsymbol{J}_x^*$.
- Baff-Refl also needs to compute fluxes on the left and right side of each finite element $\phi^{\pm}_{i\mp 1/2}$. These values are function of the Legendre coefficients of the flux $a_{k,i}$, of the left and right values of the net current $J_{i\mp 1/2}$ and of the diffusion coefficient D_i. They are also function of the type of quadrature used to obtain $\mathbb{R}_x$ and $\mathbb{A}_x$.

1. Analytical integration

$$\phi^+_{i-1/2} = \begin{cases} \frac{\Delta x_i}{3D_i} J_{i-1/2} + \frac{\Delta x_i}{6D_i} J_{i+1/2} + a_{0,i} & \text{if } K = 0 \\ \frac{\Delta x_i}{8D_i} J_{i-1/2} - \frac{\Delta x_i}{24D_i} J_{i+1/2} + a_{0,i} - \frac{5\sqrt{3}}{6} a_{1,i} & \text{if } K = 1 \\ \frac{\Delta x_i}{15D_i} J_{i-1/2} + \frac{\Delta x_i}{60D_i} J_{i+1/2} + a_{0,i} - \frac{5\sqrt{3}}{6} a_{1,i} + \frac{7\sqrt{5}}{10} a_{2,i} & \text{if } K = 2 \end{cases} \tag{14}$$

$$\phi^-_{i+1/2} = \begin{cases} -\frac{\Delta x_i}{6D_i} J_{i-1/2} - \frac{\Delta x_i}{3D_i} J_{i+1/2} + a_{0,i} & \text{if } K = 0 \\ \frac{\Delta x_i}{24D_i} J_{i-1/2} - \frac{\Delta x_i}{8D_i} J_{i+1/2} + a_{0,i} + \frac{5\sqrt{3}}{6} a_{1,i} & \text{if } K = 1 \\ -\frac{\Delta x_i}{60D_i} J_{i-1/2} - \frac{\Delta x_i}{15D_i} J_{i+1/2} + a_{0,i} + \frac{5\sqrt{3}}{6} a_{1,i} + \frac{7\sqrt{5}}{10} a_{2,i} & \text{if } K = 2 \end{cases} \tag{15}$$

2. Gauss-Legendre quadrature

$$\phi^+_{i-1/2} = \begin{cases} \frac{\Delta x_i}{4D_i} J_{i-1/2} + \frac{\Delta x_i}{4D_i} J_{i+1/2} + a_{0,i} & \text{if } K = 0 \\ \frac{\Delta x_i}{12D_i} J_{i-1/2} - \frac{\Delta x_i}{12D_i} J_{i+1/2} + a_{0,i} - \sqrt{3} a_{1,i} & \text{if } K = 1 \\ \frac{\Delta x_i}{24D_i} J_{i-1/2} + \frac{\Delta x_i}{24D_i} J_{i+1/2} + a_{0,i} - \frac{5\sqrt{3}}{6} a_{1,i} + \sqrt{5} a_{2,i} & \text{if } K = 2 \end{cases} \tag{16}$$

$$\phi^-_{i+1/2} = \begin{cases} -\frac{\Delta x_i}{4D_i} J_{i-1/2} - \frac{\Delta x_i}{4D_i} J_{i+1/2} + a_{0,i} & \text{if } K = 0 \\ \frac{\Delta x_i}{12D_i} J_{i-1/2} - \frac{\Delta x_i}{12D_i} J_{i+1/2} + a_{0,i} + \sqrt{3} a_{1,i} & \text{if } K = 1 \\ -\frac{\Delta x_i}{24D_i} J_{i-1/2} - \frac{\Delta x_i}{24D_i} J_{i+1/2} + a_{0,i} + \frac{5\sqrt{3}}{6} a_{1,i} + \sqrt{5} a_{2,i} & \text{if } K = 2 \end{cases} \tag{17}$$

3. Gauss-Lobatto quadrature

$$\phi^+_{i-1/2} = \frac{\Delta x_i}{(K+1)(K+2)D_i} J_{i-1/2} + \sum_{k=0}^{K} (-1)^k \sqrt{2k+1} \left[1 - \frac{k(k+1)}{(K+1)(K+2)}\right] a_{k,i} \quad (18)$$

$$\phi^+_{i+1/2} = -\frac{\Delta x_i}{(K+1)(K+2)D_i} J_{i+1/2} + \sum_{k=0}^{K} \sqrt{2k+1} \left[1 - \frac{k(k+1)}{(K+1)(K+2)}\right] a_{k,i} \quad (19)$$

The Baff-Refl algorithm

Theory

The Baff-Refl procedure is a legacy nodal equivalence technique available in SCIENCE and ARCADIA.

- The macro-geometry is divided into nodes over which the following reference information is available:
  - macrogroup reaction rates
  - boundary macrogroup fluxes
  - boundary macrogroup net currents
- Discontinuity factors (DF) are obtained over the node boundary so as to preserve thereference information.
- The Baff-Refl procedure is a 1D non-iterative technique.
- Macro-calculations can be performed with three numerical approaches:
  - using the Analytic Nodal Method (ANM) – SCIENCE V1
  - using the Nodal Expansion Method (NEM) – SCIENCE V2 and ARCADIA
  - using the Raviart-Thomas finite element method (RT) – DRAGON5
- The SCIENCE V2 and ARCADIA implementation for NEM is completely described in a technical note from Aldo Dall'Osso (only available at Framatome):

A. Dall'Osso, "Determination of multi-group DFs ... in APOLLO2-A for ARTEMIS-1," CONV07_0021, 2007.

The Baff-Refl algorithm

Theory

The Baff-Refl algorithm proceeds in many steps:

1. Perform the S_n reference calculation over a 1D fuel–reflector geometry

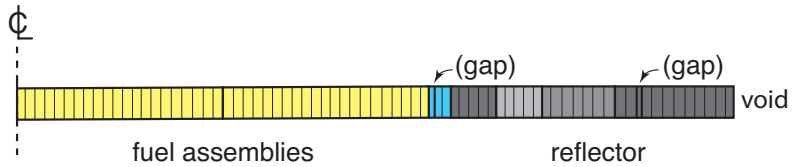


Figure 4: S_n geometry

- Use P_n scattering anisotropy with $n \geq 1$
- Homogeneous fine-group cross sections are used for representing two identical fuel assemblies (yellow). These homogeneous fine-group cross sections are recovered from a *fundamental mode* lattice calculation of the feeding assembly.
- The reflector is represented as a sandwich of water and steel slabs with gap volumes used to recover surfacic fluxes (blue and grey)
- A fixed geometric buckling is imposed to the Boltzmann equation in order to represent transverse leakage:

$$B^2 = \left(\frac{\pi}{L_z}\right)^2 + \left(\frac{\pi}{L_y}\right)^2 \tag{20}$$

where L_z and L_y are the height and thickness of the core, respectively. Currently, $B^2 = 1 \times 10^{-6}$ cm^{-2}.

1. Perform the S_n reference calculation over a 1D fuel–reflector geometry (cont'n)
   - Use a fine-group ($G \geq 99$ groups) leakage spectra $L_{i,g} = d_{i,g} B^2 \phi_{i,g}$ where the fine-group leakage coefficients are obtained from different models :
     - Golfier-Vergain model (legacy SCIENCE approach)
     - Todorova outscatter model
     - Todorova inscatter model (recommended)
   - Perform a fine-group, K_{eff} search, S_n calculation with imposed B^2
   - Homogenize and condense leakage coefficients $d_{i,g}$ and cross sections to two-group and few-region geometry
   - Recover condensed gap fluxes $\phi^*_{i\pm 1/2,g}$ and averaged fluxes

$$\langle \phi^* \rangle_{i,g} = \int_{-1/2}^{1/2} du\, \phi^*_g(u) \tag{21}$$

1. Perform the S_n reference calculation over a 1D fuel–reflector geometry (cont'n)
   - Compute corresponding gap net currents using $J^*_{1/2,g} = 0$ and

$$J^*_{i+1/2,g} = J^*_{i-1/2,g} + \frac{V_i\,\chi_{i,g}}{K_{\rm eff}} \sum_{h=1}^{G} \nu\Sigma_{{\rm f},i,h}\,\phi_{i,h} - V_i \left(\Sigma_{i,g} + B^2 D_{i,g}\right) \phi_{i,g} + V_i \sum_{h=1}^{G} \Sigma_{{\rm s},i,g\leftarrow h}\,\phi_{i,h} \qquad (22)$$

where

$V_i =$ volume of region i
$\phi_{i,g} =$ neutron flux
$K_{\rm eff} =$ effective multiplication factor
$\chi_{i,g} =$ fission spectra
$\nu\Sigma_{{\rm f},i,h} =$ ν times the macroscopic fission cross section
$\Sigma_{i,g} =$ macroscopic total cross section
$B^2 =$ imposed buckling
$D_{i,g} =$ diffusion coefficient

2. Solution of Eq. (13) to obtain Legendre coefficients of the flux as a function of reference net currents $J^*_{i-1/2,g}$ and $J^*_{i+1/2,g}$ obtained from the S_n reference calculation.
3. Calculation of interface nodal fluxes using Eqs. (14) to (19).
4. Compute the left and right discontinuity factors using Eq. (3):

$$f^-_{i,g} = \frac{\phi^*_{i-1/2,g}}{\phi^+_{i-1/2,g}} \tag{23}$$

$$f^+_{i,g} = \frac{\phi^*_{i+1/2,g}}{\phi^-_{i+1/2,g}}. \tag{24}$$

where $\phi^*_{i-1/2,g}$ and $\phi^*_{i+1/2,g}$ are the interface fluxes obtained from the S_n reference calculation.

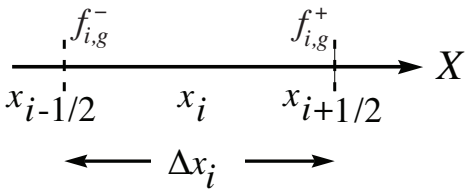


Figure 5: Single Cartesian mesh with DFs.

5. Compute the right boundary multigroup albedo

The right ($u = \frac{1}{2}^-$) P_1 albedo boundary condition in group g is written

$$\frac{D_{l,g}}{\Delta x_i} \left.\frac{d\phi_{l,g}}{du}\right|_{\frac{1}{2}^-} + \Lambda_g^+ \, \phi^-_{l+1/2,g} = -J^*_{l+1/2,g} + \Lambda_g^+ \, \phi^-_{l+1/2,g} = 0 \tag{25}$$

where the albedo function is written in terms of the albedo β_g^+ as

$$\Lambda_g^+ = \frac{1}{2} \frac{1-\beta_g^+}{1+\beta_g^+}. \tag{26}$$

The Baff-Refl procedure consists to use the following value:

$$\Lambda_g^+ = \begin{cases} \simeq 0.5 & \text{if legacy Baff-Refl} \\ \dfrac{J^*_{l+1/2,g}}{\phi^-_{l+1/2,g}} = \dfrac{J^*_{l+1/2,g} \, f^+_{l,g}}{\phi^*_{l+1/2,g}} & \text{if improved Baff-Refl} \end{cases} \tag{27}$$

6. Perform NGET normalization of the discontinuity factors

The nodal generalized equivalence theory (NGET) renormalization of the discontinuity factors is described in the paper

E. Z. Müller, "Constants for PWR Radial Reflector Regions," *Nucl. Sc.. Eng.*, **103**, 359 (1989).

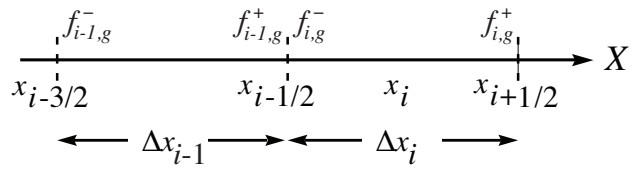


Figure 6: Two Cartesian meshes with DFs.

The nodal flux continuity equation at $x_{i-1/2}$ is written as Eq. (1):

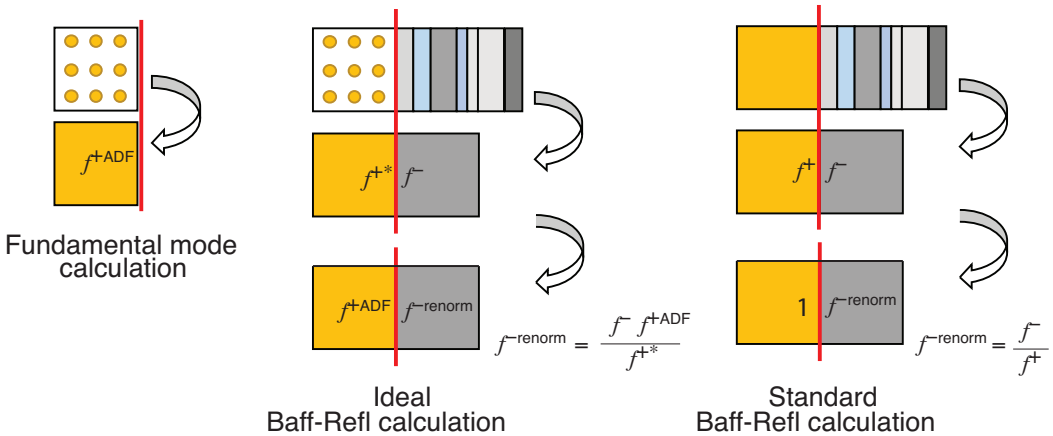
$$f^+_{i-1,g}\phi_g(x^-_{i-1/2}) = f^-_{i,g}\phi_g(x^+_{i-1/2}). \tag{28}$$

The LHS and RHS of Eq. (28) can be multiplied by a normalization constant.

NGET normalization is used for two different operations:

- Perform a decorrelation of discontinuity factors (DFs) with respect to the feeding fuel assembly.
- Ensure that $f^-_{i,g} = f^+_{i,g} = f_{i,g}$ in each node i.

6. Proceed to the decorrelation of discontinuity factors (DFs) with respect to the feeding fuel assembly. $f^{+}_{i,g} = 1$ is set for both radial and axial reflectors



In the case of an ideal Baff-Refl radial configuration, the decorrelated DFs are written

$$f^{-\mathrm{renorm}}_{i+1,g} = \frac{f^{-}_{i+1,g}\, f^{+\mathrm{ADF}}_{i,g}}{f^{+*}_{i,g}}. \tag{29}$$

Using the superposition principle, we assume $f^{+*}_{i,g} = f^{+}_{i,g}\, f^{+\mathrm{ADF}}_{i,g}$ so that the decorrelated DFs simplify to

$$f^{-\mathrm{renorm}}_{i+1,g} = \frac{f^{-}_{i+1,g}}{f^{+}_{i,g}} \tag{30}$$

corresponding to $f^{+\mathrm{renorm}}_{i,g} = 1$ in the standard Baff-Refl configuration.

7. Discontinuity factors of step 6 can be saved on the APEX file if an analytic nodal method (ANM) or nodal expansion method (NED) is used.

A Dragon mixture is a zone in Apex terminology. Discontinuity factors and equivalent albedos are written in group `miscellaneous` included in each state point of the Apex file.

Name	Type	Condition	Units	Comment
{**hadf**}	R($N_{\text{surf}} \times N_{\text{grp}}$)		1	Discontinuity factors $F^{\text{d}}_{\texttt{iq},b,g}$ on external surfaces $b \leq N_{\text{surf}}$ obtained with a nodal equivalence procedure within zone **iq**.
ALBEDO	R($N_{\text{alb}} \times N_{\text{grp}}$)	$N_{\text{alb}} \geq 1$	1	Multigroup albedos $\beta_{a,g}$ obtained with a nodal equivalence procedure.

Figure 7: Group `/calc_id/miscellaneous/` of the Apex file.

where N_{surf} is the number of surfaces where ADF information is known and N_{grp} is the number of macro energy groups.
If the Apex file contains a unique output zone, suffix `_iq` can be omitted and {*hadf*} is set to "`ADF`". Otherwise, the name of the discontinuity factor set {*hadf*} is composed using the following FORTRAN instruction:

`WRITE`(HADF,′ (`3HADF, I8`)′) *iq*

where $\texttt{iq} \leq N_{\text{mil}}$ (number of output zones).

8. If a Raviart-Thomas finite element method is used, define SPH factors as

$$\mu_{i,g} = \frac{1}{f_{i,g}} \tag{31}$$

and correct the diffusion coefficients and cross sections according to Eqs. (1) and (2):

$$\widetilde{D}_{i,g} = \mu_{i,g}\, D_{i,g}, \quad \widetilde{\Sigma}_{i,g} = \mu_{i,g}\, \Sigma_{i,g},$$
$$\widetilde{\nu\Sigma}_{\mathrm{f},i,g} = \mu_{i,g}\, \nu\Sigma_{\mathrm{f},i,g}, \quad \widetilde{\Sigma}_{\mathrm{s0},i,g\leftarrow h} = \mu_{i,h}\, \Sigma_{\mathrm{s0},i,g\leftarrow h}$$

and where the albedo function is also corrected as

$$\widetilde{\Lambda}^{+}_{g} = \mu_{I,g}\, \Lambda^{+}_{g}. \tag{32}$$

Note: Non-corrected cross sections, diffusion coefficients and albedos are written on the APEX file. The set of SPH factors is saved apart on the APEX file:

Name	Type	Condition	Units	Comment
{hequi}	R(N_{grp})		1	SPH factors in zone iq. {hequi} is a user-defined name corresponding to a specific type of SPH equivalence.

Figure 8: Group `/calc_id/xs_iq/MEDIA_SPH/` of the Apex file.

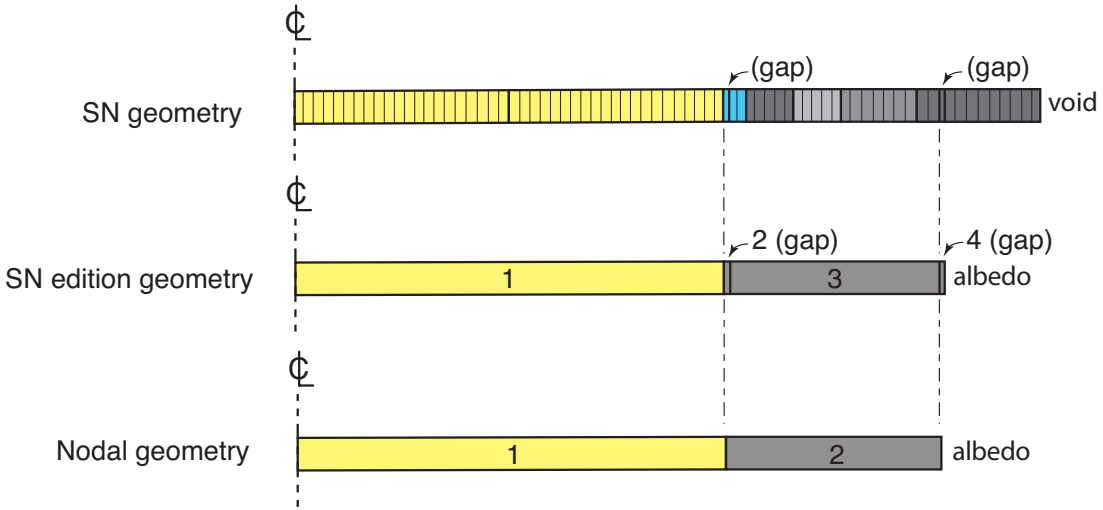


Figure 9: Geometries used by the equivalence procedure

The Baff-Refl algorithm

The Todorova inscatter approximation

Diffusion coefficients (before SPH correction) are based on the Todorova inscatter approximation:

$$d_i(u)\,\Sigma_i(u)\,\phi_i(u) - \int_0^\infty du'\, d_i(u')\,\Sigma_{s1,i}(u \leftarrow u')\,\phi_i(u') = \frac{\phi_i(u)}{3} \tag{33}$$

$$D_{i,g} = \frac{\int_{u_{g-1}}^{u_g} du\, d_i(u)\,\phi_i(u)}{\int_{u_{g-1}}^{u_g} du\,\phi_i(u)}. \tag{34}$$

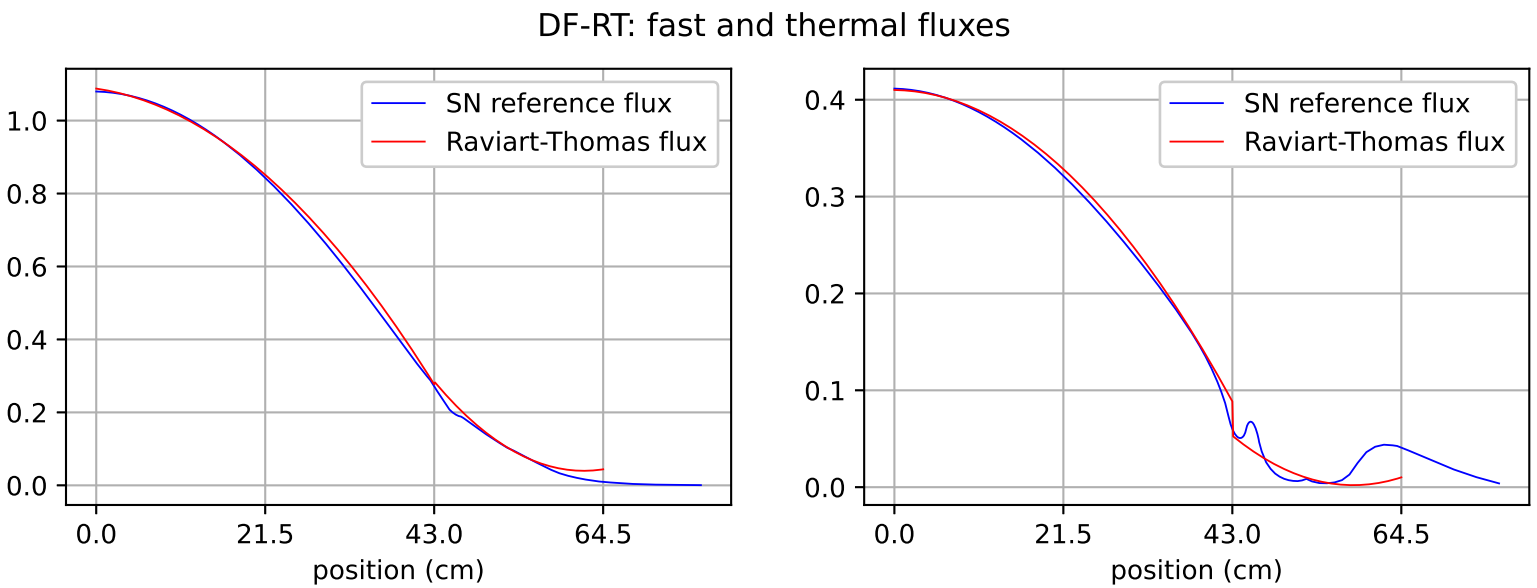


Figure 10: DF-RT: Equivalence (Todorova inscatter)

Conclusions and perspective

- The Baff-Refl equivalent reflector model offers an alternative to the legacy Lefebvre-Lebigot model for the ODYSSÉE project.
- Reflector APEX files can be processed usint the CARABAS tool to produce DKLIB data and feed COCAGNE.
- Perspectives for inprovement are:
  - Generalization of Baff-Refl to 2D geometries for use with analytic nodal method (ANM) or nodal expansion method (NED). This is the Ph. D. project of Sami Machach.
  - Study the linear equivalent reflector model (LERM) to take into account spectral effects.

Closing

Questions – Answers