Département de Mathématiques et de Génie Industriel
Polytechnique Montréal

MATHÉMATIQUES DES ÉLÉMENTS FINIS
MTH8207

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Project 1 with Comsol Multiphysics: Simulation of a column

Problem 1:

We are interested in the analysis of a column of length L, cross-sectional area A, and Young's modulus E. We assume that the column stands on a support at $x = 0$, that it is subjected to a longitudinal compression force P at $x = L$ and to the gravitational force density g. The displacement $u = u(x)$ in the column is governed by the 1D differential equation:

$$-\frac{d}{dx}\left(EA\frac{du}{dx}\right) = -\rho g A, \quad \text{in } (0, L)$$

and subjected to the Dirichlet and Neuman boundary conditions:

$$u = 0, \quad \text{at } x = 0, \qquad \text{and} \quad EA\frac{du}{dx} = -P, \quad \text{at } x = L$$

The following data will be the same for all questions: $L = 4$ m, $g = 9.81$ m/s^2, $P = 40$ kN.

1. In this question, take E, A, and ρ constant along x: $E = 20$ GPa, $\rho = 2{,}300$ kg/m^3 (concrete), and $A = A_0 = \pi \times 10^{-2}$ m^2.
   (a) Solve for the exact solution and derive the weak formulation of the problem.
   (b) Develop an application in Comsol Multiphysics to model the problem.
   (c) Compute the stress $\sigma = E du/dx$ and the relative error in the stress at $x = 0$ when using 1, 2, 4, 8, and 16 linear elements of uniform size.
   (d) Using non uniform linear elements, design, by trial and error, a mesh that yields the minimal number of degrees of freedom while reaching a relative error in the stress at $x = 0$ smaller than half a percent.
2. Keep here E and ρ constant ($E = 20$ GPa, $\rho = 2{,}300$ kg/m^3), and consider A such that

$$A = A_0\left[1 - \frac{x(L-x)}{L^2}\right]$$

   with $A_0 = \pi \times 10^{-2}$ m^2. Repeat Questions (b), (c), and (d).
3. Suppose now that the column is made of two different materials: in regions $(0, \ell)$ and $(L-\ell, L)$, with $\ell = 0.5$ m, the column is made of a material with properties $E = 10$ GPa and $\rho = 500$ kg/m^3, and in these two regions, the column has a constant square cross-section of width $a_0 = 0.20$ m; in region $(\ell, L-\ell)$, the column has material properties $E = 20$ GPa, $\rho = 2{,}300$ kg/m^3, and a constant circular cross-section with diameter $d_0 = 0.15$ m. Find the location x_s in the column where the stress is maximal. Design a mesh that should give a relative error in the maximal stress smaller than one percent.

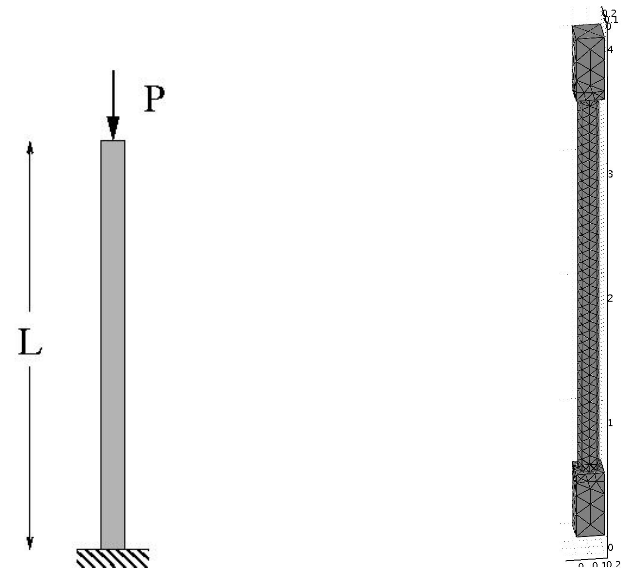

Figure 1: Description of the problem in 1D (left) and example of a mesh for the 3D model described in Problem 2 (right).

<u>PROBLEM 2:</u>

Develop a 3D FE model using linear elasticity to simulate the configuration in Problem 1.3. Suppose that the different components of the column are perfectly aligned along the centerline and that the force P is equally distributed at $x = L$. Find the maximal stress σ_s and corresponding location x_s in the column (make sure that the mesh is sufficiently refined to provide accurate solutions).

<u>PROBLEM 3:</u>

Suppose now that the circular column was imperfectly aligned with respect to the two other blocks by $\delta = 0.02$ m. Using 3D linear elasticity and assuming that the force P is equally distributed at $x = L$, compute the maximal deflection of the column.
