Aide-mémoire

Examen final

$$\oint_C f(z)\,dz = 2\pi i \cdot \sum_{i=1}^{n} \text{Rés}\,(f(z)\,;z=z_i), \qquad a_{-1} = \frac{1}{(m-1)!}\lim_{z\to z_1}\frac{d^{m-1}}{dz^{m-1}}\Big((z-z_1)^m f(z)\Big)$$

Fonction de transfert d'un SLS T : $H(z) = \sum_{k=-\infty}^{\infty} h_T(k)\,z^{-k}$ (ici $h_T(k) = T(\delta(k))$)

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \sum_{k=1}^{N} \text{Rés}(e^{s\,t}\,Y(s); s=s_k)$$

$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz$$

Transformée en Z : $Z\Big(f(n)\Big) \stackrel{\text{déf}}{=} \sum_{n=0}^{\infty} f(n)\,z^{-n} = F(z)$

$$(f*g)(t) = \int_{-\infty}^{\infty} f(\tau)\,g(t-\tau)\,d\tau, \qquad Z\Big((f_1*f_2)(n)\Big) = F_1(z)\,F_2(z)$$

$$Z\Big(n f(n)\Big) = -z\,F'(z), \qquad Z\Big(n\,c^n\,u(n)\Big) = \frac{c\,z}{(z-c)^2}$$

$$Z\Big(f(n+m)\Big) = z^m\Big(F(z) - \sum_{k=0}^{m-1} f(k)\,z^{-k}\Big), \quad m = 1,2,3\ldots$$

$$\boxed{Z\Big(f(n-m)\ \cdot u(n-m)\Big) = z^{-m}F(z)}$$

$$\boxed{Z\Big(e^{-a\,n}f(n)\Big) = F(e^a z)}$$

$$Z\Big(c^n\Big) = \frac{z}{z-c}$$

$$f(n) = \frac{1}{2\pi i}\oint_C z^{n-1}F(z)\,dz$$

Série de Laurent : $f(z) = \sum_{n=-\infty}^{\infty} a_n\,(z-z_0)^n$, où $a_n = \dfrac{1}{2\pi i}\oint_C \dfrac{f(z)}{(z-z_0)^{n+1}}\,dz$,

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Transformée de Fourier :

$$\hat{f}(\omega) = \mathscr{F}(f(t)) \stackrel{\text{déf}}{=} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-i\,\omega\,t}\,f(t)\;dt, \qquad \omega \in \mathbb{R}$$

$$f(t) = \mathscr{F}^{-1}(\hat{f}(\omega)) \stackrel{\text{déf}}{=} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(\omega)\,e^{i\,\omega\,t}\,d\omega$$

$$\mathscr{F}(\hat{f}(t)) = f(-\omega), \qquad i^n\,\frac{d^n}{d\omega^n}(\hat{f}(\omega)) = \mathscr{F}(t^n\,f(t)), \qquad \mathscr{F}(f^{(n)}(t)) = (i\omega)^n\,\hat{f}(\omega)$$

$$\mathscr{F}(f(t-t_0)) = e^{-i\omega t_0}\mathscr{F}(f(t)), \qquad \mathscr{F}(f(-t)) = \hat{f}(-\omega), \qquad \mathscr{F}(f*g) = \sqrt{2\pi}\ \mathscr{F}(f)\ \mathscr{F}(g)$$

Identité de Parseval : $\int_{-\infty}^{\infty} |f(t)|^2\, dt = \int_{-\infty}^{\infty} |\hat{f}(\omega)|^2\, d\omega$

Série de Fourier

- Pour $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(\frac{n\pi x}{L}) + b_n \sin(\frac{n\pi x}{L})\right]$ où

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\, \cos\left(\frac{n\pi x}{L}\right)\, dx, \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(x)\, \sin\left(\frac{n\pi x}{L}\right)\, dx$$

l'identité de Parseval est : $\frac{1}{L}\int_{-L}^{L}\left(f(x)\right)^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$

- Pour $f(x) = \sum_{n=-\infty}^{\infty} c_n\, e^{i\frac{n\pi x}{L}}$, $\qquad c_n = \frac{1}{2L}\int_{0}^{2L} f(x)\, e^{-i\frac{n\pi x}{L}}\, dx$

l'identité de Parseval est : $\frac{1}{2L}\int_{0}^{2L}\left|f(x)\right|^2 dx = \sum_{n=-\infty}^{\infty}\left|c_n\right|^2$

Échantillonnage : $\qquad f(t) = \sum_{n=-\infty}^{\infty} f(nT)\, \mathrm{sinc}(\Omega\,(t - n\,T))$