

# PHS6317

# NANO-ENGINEERING OF THIN FILMS

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Based on the notes first prepared in 2008 by Stéphane Larouche  
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Winter 2024

Bienvenue - Welcome

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# PHS 6317 Nanoengineering of thin films

## *Course schedule – Winter 2024*

- 12 January Introduction – Scientific and technological challenges
- 19 Fabrication methods – Vacuum physics and vapor-phase techniques
- 26\* Fabrication methods – Plasma processes and process optimization
- 2 February Fabrication methods - Plasma-surface interactions and diagnostics
- 9\*\* Fabrication methods – Thermal/Plasma spray technologies
- 16\* Optics of thin films 1, optical characterization, *Miniquiz 1 (5%)*
- 23\* **Optics of thin films 2, design of optical filters**
- 1\*\*\* March *Presentations – Emerging fabrication techniques (30%)*
- March 4-8 - Winter/Spring break***
- 15\*\* Tribomechanical properties of films and coatings
- 22\*\* Electrochemical properties – corrosion and tribo-corrosion (*filter-20%*)
- 5 April Passive functional films and coatings, *Miniquiz 2 (5%)*
- 12 Active functional films and coatings
- 16 Life cycle analysis and environmental impact
- 19\*\*\* *Presentations – Emerging applications of nanostructured films (40%)*

## Deadlines:

### Project #1 – Fabrication technique:

Choice of the subject: **26 January**

Abstract and references: **9 February**

Report and presentation: **1<sup>st</sup> March**

### Project #2 – Design of an optical filter:

Choice of the subject: **23 February**

Report: **22 March**

### Project #3 – Application of nanostructured thin films:

Choice of the subject: **16 February**

Abstract and references: **15 March**

Report and presentation: **19 April**



## Evaluation

- |  |     |
|--|-----|
| 1. <b>Project 1:</b> Bibliographic research on an emerging fabrication technique of thin films - Report and presentation               | 30% |
| 2. <b>Project 2:</b> Design of an optical filter - Report  | 20% |
| 3. <b>Project 3:</b> Bibliographic research on a specific application of the nano- engineering of thin films - Report and presentation | 40% |
| 4. <b>Miniquiz 1 and 2 ( @ 5% )</b>  | 10% |



## Project 2: Design of an optical filter (20%)

### Specific requirements:

**Deliverables:** Report, maximum 8 pages (letter size paper, 2 cm margins, Times New Roman 12 pts).

### Structure and contents:

- Introduction – describe the choice of the specific filter and its application
- Optical specifications of the filter: spectral characteristics in  $T$  and  $R$ , color coordinates, tolerances, etc.
- Methodology of the design: architecture, materials, optimization, etc.
- Discussion of the performance and sensitivity to the fabrication process
- Conclusions

### Deadlines

Choice of filter: **February 23**

Report: ..... **March 22**



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## Course schedule – Winter 2022

14 January	Introduction – Scientific and technological challenges
21	Fabrication methods – Vacuum physics and vapor-phase techniques
28*	Fabrication methods – Thermal/Plasma spray technologies
4 February	Fabrication methods – Plasma processes
11*	Fabrication methods - Plasma-surfaces interactions and diagnostics
18***	Optics of thin films 1, optical characterization, <i>Miniquiz1 (5%)</i>
25**	Optics of thin films 2, design of optical filters

## February 28 - March 4 - Winter/Spring break

11* March	<i>Presentations – Emerging fabrication techniques (30%)</i>
18***	Tribomechanical properties of films and coatings
25**	Electrochemical properties – corrosion and tribo-corrosion( <i>filter-20%</i> )
1 April	Passive functional films and coatings, <i>Miniquiz 2 (5%)</i>
8	Active functional films and coatings
15	Life cycle analysis and environmental impact
19***	<i>Presentations – Emerging applications of nanostructured films (40%)</i>

## Deadlines:

### Project #1 – Fabrication technique:

Choice of the subject: **28 January**

Abstract and references: **11 February**

Report and presentation: **11 March**

### Project #2 – Design of an optical filter:

Choice of the subject: **25 February**

Report: **25 March**

### Project #3 – Application of nanostructured thin films:

Choice of the subject: **18 February**

Abstract and references: **18 March**

Report and presentation: **19 April**



## Project 2: Design of an optical filter (20%)

### Specific requirements:

**Deliverables:** Report, maximum 10 pages (letter size paper, 2 cm margins, Times new roman 12 pts)

### **Structure and contents:**

- Introduction – describe the choice of the specific filter
- Optical specifications of the filter: spectral characteristics in  $T$  and  $R$ , tolerances, etc.
- Methodology of the design (architecture, materials, optimization,...)
- Discussion of the performance and sensitivity to the fabrication process
- Conclusions

### **Deadlines:**

Choice of the filter: .. **25 February**

Report: ..... **25 March**



# Outline

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Chapter 8B

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## Optical metrology

1. Spectrophotometry
2. Data analysis methodology for spectrophotometric data
  - a. The envelope method
3. Spectroscopic ellipsometry
4. Data analysis methodology of ellipsometric data
5. *OpenFilters* design software

# Spectrophotometry



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- When designing an optical filter, we are interested in determining :
  - the optical properties of the substrate ( $n, k$ );
  - the optical properties of each material that is used ( $n, k$ ) and their deposition rate, typically through the deposition of a film with thickness  $X$  in a given time.
- We also want to know :
  - if the deposited layers are homogeneous;
  - if the layers diffuse light;
  - if they possess a surface or interface roughness.
- Finally, once the filter is fabricated, we want to check its performance.

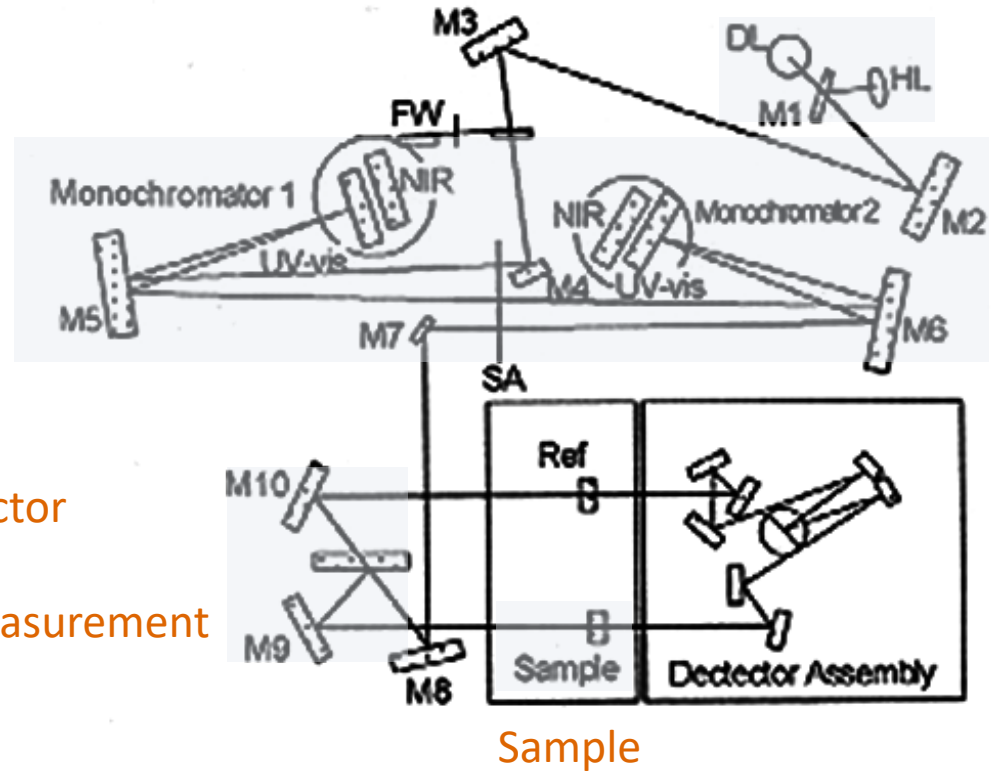
All these steps can be performed by using spectrophotometry.



# Spectrophotometer

- A spectrophotometer is a system which allows one to measure the transmission and reflection of a sample.
- The measurements may be specular, diffusive or total.

Lamp sources



Monochromators

Beam selector

Reference and measurement

Sample

# Substrate characterization

How can we obtain the substrate's refractive index using its transmission spectrum?

One can demonstrate that the transmission in the low absorption region is given by:

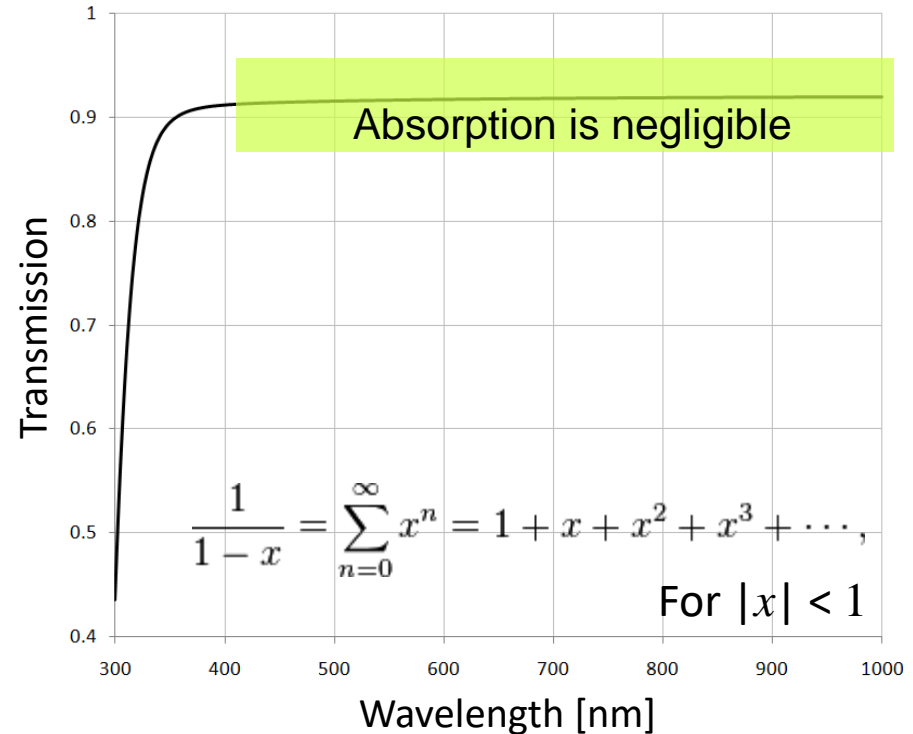
$$T = \frac{(1-R)^2}{(1-R^2)}$$

with  $R$ , the reflection coefficient in intensity at an interface:

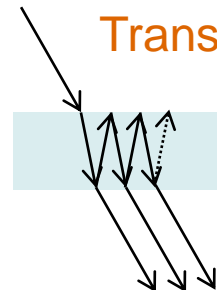
$$R = \left( \frac{n-1}{n+1} \right)^2$$

We can then demonstrate:

$$n = \left( \frac{1 + \left( \frac{1-T}{1+T} \right)^{1/2}}{1 - \left( \frac{1-T}{1+T} \right)^{1/2}} \right)$$



Transmission of a B270 substrate



$$T = (1-R)^2 (1 + R^2 + R^4 + \dots + R^{2i})$$

where  $i$  is an integer



# Substrate characterization (2)

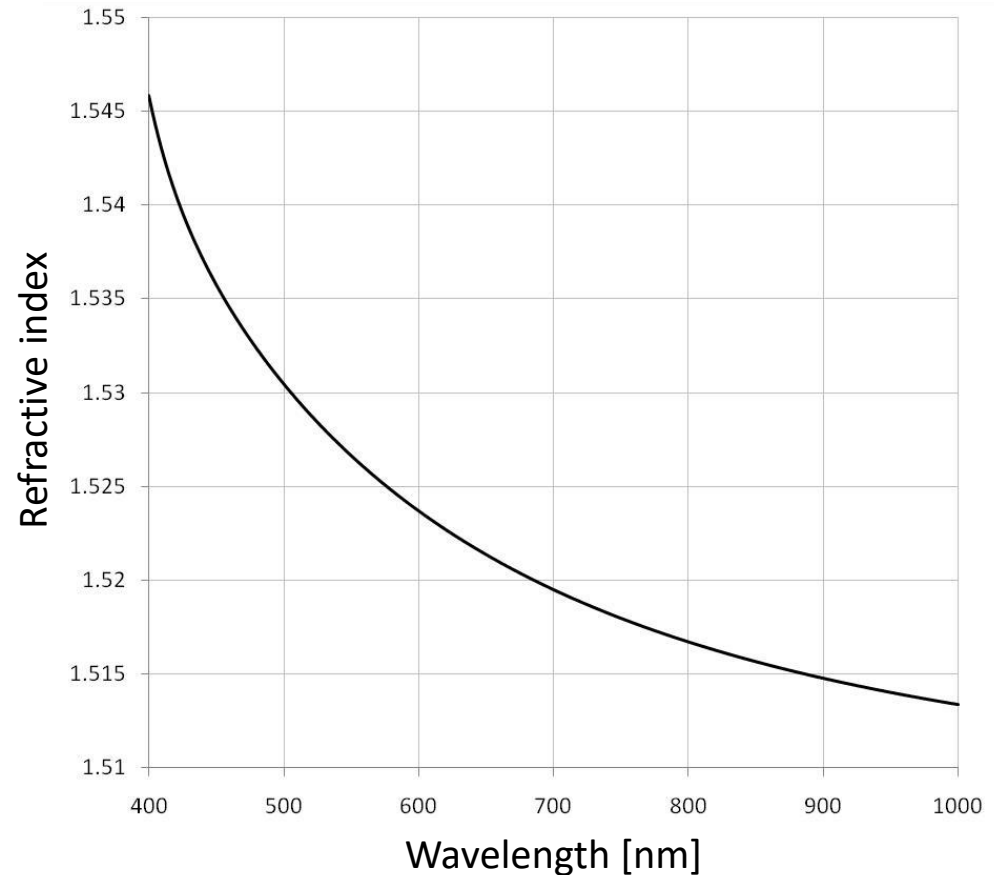


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Using the previously defined equation and the transmission spectrum of a B270 glass substrate, we obtain the following refractive index dispersion curve.

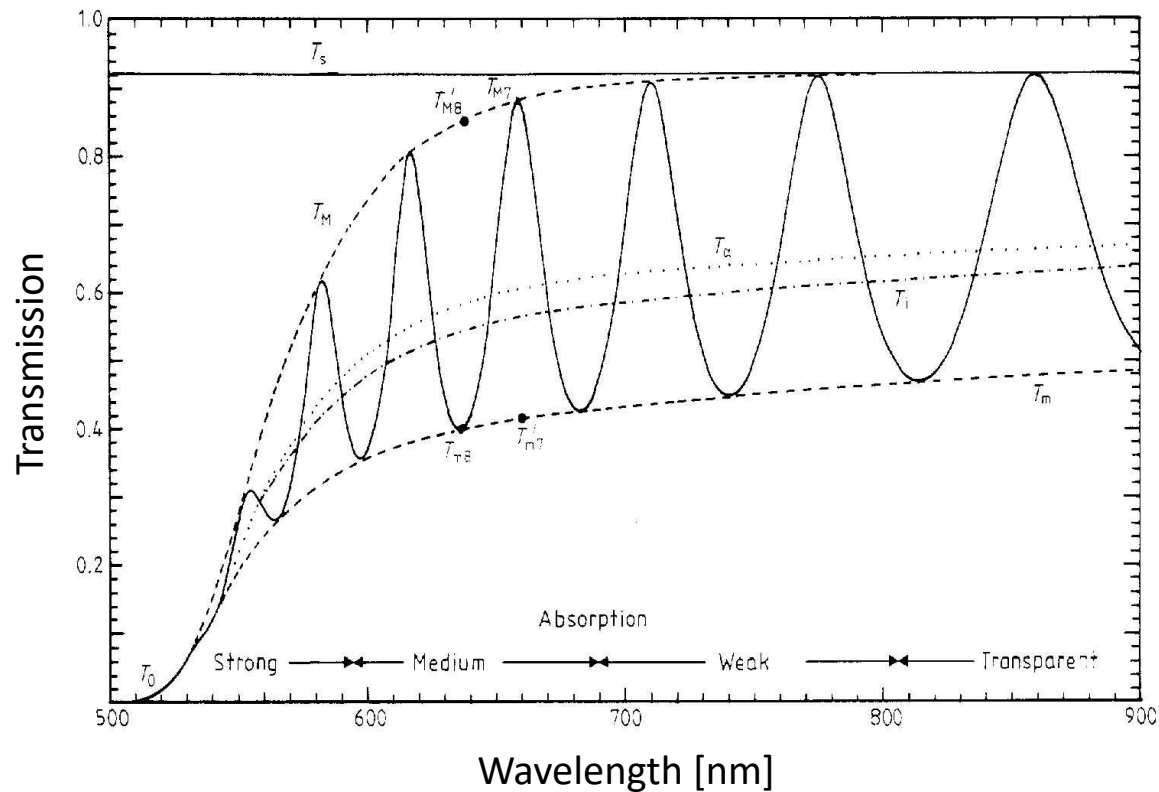


Refractive index of a B270 glass substrate.



# The envelope method

When studying the transmission or reflection spectra of a thin film on a substrate, one can observe a series of minima and maxima. The traced lines which join all these points together are termed envelopes.



R. Swanepoel, « Determination of the thickness and optical constants of amorphous silicon », *J. Phys. E.*, vol. 16, 1983, p. 1214–1223.



# The envelope method (2)

- Let us demonstrate the envelope method in the simplest case: a non-absorbing thin film on a semi-infinite substrate.
- We have seen during the previous course that:

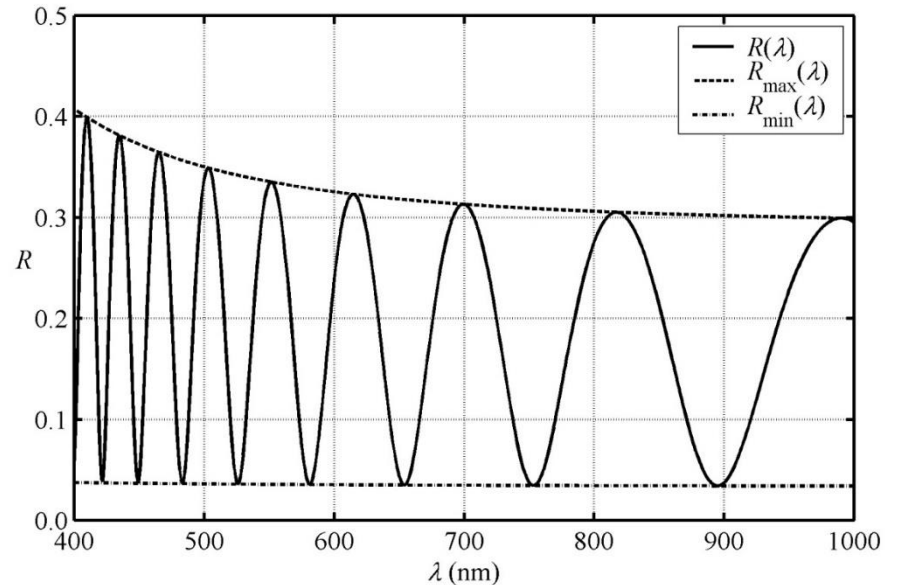
$$r = \frac{r_{2 \rightarrow 1} + r_{1 \rightarrow 0} e^{i2\phi}}{1 + r_{2 \rightarrow 1} r_{1 \rightarrow 0} e^{i2\phi}}$$

where

$$r_{2 \rightarrow 1} = \frac{n_2 - n_1}{n_2 + n_1},$$

$$r_{1 \rightarrow 0} = \frac{n_1 - n_0}{n_1 + n_0} \quad \text{and}$$

$$\phi = 2\pi \frac{N_1 d}{\lambda}.$$



Reflection of a thin film on a glass substrate.



# The envelope method (3)

- The minimum envelope  $R_{\min}(\lambda)$  corresponds to half-wave points, i.e. the coating is « absent », and therefore, we obtain the reflection spectrum of the substrate.
- The maximum envelope corresponds to quarter-wave points. At these points,  $\exp(i2\phi) = -1$ , and consequently

$$r_{\max} = \frac{r_{2 \rightarrow 1} - r_{1 \rightarrow 0}}{1 - r_{2 \rightarrow 1} r_{1 \rightarrow 0}}$$

- By inserting the expressions for the various interface reflections, we obtain:

$$\begin{aligned} r_{\max} &= \frac{\frac{n_2 - n_1}{n_2 + n_1} - \frac{n_1 - n_0}{n_1 + n_0}}{1 - \frac{n_2 - n_1}{n_2 + n_1} \frac{n_1 - n_0}{n_1 + n_0}} = \frac{\frac{(n_2 - n_1)(n_1 + n_0)}{(n_2 + n_1)(n_1 + n_0)} - \frac{(n_2 + n_1)(n_1 - n_0)}{(n_2 + n_1)(n_1 + n_0)}}{\frac{(n_2 + n_1)(n_1 + n_0)}{(n_2 + n_1)(n_1 + n_0)} - \frac{(n_2 - n_1)(n_1 - n_0)}{(n_2 + n_1)(n_1 + n_0)}} \\ &= \frac{(n_2 - n_1)(n_1 + n_0) - (n_2 + n_1)(n_1 - n_0)}{(n_2 + n_1)(n_1 + n_0) - (n_2 - n_1)(n_1 - n_0)} = \frac{2n_2n_0 - 2n_1^2}{2n_2n_0 + 2n_1^2} \end{aligned}$$





# The envelope method (4)

- To obtain the reflection in intensity, we multiply  $r$  by its complex conjugate:

$$R_{\max} = r_{\max} r_{\max}^* = \left( \frac{2n_2 n_0 - 2n_1^2}{2n_2 n_0 + 2n_1^2} \right)^2 = \frac{4n_2^2 n_0^2 - 8n_2 n_1^2 n_0 + 4n_1^4}{4n_2^2 n_0^2 + 8n_2 n_1^2 n_0 + 4n_1^4}$$

which allows us to obtain:

$$\begin{aligned} R_{\max} (4n_2^2 n_0^2 + 8n_2 n_1^2 n_0 + 4n_1^4) &= 4n_2^2 n_0^2 - 8n_2 n_1^2 n_0 + 4n_1^4 \\ \rightarrow 4n_2^2 n_0^2 (R_{\max} - 1) + 8n_2 n_0 (R_{\max} + 1)n_1^2 + 4(R_{\max} - 1)n_1^4 &= 0 \end{aligned}$$

which is a quadratic equation in  $n_1^2$ .



# The envelope method (5)

- The solution of this quadratic equation is:

$$\begin{aligned}
 n_1^2 &= \frac{-8n_2n_0(R_{\max} + 1) \pm \sqrt{64(R_{\max} + 1)^2 n_2^2 n_0^2 - 64(R_{\max} - 1)^2 n_2^2 n_0^2}}{8(R_{\max} - 1)} \\
 &= \frac{-8n_2n_0(R_{\max} + 1) \pm 8n_2n_0 \sqrt{(R_{\max} + 1)^2 - (R_{\max} - 1)^2}}{8(R_{\max} - 1)} \\
 &= n_2n_0 \frac{-(R_{\max} + 1) \pm \sqrt{(R_{\max} + 1)^2 - \frac{(R_{\max} + 1)^2 (R_{\max} - 1)^2}{(R_{\max} + 1)^2}}}{(R_{\max} - 1)} \\
 &= n_2n_0 \frac{-(R_{\max} + 1) \pm (R_{\max} + 1) \sqrt{1 - \frac{(R_{\max} - 1)^2}{(R_{\max} + 1)^2}}}{(R_{\max} - 1)} \\
 &= n_2n_0 \left( \frac{-(R_{\max} + 1)}{(R_{\max} - 1)} \pm \frac{(R_{\max} + 1)}{(R_{\max} - 1)} \sqrt{1 - \frac{(R_{\max} - 1)^2}{(R_{\max} + 1)^2}} \right) = n_2n_0 \left( \frac{-(R_{\max} + 1)}{(R_{\max} - 1)} \pm \sqrt{\frac{(R_{\max} + 1)^2}{(R_{\max} - 1)^2} - 1} \right)
 \end{aligned}$$

where the sign is chosen in order to obtain a physically acceptable solution.

# The envelope method (6)



- To determine the thickness, one only needs to observe the spacing between the oscillations.
- It is possible to demonstrate that if  $\lambda_1$  and  $\lambda_2$  are the positions of two consecutive maxima or minima, the thickness of the coating is given by:

$$d = \frac{\lambda_1 \lambda_2}{2(\lambda_1 n_1(\lambda_2) - \lambda_2 n_1(\lambda_1))}$$

where  $n_1(\lambda_1)$  and  $n_1(\lambda_2)$  are the refractive indices at  $\lambda_1$  and  $\lambda_2$ .

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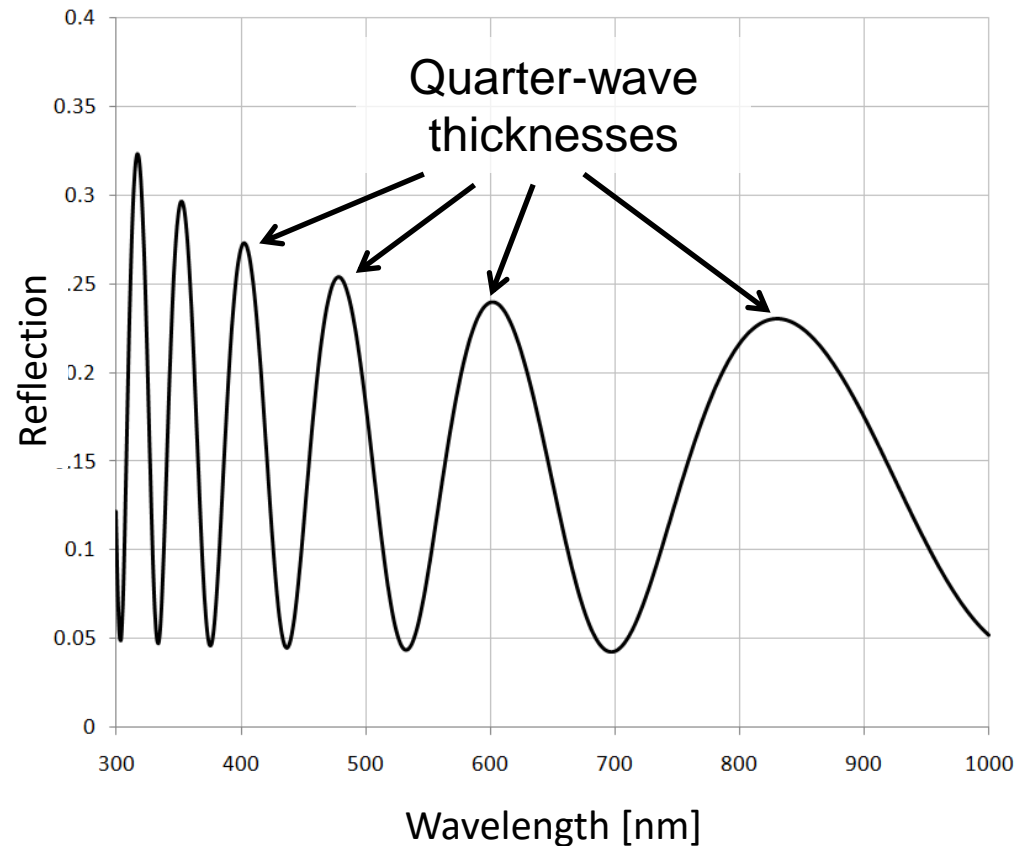
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# The envelope method – an example

Here is an example of the reflection spectrum of a  $\text{Ta}_2\text{O}_5$  thin film deposited by DIBS (dual ion beam sputtering) on a semi-infinite B270 substrate.



Reflection of a  $\text{Ta}_2\text{O}_5$  thin film on a semi-infinite substrate of B270 glass.

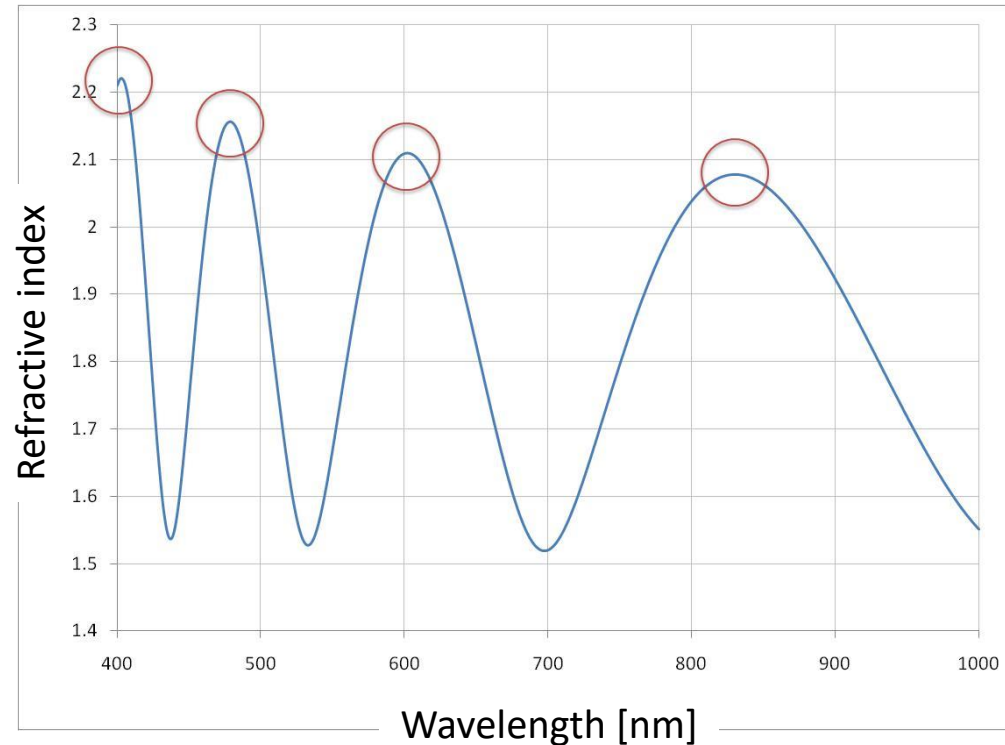


# The envelope method – an example (2)

Here is what we obtain if we calculate  $n$  using the reflection spectrum.

The maxima (red circles) correspond to the refractive index of the  $\text{Ta}_2\text{O}_5$  thin film.

Calculating the thickness by using the indices at 602 nm and 830 nm, we obtain 499.6 nm.



Refractive index calculation for a  $\text{Ta}_2\text{O}_5$  thin film using the envelope method.

This method works very well for most conditions; however, there are instances when it does not apply as well. For example, when the deposited film is too thin or when its refractive index is too close to the one of the substrate.



# The envelope method – the impact of inhomogeneities

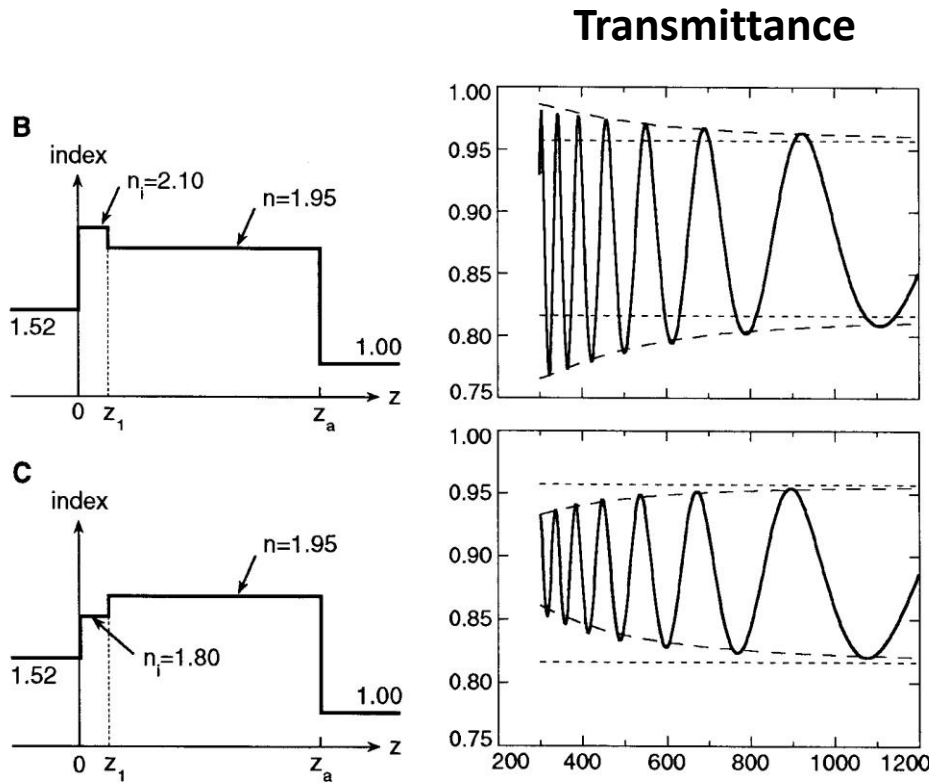


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Chapter 8B

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## Envelopes of transmittance

$$\Gamma^{\max}(\lambda) = \frac{4n_a n_s}{(n_a + n_s)^2} \left\{ 1 + \frac{\eta}{n} \frac{n_s - n_a}{n_s + n_a} \left[ 1 - \cos \frac{4\pi n z_1}{\lambda} \right] \right\},$$

$$\Gamma^{\min}(\lambda) = \frac{4n_a n_s n^2}{(n_a n_s + n^2)^2} \left\{ 1 - \frac{\eta}{n} \frac{n^2 - n_a n_s}{n^2 + n_a n_s} \left[ 1 - \cos \frac{4\pi n z_1}{\lambda} \right] \right\},$$

where  $n = n_o$  and  $\eta = n_i - n$

A. V. Tikhonravov, M. K. Trubetskov, Brian T. Sullivan et J. A. Dobrowolski, « Influence of small inhomogeneities on the spectral characteristics of single thin films », *Appl. Opt.*, vol. 36, 1997, p. 7188-7199



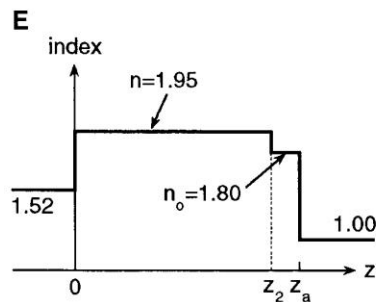
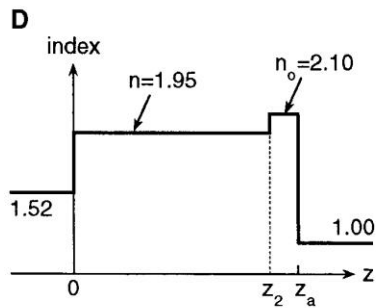
# The envelope method – the impact of inhomogeneities (2)



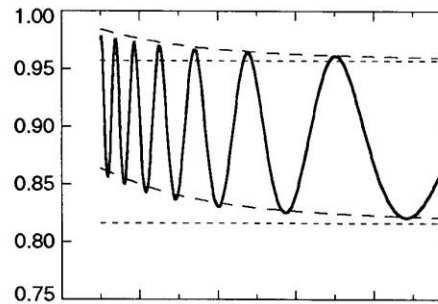
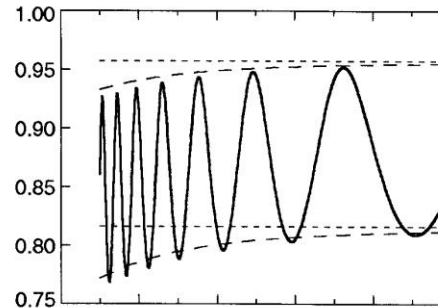
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**Transmittance**



**Envelopes of transmittance**

$$\Gamma^{\max}(\lambda) = \frac{4n_a n_s}{(n_a + n_s)^2} \left\{ 1 + \frac{\eta n_s - n_a}{n n_s + n_a} \left[ \cos \frac{4\pi n(z_a - z_2)}{\lambda} - 1 \right] \right\},$$

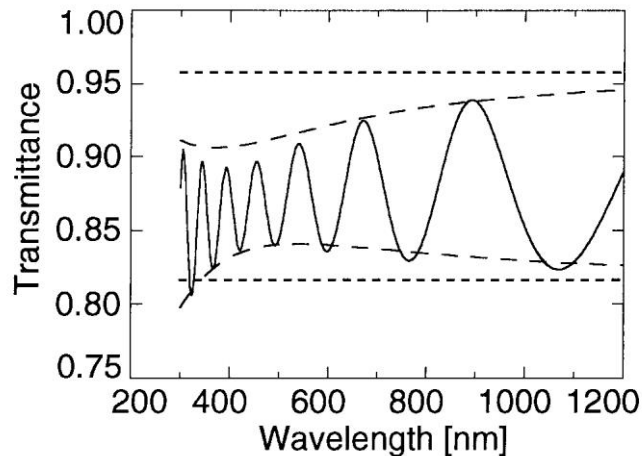
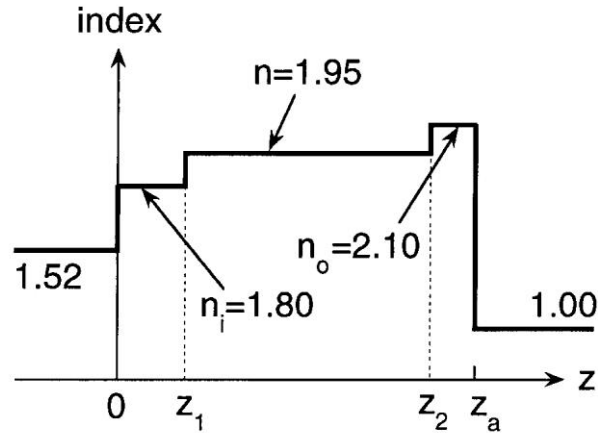
$$\Gamma^{\min}(\lambda) = \frac{4n_a n_s n^2}{(n_a n_s + n^2)^2} \left\{ 1 + \frac{\eta n^2 - n_a n_s}{n n^2 + n_a n_s} \left[ \cos \frac{4\pi n(z_a - z_2)}{\lambda} - 1 \right] \right\},$$

where  $n = n_i$  and  $\eta = n_o - n$

A. V. Tikhonravov, M. K. Trubetskov, Brian T. Sullivan et J. A. Dobrowolski, « Influence of small inhomogeneities on the spectral characteristics of single thin films », *Appl. Opt.*, vol. 36, 1997, p. 7188-7199



# The envelope method – the impact of inhomogeneities (3)

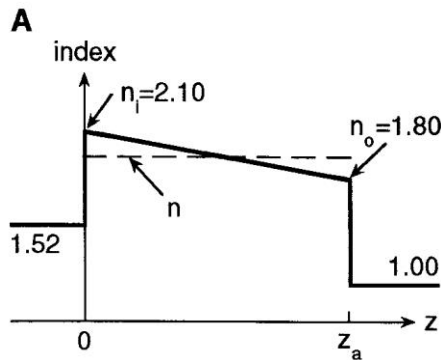


A. V. Tikhonravov, M. K. Trubetskov, Brian T. Sullivan et J. A. Dobrowolski, « Influence of small inhomogeneities on the spectral characteristics of single thin films », *Appl. Opt.*, vol. 36, 1997, p. 7188-7199

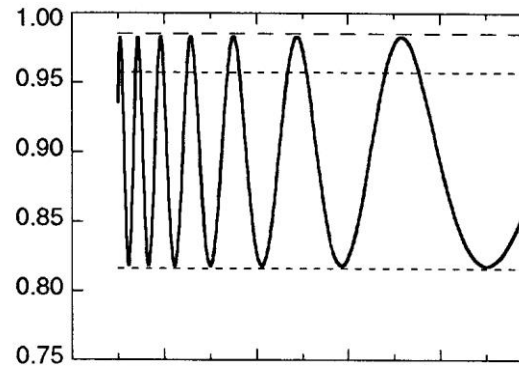




# The envelope method – the impact of inhomogeneities (4)



Transmittance



Envelopes of transmittance

$$\Gamma^{\max}(\lambda) = \frac{4n_a n_s}{(n_a + n_s)^2} \left\{ 1 + \frac{\eta n_s - n_a}{n n_s + n_a} \right\},$$

$$\Gamma^{\min}(\lambda) = \frac{4n_a n_s n^2}{(n_a n_s + n^2)^2},$$

where  $n = \frac{1}{2}(n_o + n_i)$  and  $\eta = n_i - n_o$

A. V. Tikhonravov, M. K. Trubetskov, Brian T. Sullivan et J. A. Dobrowolski, « Influence of small inhomogeneities on the spectral characteristics of single thin films », *Appl. Opt.*, vol. 36, 1997, p. 7188-7199



# The envelope method – additional references



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- J. C. Manifacier, J. Gasiot and J. P. Fillard, « A simple method for the determination of the optical constants  $n$ ,  $k$  and the thickness of a weakly absorbing thin film », *J. Phys. E*, vol. 9, 1976, p. 1002–1004
- R. Swanepoel, « Determination of the thickness and optical constants of amorphous silicon », *J. Phys. E*, vol. 16, 1983, p. 1214–1223.
- Dirk Poelman and Philippe Frederic Smet, « Methods for the determination of the optical constants of thin films from single transmission measurements : a critical review », *J. Phys. D*, vol. 36, 2003, p. 1850–1857.
- A. V. Tikhonravov, M. K. Trubetskov, Brian T. Sullivan and J. A. Dobrowolski, « Influence of small inhomogeneities on the spectral characteristics of single thin films », *Appl. Opt.*, vol. 36, 1997, p. 7188-7199.
- Daniel Poitras, «Admittance diagrams of accidental and premeditated optical inhomogeneities in coatings », *Appl. Opt.*, vol. 41, 2002, p. 4671-4679.

# Spectrophotometry – Analysis methodology

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- The envelope method allows one to gain insight into the structure of a thin film.
- In reality, we rarely evaluate the optical properties of a thin film « manually ». Indeed, this is typically done using software.
- The data analysis is performed very similarly as the refinement process during designing an optical filter. In fact, one must first specify an appropriate starting design, and the software then optimizes the model's parameters to better represent the measured data.
- Contrary to a design process, all the solutions given by the software are not necessarily “real”. One must, therefore, have a good idea of the layers which are present before starting the analysis.

# Spectrophotometry – Analysis methodology (2)



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- To determine the optical properties of a thin film, one should:
  1. Measure the transmission or reflection spectrum of the substrate.
  2. Measure the transmission or reflection spectrum of the thin film on the same substrate.
  3. Determine the approximate refractive index of the film using the envelope method.
  4. Evaluate the approximate thickness of the film using the spectrum.
  5. Adjust the thickness and refractive index using a data analysis software.
  6. Add absorption, non-homogeneity, interfaces, etc. to the model.
  
- Spectrophotometry allows the measurement of two data points per wavelength ( $T$  et  $R$ ), which, therefore, allows for the determination of two parameters. If one supposes that the refractive index does not vary or that it varies according to a dispersion function, one can determine a larger number of parameters.



# Polarization of light



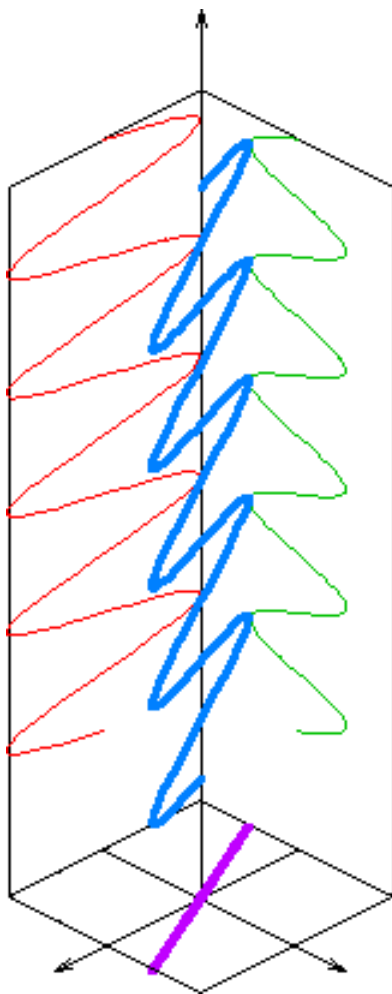
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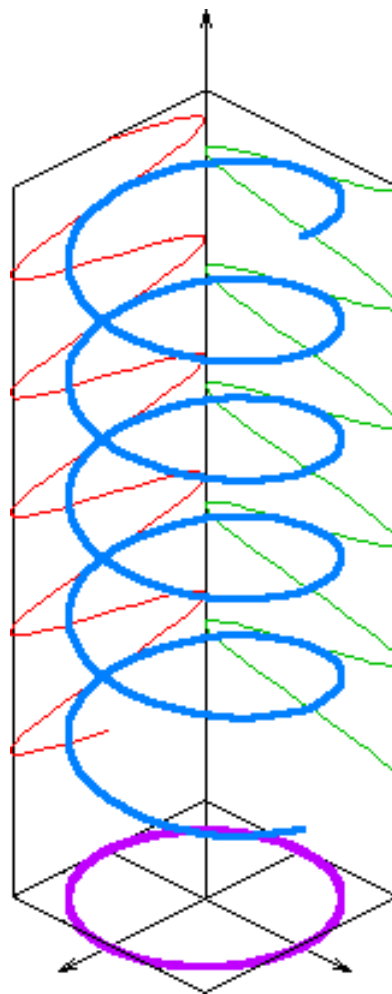
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Stéphane Larouche  
Department of  
Engineering Physics  
2022

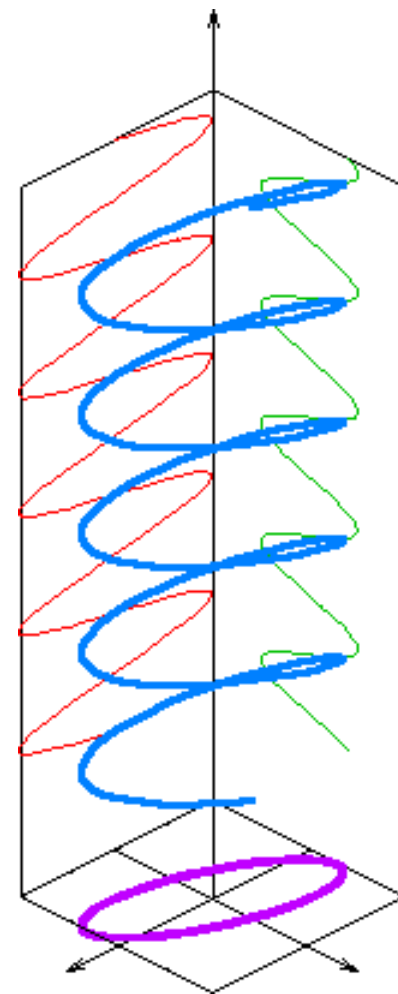
Linear



Circular



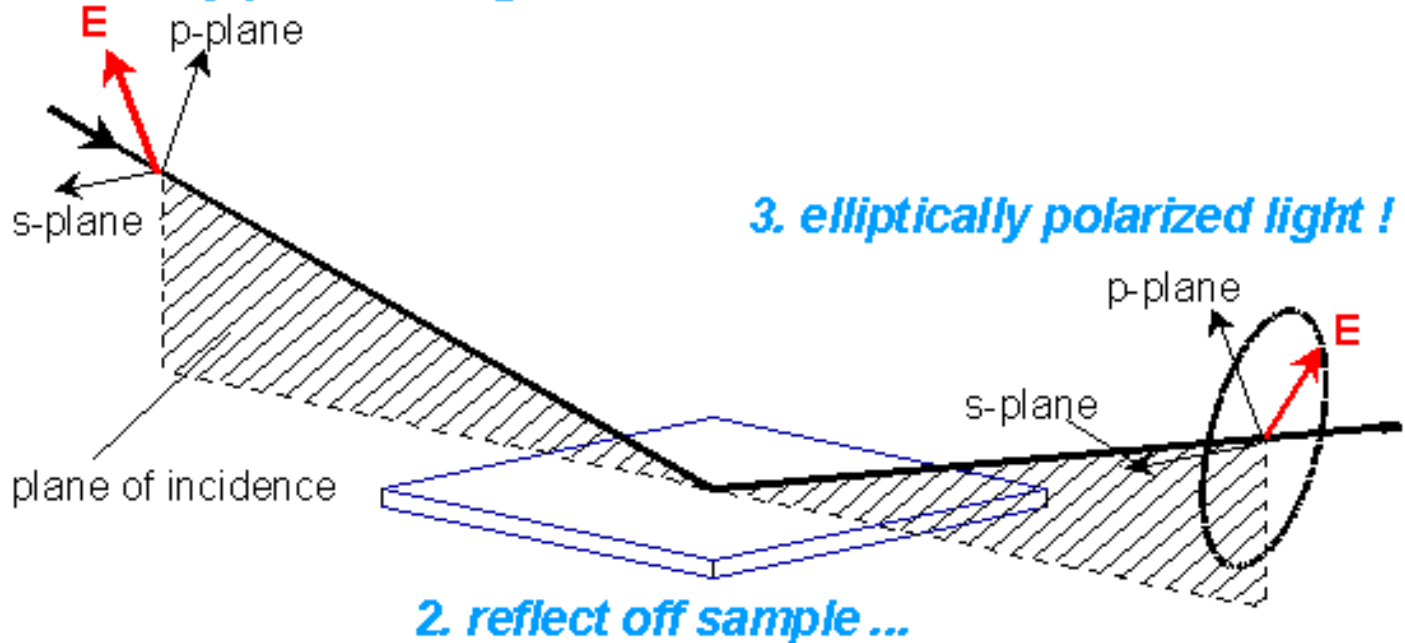
Elliptical





# Why is it called ellipsometry?

**1. linearly polarized light ...**



J. A. Woollam Co., « Ellipsometry tutorial », [www.jawoollam.com](http://www.jawoollam.com).



# $\Psi$ and $\Delta$ : definitions

- We know that the reflection coefficients in amplitude,  $r_s$  and  $r_p$ , and in intensity differ depending on polarization. We can define:

$$\rho = \frac{-r_p}{r_s}$$

the ratio between two polarizations. We can express this ratio in polar coordinates to obtain:

$$\rho = \frac{-r_p}{r_s} = \tan \Psi e^{i\Delta}$$

where the two ellipsometric variables  $\Psi$  et  $\Delta$  are defined. The variable  $\Psi$  corresponds to the amplitude ratio between the two polarizations, and  $\Delta$  corresponds to the phase shift between the two.

- The – sign allows one to obtain  $\Delta = 180^\circ$  at normal incidence. It is simply an ellipsometric convention which many forget to mention. For more information on the different ellipsometric conventions consult Rolf H. Muller, « Definitions and conventions in ellipsometry », *Surf. Sci.*, vol. 16, 1969, p. 14–33.



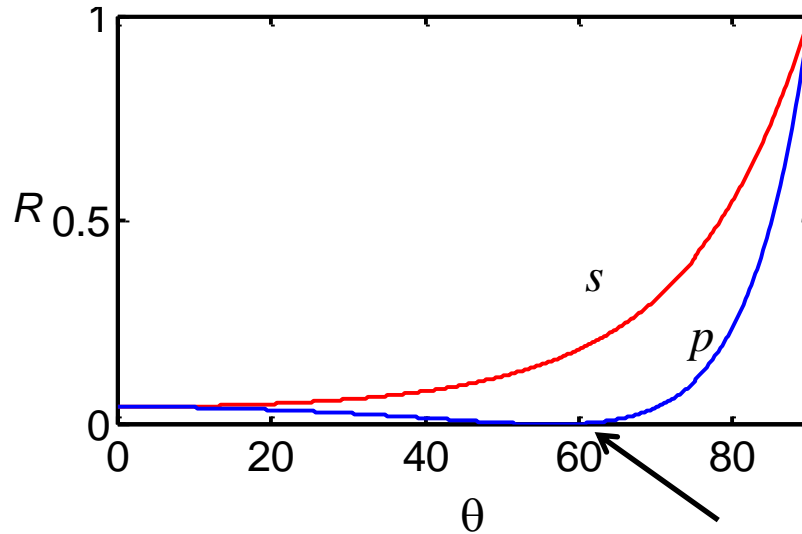
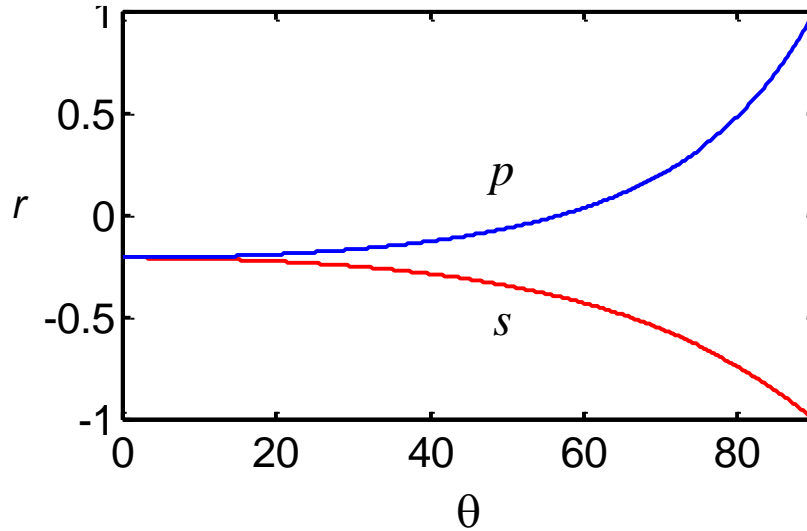
# $\Psi$ and $\Delta$ for a semi-infinite glass substrate ( $n = 1.52$ )



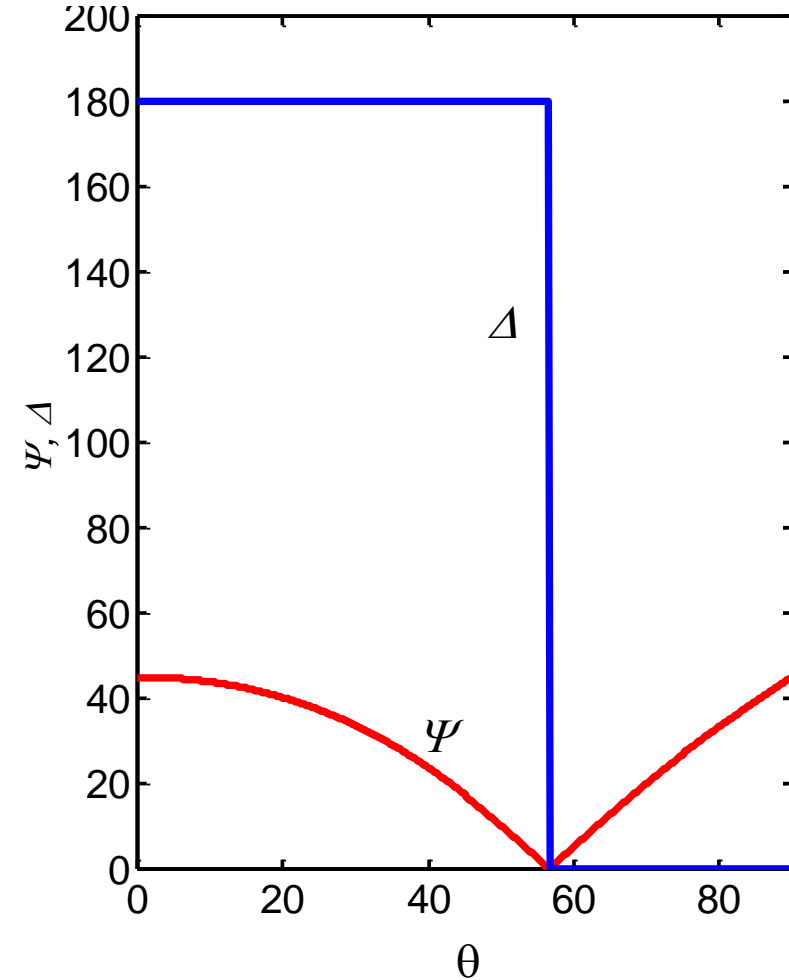
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Brewster angle

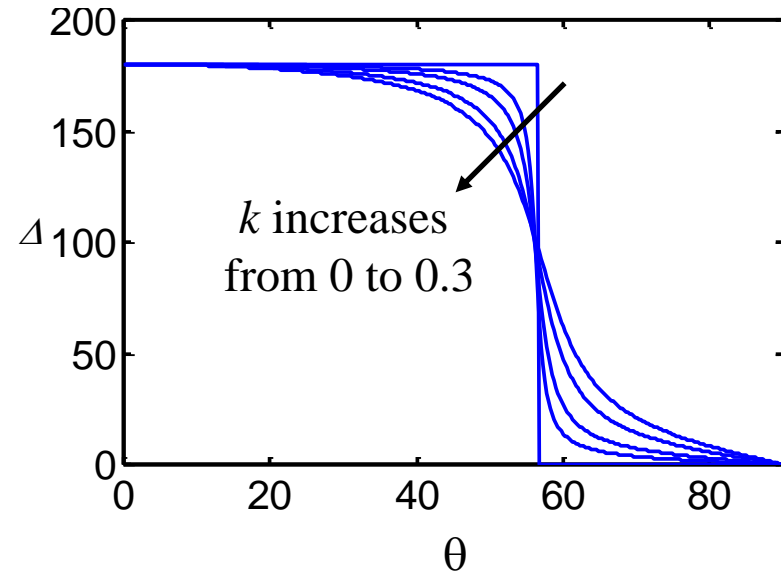
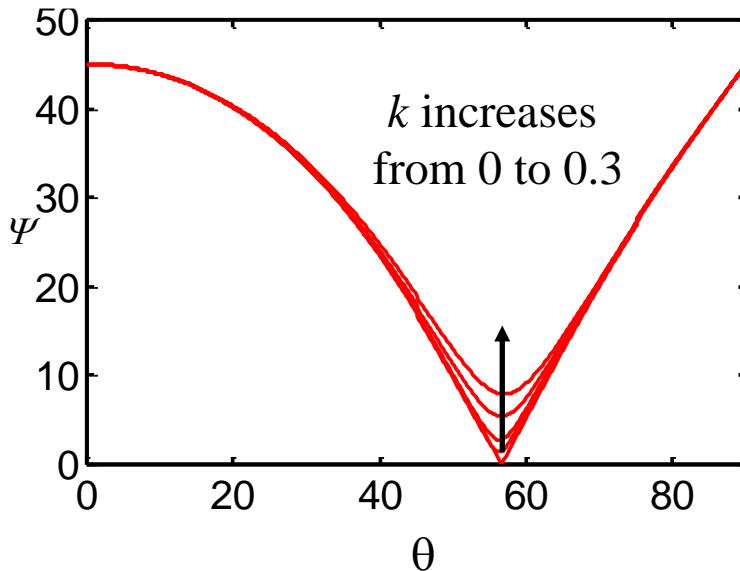






# The impact of absorption

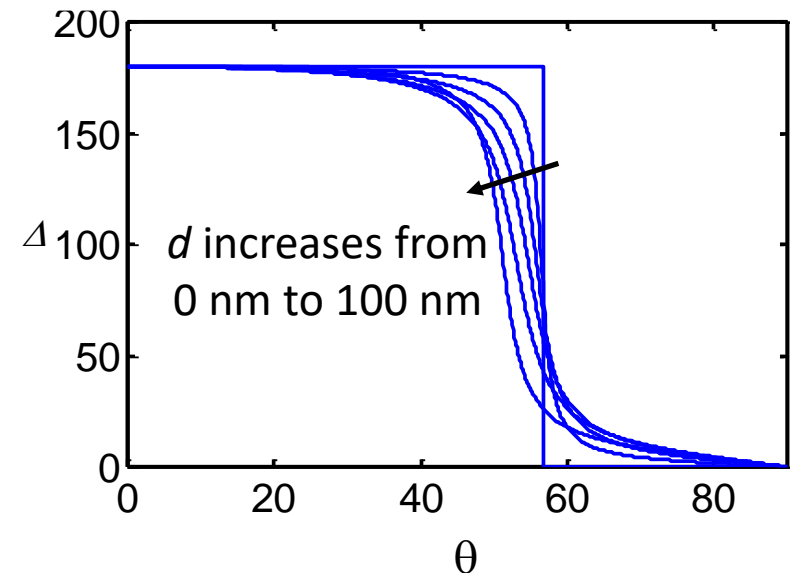
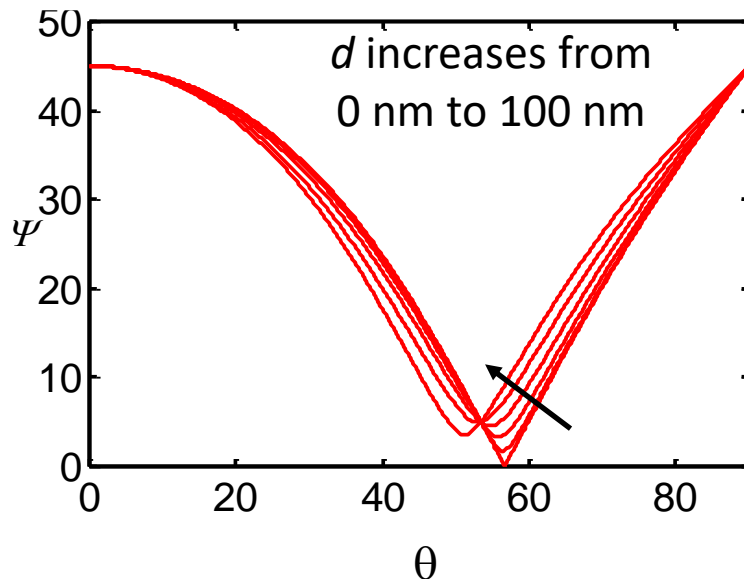
- When the substrate is absorbing, the  $p$ -polarized reflection no longer drops to zero and consequently neither does  $\Psi$ . In addition,  $r_s$  and  $r_p$  are no longer real,  $\Delta$  can thus take on values different than  $0^\circ$  and  $180^\circ$ .
- The sensitivity of  $\Psi$  is the highest at the Brewster angle whereas  $\Delta$  is most sensitive near the Brewster angle.





# The addition of a $\text{MgF}_2$ thin film

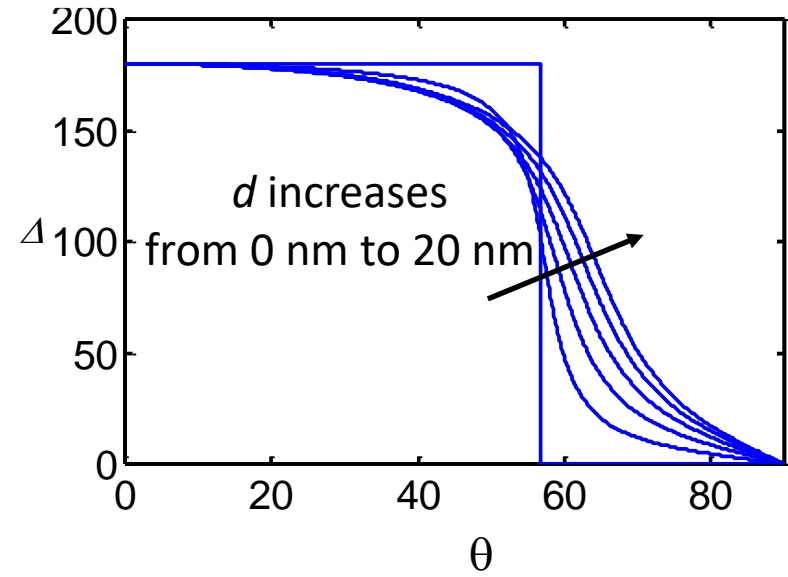
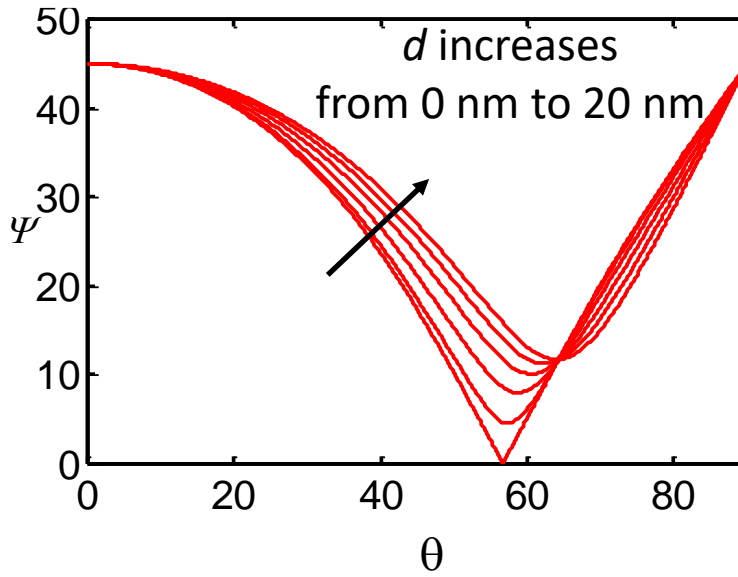
- The addition of a thin film of  $\text{MgF}_2$  ( $n = 1.38$ ) with a lower refractive index than the substrate displaces the minimum in  $\Psi$  towards lower angles.
- Since we are no longer in the case of a reflection from a single interface, we can no longer speak of a Brewster angle but of a pseudo-Brewster angle.





# The addition of a $\text{TiO}_2$ thin film

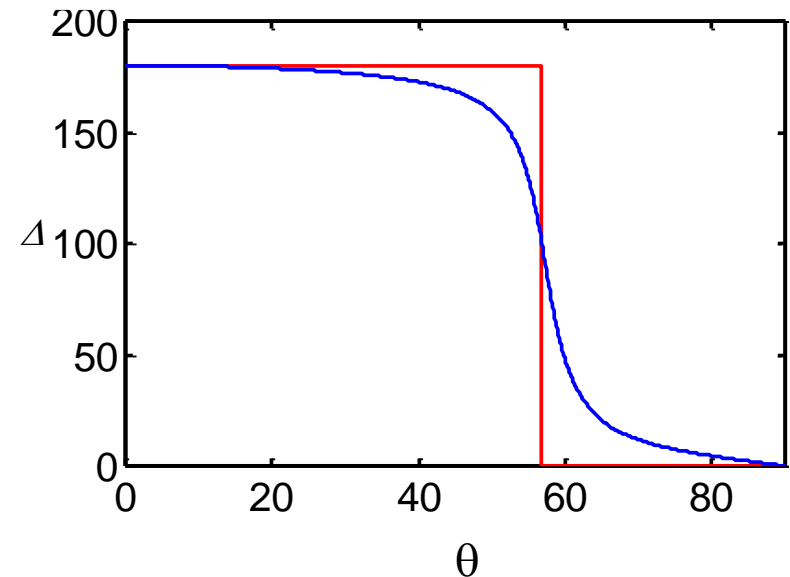
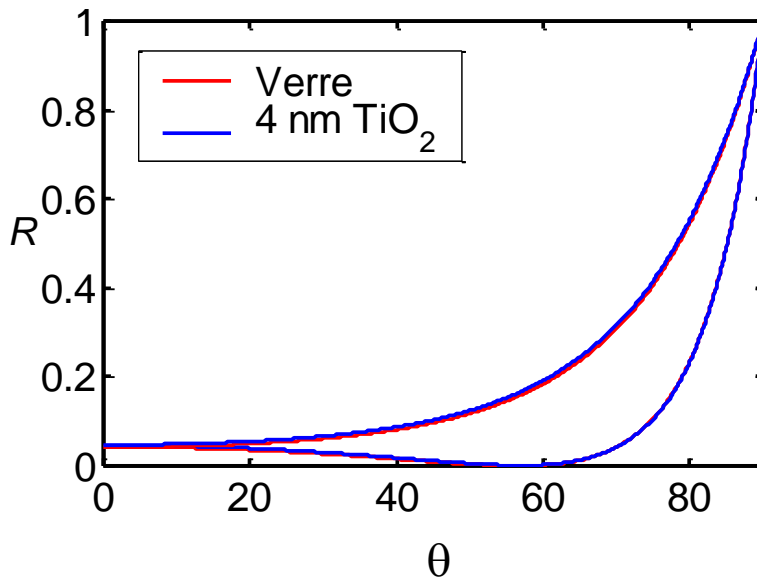
- The addition of a thin film of  $\text{TiO}_2$  ( $n = 2.35$ ) with a higher refractive index than the substrate displaces the minimum in  $\Psi$  towards higher angles (higher pseudo-Brewster angle).
- Since there is a larger optical contrast between the glass substrate and  $\text{TiO}_2$ , the addition of a very thin film generates significant changes in  $\Psi$  and  $\Delta$ .





# Comparing spectrophotometry with ellipsometry

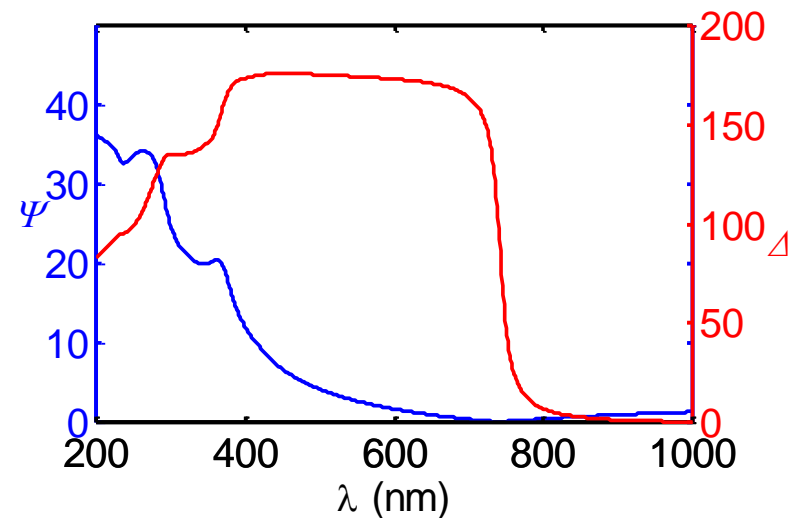
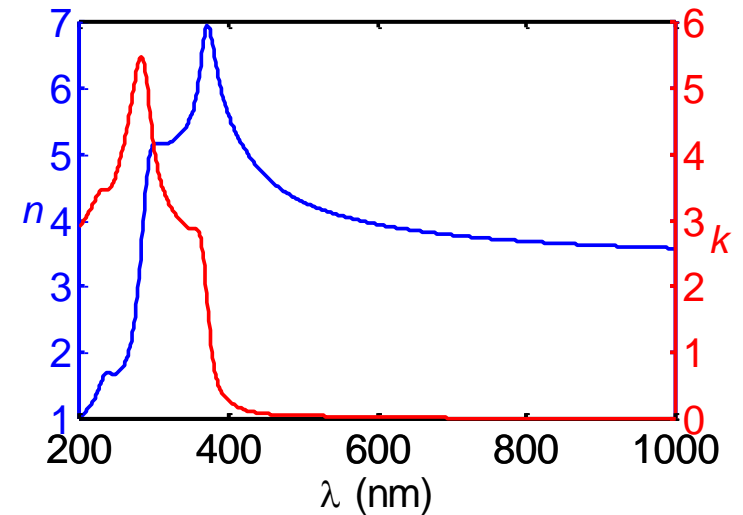
- While the reflection spectrum practically does not change following the addition of a 4-nm-thick  $\text{TiO}_2$  film, the ellipsometric spectrum (in particular  $\Delta$ ) changes quite substantially.
- Ellipsometry is, therefore, advantageous for thin and ultra-thin films.





# Spectroscopic ellipsometry on Si

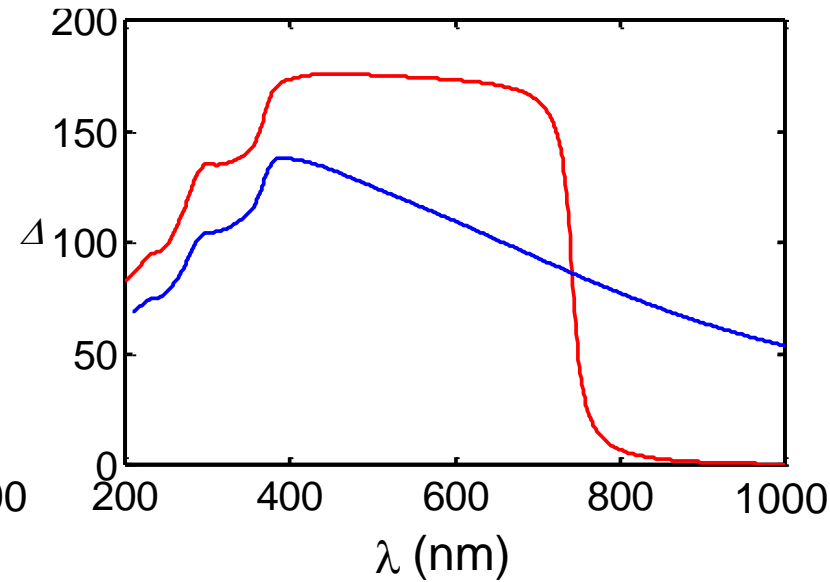
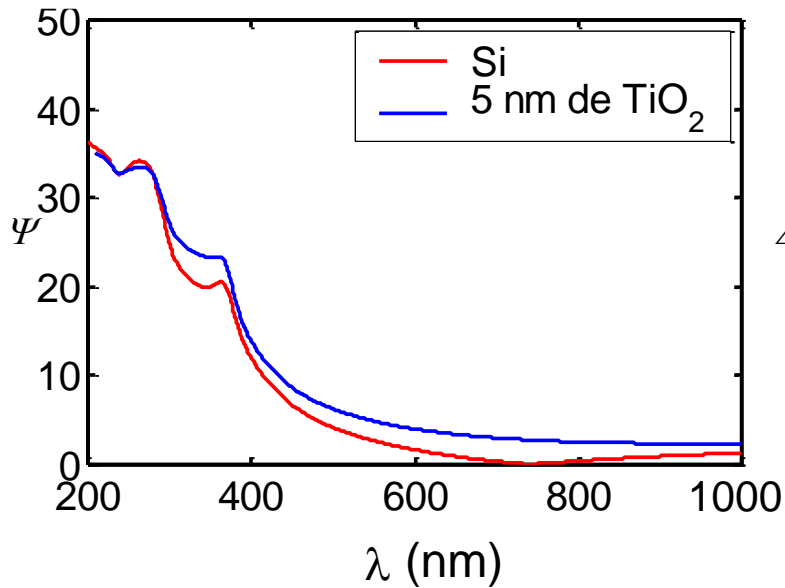
- The refractive index of a material changes with wavelength and so does the Brewster angle.
- If the absorption coefficient varies as well, the widening of the transition of  $\Delta$  will also vary vs. wavelength.
- Taking a measurement close to the Brewster angle ( $75^\circ$  in the present case) results in large  $\Psi$  and  $\Delta$  differences as a function of wavelength; this allows for a precise assessment of  $n$  and  $k$ .
- Note that we are using two variables to determine two others, the system is, therefore, well defined.





# Spectroscopic ellipsometry (5 nm of $\text{TiO}_2$ on Si)

- Since we are close to the Brewster angle, we can observe quite large variations.



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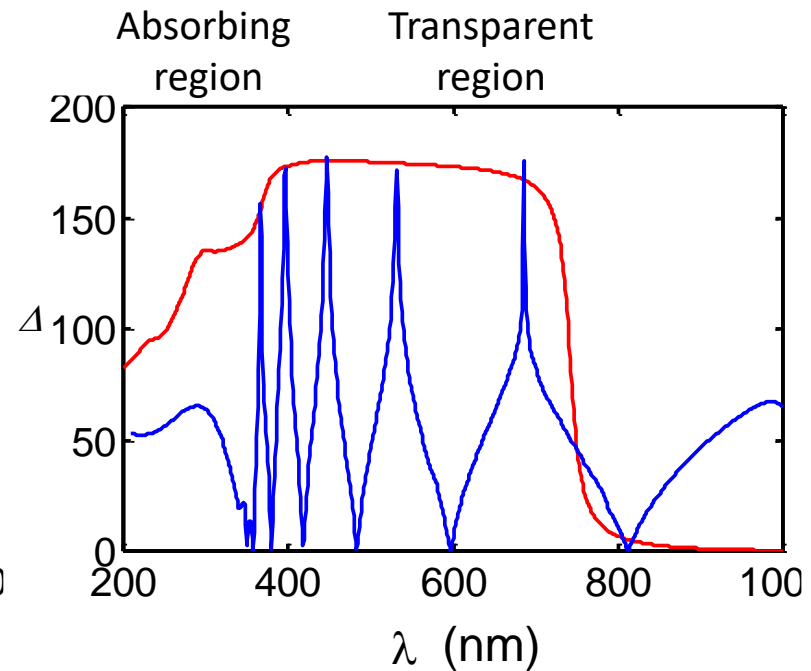
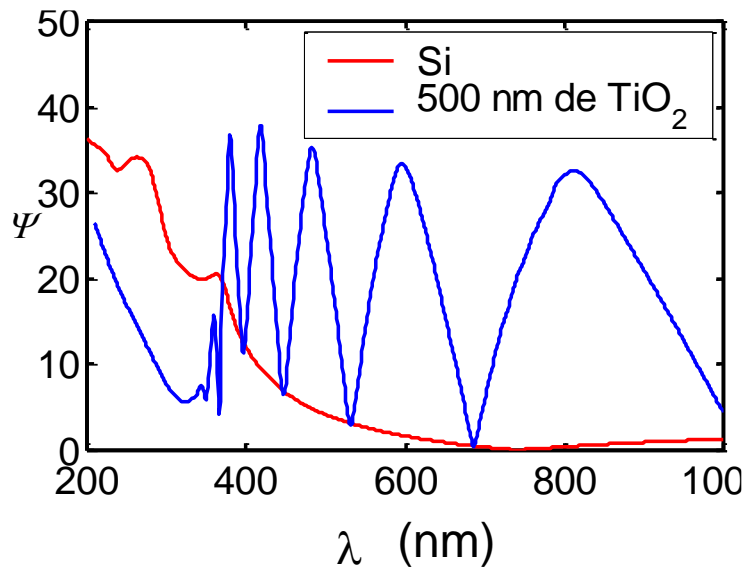
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# Spectroscopic ellipsometry (500 nm of TiO<sub>2</sub> on Si)

- With the addition of a thick TiO<sub>2</sub> film, one can observe the appearance of interference fringes, just like in transmission and reflection spectra.
- In the transparent region, one can use a method similar to the envelope method to determine the refractive index and thickness of the film.
- In the absorbing region, we may consider the film as semi-infinite to determine its refractive index.

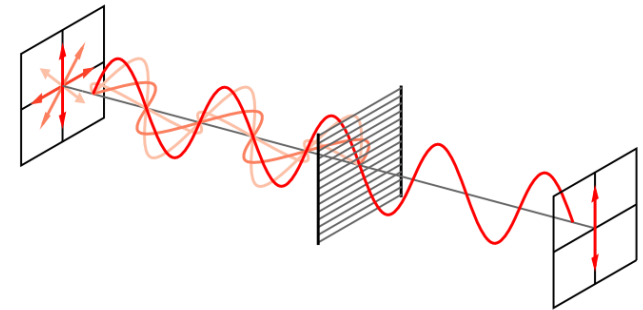




# Polarizers and compensators

## Polarizers

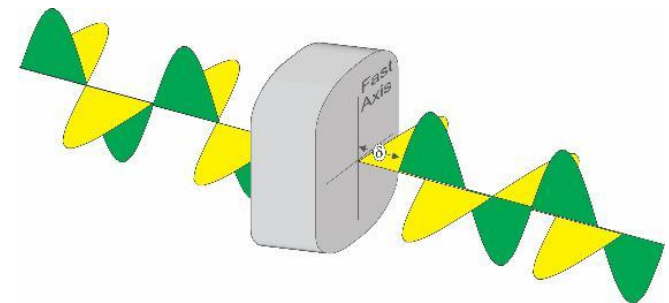
- They transmit light of a single polarization.
- It is impossible to differentiate a polarizer positioned at an angle  $\theta$  from a polarizer at an angle  $\theta + 180^\circ$ .
- The transmitted light is always linearly polarized.



[www.wikipedia.org](http://www.wikipedia.org)

## Compensators

- They introduce a phase shift of  $90^\circ$  between both components of the incident light beam according to a slow and fast axis.
- They thus allow one to change the polarization state of the incident light. A light beam linearly polarized at an angle of  $45^\circ$  will be transformed into a circularly polarized beam.



J. A. Woollam Co., « Compensators »,  
[www.jawoollam.com](http://www.jawoollam.com).





# Null ellipsometer

## Advantages

- Configuration with the most accurate and precise results.
- One can determine  $\Psi$  and  $\Delta$  over their full range.

## Disadvantages

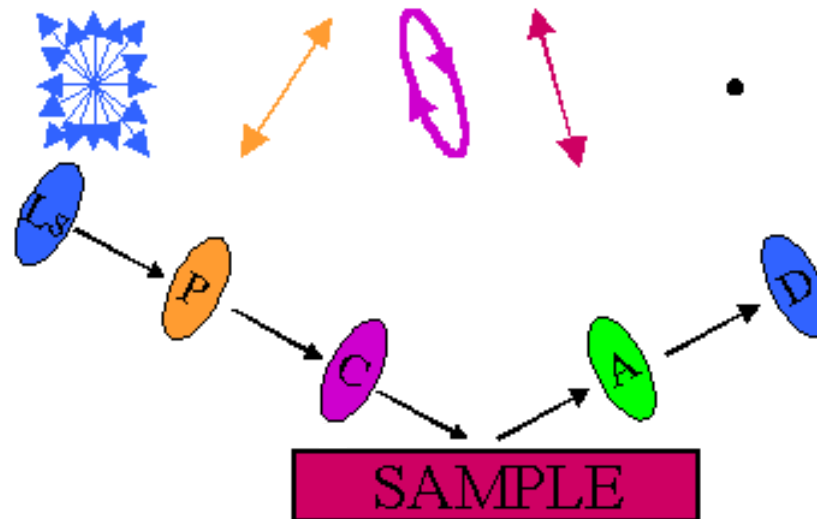
- Measurements are long and hard to automate.



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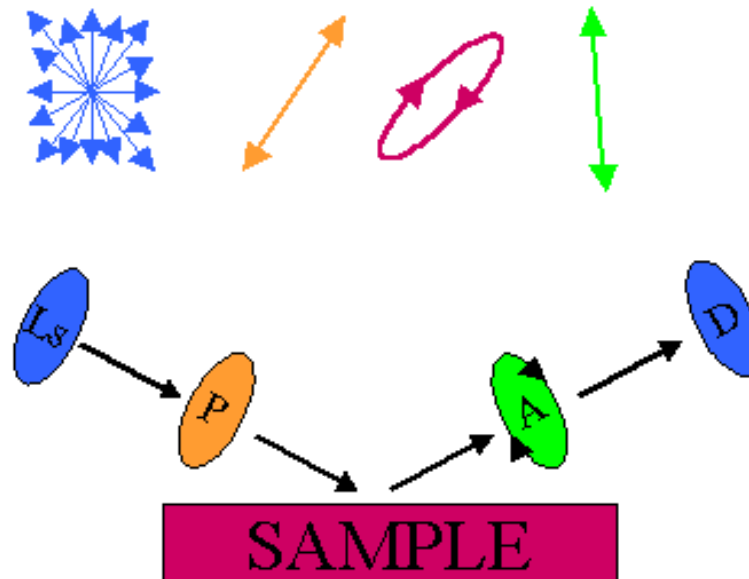
# Rotating analyser ellipsometer, RAE

## Advantages

- There are only two polarizers.
- It is easy to fabricate polarizers with very high extinction coefficients.
- It is easy to fabricate polarizers with a very low spectral dependence.

## Disadvantages

- Impossible to distinguish  $\Delta$  over its full range from  $0^\circ$  to  $360^\circ$ .  $\Delta$  is limited between  $0^\circ$  and  $180^\circ$ .
- Very low sensitivity when  $\Delta = 0^\circ$  or  $\Delta = 180^\circ$ .
- The detector's sensitivity to changes in polarization can lead to errors.





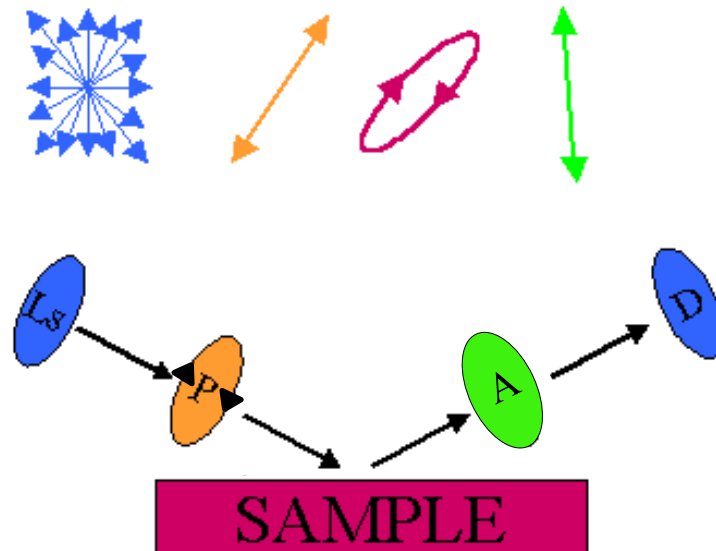
# Rotating polarizer ellipsometer, RPE

## Advantages

- There are only two polarizers.
- It is easy to fabricate polarizers with very high extinction coefficients.
- It is easy to fabricate polarizers with a very low spectral dependence.

## Disadvantages

- Impossible to distinguish  $\Delta$  over its full range from  $0^\circ$  to  $360^\circ$ .  $\Delta$  is limited to between  $0^\circ$  and  $180^\circ$ .
- Very low sensitivity when  $\Delta = 0^\circ$  or  $\Delta = 180^\circ$ .
- The light source's polarization can lead to errors.





# Rotating compensator ellipsometer, RCE

## Advantages

- No region of low sensitivity.
- One can distinguish  $\Delta$  from  $0^\circ$  to  $360^\circ$ .
- It is possible to perform generalized ellipsometry measurements.

## Disadvantages

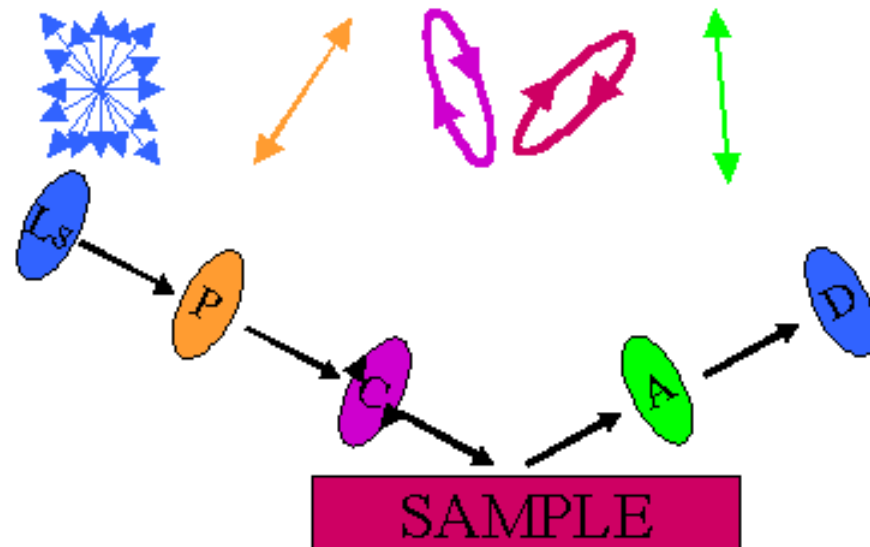
- It is difficult and costly to fabricate a compensator which performs ideally over a large wavelength spectrum.



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# Jones vector

- To study how an ellipsometer functions, we can express the electric field by:

$$\vec{E} = \begin{bmatrix} \tilde{E}_p \\ \tilde{E}_s \end{bmatrix} = \begin{bmatrix} E_p e^{i\phi_s} \\ E_s e^{i\phi_p} \end{bmatrix} \equiv \begin{bmatrix} E_p \\ E_s e^{i(\phi_p - \phi_s)} \end{bmatrix}$$

where  $E_p$  and  $E_s$  are the  $p$  and  $s$  components of the electric field; the final relation is possible since we are only interested in the relative phase difference between both polarizations.

- The linear polarizations  $p$  and  $s$  are therefore expressed by:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

whereas left and right circularly polarized light are expressed by:

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} \quad \text{and} \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}.$$





# Jones matrices

- The various optical elements which modify the electric field can be represented by a 2 x 2 matrix.
- Polarizing filters oriented in  $p$  or in  $s$  can be respectively represented by:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

whereas a polarizer oriented at  $\pm 45^\circ$  will be represented by:

$$\frac{1}{2} \begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix}.$$

- The reflection upon an isotropic sample is represented by:

$$\begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix}.$$

- Finally, the rotation of an element by an angle  $\theta$  is represented by:

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$



# Jones formalism for a RAE

- Using the Jones formalism, the electric field at the detector of a RAE is given by:

$$\begin{bmatrix} E_p \\ E_s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos \theta_A & \sin \theta_A \\ -\sin \theta_A & \cos \theta_A \end{bmatrix} \begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix} \begin{bmatrix} \cos \theta_P & -\sin \theta_P \\ \sin \theta_P & \cos \theta_P \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

*H. Fujiwara, Spectroscopic Ellipsometry: Principles and Applications, 2007.*

where  $\theta_P$  and  $\theta_A$  are the angles at which are positioned the polarizer and analyzer.

- The intensity which will be measured is therefore given by isolating the term  $\theta_A$  since it is the analyzer which is rotating:

$$I = E_p^2 + E_s^2 \propto 1 + \frac{|\rho|^2 - \tan^2 \theta_P}{|\rho|^2 + \tan^2 \theta_P} \cos(2\theta_A) + \frac{2\Re(\rho)\tan \theta_P}{|\rho|^2 + \tan^2 \theta_P} \sin(2\theta_A)$$

where  $\rho = -r_p/r_s = \tan \Psi e^{i\Delta}$ . This equation can also be expressed as:

$$I \propto 1 + \alpha \cos(2\theta_A) + \beta \sin(2\theta_A)$$

where

$$\alpha = \frac{|\rho|^2 - \tan^2 \theta_P}{|\rho|^2 + \tan^2 \theta_P} \quad \text{and} \quad \beta = \frac{2\Re(\rho)\tan \theta_P}{|\rho|^2 + \tan^2 \theta_P}.$$

# Jones formalism for a RAE (2)

- Since the analyzer is rotating,

$$\theta_A(t) = 2\pi f_A t + \theta.$$

- Finally, using Fourier analysis, it is possible to demonstrate that:

$$\tan \Psi = \sqrt{\frac{1+\alpha}{1-\alpha}} |\tan \theta_P|$$

and

$$\cos \Delta = \frac{\beta}{\sqrt{1-\alpha^2}} \frac{\tan \theta_P}{|\tan \theta_P|}.$$



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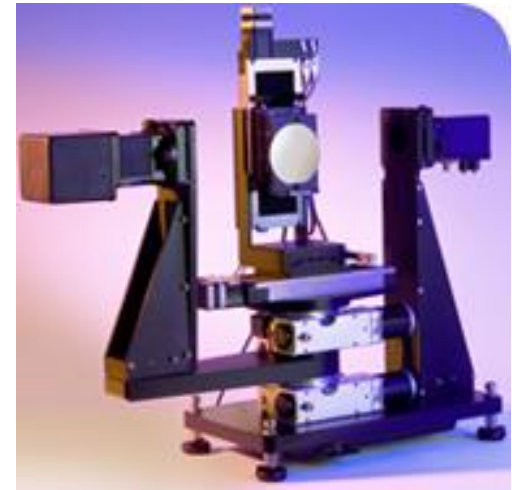
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# The RC2 by J.A. Woollam Co.

- One of the ellipsometers which the FCSEL possesses is a RC2-XI (Dual Rotating Compensators) from the company J.A. Woollam Co.
- It is a variable angle spectroscopic ellipsometer with two rotating compensators.
- It possesses a high accuracy over the whole range of  $\Psi$  and  $\Delta$ . It offers the complete characterization of the Jones matrix (anisotropic samples) and the Mueller matrix (anisotropic and depolarizing samples).
- The measured wavelength range is from 210 nm to 2500 nm.
- The measurements are very fast due to the presence of a CCD detector.
- This allows for various types of measurements: rapid  $xy$  mapping, as a function of temperature using a heat cell, during electrochemical testing, etc.

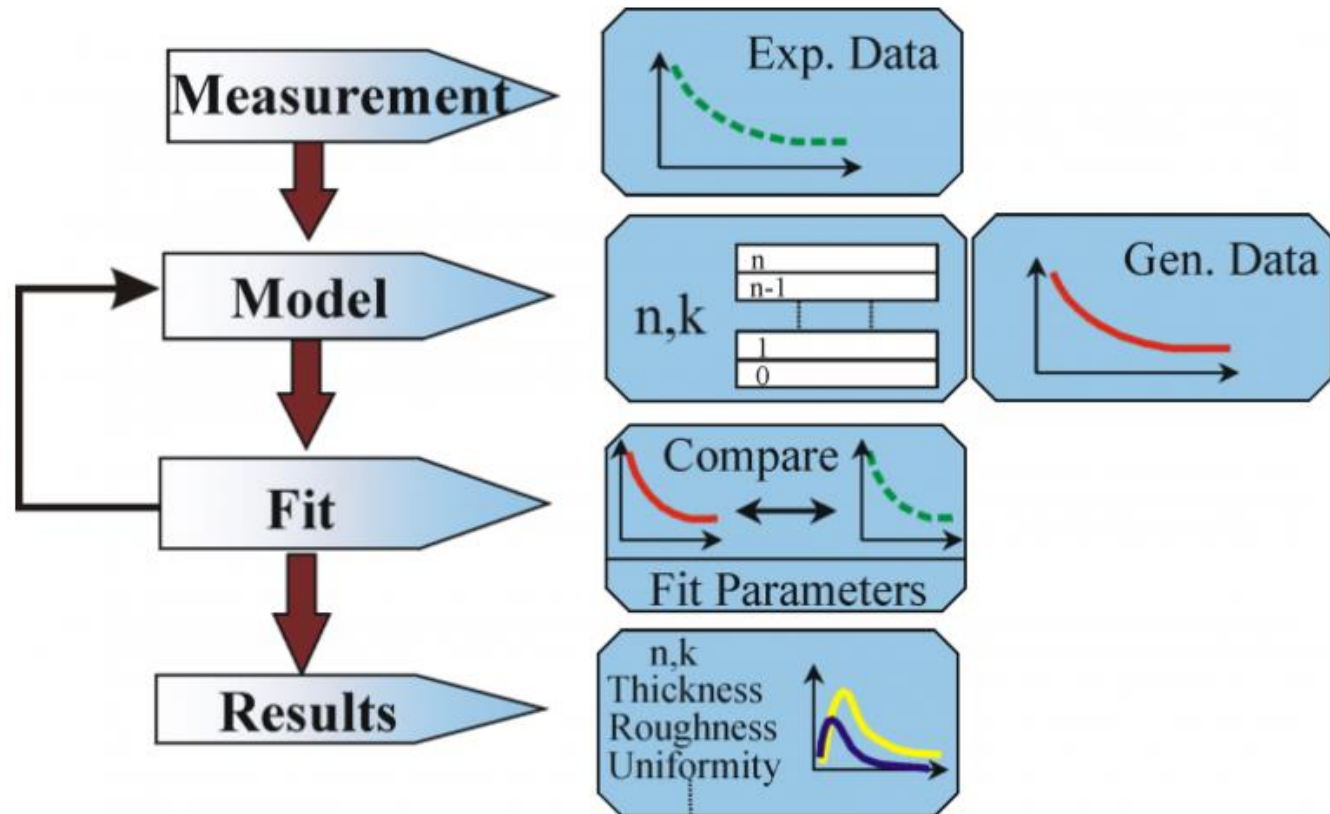


www.jawoollam.com

$$\begin{bmatrix} r_{pp} & r_{sp} \\ r_{ps} & r_{ss} \end{bmatrix} \quad \text{Jones matrix}$$

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \quad \text{Mueller matrix}$$

# Analysis process



J. A. Woollam Co., « Ellipsometry tutorial », [www.jawoollam.com](http://www.jawoollam.com).



# Analysis process – The model

- The analysis of the ellipsometric data requires the development of an optical model. This model contains:
  - the substrate's optical properties and thickness;
  - the total number of layers;
    - their individual thicknesses;
    - their optical properties;
  - the presence of surface roughness or of interfaces;
  - the presence of inhomogeneities;
  - the presence of non-uniformity;
  - the influence of the substrate's backside reflection;
  - ...
- If specific parameters are already known, they can be set; for the other parameters, it is preferable to supply reasonable starting values before optimizing the model.



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# Analysis process - MSE

- The optimization process of the model's parameters will minimize the mean square error (MSE) between the model and experimental data:

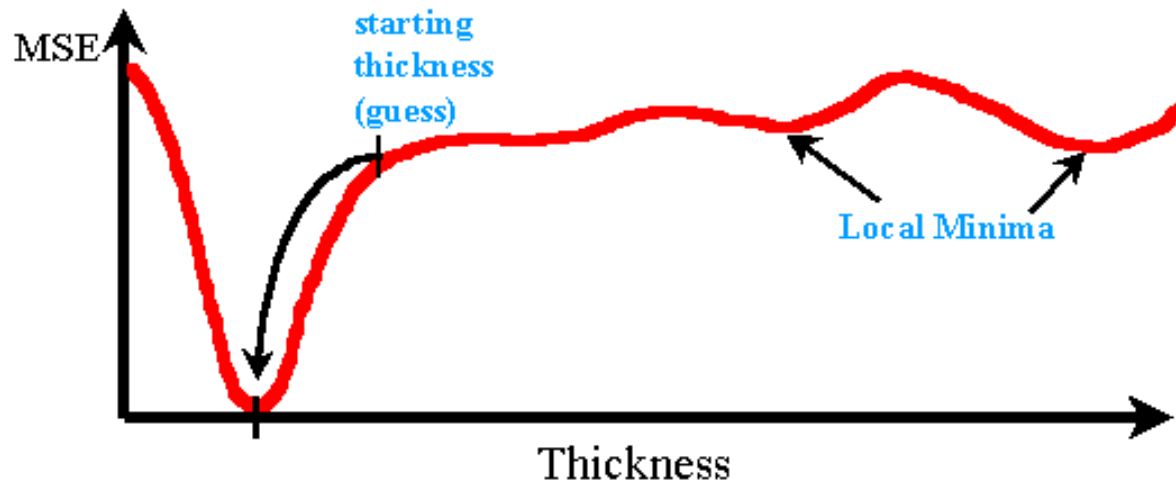
$$MSE = \frac{1}{2N - M} \sum_{i=1}^N \left[ \left( \frac{\psi_i^{\text{mod}} - \psi_i^{\text{exp}}}{\sigma_{\psi,i}^{\text{exp}}} \right)^2 + \left( \frac{\Delta_i^{\text{mod}} - \Delta_i^{\text{exp}}}{\sigma_{\Delta,i}^{\text{exp}}} \right)^2 \right] = \frac{1}{2N - M} \chi^2,$$

where  $2N$  is the number of data points,  $M$  is the number of model parameters, the exponent *exp* identifies the experimental data, the exponent *mod* identifies the model and  $\sigma$  is the experimental uncertainty of each data point.



# Choice of the appropriate initial values

- The Levenberg-Marquart algorithm, used to optimize the model parameters, is a local optimization algorithm; it will, therefore, find the minimum MSE closest to the initial values (or one of the closest).
- To obtain accurate parameter values, it is important to pay attention to the initial parameter values.



J. A. Woollam Co., « Ellipsometry tutorial », [www.jawoollam.com](http://www.jawoollam.com).

# Choice of the appropriate initial values – An example



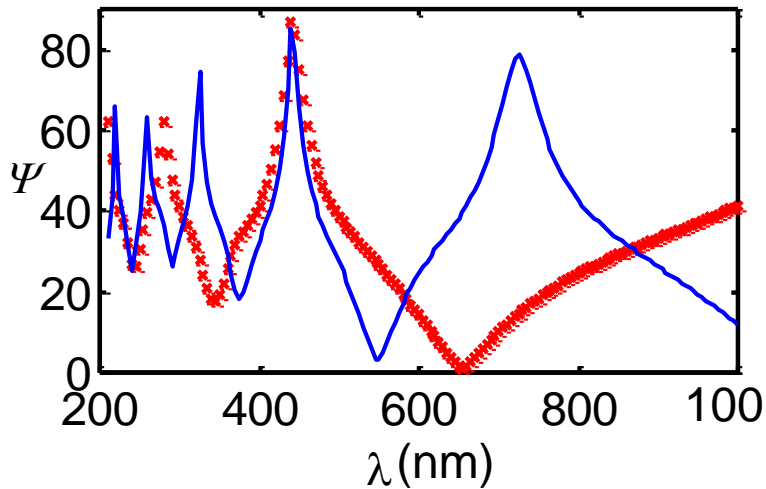
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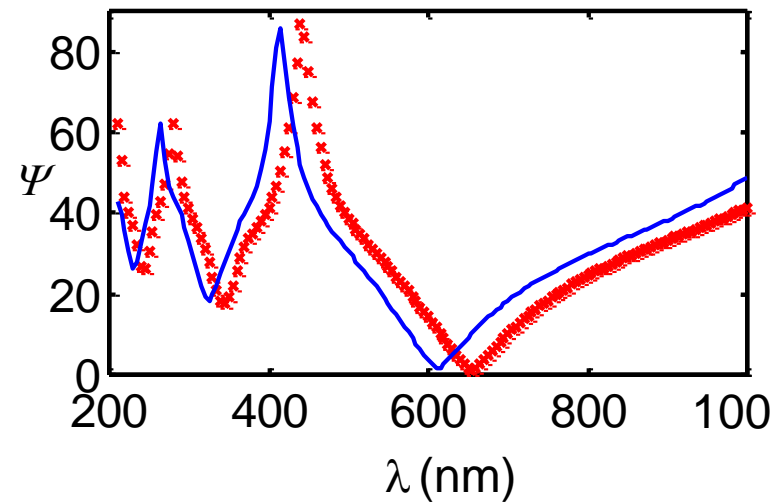
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Stéphane Larouche  
Department of  
Engineering Physics  
2022

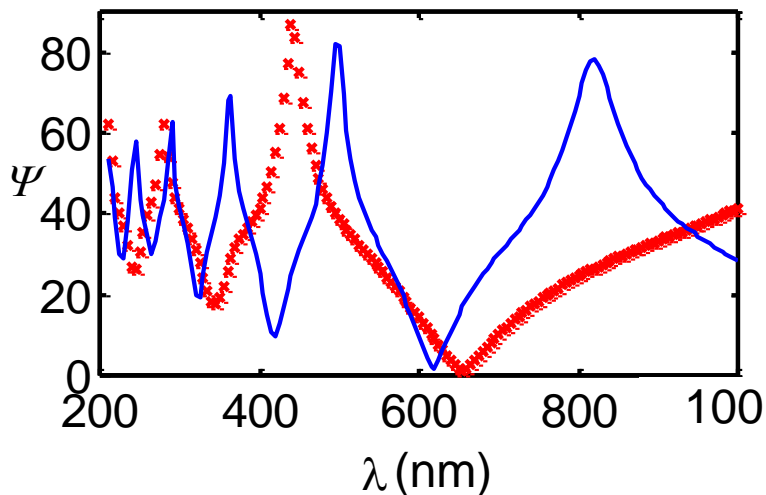
Bad starting point



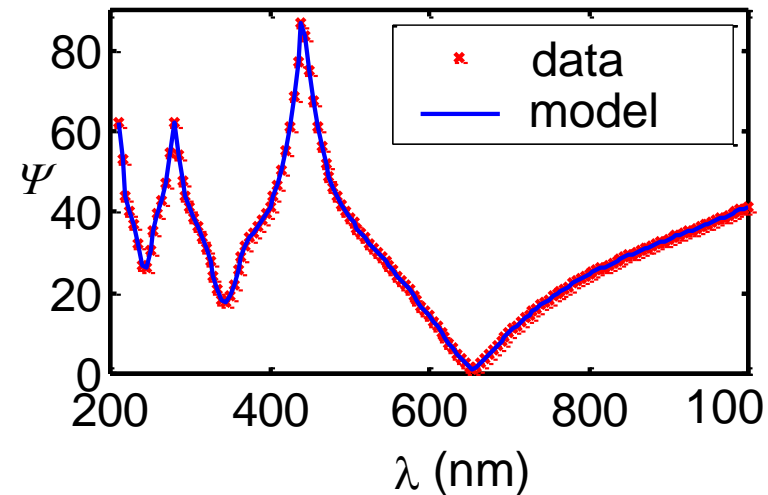
Good starting point



Bad optimization



Good optimization



# Evaluating the results



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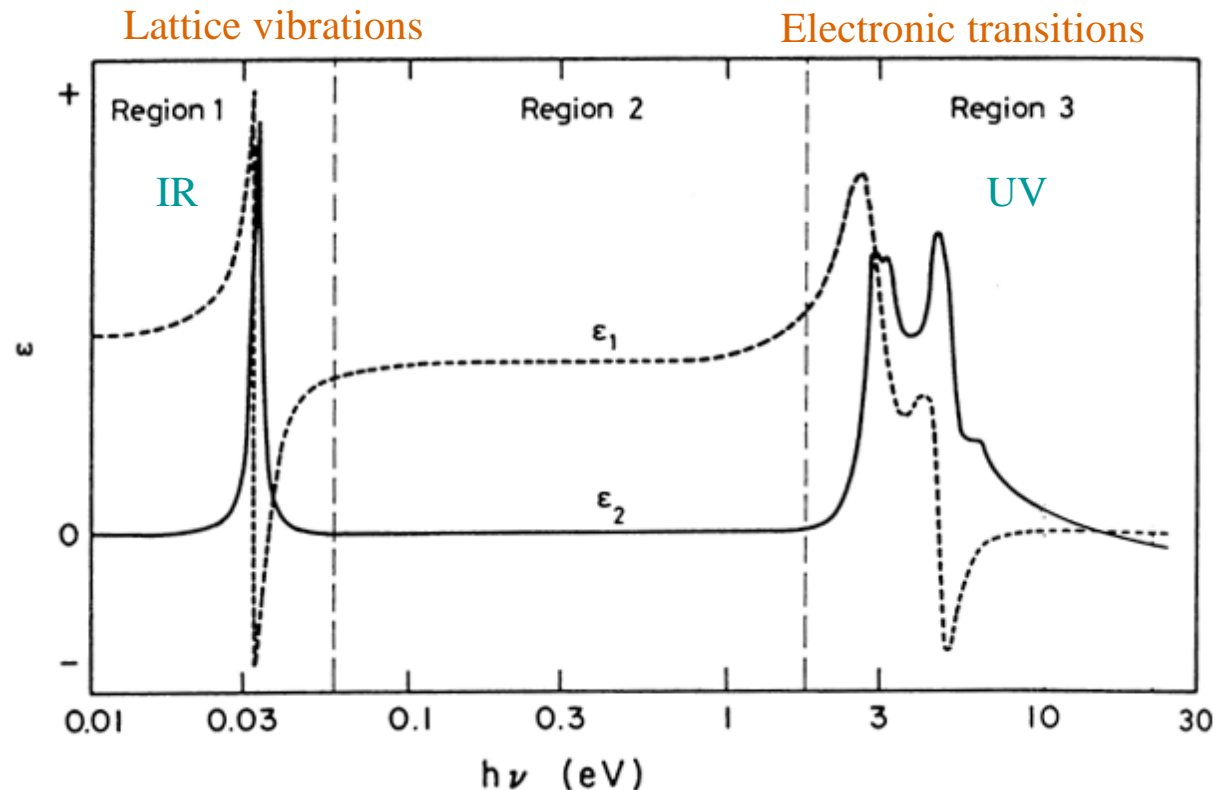
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- Before accepting the obtained results following optimization, it is important to ask oneself the following questions:
  - Does the model reproduce the experimental data?
  - Are the obtained parameter values reasonable?
    - Positive thicknesses;
    - Normal dispersion curves;
    - Positive absorption coefficients;
    - ...
  - Is the MSE sufficiently low? Can it be significantly reduced by the addition of additional parameters?
  - Is there any correlation between parameters (see the correlation matrix)?



# Using dispersion models

- The refractive index varies as a function of the wavelength; in addition, it varies in a continuous fashion. This dispersion is the result of the interaction of light with the material; at specific frequencies, absorption bands will appear; these can be modeled using oscillators (lorentzian, gaussian, ...) and adjusting their position, amplitude and broadness. It then becomes possible to model the refractive index using a limited number of parameters.







# Cauchy/Urbach dispersion models

- In the low absorption region, the refractive index varies quite slowly and is often modeled using the empirical Cauchy formula:

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

where  $\lambda$  is given in micrometers.

- The beginning of the absorption band (absorption tail), can be modeled using the Urbach model:

$$k(\lambda) = A_k \exp\left(12400 E \left[ \frac{1}{\lambda} - \frac{1}{\lambda_{\text{Edge}}} \right]\right).$$

amplitude                      exponent                      band edge



# Acquisition strategies

- To determine the optical properties of a **semi-infinite transparent substrate**:
  - Measure at multiple angles to determine the Brewster angle;
  - Measure at several wavelengths to determine the dispersion curve;
  - Use transmission measurements to decorrelate the effect of absorption of that of the surface roughness.
- To determine the optical properties of an **absorbing substrate**:
  - Make a spectroscopic measurement at an angle near the Brewster angle.
- To determine the properties **of a single layer** on a substrate:
  - Determine the substrate's properties;
  - Make a spectroscopic measurement at an angle close to the Brewster angle of the substrate;
  - Make a measurement at a second angle if there are signs of correlation between some of the model's parameters.
- To determine the properties of **multiple layers** on a substrate:
  - Determine the substrate's properties;
  - If possible, characterize each layer separately;
  - Make spectroscopic measurements at several angles below, near and above the Brewster angle; this will vary the optical path thus helping to decorrelate the thicknesses and refractive indices of the different layers.



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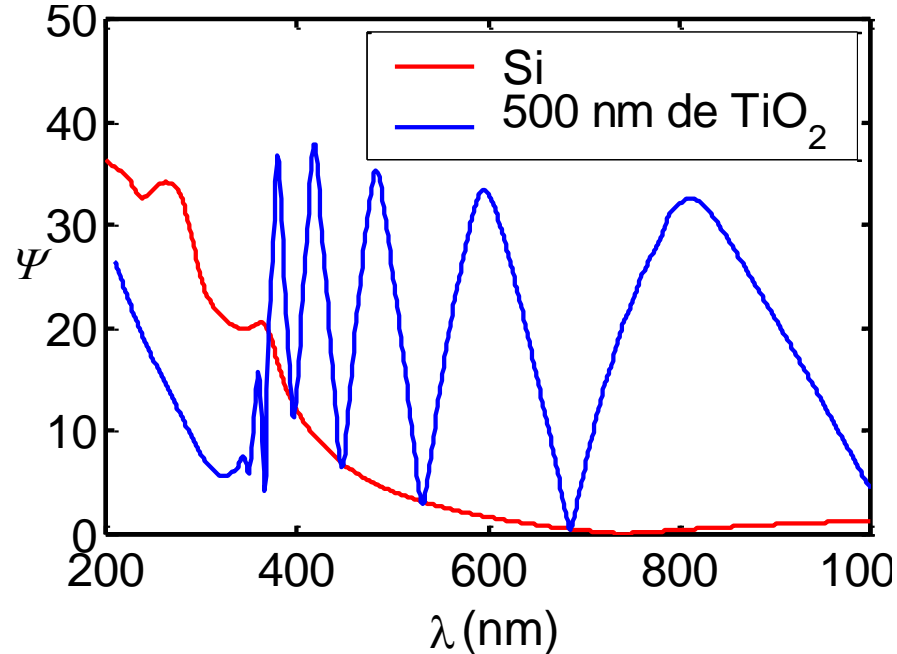
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# Which wavelength region should I choose?

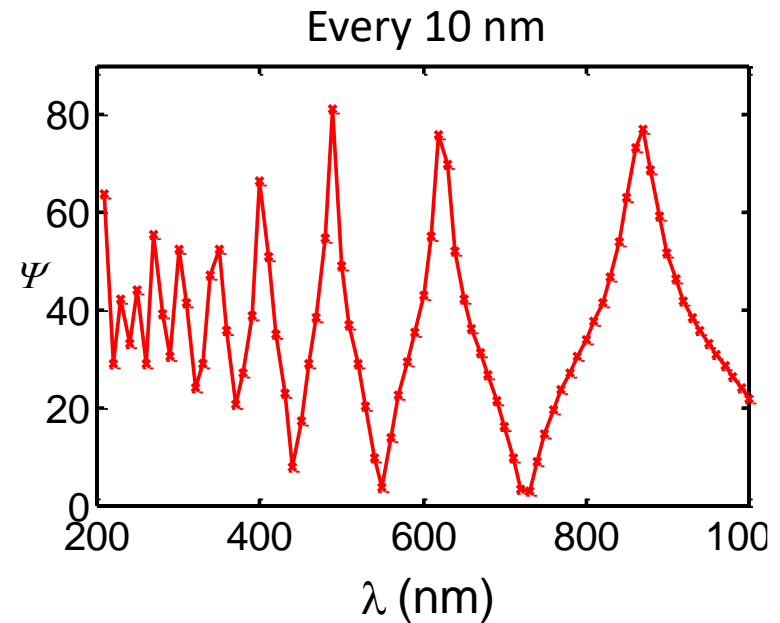
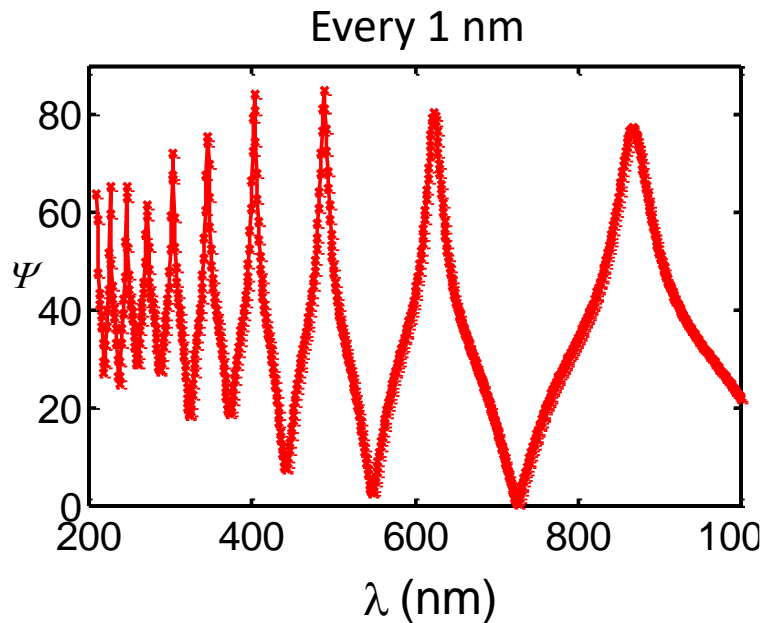
- Obviously, the region where we wish to obtain the optical properties.
- If we want to obtain the film's thickness, we must choose a region where the film is transparent.





# Resolution

- The thicker the film, the closer the interference fringes; one must therefore measure at a higher number of wavelengths.
- Keep the system's spectral resolution in mind.





# Applications: *In situ* measurements using a heat cell

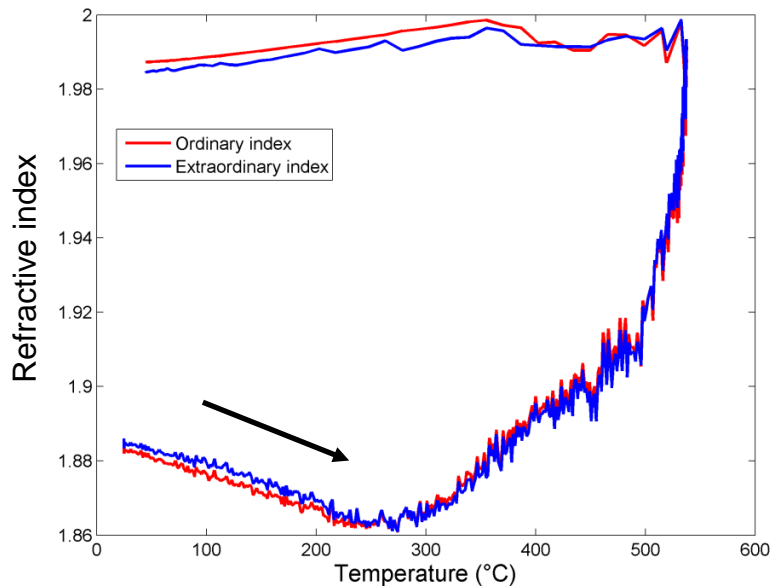
Example of *in situ* measurements as a function of temperature using a heat cell on the RC2.



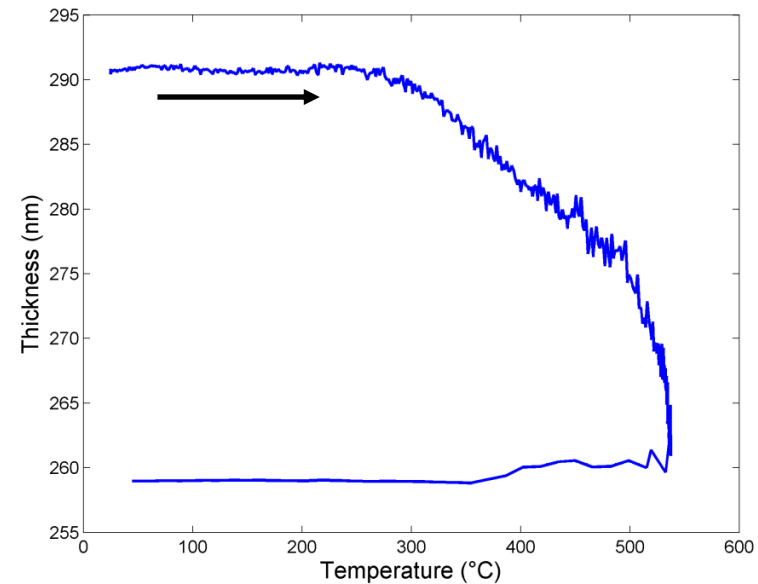
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Refractive index of a  $\text{SiO}_2\text{-WO}_3$  thin film as a function of temperature.

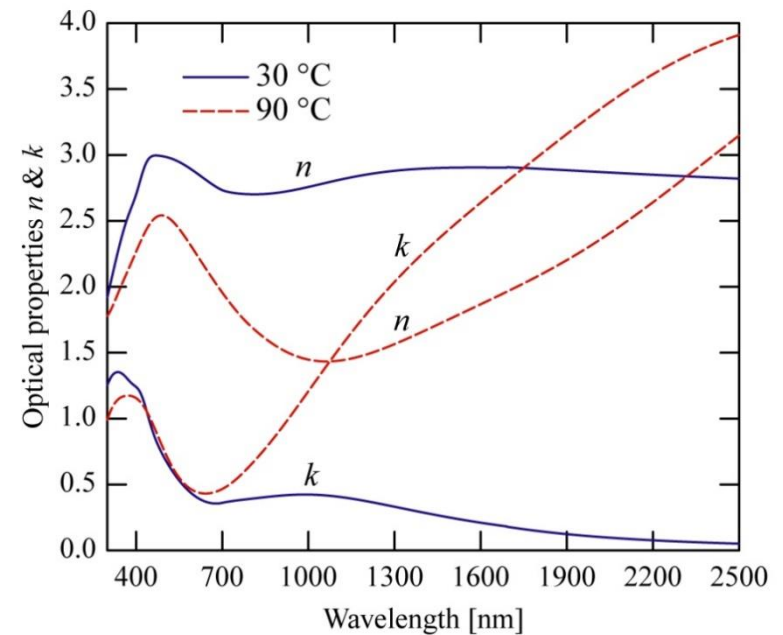
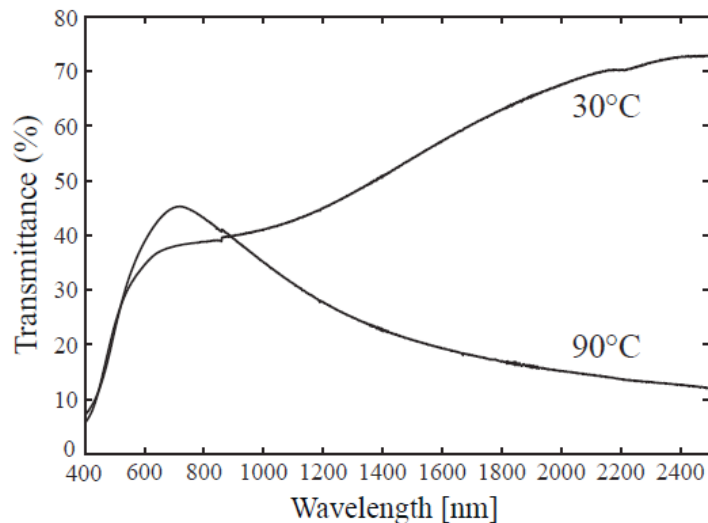


Thickness vs temperature.



# Applications: *In situ* measurements using a heat cell – Thermochromic VO<sub>2</sub>

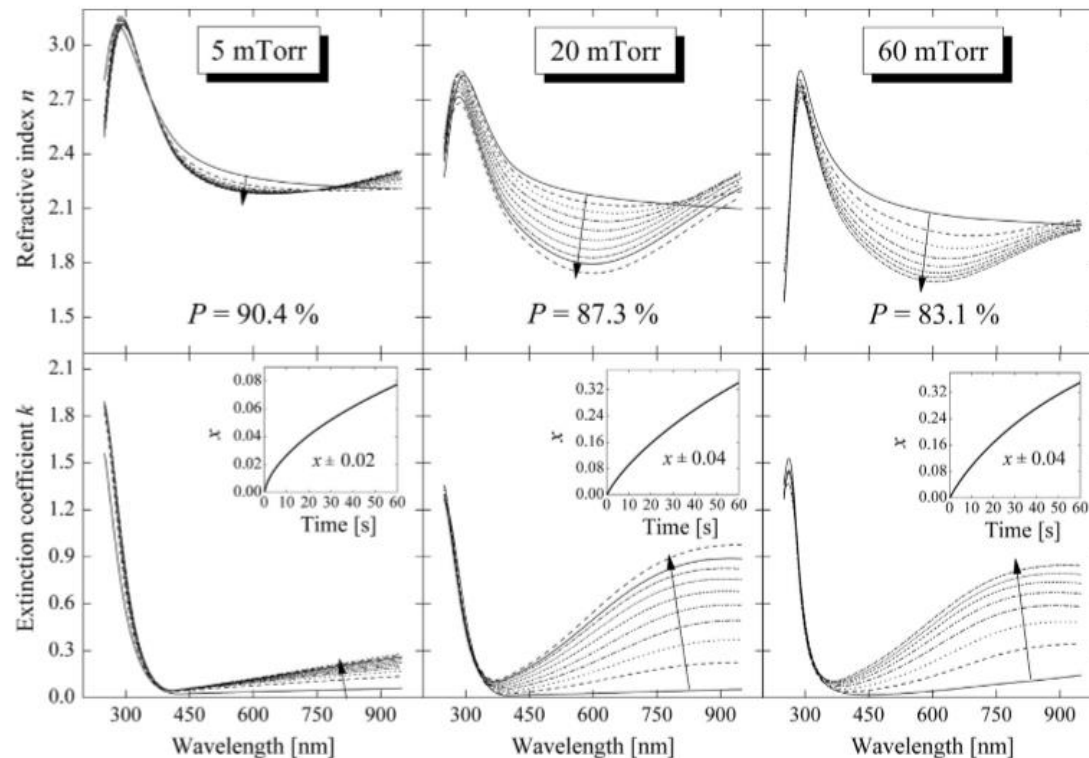
Example of a thermochromic VO<sub>2</sub> coating.





# Applications: *In situ* measurements using an electrochemical cell – Electrochromic $\text{WO}_3$

Optical properties of electrochromic  $\text{WO}_3$  thin films deposited at different pressures under coloration via  $\text{H}^+$  insertion.



**Fig. 4.** Dispersion curves ( $n$  and  $k$ ) for the 5, 20 and 60 mTorr samples. The insets contain the evolution of  $x$  vs time during coloration when passing from 1.0 to  $-0.6$  V vs SCE. The dispersion curves are separated by a  $\Delta t$  of 2 s.



# Applications: *In situ* measurements during deposition - Dynamics of plasma-induced modifications

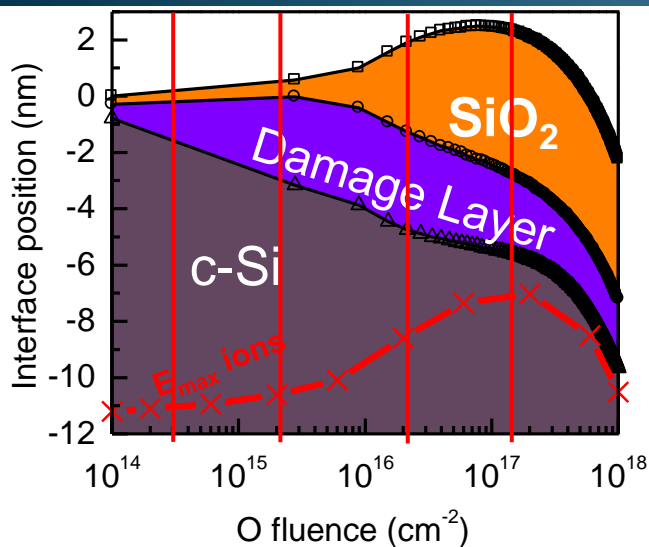
## Experiment

$$V_B = -600 \text{ V}$$

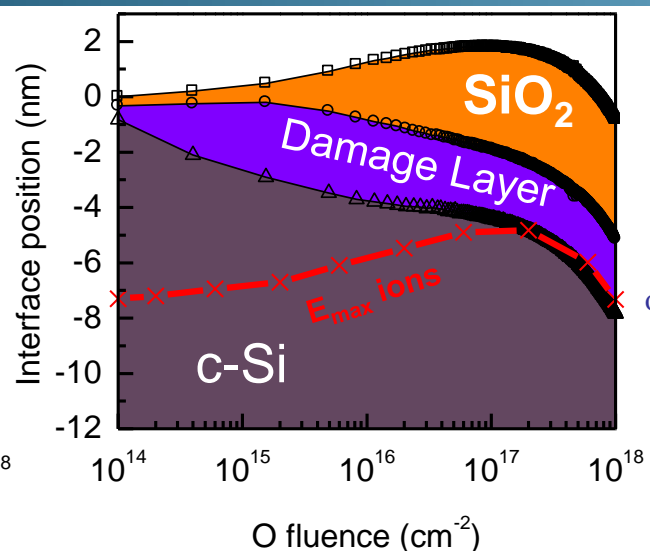
$$E_{max} \approx 690 \text{ eV}$$

$$E_m \approx 180 \text{ eV}$$

$$\phi_i = 2 \times 10^{15} \text{ O cm}^{-2}$$



**0.1 s 1 s 10 s 1 min**



$$V_B = -300 \text{ V}$$

$$E_{max} \approx 345 \text{ eV}$$

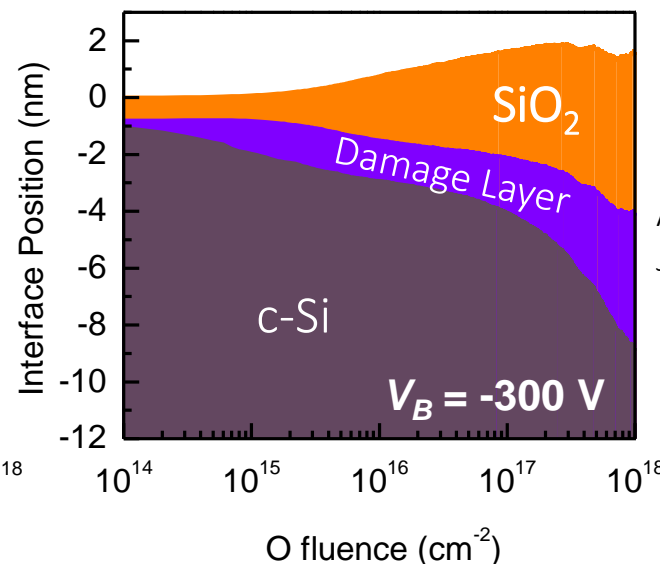
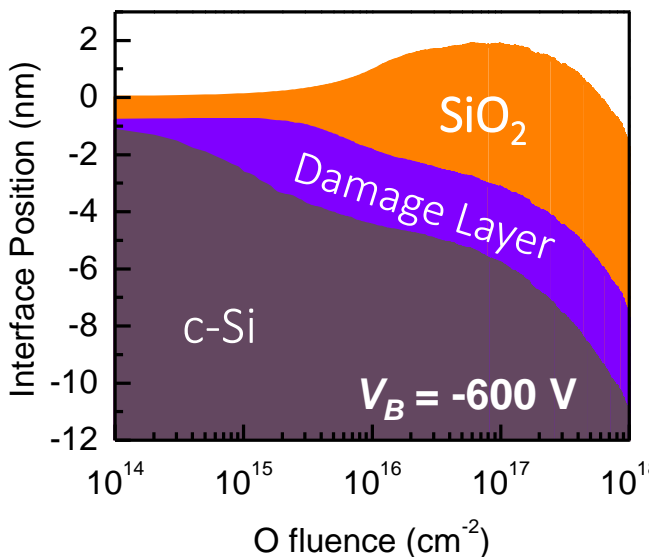
$$E_m \approx 90 \text{ eV}$$

$$\phi_i = 1 \times 10^{15} \text{ O cm}^{-2}$$

## Simulation

TRIDYN-FZR

No Fitting!



Amassian *et al.*  
JAP, 2006





# Live example

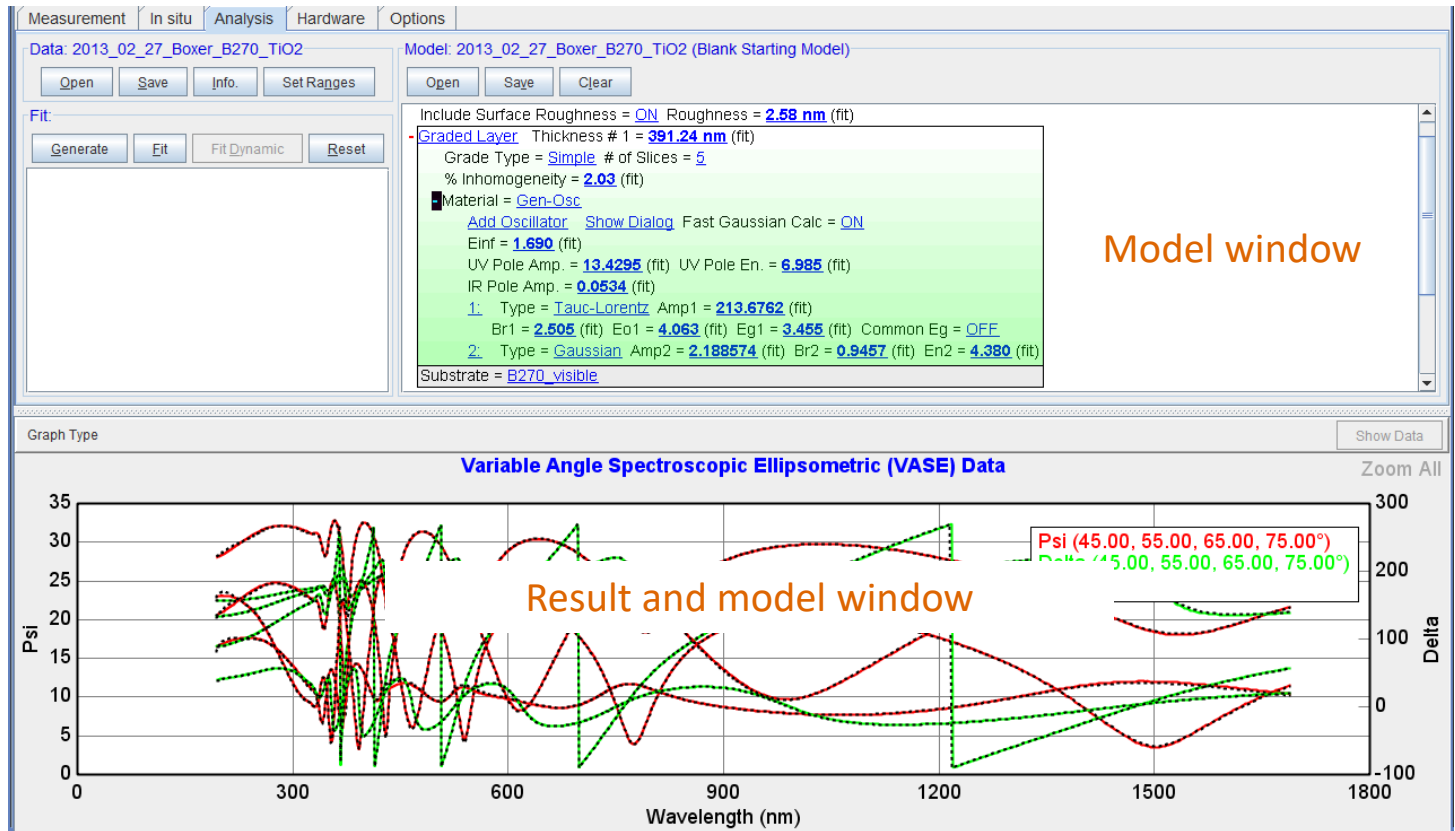
Live example of using an ellipsometry modeling software (*CompleteEase* from J.A. Woollam Co.).



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# Using *OpenFilters*

In this example, I will be using *OpenFilters*, an Open Source software developed by a former PhD student of Polytechnique Montreal (S. Larouche).

Available at:

<https://www.polymtl.ca/larfis/en/links>

**To start:** Create a project and add a filter

- Choice of substrate (1)
- Reference wavelength (2)
- Wavelength range (3)
- Backside reflection (4)
- Etc.

**Properties**

**Substrate and mediums**

Substrate: FusedSilica (1) Thickness: 1.0 mm  
Front medium: void Back medium: void  
 Don't consider substrate and mediums

**Wavelengths**

Reference wavelength: 550.0 nm (2)  
Wavelengths: 300.0 to 1000.0 every 1.0 nm (3)

**Graded-index layers**

Steps spacing:  deposition  0.01  
Sublayer minimum optical thickness: 0.0 nm

**Color**

Illuminant: CIE-D65 Observer: CIE-1931

**Analysis**

Consider backside (4)  
Ellipsometer type:  RAE  RPE  RCE  
Delta min: -90.0 degrees

**Monitoring**

Consider backside  
Ellipsometer type:  RAE  RPE  RCE  
Delta min: -90.0 degrees  
Sublayer thickness: 1.0 nm

OK Cancel

# Using *OpenFilters* – Adding layers

**Next:** Add layers

- Choice of material (1)
  - **Note:** It is possible to add your own materials.
- Specify thickness (2)
- Choice of thickness definition: nm, quarter-wave, etc. (3)

**Simple layer**

Layer

Material: Ta2O5\_2009 (1) Thickness: 0.0 (2) nm (3)  
Index:

Side/Position

Side:  front  back  
Position:  top  bottom  at position

OK Cancel

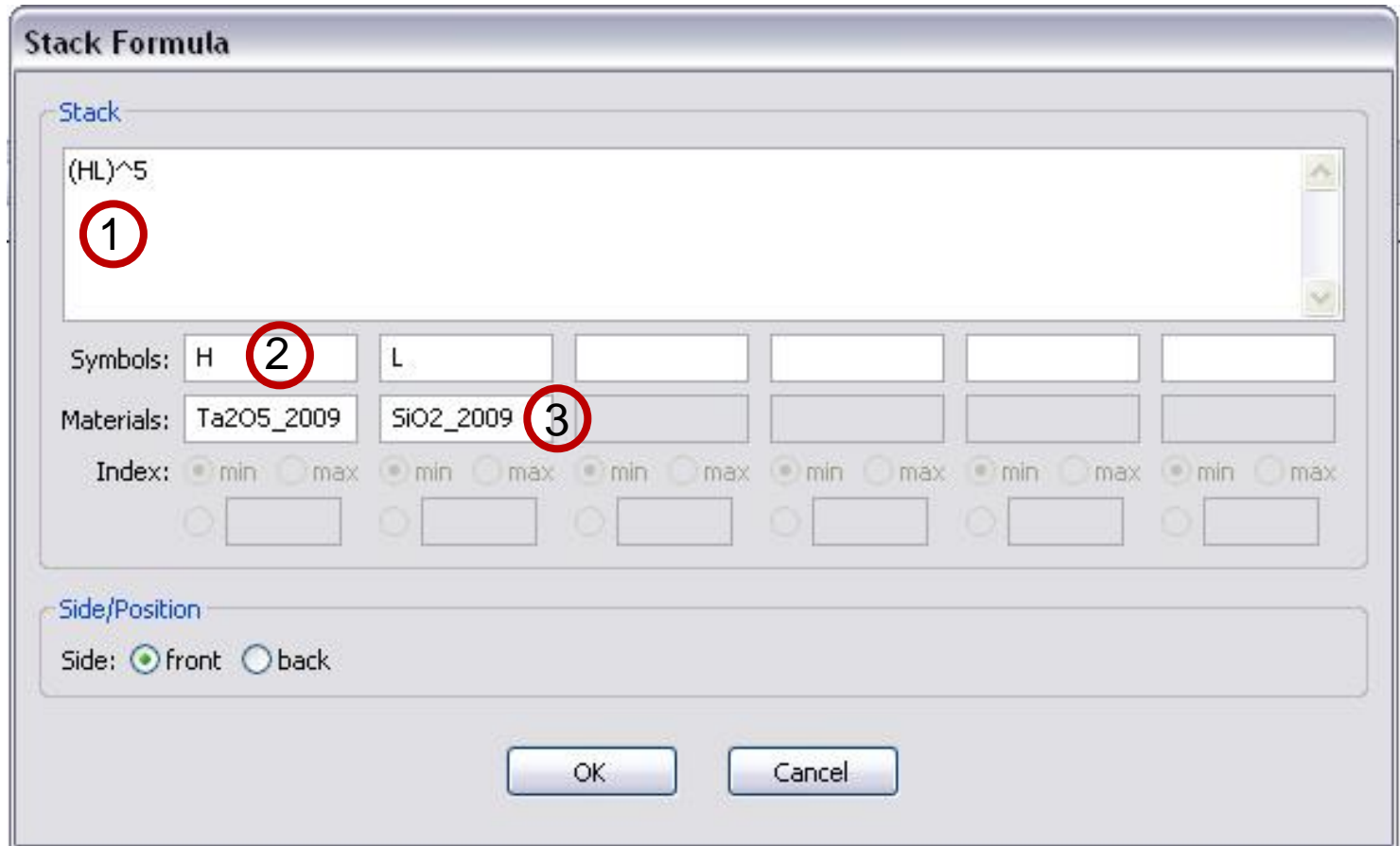




# Using *OpenFilters* – Stack formula

**To go faster:** Adding multiple layers using a stack formula

Structure of the filter (1), define symbols (2) and choice of materials (3)



**Stack Formula**

Stack

(HL)^5

1

Symbols: H 2 L

Materials: Ta2O5\_2009 SiO2\_2009 3

Index:  min  max  min  max  min  max  min  max  min  max  min  max

Side/Position

Side:  front  back

OK Cancel

# Using *OpenFilters* – Filter optimization

To optimize a filter, one must define targets.

Add Reflection Target	Ctrl+R
Add Transmission Target	Ctrl+T
Add Absorption Target	
Add Reflection Spectrum Target	Shift+Ctrl+R
Add Transmission Spectrum Target	Shift+Ctrl+T
Add Absorption Spectrum Target	
Add Reflection Phase Target	
Add Transmission Phase Target	
Add Reflection GD Target	
Add Transmission GD Target	
Add Reflection GDD Target	
Add Transmission GDD Target	
Add Reflection Phase Spectrum Target	
Add Transmission Phase Spectrum Target	
Add Reflection GD Spectrum Target	
Add Transmission GD Spectrum Target	
Add Reflection GDD Spectrum Target	
Add Transmission GDD Spectrum Target	
Add Reflection Color Target	
Add Transmission Color Target	
Read Target from File	

- Spectral and wavelength specific targets
- Color targets
- Etc.
- It is possible to read targets from a file.

The optimization process can then be launched:

- Refine the thicknesses of the layers already present.
- “Needle” method: Add infinitely thin layers at the most appropriate positions and refine.

Design/Optimize	Materials	About
Refine		F1
Needles / Refine		F2
Steps / Refine		F3
Fourier transform method		F4



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# Using *OpenFilters* – Filter analysis

## Possibilities: Analyse filter

- Spectral reflection
- Spectral transmission
- Spectral absorption
- Color calculations
- Angular color variations
- Optical monitoring
- Ellipsometry
- Electric field intensity
- Etc.

Analyse	Design/Optimize	Materials	About
Calculate Reflection			Alt+R
Calculate Transmission			Alt+T
Calculate Absorption			
Calculate Reflection Phase			
Calculate Transmission Phase			
Calculate Reflection GD			
Calculate Transmission GD			
Calculate Reflection GDD			
Calculate Transmission GDD			
Calculate Ellipsometry			Alt+E
Calculate Color			Alt+C
Calculate Color Trajectory			Shift+Alt+C
Calculate Admittance Diagram			
Calculate Circle Diagram			
Calculate Electric Field			
Calculate Reflection Monitoring			Shift+Alt+R
Calculate Transmission Monitoring			Shift+Alt+T
Calculate Ellipsometric Monitoring			Shift+Alt+E
Reverse direction			▶
Show targets			
Show all results			
Export results			



# Using *OpenFilters* – Preproduction

## Random error analysis – Filter sensitivity to deposition errors

Random error analysis

**Data type**

Data types:  reflection  transmission  absorption  
 reflection phase  transmission phase  reflection GD  transmission GD  reflection GDD  transmission GDD  
 color

Angle:  degrees Polarization:  s  p  unpolarized

Illuminant:  Observer:

**Errors**

Vary thickness by:   %   nm

Distribution:  uniform  normal Nb. tests:

Show:  all results  mean  +/-  standard deviations  worst case  
 design  targets



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# References

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- Harland G. Tompkins, *A user's guide to ellipsometry*, San Diego, Academic Press, 1993.
  - Application examples of ellipsometry. Library (QC 443 T65 1993).
- R. M. A. Azzam and N. M. Bashara, *Ellipsometry and polarized light*, Amsterdam, North-Holland, 1977.
  - Interesting for an in-depth analysis of the mathematics behind ellipsometry.
- *WVASE32 User Guide* from J.A. Woollam Co.