Feb 28, 2017

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$$g(\rho,\theta)=\iint f(x,y)\,\delta(x\cos\theta+y\sin\theta-\rho)\,dx\,dy$$

TRANSFORMÉE DE RADON

$$f_{\theta_k}(x,y)=g(\rho,\theta_k)=g(x\cos\theta_k+y\sin\theta_k,\theta_k)$$

RÉTROPROJECTION

$$b(x,y)=\int_0^{\pi} g(\rho,\theta)\Big|_{\rho=x\cos\theta+y\sin\theta}\,d\theta$$

RECONSTRUCTION PAR RÉTROPROJECTION

$$\hat{f}(x,y)=\int_0^{\pi}\left[\int_{-\infty}^{\infty}|\omega|\,G(\omega,\theta)\,e^{j2\pi\omega\rho}\,d\omega\right]_{\rho=x\cos\theta+y\sin\theta}d\theta$$

RECONSTRUCTION PAR RÉTROPROJECTIONS FILTRÉES

THÉORÈME DE LA TRANCHE DE FOURIER

$$G(\omega,\theta)=\int_{-\infty}^{\infty}g(\rho,\theta)\,e^{-j2\pi\omega\rho}\,d\rho=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\,\delta(x\cos\theta+y\sin\theta-\rho)\,e^{-j2\pi\omega\rho}\,dx\,dy\,d\rho=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\left[\int_{-\infty}^{\infty}\delta(x\cos\theta+y\sin\theta-\rho)\,e^{-j2\pi\omega\rho}\,d\rho\right]dx\,dy=$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\,e^{-j2\pi\omega[x\cos\theta+y\sin\theta]}\,dx\,dy=$$

CHANGE LE VARIABLES: $u=\omega\cos\theta$ $\quad v=\omega\sin\theta$

$$=\underbrace{\iint f(x,y)\,e^{-j2\pi(ux+vy)}\,dx\,dy}_{F(u,v)}\Bigg|_{\substack{u=\omega\cos\theta\\ v=\omega\sin\theta}}$$

$du\,dv = \omega\, d\omega\, d\theta$

RECONSTRUCTION PAR RÉTROPROJECTION

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv =$$

$$= \int_0^{2\pi}\int_0^{\infty} F(\omega\cos\theta, \omega\sin\theta)\, e^{j2\pi\omega(x\cos\theta + y\sin\theta)}\, \omega\, d\omega\, d\theta =$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} |\omega|\, G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\, d\omega\, d\theta =$$

$$= \int_0^{\pi}\left[\int_{-\infty}^{\infty} |\omega|\, G(\omega,\theta)\, e^{j2\pi\omega\rho}\, d\omega\right]_{\rho = x\cos\theta + y\sin\theta} d\theta$$

POURQUOI $|\omega|$?

5.32.

$$f(x,y) = \int_0^{2\pi}\int_0^{\infty} G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\, \omega\, d\omega\, d\theta =$$

$$= \int_0^{\pi}\int_0^{\infty} G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\, \omega\, d\omega\, d\theta +$$

$$+ \int_0^{\pi}\int_0^{\infty} G(\omega,\theta+\pi)\, e^{j2\pi\omega(x\cos(\theta+\pi)+y\sin(\theta+\pi))}\, \omega\, d\omega\, d\theta$$

$$\boxed{\int_0^{\pi}\int_0^{\infty} G(-\omega,\theta)\, e^{-j2\pi\omega(x\cos\theta+y\sin\theta)}\, \omega\, d\omega\, d\theta = \int_0^{\pi}\int_{-\infty}^{0} G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\, (-\omega)\, d\omega\, d\theta}$$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} |\omega|\, G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\, d\omega\, d\theta$$

5.29. $f(x,y) = A e^{-x^2-y^2}$