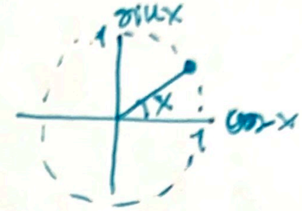
Example 4.4

$$F_K = \sum_{n=0}^{N-1} f_n e^{-2i\pi Kn/N}$$

$$\left\{\begin{matrix} f_0 = 1 \\ f_1 = 2 \\ f_2 = 4 \\ f_3 = 4\end{matrix}\right\} \quad \{N = 4\} \quad \{e^{-ix} = \cos x - i\sin x\}$$



$k=0 \Rightarrow F_0 = f_0 e^{-2i\pi(0)(0)/4} + f_1 e^{-2i\pi(0)(1)/4} + f_2 e^{-2i\pi(0)(2)/4}$

$+ f_3 e^{-2i\pi(0)(3)/4} = f_0 + f_1 + f_2 + f_3 = \boxed{11}$

$k=1 \Rightarrow F_1 = f_0 e^{-2i\pi(1)(0)/4} + f_1 e^{-2i\pi(1)(1)/4} + f_2 e^{-2i\pi(1)(2)/4}$

$+ f_3 e^{-2\pi(1)(3)/4} = 1 + 2e^{-i\pi/2} + 4e^{-i\pi} + 4e^{-3i\pi/2}$

$= 1 + 2(\overset{0}{\cancel{\cos}}(\pi/2) - i\overset{1}{\cancel{\sin}}(\pi/2))$
$+ 4(\overset{-1}{\cancel{\cos}}(\pi) - i\overset{0}{\cancel{\sin}}(\pi))$
$+ 4(\overset{0}{\cancel{\cos}}(3\pi/2) - i\sin(\overset{-1}{\cancel{3\pi/2}}))$

$= 2 - 2i - 4 + 4i = \boxed{-3+2i}$

$k=2 \Rightarrow F_2 = f_0 e^{-2i\pi(2)(0)/4} + f_1 e^{-2i\pi(2)(1)/4} + f_2 e^{-2i\pi(2)(2)/4}$

$+ f_3 e^{-2i\pi(2)(3)/4}$

$= 1 + 2e^{-i\pi} + 4e^{-2i\pi} + 4e^{-3i\pi}$

$= 1 + 2(\overset{-1}{\cancel{\cos}}(\pi) - i\overset{0}{\cancel{\sin}}(\pi)) + 4(\cos(\overset{1}{\cancel{2\pi}}) - i\sin(\overset{0}{\cancel{2\pi}}))$
$+ 4e(\cos(\overset{-1}{\cancel{3\pi}}) - i\overset{0}{\cancel{\sin}}(3\pi))$

$= 1 - 2 + 4 - 4 = \boxed{-1}$

$k=3 \Rightarrow F_3 = f_0 e^{-2i\pi(3)(0)/4} + f_1 e^{-2i\pi(3)(1)/4} + f_2 e^{-2i\pi(3)(2)/4}$

$+ f_3 e^{-2i\pi(3)(3)/4}$

$= 1 + 2e^{-3i\pi/4} + 4e^{-3i\pi} + 4e^{-9i\pi/2}$

$= 1 + 2(\overset{0}{\cancel{\cos}}(3\pi/4) - i\sin(\overset{-1}{\cancel{3\pi/4}})) + 4(\overset{-1}{\cancel{\cos}}(3\pi) - i\sin(\overset{0}{\cancel{3\pi}}))$
$+ 4(\cos(9\pi/2) - i\sin(\overset{1}{\cancel{9\pi/2}})) = 1 + 2i - 4 - 4i = \boxed{-3-2i}$

$$TF\{x_1(t)\} = X_1(f) = \int_{-\infty}^{+\infty} x_1(t)\, e^{-2i\pi f t}\, dt$$

$$FT\{x_2(t)\} = X_2(f) = \int_{-\infty}^{+\infty} x_2(t)\, e^{-2i\pi f t}\, dt$$

Si la TF est linéaire :

$$TF\{c_1 x_1(t) + c_2 x_2(t)\} = c_1 X_1(f) + c_2 X_2(f)$$

$$TF\{c_1 x_1(t) + c_2 x_2(t)\} = \int_{-\infty}^{+\infty} (c_1 x_1(t) + c_2 x_2(t)) e^{-2i\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} c_1 x_1(t) e^{-2i\pi f t}\, dt + \int_{-\infty}^{+\infty} c_2 x_2(t) e^{-2i\pi f t}\, dt$$

$$= c_1 X_1(f) + c_2 X_2(f)$$