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THE FRANZ EDELMAN AWARD
Achievement in Operations Research

Optimizing Network Designs for the World's Largest Broadband Project

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The national broadband network (NBN) is the largest public infrastructure project undertaken in Australia, and NBN Co is the government-owned company responsible for building the network. By using operations research, NBN Co expects to avoid more than $AUD1.7 billion in unnecessary construction and design costs on this $AUD36 billion project. At the beginning of this 10-year project, NBN Co divided the country into more than 3,000 fiber-serving-area modules (FSAMs), each covering approximately 2,500 premises, and will design and construct one FSAM each day. NBN Co contracted with Biarri Networks, an Australian commercial mathematics company, to optimize the design task. To accomplish this, Biarri created a fiber-optic network design (FOND) software product based on a network-flow mixed-integer programming engine. This engine minimizes the cost of materials and labor for each FSAM, subject to a variety of constraints, and provides a solution in less than five minutes. To date, more than 650 FSAM designs have been completed using FOND. This has saved NBN Co an estimated $AUD325 million in avoided construction cost, and the planning time per FSAM has decreased from 145 to 16 days.

Keywords: networks; networks: flow; Internet; government; engineering.

Between 2003 and 2007, the Australian Federal Government evaluated several options for building a national high-speed broadband network. In these evaluations, it considered several technologies: wireless (small radio towers installed on power poles on alternating streets); fiber to the node (FTTN) (optical fiber connecting exchanges to cabinets (nodes) on each street with the existing copper network used to connect the node to premises); and fiber to the home or premises (FTTH or FTTP) (optical fiber running from the exchange directly to the premises). The Australian Government looked at a variety of commercial models to determine how each would handle the technology options; in 2010, it made a decision to initiate a project to build and operate a wholesale FTTH network. Private enterprise would provide retail products (e.g., telephony and broadband) over this network.

To realize this vision, the Australian Federal Government established NBN Co, a government-owned company that it charged with building and operating the national broadband network (NBN). The NBN, the largest infrastructure project Australia has ever undertaken, would cover almost 13 million premises, 93 percent with fixed-line connections and the remainder through wireless or satellite technologies. The project would require the installation of 181,000 kilometers (km) of gigabit-capable passive optical network fiber and 57,000 km of transit optical fiber. At the beginning of the project, NBN Co divided the country into more than 3,000 fiber-serving-area modules (FSAMs).

In late 2009, Biarri, an Australian mathematics company, began proof-of-concept studies by applying operations research (OR) techniques to optimize aspects of the FSAM designs. In early 2011, as a result of the success of these preliminary studies, NBN Co contracted with Biarri to use its fiber-optic network design (FOND) software to produce the NBN designs; NBN Co now uses FOND is all NBN designs. During the second half of 2011, Biarri and NBN Co refined FOND, established end-to-end workflows, and commenced production of the designs. In the first half of 2012, the production rate increased from five to 40 FSAM designs per month; as of the end of 2013 more than 650 designs had been completed.

Designs are produced by expert planners, each of whom uses FOND and works on one FSAM at a time. Each planner must attend FOND training, which is a two-day intensive course. After successfully completing this training, a planner can design a FSAM in approximately 16 working days, a significant reduction in the planning times of even the best planners using the manual or computer-aided design plug-in tools they had available previously.

The change of government in 2013 triggered a review of the program; based on this review, the program's objectives were modified in December 2013. In place of the previous target of serving 93 percent of premises with FTTH, the new objective was to deliver a mix of newly constructed FTTH or FTTN to 68 percent of the premises, and reuse the existing hybrid fiber coaxial (HFC) cable where private network operators had already installed it. This change in policy triggered the need for another round of analytics to determine the optimal fiber service footprints—the homes and areas to be serviced by a particular technology. Nevertheless, the challenge of designing hundreds of fiber networks still remains. Furthermore, all 650 designs completed using FOND prior to the change in government have been constructed or are scheduled for construction.

This paper describes the challenge of building a national broadband network in a country as large and as sparsely populated as Australia, which is the sixth-largest country in the world, has approximately 80 percent of the surface area of the United States, but has only seven percent of its population.

The Technical Problem

NBN Co selected passive optical network (PON) architecture. In a PON architecture, no active electronic components lie between the premises and the exchange, which contains the switching equipment and connections to the rest of the communications network. Instead, a continuous path through glass fiber runs from the premises to the exchange. This decision set the fundamental constraints for the FTTH designs, but left numerous details to be determined. The propagation properties in the final architecture allow a maximum network length of 15 km between the exchange and the premises; in comparison, the legacy copper network has a maximum network length of 5 km.

The NBN architecture uses a range of cable types and a variety of installation options. The cables are single fibers, which are sheathed in protective plastic, or cables consisting of multiples of 12 fibers, including 12, 72, 144, 256, and 576 fibers. These cables can be installed in the following ways: reusing existing overhead telecommunications and power poles (aerial); reusing existing underground cable ducts (where space is available); and installing new underground cable ducts. The cables are then connected in splice enclosures, which have different constraints depending on whether they can be installed underground in pits that connect the ducts, at ground level, or on the aerial poles. The cost of constructing the network includes the cost of materials (relatively low per meter or item), installation (a moderate per-meter cost for cable plus a per-connection cost for splicing fibers together), new underground ducts where required (very high cost per meter), and new local fiber hubs. NBN Co needed an approach that minimized total costs.

Working from the premises toward the exchange, from bottom to top in Figure 1, the layers of the network proceed as shown (this discussion focuses on a more-or-less pure network architecture; actual architectural details vary slightly).

(1) Premises in close proximity to each other are grouped together and connected to the same

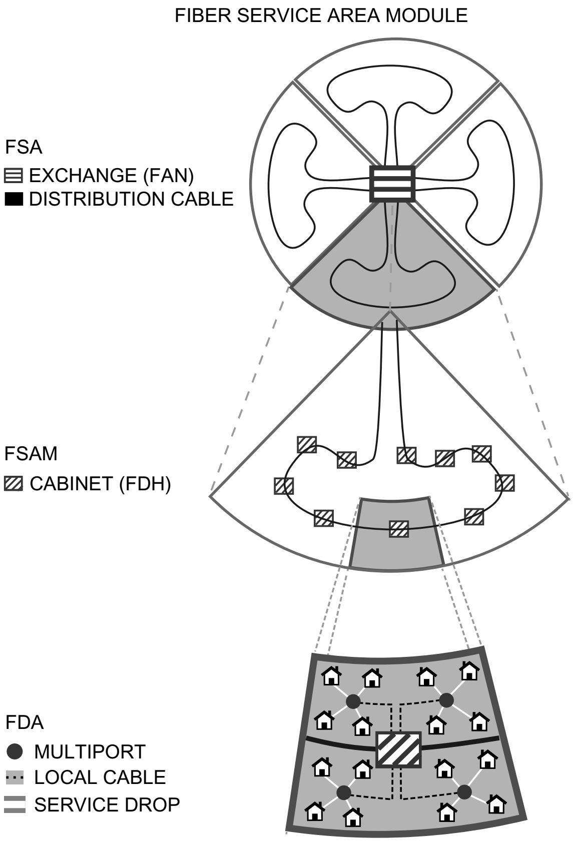

Figure 1: The NBN architecture is based on three tiers of areas. The top two tiers comprise several areas in the tier immediately below. The dark-shaded region of the top tier is shown in more detail in the center; likewise, the dark-shaded region of the center tier is shown in more detail in the bottom tier. The largest fiber-serving areas (FSAs) serve up to 30,000 premises; each FSAM serves approximately 2,500 premises, and each fiber distribution area (FDA) serves approximately 200 premises.

multiport. A multiport, which is a small device connected near a fiber-optic cable, allows individual fibers to be removed from the cable and then connected to 1–12 service-drop cables, each of which connects to a premise. The upper bound on the number of service drops is limited by the design rules chosen for a particular area; for example, if each premise is allocated three fibers, four active service drops would be assigned. This ensures that enough fiber is available at the endpoints of the network to accommodate future growth in the area.

(2) Multiports are connected by local cables back to a cabinet, that is, a fiber distribution hub (FDH). The collection of premises connected to the same FDH—perhaps 200 premises—make up a fiber distribution area (FDA). The cables in this part of the network can be aerial or underground, with a preference for underground to reduce the likelihood of accidental damage. A FDA is usually connected as a tree, with limited branching choices.

(3) FDHs are connected by a distribution ring. The distribution ring is a cable that runs from an exchange, through a collection of FDHs, and back to the exchange. This part of the network must be underground to minimize damage and consequent disruption to service. The FDAs that are grouped together by a distribution ring make up a FSAM, which contains approximately 2,500 premises.

(4) Each exchange or fiber access node (FAN) serves several FSAMs. The total area served by an exchange is called a fiber service area (FSA). The exchanges are connected to each other by the national transit (i.e., backbone) network. This largely exists already, although it must be augmented to build in redundancy and carry the anticipated increase in data.

To build the NBN, NBN Co created a design team, which works through the layers in the network beginning in the sequence of top to bottom shown in Figure 1. The first logical step in designing the NBN was to determine which areas will be served by the optical-fiber network described in items (1)–(4) above. The remaining areas, which have insufficient population density to justify a fiber rollout, will be served by wireless and satellite technology. We call the set of locations covered by fiber the saturation footprint.

The design team subdivided the country into approximately 3,000 FSAMs. This subdivision was based on a mixture of natural demographic subdivisions (e.g., small towns) and conditions resulting from the existing telecommunications infrastructure, particularly the locations of existing telephone exchanges, which had to be refitted with optical networking equipment. NBN Co manually determined the first FSAMs, but then engaged Biarri to develop a service-footprint optimization process (discussed later) to determine the boundaries of the remaining FSAMs.

Once the FSAMs had been determined, each required a separate design. This was a huge task that required a high level of automation and was an excellent candidate for optimization. As a further complication, during 2011, NBN Co contracted with Telstra, the largest telecommunications operator in Australia, to gain access to Telstra's existing pit and duct network and exchanges. This network spanned much of the target fiber service areas; however, it was capacitated and built to serve a 5 km star topology. Nevertheless, using the existing ducts was substantially cheaper than building new ducts or aerially installing cables. Therefore, a goal of the optimization was to effectively use the remaining capacity of the existing duct network. Although many of the existing ducts were used, all the fiber-optic cables and multiports are new.

Through its exposure to the NBN Co data and proposed architecture, Biarri realized the potential for applying optimization to the design process, particularly the process of designing the network within a FSAM. Literature searches indicated that solutions to problems of this type had not been attempted previously, at least not on the scale of NBN Co's FSAMs. Following this preliminary work and sales meetings, NBN Co engaged Biarri to complete a series of proof-of-concept studies. Since then, Biarri's algorithms, incorporated into the FOND software package, have been used for all designs within NBN Co, producing substantial savings in design effort and build cost. The uniqueness of the solution was reinforced in 2014 when Biarri was granted a patent for the FOND algorithms and methodology (Forbes and Hollis 2014), essentially confirming that no prior instance of an equivalent approach to FOND existed.

The Organizational Challenge

Once the decision was made that 93 percent of Australian homes would be connected to the NBN by fiber, the scale of NBN Co's design and planning task became clear. The network was envisioned originally to comprise more than 4,100 FSAMs, each containing approximately 2,500 premises; using a manual process, each FSAM would take more than 145 person-days to design. With the entire project scheduled to take 10 years, NBN Co would need to complete more than one FSAM each day. It estimated that completing the FSAM installations would require 600 trained network planners; for comparison, the team currently has 50 planners. Ensuring planners would produce network designs that were both efficient and compliant with the architecture would put massive strain on NBN Co's resources. The company realized that even a partial automation of this process would provide it with immense benefits.

The agreement between NBN Co and Telstra to use spare capacity in Telstra's existing cable duct network accentuated the difficulty of manual planning, which is difficult even in the best circumstances, when all candidate arcs (i.e., new underground ducts or above ground cabling) are essentially uncapacitated. Dealing with effectively free, but strictly capacitated, options is beyond the ability of all but the most expert human planners.

Solutions Considered and Selected

During the initial phases of the NBN proof-of-concept studies, Biarri researched existing techniques and approaches to multitier network-flow problems. Although some approaches it found (e.g., the distribution ring) could solve parts of the problem, they had idiosyncrasies that required a different approach (Yoon and Current 2008). For example, models that solve multitier networks are well documented; however, a combination of unique arc-costing, capacity, and network-branching constraints meant that adapting an existing approach would likely require the same effort as developing a new approach (Ali 2006, Lee et al. 1993, Yoon et al. 1998).

As the first option, the Biarri team considered a mixed-integer programming (MIP) formulation based on the flow of individual fibers from the FDH to the premises (or vice versa), because the team had considerable experience with network-flow models. This model essentially divides a FSAM into a collection of capacitated spanning trees; depending on the architecture of the trees, their design has several constraints. It is a fixed-charge network-flow problem with many side constraints.

Once the model had been fully developed and tuned, this solution approach produced excellent results with the prototype architecture that NBN Co was using at the time, and has proved adaptable enough to handle multiple architecture changes throughout the project. However, several architecture changes made the model progressively more complicated and resulted in increased solution times, especially if the models were run to provable optimality.

Prior to commissioning Biarri, NBN Co had conducted an extensive global search for other technological solutions, but found none capable of handling the scale and complexity of its problem. It had also considered training the projected 600 planners, but decided that to recruit and train a sufficient number in the available time would be almost impossible, particularly because of the large number of consistency and compliance requirements these planners would need to meet. Moreover, the proof-of-concept studies had convinced NBN Co that the quality of the solution that a manual planner could generate could not compete with that of a solution produced by optimization.

In developing FOND, Biarri kept in mind two critical success factors: produce a technically accurate design within a reasonable amount of time, and produce a design that is cheaper to construct than designs generated manually.

Biarri realized that it could meet these goals by using a series of highly specialized mixed-integer programs. Throughout the initial phase of the project, it compared many FSAM designs (i.e., FOND-generated designs and those submitted by external design houses) across multiple architectures (e.g., aerial and underground). The most rigorous was a set of five FSAMs, each of which contains a mixture of geotypes (e.g., old inner urban, rural, town, new estate). Biarri did its comparisons in a like-for-like manner by manipulating the tool to produce designs equivalent to those done by NBN Co and other external design houses, and comparing them to optimized designs produced with minimal planner input. In all comparisons, FOND was superior, providing savings of approximately 10 percent in construction costs for all areas.

Over the project's duration, the equipment manufacturers created new products, including smaller-diameter cables, cables with higher tensile strength, and network hubs that fit into smaller pits (vaults). Each new product gave NBN Co an opportunity to reconsider the architecture. In most cases, the product provided a simple reduction in new underground ducts because it enabled the previously unsuitable infrastructure to hold the smaller elements.

As the architecture rules changed, NBN Co continued to do comparisons where practical to ensure the savings benefits were being maintained. Because each comparison required a significant time investment from either NBN Co or a specialist design contractor, NBN Co did a comparison only after implementing a significant architecture change (i.e., a change that has a high possibility of providing design savings). With Biarri's assistance, NBN Co has repeatedly shown that FOND can rapidly produce high-quality solutions. The planners were increasingly and understandably reluctant to put the effort into producing manual designs; however, because FOND repeatedly performed better than the manual planners, NBN Co has had little need to continue these comparisons. The company still utilizes FOND to price and analyze architecture changes, and has used the tool to test and prove all recent architecture changes aimed at saving construction costs.

The optimization tools in FOND typically include the three following steps used in this order; the actual process is iterative for many practical reasons.

(1) Group individual premises into multiports within a FSAM. This is solved using a hub-selection mixed-integer program (Appendix A shows the formulation) to determine the location of each multiport and the connection between each dwelling and a multiport. Each multiport hub is small (four to six homes depending on the number of fibers allocated to each home), and placement and service areas depend on local infrastructure. A cable-length constraint means that few choices are available at this level, and experience suggests that minimal global-optimality losses result from the decisions made at this level. This process reduces the number of nodes to be considered in step (2) next from approximately 2,500 premises to approximately 600 multiports.

(2) Solve the core design problems of positioning the FDHs within a FSAM and proposing a candidate connection of all multiports back to FDHs. We also implemented this solution using a mixed-integer program (Appendix B shows the formulation). The candidate location set for these FDH hubs is restrictive because placement is subject to local council approval. Each tool permits user intervention after it has completed. Thus, planners can make small changes based on details that are unique to the area, but cannot be conveyed easily via automation. For example, after a planner has received a candidate solution, that planner may notice that a simpler and more maintainable design can be achieved if the solver can use a small

amount of new infrastructure; therefore, the planner might add this new infrastructure to the candidate network and regenerate the solution. This tool simplifies some architectural rules, without undermining the legality of the proposed solutions.

(3) Run the optimization models, as we explain next, to complete the detailed design for the FSAM.

(a) Generate a distribution ring that connects all the FDHs in a diverse path to the exchange. The distribution cable may also take the form of a double ring—a ring that has a point at which the cable splits into two and another point at which the two cables merge. Note that this distribution ring is built using dedicated underground cabling (Appendix C shows the formulation).

(b) Run an optimization using a fully detailed FDA architecture to connect all multiports to FDHs. The assignment of multiports to FDHs generated during the prior FDH positioning optimization is discarded because that optimization is not constrained by the service details of a FDA. This optimization can use spare capacity in the distribution ring to save costs (Appendix D shows the formulation).

(c) Run a final cable-allocation optimization to allocate the specific cable types in each section of the multiports to FDHs network. The difference in cable costs is at least an order of magnitude smaller than the cable installation costs; therefore, earlier optimizations ignore it (Appendix E shows the formulation).

Implementation and Challenges Conquered

The implementation of FOND for NBN Co has been an iterative process over several years. The genesis was an initial prototype, written in a modeling language and built as part of a proof-of-concept study, to optimize the design of a FSAM (i.e., position the FDHs and determine which cables connect FDHs to multiports). Biarri implemented this for one trial FSAM that had previously been designed manually, and gave NBN Co the allocation of premises to the FSAM and grouping of premises into multiports. The first model used produced solutions within one percent of optimality in overnight optimization runs.

Given the iterative nature of the use NBN Co planned for the tool, however, a run time of this length was unacceptable. (Iterations are required because of the interdependencies of the models and the practical impossibility of correctly specifying all model inputs prior to the first iteration of a FSAM model.) Biarri tried to tighten the formulation and investigate specific solver parameters to decrease the solve time. To tighten the formulation, it added two significant branching variables to constrain the underlying undirected graph in the production of directed trees (see Appendix D). After modifying these branching variables and determining solver settings that were more suitable to the particular formulation, this prototype produced a design whose construction costs were at least 10 percent lower than the costs of the manual designs, and ran to optimality in less than two minutes on a basic laptop computer. It took data from Excel files and produced a keyhole markup language file (i.e., map layer) as output.

We then converted the prototype to a formal software project, in which development included three main elements:

(1) Ongoing broadening of the scope of the software package, particularly into upstream and downstream data, pre- and postprocessing. Some of the pre- and postprocessing steps were in the form of optimizations (e.g., optimization of the multiport locations and connections and optimization of the detailed cable locations). As a result, the core optimization engines had a much more predictable environment. For example, Biarri could guarantee that all multiports entered into the optimization model were connected by candidate network arcs; therefore, the optimization engine did not need to consider them.

(2) Formal software development to create logging and user diagnostics and simplify the workflow to make FOND easier to use. In particular, when a feasible solution cannot be found, providing as much diagnostic information as possible is important, even if doing so requires automatically running several relaxed models. NBN Co does not want to hire mathematicians to design the network or explain why a result is infeasible.

(3) Continuous development of the optimization to reduce construction costs and match actual or proposed changes in the architecture.

In the process of developing FOND and the underlying algorithms, Biarri faced several significant hurdles, which we categorize as: (1) producing reasonable optimization run times without excessively degrading solution quality, (2) ensuring that end-to-end planning time is minimized, and (3) ensuring quality, while developing a complex rapidly evolving optimization and software product.

In 2010 and 2011, NBN Co established 145 days as the benchmark time to manually create designs. During 2012 and 2013, it then measured the duration of the end-to-end design process, including subprocesses, which was initially 37 days, and validated the design-cost benefits of using FOND rather than a manual design. The data were also used to drive performance improvements through additional functionality and streamlined workflow in FOND, improved training for planners unfamiliar with using optimization, and more complete integration into prior and subsequent processes. This continuous improvement saved an additional 21 days over the 37 days measured at the start of the deployment, thus reducing the time to 16 days.

Several designs produced during the initial proof-of-concept studies became the base for a set of regression tests that were run to test each change to the FOND software. This allowed developers to be confident that modifications to the tool did not adversely affect FOND solutions or run times. It also allowed them to be confident in refactoring the code base when needed and provided a baseline on which they could compare optimization improvements. This set of tests was crucial to maintaining rapid development, while also maintaining consistency in FOND output and speed.

By March 2013, NBN Co had completed designs for 1,000,000 premises using FOND. FOND has an extremely high profile within NBN Co's executive team and its sole shareholder, the Federal Government, which is represented by the Minister for Communications, Malcolm Turnbull. The executives and the minister are aware of the tool's value and the impact it has had on NBN Co's planning and design efforts. The success of FOND has also led to other major projects, which we list next, executed using FOND or other OR tools.

- Design of the point-to-point network: The NBN has numerous pairwise connections (e.g., between FANs and from FANs to the fixed wireless towers). Biarri created the FOND P2P solver to generate these designs using an A* algorithm (Hart et al. 1968) and the network read and write functionality in the primary FOND tool. The P2P solver has reduced the design time from hours to minutes, and we expect that it will save NBN Co several years of design time over the more than 500 designs still required.
- Analysis of the service footprint: This is the determination of which areas will be served by fiber and which will be served by wireless. The service-footprint analysis comprises three engines. The first uses the density-based spatial clustering of applications with noise algorithm (Ester et al. 1996) to cluster premises; it then selects the clusters based on a cost model that reflects the tiered architecture. The most cost-effective 93 percent constitute the fiber footprint. The second engine assigns each premises to a possible FAN (exchange) using a hub-selection mixed-integer program. The premises assignments define the FSAs. Finally, each FSA is subdivided into FSAMs using a graph-partitioning algorithm to minimize interleaving between FSAM boundaries. Although estimating the time that NBN Co saves is difficult, the objectivity of the approach is important, given the public scrutiny of the results. The decision to move some areas to FTTN and reuse existing high-speed HFC cables means that this exercise will be revisited.
- Render: Automation and optimization of the fiber network construction schedule. In 2013, Biarri and its partner, Make Ready Australia, created Render, a software tool that uses FOND's design output to generate and optimize the schedule of jobs to physically build each FSAM. Proof-of-concept studies show that using Render results in substantial administrative savings because of the automation and substantial construction productivity improvements optimization provides. The implementation of Render was quick partly because the standardization of designs generated by FOND simplified the network interpretation within Render, and because the Biarri development team was intimately familiar with the details of the NBN through its work on FOND. In field trials, Render has demonstrated that it can provide significant savings.
- Reloaded: This project, which has completed the proof-of-concept stage, combines a FOND-generated FSAM design with data feedback from the field. After

a design has been produced, a team of external contractors physically inspects the location at which the network is to be installed and nearby alternative locations, and corrects or updates the data. For example, a duct may not have the anticipated amount of spare capacity, or a candidate joint location may contain asbestos, making its reuse expensive. These data updates are then entered back into FOND and the design is regenerated, with a preference for making minimal changes to the plan.

Summary of Benefits

In this section, we summarize the benefits of FOND.

• Substantial reduction in planning cost and time: To manually plan and design a FSAM requires more than 145 person-days. To date, we have reduced this to 16 person-days because of using FOND, and we expect to be able to make additional reductions. Moreover, this reduction in planning time means that whenever we define a new architecture, we can replan the FSAMs. Previously, NBN may not have been able to justify the incremental savings in build cost because of the additional planning effort required.

• Short run times that facilitate strategic decision making: The run time for each optimization step is a few minutes. As a result, planners have evolved from being tacticians to being strategic decision makers. By removing the need for planners to control and create the detail within a design, they can focus on the large-impact options available in that area, such as investigating the construction of new road and rail crossings. Traditionally, these network modifications occur at the start of the design process. FOND allows fast, objective analysis of these options.

• Large reductions in build cost: In all proof-of-concept and comparison studies conducted, FOND has consistently produced designs with build-cost estimates that are approximately 10 percent lower (20 percent lower in some cases) than the best manual designs, because computer optimization allows a planner to implicitly consider a vast number of choices and trade-offs. This is particularly apparent in the optimization tool's ability to reuse inexpensive, but capacitated, existing ducts in an efficient way.

• Consistency of design choices: Human planners may resolve similar design problems in different ways. An optimization-based design tool will consistently solve similar problems in the same way, resulting in designs that look similar in most important respects. Standardizing procedures across multiple sites reduces NBN Co management overhead and provides economies of scale for training. Standardizing designs means that the downstream data recipients (e.g., GIS software) require less flexibility; it also allows construction companies (and all other downstream organizations) to standardize their procedures and introduce other automation and optimization processes. Render, which we describe in the *Implementation and Challenges Conquered* section, is an example.

• Quick evaluation of the impact of proposed architectural changes: Throughout the project, many alterations to the architecture were suggested and evaluated. For example, at a multiport, we could remove from the cable the exact number of fibers required for that multiport; alternatively, we could extract a ribbon (a small subcable with 12 fibers) from the cable. The second approach reduces the complexity and cost of the installation, but can also waste the cable's fiber capacity, which creates a problem if the cable is already at the maximum available size. Alternatives such as this can be investigated quickly by running the reconfigured, or perhaps slightly modified, optimization steps over a representative collection of data sets.

To date, we have designed more than 650 FSAMs using the optimization methods described. This has saved NBN Co an estimated \$AUD30 million in design costs (all dollar figures are in \$AUD, with a rough conversion ratio of 0.8 \$US to 1 \$AUD), with the planning time per FSAM reduced from 145 to 16 person-days (see Table F.1 in Appendix F). NBN Co estimates the construction savings from these FSAMs to be \$AUD325 million. This estimate is based on a comparison of the construction costs of manually generated versus FOND-generated FSAM designs. As the rate of design and construction of the more than 4,100 FSAMs increased, NBN Co forecast the savings from the optimization tool to be \$AUD250 million per annum—\$AUD225 million in constructions costs and \$AUD25 million in design costs. Over the life of the project, this will result in \$AUD1.7 billion in cost avoidance. We base this on the initial project estimate of $AUD36 billion and the reduction in the number of FSAMs designed from the original 4,100 to the 3,000 FSAM designs now anticipated, as we explain later.

As a result of the September 2013 Australian national elections, the government changed. The current government is still committed to implementing a national broadband network; however, to reduce the cost of rolling out the network, it will convert many FSAMs from a FTTH to FTTN model and will make much greater use of existing HFC high-speed Internet cabling, although it is not optical fiber. No planning will be required for the existing HFC cabling, reducing the total savings that can be achieved from using FOND. Nevertheless, the introduction of a second architecture choice (FTTN) will require design optimization and will reinforce FOND as a strategic tool in the planning exercise. We anticipate that the FTTN architecture will be implemented by inferring premises and node combinations from the existing copper network and then representing the nodes as demand points in the FSAM solver (Appendix D shows the formulation). The FSAM solver will then connect these demand points to the distribution ring with the FTTH cabinets.

The construction cost savings were determined via a series of comparisons of FOND-generated designs and those of fiber design experts. One of the first designs was for a FSAM in Townsville, Queensland. The area required extensive amounts of new network infrastructure to accommodate the distribution ring. The planners were able to determine a close-to-optimal feeder route to minimize this network construction; however, the major cost savings occurred when the planners considered the new network required for the cables from the cabinet to the home.

The manual planning approach to a FSAM is modular; in each FDA, a FSAM is planned relatively independently of other FSAMs. Although each FDA design can seem reasonable when viewed in isolation, when we consider the entire network, we can achieve large savings through a global optimization process. FOND can synchronize the network augmentation required by both the distribution and local networks, resulting in large overall savings. As Table 1 shows, FOND used more cable to ensure that network augmentation and construction were kept to a minimum. In Townsville, it achieved savings of more than 2 km of new build, which is the majority of the 10.8 percent construction savings achieved.

Lessons Learned

From the outset, NBN Co was open to a new approach; its management could see how manually intensive the design process was, and how much of a bottleneck it would be in constructing the NBN. As a result of this project, the company has learned to distinguish between the benefits gained from automation and those gained from optimization, and now recognizes the substantial difference.

Additionally, all stakeholders have become aware of the need for quality data, whether the designs are generated manually or by using a tool such as FOND. Because FOND designs the network to a fine level of detail, the data required must be at a similar level.

During our early work on this project, Biarri and NBN Co clearly understood that using agile software development practices would be necessary to ensure that the changes NBN Co deemed most valuable would be quickly made to the software. Additionally,

	Design expert solution		Optimized solution		Reduction		
Component	Meters	$AUD	Meters	$AUD	Meters	$AUD	%
Distribution new build	9,070	997,775	8,835	971,861	235	25,850	2.59
Local new build	3,285	361,438	1,255	138,150	2,030	223,300	61.7
Local cable	28,190	704,770	28,583	714,576	−393	−9,825	−1.39
Total cable	54,750	273,750	51,814	259,070	2,936	14,680	5.36
Total cost		2,337,733		2,083,657			10.8

Table 1: The table shows a detailed overview of the construction savings for a FSAM in Townsville, Queensland. FOND's global approach to optimization provides significant savings to the most expensive portions of this network—network construction and augmentation for local cables.

NBN Co and Biarri each needed to have small, fast, and flexible teams to cope with the rapid changes in the FOND product. Once the workflow had stabilized, NBN Co was able to ramp up its design team that uses FOND. It advocates Biarri's agile software development methodology to other vendors, recognizing the importance of this approach in the FOND deployment.

OR, and specifically optimization, was the primary reason for the project's success. The results of the early proof-of-concept studies were compelling. Because of the culture at NBN Co, management was open to new ideas, which made presenting the concepts and prototypes easy for us. Biarri was responsive and consultative, ensuring that it would develop the right solution and allowing the math to speak for itself.

Future Developments

Our future FOND development includes three main initiatives:

(1) Extension to other processes within NBN Co: This is ongoing, particularly relative to the Render and Reloaded projects discussed previously. Additionally, considerable consultancy work, and perhaps some modifications to the tool, will be required as a result of the changes to the NBN architecture that will arise because of the change of government.

(2) Deployment of the tool into other countries and (or) networks: The generic implementation of the product makes it applicable to many large-scale networks, especially optical-fiber networks. Biarri has already signed a contract with Chorus New Zealand to use the tool to design an optical-fiber network to which 575,000 houses will connect. Small but important changes were required to the FSAM optimization tools. As of February 2014, Biarri was in negotiations to use FOND as part of the design of Indonesia's broadband network, and anticipates that this network will connect 40 million premises. To date, Biarri has conducted FOND proof-of-concept consultancies in four countries.

(3) Improvement of the optimization processes within FOND: Work on the optimization algorithms used in FOND is the subject of one author's PhD research. In addition to a general refinement of the models, Biarri is evaluating a one-click solution process for designing a FSAM; the process should be flexible enough to cope with different architectures. It is also investigating the use of lazy-constraint or delayed-column generation, where only the constraint or column generator varies with the architecture. This would greatly assist a user in applying the tool to slightly different architectures.

Appendix A. MIP Formulation for Multiport Hub Selection

This formulation determines the multiport locations within a FSAM and the premises (land parcels) that each multiport will service. We use the following notation:

- $P \in N^P$ indexed by p: the set of all pits and (or) poles in the FSAM that are candidates for multiport placement; $C \in N^C$ indexed by c is the set of all land parcels in the FSAM; D_C is the fiber demand (i.e., number of fibers required) at land parcel c.
- M^f: the fiber capacity of a single multiport.
- C^A_{cp}: the cost of connecting land parcel c to a multiport at pit or pole p.
- C^M_p: the cost of installing a multiport at pit or pole p.
- C^R_p: the cost of remediating pit p so that it can contain additional multiports.
- $U \in N^P$: the set of all pits within the FSAM.
- M^{norm}_p: the number of multiports allowed in pit p prior to remediation; M^{rem}_p is the number allowed after remediation. Remediation is used to upgrade pits of insufficient size to accommodate additional multiports.

Define the following:

- $z_p \in \mathbb{Z}^+$: the number of multiports placed at pit or pole p.
- $x_{cp} \in \{0,1\}$: 1 if land parcel c is to be serviced by a multiport at pit or pole p.
- $r_p \in \{0,1\}$: 1 if pit $p \in U$ is remediated to support the installation of additional multiports.

$$\text{Minimize} \quad \sum_{p\in P}[C^M_p z_p + C^R_p r_p] + \sum_{c\in C}\sum_{p\in P} C^A_{cp} x_{cp}$$

$$\text{subject to:} \quad \sum_{p\in P} x_{cp} = 1 \quad \forall c \in C, \tag{A1}$$

$$\sum_{c\in C} D_c x_{cp} \le M^f z_p \quad \forall p \in P, \tag{A2}$$

$$z_p \le M^{\text{norm}}_p + M^{\text{rem}}_p r_p \quad \forall p \in U. \tag{A3}$$

Constraint (A1) ensures that each land parcel in the area is serviced by a multiport. Constraint (A2) restricts the service capacity of a pit or pole depending on the number of multiports installed there. Constraint (A3) enforces rules about how many multiports can legally fit within certain pits, while allowing the possibility of pits being upgraded (remediated) to fit additional multiports.

Appendix B. MIP Formulation for FDH Hub Selection

This formulation positions the FDHs within a FSAM and proposes a candidate connection of all multiports back to the FDHs. We use the following notation:

- A set of N^P pits and (or) poles P in the FSAM indexed by p.
- The set $H \subset N^P$ of pits and (or) poles that are potential FDH locations.
- M^F: the maximum active ribbon capacity (expressed as number of fibers) of a FDH.
- Demand D_i: Each pit has a demand D_i, which is the number of multiports required at this pit and (or) pole i.
- A distance L, which is the maximum length a pit and (or) pole can be from its servicing FDH.
- γ_{ij}: the distance from multiport pit and (or) pole i to FDH pit and (or) pole j.
- C^H: the fixed cost of a FDH installation.
- C^c: the fixed cost per meter of assignment.

Define:

- $z_j \in \{0,1\}$: 1 if pit and (or) pole j is used as an FDH; 0 otherwise.
- $x_{if} \in \{0,1\}$: 1 if mulitport pit and (or) pole i is serviced by the FDH at pit and (or) pole j used; 0 otherwise. Note that these assignments are created if and only if $\gamma_{ij} \le L$.

$$\text{Minimize} \quad C^H \sum_{p \in H} z_p + C^c \sum_{i \in P \mid D_i > 0} \sum_{j \in H} \gamma_{ij} x_{ij}$$

$$\text{subject to:} \quad \sum_{i \in P \mid D_i > 0} D_i x_{ij} \le M^F z_j \quad \forall j \in H, \tag{B1}$$

$$\sum_{j \in H} x_{ij} = 1, \quad \forall i \in P \mid D_i > 0, \tag{B2}$$

$$x_{ij} \le z_j, \quad \forall i \in P \mid D_i > 0, \forall j \in H. \tag{B3}$$

The formulation described is a simple hub-selection mixed-integer program with very few idiosyncrasies. Constraint (B1) ensures that each FDH is not oversubscribed, Constraint (B2) ensures that every multiport demand point is assigned to a FDH, and constraint (B3) is a well-known hub-selection constraint, which tightens the LP relaxation.

Appendix C. MIP Formulation for Distribution Network

This formulation connects the FDHs in a ring (or double ring) and then back to the exchange. Multiple cable types may be used to form this ring; because of the split allowed to form a double ring, multiple cable types can be used in constructing a single distribution network.

We define the following sets and data:

- Set of all arcs A within a FSAM indexed by a. Note that arcs are directional and traverse from node F_a to T_a.
- Set containing all pits and poles P within a FSAM indexed by p.
- Set of cable types T available for use in the distribution network indexed by t. These are typically the larger cables available (i.e., 564, 288, 144 fiber cables).
- P^F: the set of nodes that are FDHs.
- An entry point x into the FSAM for the distribution cable.
- An exit point w from the FSAM for the distribution cable.
- D_p^1 and D_p^2: each are 1 for all $p \in P^F$; D_x^1 and D_w^2 are each equal to the total number of FDHs in the FSAM ($|P^F|$); D_n^1 and D_n^2 are 0 for all other $n \in P$.
- $\deg_n$: the degree of node $n \in P$.

Define the following variables:

- $y_a^t \in \{0,1\}$: 1 if the distribution cable uses arc a, and cable size t; 0 otherwise.
- $c_a^t \in \mathbb{R}^+$: the cost of using arc a for distribution cable of cable size t.
- $\gamma^t \in \mathbb{Z}^+$: the FDH capacity of cable t (i.e., how many FDHs can be connected using a cable of a particular size).
- $\mathrm{d}_a \in \mathbb{R}^+$: the additional cost of using arc a for distribution cables in a second direction, given that it is already being used for distribution in a first direction.
- $\Delta_a \in \{0,1\}$: 1 if an arc and its reverse direction are used; 0 otherwise.
- $s_p^o \in \{0,1\}$: 1 if a branching point is placed at pit p, allowing two cables to leave pit p with a single cable arriving into pit p.
- $s_p^i \in \{0,1\}$: 1 if a splice point is placed at pit p, allowing two cables to enter pit p with a single leaving.
- $c_p \in \mathbb{R}^+$: the cost of placing a branch or splice point in pit p.
- $f_a^1 \in \mathbb{Z}^+ \cup \{0\}$: the flow of commodity 1 along arc a (commodities are hypothetical constructs of convenience and are described below).
- $f_a^2 \in \mathbb{Z}^+ \cup \{0\}$: the flow of commodity 2 along arc a.
- $\tau_n^t \in \{0,1\}$: 1 if cable type t is incident on node n.

$$\text{Minimize} \quad \sum_{a \in A,\, t \in T} c_a^t y_a^t + d_a \Delta_a + \sum_p c_p (s_p^o + s_p^i)$$

subject to:

$$\sum_{a \in A \mid p = T_a} f_a^1 \ge \sum_{a \in A \mid p = F_a} f_a^1 + D_p^1 \quad \forall p \in P, \tag{C1a}$$

$$D_p^2 + \sum_{a \in A \mid p = T_a} f_a^2 \ge \sum_{a \in A \mid p = F_a} f_a^2 \quad \forall p \in P, \tag{C1b}$$

$$D_x^1 \ge \sum_{a \in A \mid x = F_a} f_a^1, \tag{C1c}$$

$$\sum_{a \in A \mid w = T_a} f_a^2 \ge D_w^2, \tag{C1d}$$

$$\sum_{a \in A \mid w = T_a} \sum_{t \in T} y_a^t = 1, \tag{C2a}$$

$$\sum_{a \in A \mid w = F_a} \sum_{t \in T} y_a^t = 0, \tag{C2b}$$

$$\sum_{a \in A \mid x = T_a} \sum_{t \in T} y_a^t = 0, \tag{C2c}$$

$$\sum_{a \in A \mid x = F_a} \sum_{t \in T} y_a^t = 1, \tag{C2d}$$

$$\sum_{p\in P} s_p^o \le 1, \tag{C3a}$$

$$\sum_{p\in P} s_p^i \le 1, \tag{C3b}$$

$$s_p^o + \sum_{a\in A|p=T_a} \sum_{t\in T} y_a^t = s_p^i + \sum_{a\in A|p=F_a} \sum_{t\in T} y_a^t \quad \forall p \ne x, y, \tag{C3c}$$

$$2s_p^o + s_p^i + \sum_{a\in A|p=T_a} y_a^t \ge \sum_{a\in A|p=F_a} y_a^t \quad \forall p \ne x, y, \forall t \in T, \tag{C3d}$$

$$\sum_{t\in T} [y_a^t + y_{rev(a)}^t] \le 1 + \Delta_a \quad \forall a \in A,\ T_a < F_a, \tag{C4}$$

$$f_a^1 + f_a^2 \le \sum_{t\in T} \gamma^t y_a^t \quad \forall a \in A, \tag{C5}$$

$$y_a^t \le f_a^1 + f_a^2 \quad \forall a \in A, \tag{C6}$$

$$\sum_{a\,|\,n=F_a \text{ or } n=T_a} y_a^t \le \deg_n \tau_n^t \quad \forall n \in N,\ \forall t \in T, \tag{C7a}$$

$$\sum_{t\in T} \tau_n^t \le 1 + 2(s_p^o + s_p^i) \quad \forall n \in N. \tag{C7b}$$

Constraints (C1a) and (C1b) ensure that flows to and from each FDH are conserved; constraints (C1c) and (C1d) set up the source of flow commodity 1 and sink of flow commodity 2. By doing this, the desired property of the network is set up as the solution and is now forced to travel from the entry point to the exit point, visiting each FDH along the way and also having an independent path from both the entry and exit point back to each FDH. Both flow commodities are hypothetical and do not have any ties to physical elements of the network. Constraints (C2a)–(C2d) ensure that the entry and exit nodes exhibit the desired network connectivity properties, that is, there is exactly one arc coming from the entry point and one arc going to the exit point. Constraints (C3a) and (C3b) describe the double-ring properties of the network; constraint (C3a) allows the network to branch at, at most, a single point, and constraint (C3b) allows two paths to combine at, at most, one point. When a double ring (dual loop) is formed, the left side of both constraints will be 1. Constraint (C3c) ensures the correct number of cables enter and leave each pit; for most pits, this is a single cable in and a single cable out; however, for the branch points it is one cable in and two cables out; for the splice point, it is two cables in and one cable out. Constraint (C3d) ensures that the correct cable type is used on each arc; the network cannot arbitrarily switch between different cable types; this can occur only at a branching point. Constraint (C3d) explicitly allows only as many incoming arcs using cable type t as there are outgoing arcs of cable type t, unless the node is a branching point; in this case, the outgoing cables do not have to match the incoming ones.

Constraint (C4) ensures that an additional cost is incurred for using an arc and its direct reverse, because the path must be diverse; therefore, although different arcs are conceptually being used to physically make the path diverse, new network will have to be installed on the opposite side of the road to the existing arc. Path diversity refers to the requirement for two paths from any particular point in the network back to the exchange; these paths do not share any network. The cost of creating new network is one of the largest costs in creating a distribution network; hence, modeling it correctly with this constraint and Δ_a variable is important.

Constraint (C5) ensures that the flow never exceeds a cable's capacity; although the flow in this formulation is purely conceptual, it does represent the number of FDHs that a particular cable is actively servicing; larger cables can service more FDHs than smaller ones. Constraint (C6) is a branching constraint in the MIP sense—not in the network sense. Constraints (C7a) and (C7b) specify that the cable being used may only change at a branching point.

Appendix D. MIP Formulation for Local Fiber Network

This formulation connects each FDH to a set of multiports subject to the NBN architecture.

Let the model assume the following data:

- A set of pits and (or) poles P in the FSAM indexed by p.
- Each pit or pole has a demand μ_p, which is the number of multiports placed at pit or pole p.
- A set of possible directional arcs A in the FSAM indexed by a, each going from pit and (or) pole F_a to pole T_a. Each physical connection between pits or poles will result in two arcs. The length of arc a is given by l_a.
- V^A subset of A, which is the set of arcs that can only be used for trenched cable.
- Δ^A subset of A, which is the set of arcs that contains an existing Telstra duct.
- The degree of each pole d_p, which is the number of arcs that start at (and end at) pole p.
- The set H of poles that are potential FDHs or splice locations.
- The fixed cost of a FDH installation—C^H.
- A set of cable types T indexed by t. Each cable type has a maximum ribbon capacity of P_t. Each arc a has a known cost for being connected by cable type t—C_{at}^A. We calculate this from the length of the arc.
- C_a^U: the cost of installing an underground cable along arc a.
- $C_a'^U$: the cost of installing an additional underground trench along arc a. Note that for most arcs, this cost will be the same as C_a^U; however, we define it in the formulation for clarity.
- T^H: the maximum ribbon capacity for a FDH.
- τ_a: the maximum number of ribbons allowed in underground arc a (used to capacitate ducts).

Define the following variables:

- $y_{at} \in \mathbb{R}^+$, $y_{at} \le 1$: 1 if arc a has a cable of type t installed.
- $y_a^u \in \mathbb{R}^+$, $y_a^u \le 1$: 1 if the underground portion of arc a is a utilized trench. Note that cables may be placed in the

ground below an aerial span; this underground portion is commonly referred to as a trench.

- $y_a^{'u} \in \{0,1\}$: 1 if arc a exceeds duct capacity (this is created only for arcs with ducts).
- $\pi_a \geq 0$: the ribbon flow on arc a (i.e., the number of free ribbons that will be available at the end pole).
- $\pi_a^u \geq 0$: the ribbon flow on the trench on arc a.
- $z_p \in \mathbb{R}^+$, $z_p \leq 1$: indicates if pit or pole p is used as a FDH. By definition, $z_p = 0$ if $p \notin H$.
- $w_p \in \mathbb{R}^+$, $w_p \leq 1$: indicates if pit or pole p is used as a branching point. Note that fractional values of w_p and z_p are allowed to collapse the symmetry that occurs in the formulation, because hub and branching locations can be swapped with no change to the cost of the solution.

The following are branching variables used in the formulation:

- $r_p \in \{0,1\}$: 1 if a pit or pole p with no demand is not used in the solution.
- $u_p \in \{0,1\}$: 1 if a pit or pole is used as a FDH or branching point.
- δ_p: an integer (possibly negative), is the difference between the total arc in flow and the total arc outflow for $p \in H$.
- $s_a \in \{0,1\}$: 1 if the span corresponding to arc a is used and exists only if $F_a < T_a$ (this is defined by the unique index assigned to pits or poles in the application). Define inv(a) as the arc that flows in the opposite direction to arc a.
- $s_a^u \in \{0,1\}$: 1 if the span corresponding to arc a is being trenched and only exists if $F_a < T_a$. Define inv(a) as the arc that flows in the opposite direction to arc a.

$$\text{Minimize } \; C^H \sum_{p\in H} z_p + \sum_{a\in A}\Big[C_a^U y_a^u + C_a^{'u} y_a^{'u} + \sum_{t\in T} C_{at}^A y_{at}\Big]$$

subject to:

$$T^H z_p + \sum_{a\in A \mid T_a=p} (\pi_a + \pi_a^u) \geq \mu_p + \sum_{a\in A\mid F_a=p} (\pi_a + \pi_a^u) \quad \forall p, \tag{D1}$$

$$z_p + \sum_{a\in V\mid T_a=p} y_a^u + \sum_{t\in T}\sum_{a\in A\mid T_a=p} y_{at} = 1 \quad \forall p, D_p > 0, \tag{D2a}$$

$$z_p + \sum_{a\in V\mid T_a=p} y_a^u + \sum_{t\in T}\sum_{a\in A\mid T_a=p} y_{at} \leq 1 \quad \forall p, D_p = 0, \tag{D2b}$$

$$\sum_{a\in V\mid T_a=p} y_a^u + \sum_{t\in T}\sum_{a\in A\mid T_a=p} y_{at} + r_p = 1 \quad \forall p, D_p = 0,\, p \notin H, \tag{D2c}$$

$$z_p d_p + \sum_{a\in A\mid T_a=p} y_a^u (d_p - 2) + \sum_{a\in V\mid T_a=p} y_a^u + \sum_{a\in A\mid T_a=p} y_{at} \geq \sum_{a\in A\mid F_a=p} y_{at} + \sum_{a\in A\mid F_a=p} y_{at} \quad \forall p, \forall t, \tag{D3a}$$

$$z_p d_p + \sum_{a\in A\mid T_a=p} y_a^u (d_p - 2) + \sum_{a\in V\mid T_a=p} y_a^u + \sum_{t\in T}\sum_{a\in A\mid T_a=p} y_{at} \geq \sum_{t\in T}\sum_{a\in A\mid F_a=p} y_{at} + \sum_{a\in V\mid F_a=p} y_a^u \quad \forall p \in H, \tag{D3b}$$

$$\pi_a \leq \sum_{t\in T} P_t y_{at} \quad \forall a, \tag{D4a}$$

$$\pi_a^u \leq y_a^u T^H \quad \forall a \notin \Delta^A, \tag{D4b.1}$$

$$\pi_a^u \leq \tau_a y_a^u + y_a^{'u} T^H \quad \forall a \in \Delta^A, \tag{D4b.2}$$

$$y_a^u \leq \sum_{t\in T} y_{at} \quad \forall a \notin V^A, \tag{D5}$$

$$u_p = z_p + w_p \quad \forall p, \tag{D6}$$

$$s_a = \sum_{t\in T}\sum_{a\in A\mid F_a=p} (y_{at} + y_{\mathrm{inv}(at)}) \quad \forall a, F_a < T_a, \tag{D7a}$$

$$s_a^u = y_a^u + y_{\mathrm{inv}(a)}^u \quad \forall a, F_a < T_a, \tag{D7b}$$

$$\sum_{t\in T}\sum_{a\in A\mid T_a=p} y_{at} = \delta_p + \sum_{t\in T}\sum_{a\in A\mid F_a=p} y_{at} \quad \forall p, \tag{D8}$$

$$z_p d_p + (d_p - 1) \sum_{a\in A\mid T_a=p} y_a^u \geq \sum_{a\in A\mid F_a=p} y_a^u \quad \forall p. \tag{D9}$$

Constraint (D1) ensures that each multiport is serviced and that FDH capacity is respected; note that each multiport is serviced by one ribbon. Constraint (D2a) ensures that a pole with demand is either a FDH or it has exactly one cable connecting into it. If a pole has no demand, it can have a FDH, an incoming cable, or neither as constraint (D2b) describes. Constraint (D2c) introduces the first branching variable used in the model; here the constraint states that every node with no demand may have at most one incoming arc. The branching variable in constraint (D2c) provided a small increase in solve speed. Constraints (D3a) and (D3b) ensure that the local cable network includes no branching at a pole. Branching may only occur in the local network at FDHs and pits. In addition, constraint (D3b) tightens the relaxation of the formulation by describing constraint (D3a) over all cable types.

Constraints (D4a), (D4b.1), and (D4b.2) ensure that the ribbon flow on an arc (trenched or aerial) is less than the maximum allowed by the particular cable type installed. Constraint (D5) ensures that new build can only be created below installed aerial cables; note that the constraint contains only aerial arcs and does not constrain the construction of new build in underground areas. Constraint (D6) adds the second branching variable, which merges the FDH and branching variables. This allows the z_p and w_p variables to be linear removing all symmetry caused by these variables in feasible solutions. The symmetry being discussed here is because each FDH and splice point is interchangeable in any feasible solution. Constraints (D7a) and (D7b) introduce the third branching variable of the model, which significantly improves the solution speed by removing the restriction that each tree is a directed graph. Direct-flow variables plus the constraints restricting tree structure will ensure that solution integrity is preserved and will allow the model to be significantly faster by letting the y_a and y_a^u variables be linear. Constraint (D8) contains the final branching constraint of the model—an integer variable representing the number of branches at any particular branch point. This constraint offers some slight improvements to solution speed. Constraint (D9) constrains the construction of additional underground network by stating that a pit or pole must have at least one incoming new build arc for there to be an outgoing new build arc or for the location to have a FDH installed. This ensures that the model correctly models real-world behavior with regard to underground build and avoids it by placing only underground build when required for branching reasons; essentially, each aerial branching point must have an underground link back to the FDH.

Appendix E. MIP Formulation for Detailed FDH to Multiports Network Layout

The formulation in Appendix D does not address the details of where particular cables and joints are located in the local network. It determines the overarching layout of the network, but not the specifics of how the cables connect from the FDH to the multiport. Two tiers of cable lie between the FDH and multiport; the FDH connects to multiple cables that we call local sheath segments (LSSs). These connect to joints known as access joint locations (AJLs), which in turn service (one or more) multiport sheath segments (MSSs). The MSSs connect to and ultimately service each multiport.

Let the model assume the following data:

- A set of pits and poles in the FSAM P indexed by p.
- A set of connections between arcs A indexed by a, which have a start pit or pole S_a, an end pit or pole E_a, and an available cross-sectional capacity π_a.
- Each pit or pole $p \in P$ has a demand d_p, which is the number of MSS cables required at that point.
- A set of potential underground LSS cables L. Each LSS cable $l \in L$ starts at node F_l, ends at node T_l, and has a length given in meters by Len_l. Each LSS cable also spans a set of arcs A_l, has a cost per meter of C^L, and a cross-sectional area π^L.
- A LSS cable can service Cap^L MSS cables.
- A set of potential underground MSS cables M. Each MSS cable $m \in M$ starts at node F_m, ends at node T_m, and has a length given by Len_m. Each MSS cable spans a set of arcs A_m, and has a cost per meter of C^M and a cross sectional area π^M.

Define the following:

- $l_i \in \{0,1\}$: 1 if LSS i is used, and 0 if not.
- $m_i \in \{0,1\}$: 1 if MSS i is used, and 0 if not.

$$\text{Minimize} \quad C^L \sum_{i\in L} \text{Len}_i l_i + C^M \sum_{i\in M} \text{Len}_i m_i$$

subject to:

$$\sum_{i\in M \mid T_i = p} m_i = 1 \quad \forall p \in P \mid d_p > 0, \tag{E1}$$

$$\sum_{j\in L \mid T_j = F_i} l_j \ge m_i \quad \forall i \in M, \tag{E2}$$

$$\sum_{i\in M \mid T_j = F_i} d_{T_i} m_i \le \text{Cap}^L l_j \quad \forall j \in L, \tag{E3}$$

$$\pi^L \sum_{i\in L \mid a\in A_i} l_i + \pi^M \sum_{j\in L \mid a \in A_j} m_j \le \pi_a \quad \forall a \in A. \tag{E4}$$

Constraints (E1) ensure each multiport is serviced by a MSS cable, and constraint (E2) requires that each MSS cable is connected to a LSS cable. Constraint (E3) restricts the service capacity of the MSS cables connecting to the LSS cable from the FDH. Constraint (E4) limits the cross-sectional capacity of each arc to ensure that no additional network is built to accommodate the cables.

Appendix F. Savings Calculations

				FSAM designs	
				Completed (to December 2013)	Total (2013 revision)
Category	Units per FSAM	Unit saving	Savings per FSAM	650	3,000
Design time savings (days)	129	\$500	\$64,500	\$41,925,000	\$193,500,000
Construction savings (premises)	2,500	\$200	\$500,000	\$325,000,000	\$1,500,000,000
Total savings			\$564,500	\$366,925,000	\$1,693,500,000

Table F.1: The data in the table illustrate the savings we achieved.

Notes. **In Table 2: (1) The number of days required to produce a design decreased from 145 to 16, as determined through benchmarking before and during the FOND deployment. We use a conservative daily labor rate of \$500 to determine the design time savings. (2) We calculated the construction savings by comparing the estimated construction costs of the manual designs and those produced by FOND. These showed FOND designs were \$200 per premises lower than the manual designs that expert planners generated. (3) The 2013 strategic review revised the fiber footprint down to 68 percent of premises. The revised FSAM target is 3,000. (4) All \$ figures are in Australian dollars (AUD).**

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Joe Forbes believes that commercial mathematics is only truly powerful when it is simple to use and produces real-world results. Joe is creating a group of Biarri businesses that blend good design and mathematics, providing accessible business analytics and optimisation. Most importantly, he wants to work with smart people who build clever solutions that Biarri's clients love to use.

Michael Forbes has more than 25 years of experience as a practitioner in many countries and many application areas. As well as consulting for Biarri, he lectures third- and fourth-year courses in operations research at the University of Queensland, where he also supervises several postgraduate students.

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