Interfaces
Vol. 41, No. 6, November–December 2011, pp. 521–533
ISSN 0092-2102 | EISSN 1526-551X | 11 | 4106 | 0521

informs®
http://dx.doi.org/10.1287/inte.1110.0563

Building Cyclic Schedules for Emergency Department Physicians

Yann Ferrand, Michael Magazine, Uday S. Rao
Department of Operations and Business Analytics, University of Cincinnati, Cincinnati, Ohio 45221
{ferrany@mail.uc.edu, mike.magazine@uc.edu, uday.rao@uc.edu}

Todd F. Glass
Cincinnati Children's Hospital Medical Center, Liberty Campus, Cincinnati, Ohio 45229, todd.glass@cchmc.org

Physicians at a branch of the emergency department at Cincinnati Children's Hospital Medical Center complained that their schedules were too erratic because of the multitude of operating requirements, regulatory constraints, physician preferences, and holiday requests. We addressed this issue by using integer programming to build cyclic schedules that can be repeated throughout the year. These schedules are flexible enough to handle incorporating holidays, work assignments, and vacation requests ex post. After we rolled out the calendar-year-based cyclic schedule, we captured statistics to assess the viability and the quality of the yearly schedule generated. Surveys of the physicians and the scheduler after implementation showed that the new schedule provides predictability and well-balanced work patterns.

Key words: health care: emergency department physician scheduling; cyclic schedule; integer programming applications.
History: This paper was refereed. Published online in *Articles in Advance* September 30, 2011.

Cincinnati Children's Hospital Medical Center (http://www.cincinnatichildrens.org), a premier children's hospital, recently opened an additional emergency department in a suburban area. Typical of emergency departments, this facility operates 24/7, and must be staffed accordingly. In this environment, the emergency physician is one of the key personnel, and his/her work schedule must be built carefully to meet operating requirements, regulatory constraints, and physician preferences. Operating requirements pertain to physicians' workloads at a given location; regulatory constraints include, for example, the number of hours of rest required between two working shifts; physicians need at least a 16-hour break between working shifts. Physician preferences comprise the work patterns, especially as they relate to the circadian rhythm of the human body; changing a work pattern from late in the day to early the next day is more strenuous on the human body than changing it from early to late in the day. These requirements include a multitude of constraints and render the work of the scheduler (i.e., the person responsible for developing the schedule) a challenging task, even when software support is available.

Physicians' schedules have traditionally been built around their requests for time off. The ruling paradigm is that physicians' requests for time off (i.e., holidays and vacations) are guaranteed to be honored, provided they are made with reasonable notice and respect for administrative rules of seniority and fairness. After determining the time off for each physician, the scheduler assigns the working shifts, ensuring that all the necessary constraints are met. At Cincinnati Children's Hospital Medical Center, this effort, albeit tedious, is accomplished by using Web-based scheduling software, Peake Software Lab's Tangier Web (http://www.peakesoftware.com). This software keeps track of operating requirements and regulatory constraints, which helps the scheduler as she develops a schedule that aims at accounting for physician preferences.

The scheduler develops schedules for the group of physicians working at the branch emergency department (the branch) and for all other physicians working at the base emergency department (the base) of the main hospital. These two schedules are interdependent because the branch's current staff is insufficient to cover all the shifts required to operate the facility 24/7. Therefore, emergency physicians from the base must be assigned to work any shift that is left unassigned after the physicians working at the branch have been assigned their shifts.

In practice, the schedules have been built as follows for both base and branch sites, a process that the base still uses. Shifts have an eight-hour duration, and the days are split into three shifts, the AM shift (8 AM–4 PM), the PM shift (4 PM–12 AM), and the *overnight* shift (12 AM–8 AM); each shift must be staffed with exactly one emergency department physician. In assigning legal-holiday shifts to physicians, the scheduler assigned the entire calendar year starting July 1 based on fairness rules, which include preference rankings for legal holidays not worked by the physicians, seniority, and legal holidays assigned for the previous years. The scheduler next assigned the weekend and weekday shifts the physicians would work, considering their requests for vacations and conferences. The weekend shifts were assigned for a six-month period; the weekday shifts were assigned for a three-month period (i.e., a quarter). Complete schedules were built four times each year for three-month periods. This provided the physicians with flexibility in choosing their time-off requests, because they could make requests during the year. However, the physicians did not learn their working schedules until the schedule was published shortly before the quarter began. In addition, the schedules did not display any repeating pattern over time, because they were built around the requests for time off. These attributes of the scheduling process made it impractical for the physicians to plan their time off in advance, except for their official vacation requests.

When the new branch opened, the physicians who staff it expressed their desire to try a new approach to developing their work schedules. They wanted their work schedules to be more predictable by using repeating patterns; they also wanted more consideration given to both their working preferences and their need for schedules that respect the human body's circadian rhythm. In the previous scheduling approach, each period was unique, and the physicians felt that the schedules were not published sufficiently in advance. This lack of visibility into the future prevented the physicians from making long-term plans and tended to generate physician burnout. The literature on nurse scheduling documents the importance of work predictability to avoid burnout. Mueller and McCloskey (1990) find that reliable schedules are a component of nurses' job satisfaction; Shader et al. (2001) study the relationship between schedule stability and nurses' job satisfaction, stress, and turnover; Stachota et al. (2003) show that working hours and schedules are a primary reason that nurses terminate their employment. Note that in our research problem we develop the schedule for the physicians working primarily at the branch, and not for the physicians working primarily or exclusively at the base.

One way to achieve predictability is to build cyclic schedules that repeat period after period. The downside is that it is impossible to build a cyclic schedule that considers diverse requests for time off and the assigned legal holidays, because the uniqueness of each period conflicts with the repetition of a predetermined cycle. However, the physicians noted that knowing their work schedules in advance would allow them to better organize their lives during their time off. They hoped that specific vacations requests could still be granted because they would give the scheduler their requests with as much lead time as they had previously given. Therefore, a cyclic-scheduling approach represented a paradigm shift for the physicians who were willing to organize their time off around their work schedule, as opposed to having a work schedule built around their time off. Handling the legal-holiday work assignments remained a major difficulty, because we had to adapt the cyclic schedule to fit those assignments. In our solution, we build a schedule, which assigns each physician to work specific shifts, for all the emergency physicians working at the branch facility; we can then remove some of these shift assignments for a particular physician and assign a physician from the base emergency department to these shifts. Assigning additional shifts to a physician from the branch after the schedule has been built is difficult because of all the restrictions that constrain the feasibility of a particular schedule. Thus, assigning more shifts than the physicians are required to work provides us with flexibility to handle holiday assignments and vacation requests by removing shifts. These alterations somewhat perturb the repeating nature of the cyclic schedule, but they cause little reduction in predictability of work assignments, which is the primary goal for the physicians.

Literature Review

Although a large body of literature addresses cyclic scheduling, it does not include a significant amount that relates to health care. Diverse applications of cyclic scheduling can be found in workforce staffing. Baker (1976) provides a review of various instances in which mathematical models have been used in workforce scheduling; Baker and Magazine (1977) devise algorithms to schedule days off in environments of continuous operations, and Bartholdi et al. (1980) study the cyclic-staffing problem and its structure when modeled as an integer program. In health-care applications, a large section of this research on cyclic scheduling addresses nurse scheduling, a topic that has been studied for more than 40 years and has similarities to physician scheduling. Ernst et al. (2004) compile an extensive bibliography of personnel scheduling and rostering, which includes health-care applications. Maier-Rothe and Wolfe (1973) use heuristics to develop cyclic schedules for nurses; we note that the nurses were initially resistant to using cyclic schedules because of the perceived inflexibility of a repeating pattern. The authors overcame this concern by providing the nurses with several feasible alternatives from which they could choose. Their heuristics, albeit appropriate for their problem cannot be adapted to solve our problem, because these problems differ considerably in terms of staffing requirements and policies. Typically, the differences in contractual requirements, worker preferences, and specialties also distinguish physician scheduling from nurse scheduling.

Carter and Lapierre (2001), who classify physician scheduling into three general approaches (i.e., acyclic, cyclic with rotation, and cyclic without rotation), provide a general mathematical formulation of the physician-scheduling problem and describe an application for each approach. They discuss the need to find a way to handle vacation requests and to avoid disrupting the circadian rhythm in sequencing day and night shifts. Because an acyclic schedule is nonrepetitive, it can consider holidays; however, creating such a schedule is generally more time intensive than creating a cyclic schedule and does not have the desired repeatability. Under a cyclic schedule with rotation, the physicians all follow the same cycle; however, each starts at a different point in the pattern. Although this approach has the advantage of being equitable, it does not accommodate personal preferences. A cyclic schedule without rotation provides each physician with his/her own work pattern, and can therefore accommodate individual preferences and workload requirements. The general formulation of common constraints in Carter and Lapierre (2001) allows us to build upon this work to develop a model that fits our working environment. In our context, each physician works a different number of hours, thus requiring that we develop individual schedules.

The study on normal sleep patterns and circadian rhythms in Nelson (2008) shows that work schedules that respect circadian rhythms could reduce sleep deprivation, which in turn helps to prevent job burnout. Dittus et al. (1996) use simulation to evaluate the quality of medical-resident work schedules prior to implementation, and pay particular attention to the effects of the potential schedules on sleep and activity patterns.

Beaulieu et al. (2000) also devise a physician schedule by solving an integer program. They formulate the problem for a six-month period, and then solve it month by month to reduce the problem size; the solution for a particular month considers the schedules for the previous five months. We can also draw upon their work in formulating our problem, because their constraints, which are set in terms of work requirements and work patterns, resemble some of our physician work preferences. More recently, Brunner et al. (2009) use mixed-integer programming to construct physicians' working shifts in which the start time of the shift is within a time window, and the shift's duration is flexible and does not exceed a specific length of time; in contrast, shift start and finish times are fixed in our problem.

Another approach to scheduling physicians is to combine integer programming or constraint programming with heuristics for local search, as Rousseau et al. (2002) and Bourdais et al. (2003) discuss. Other solution approaches include constraint programming (Laporte and Pesant 2004) and tabu search methods (Buzon 2001). Gendreau et al. (2007) also present a generic formulation of the usual constraints associated with the physician-scheduling problem, and examine four solution approaches and their appropriateness depending on the particularities of the problem. Scheduling medical residents also has similarities to our problem (Franz and Miller 1993, Cohn et al. 2009). However, it differs from physician scheduling in that scheduling medical residents has additional requirements, such as the need to schedule rotations through various specialties within the hospital.

This paper contributes to the body of knowledge on emergency-physician scheduling by formulating specific constraints associated with work patterns that are desirable to the physicians in our problem environment, but that can also be important for physicians working in other emergency departments. To the best of our knowledge, these constraints have not been formulated in other papers. This paper also illustrates the iterative nature of model development and reinforces the notion that successful implementation of new schedules depends greatly on this iterative negotiation in constraint formulation and evaluation of the corresponding solution. Finally, it shows that even if two facilities share some staff, in this case the branch and base emergency departments, they can still be operated with two distinct scheduling methods.

Problem Analysis and Formulation

Main Formulation

At our initial meeting with the physicians and the scheduler, we identified the objective as the development of a schedule for the physicians working at the branch facility. This schedule would meet their work requirements, provide them with predictability over their work assignments, consider their work preferences, and respect the circadian rhythm of the human body.

Based on our literature review and goal definition with the physicians, we decided to address their need for a new schedule by building a cyclic schedule using integer programming. With this approach, we could account for their work requirements and preferences; the generated cycle could then be repeated over the course of the year with some minor perturbations. Through an iterative process described in the next subsection, we developed an integer program that captures all work requirements and preferences. Drawing from goal programming, we formulated constraints as either hard or soft by adding a slack variable that captures the deviation from the right-hand side of the constraint; hard constraints must be met, whereas soft constraints can be violated with a penalty. The objective function minimizes the sum of all deviations; this can be a simple or weighted sum, reflecting different priorities. We modeled regulatory constraints and work requirements as hard constraints, and we classified physician preferences as either hard or soft constraints based on how important the physicians considered each preference to be.

The appendix includes the detailed formulation of the model with the most current constraints and cycle length. It also shows the problem's complexity, which results from the large number of idiosyncratic rules. We programmed the models in AMPL and solved them with CPLEX 11.0 running on a 2.33 GHz personal computer with 2.98 GB of RAM. The run times, which varied across the different models we solved, ranged from a few minutes to several hours. The run time depends on the cycle length, which has an effect on problem size, and on the constraints we formulated. Specifically, the weekend assignment rules we adopted at a later stage in the model development largely increased the run time. Solving this model for the set of five physicians (labeled MD1–MD5 in the appendix) working at the branch has approximately 1,600 variables and 4,700 constraints. We obtained an optimal solution in less than six hours.

A summary of the general rules we used for regulatory constraints, work requirements, and work preferences follows.

- Regulatory constraints
  - R1: Give at least 16 hours off between two working shifts (hard constraint)
  - R2: Assign one physician on a shift (soft constraint)
- Work requirements (hard constraints)
  - W1: The total number of shifts assigned to each physician is fixed

- W2: The proportion of shifts (i.e., AM, PM, and *overnight*) is one-third for each type of shift
- W3: At least 20 percent of the AM shifts must be assigned on Friday
- W4: The total number of "Friday" shifts (i.e., Friday PM and *overnight*) and "weekend" shifts (i.e., from Saturday AM to Sunday *overnight* included) assigned is fixed

• Physician preferences (hard constraints)
- P1: Batch weekend shifts together and ensure a fair distribution of weekend shifts
- P2: Never assign a physician to work more than two consecutive weekends
- P3: Assign at most four shifts in any given week, and assign shifts to group consecutive working days
- P4: If the last worked shift is an *overnight*, the break must be at least two days
- P5: If the last worked shift is PM, the break could be one day off followed by either a PM or *overnight* shift, or at least two days off if the next assigned shift is an AM shift
- P6: If the last worked shift is an AM, then the break could be only one day off

• Physician preferences (soft constraints)
- P7: If a weekend is off, then the preceding Friday or the following Monday is off
- P8: Assign no more than two consecutive *overnight* shifts during the Monday–Thursday period.

Development Process

To obtain an appropriate schedule, we needed to know work requirements and preferences, and the length of the repeating cycle. We gathered the information on work requirements, outlined in the main model formulation in the appendix, from the hospital's scheduler. We then developed the rules that we would use to create the schedule. There was consensus among the physicians about several attributes that would constitute a good schedule, such as respecting the circadian rhythm and avoiding single shifts, in which a working shift is preceded and followed by a day off. However, other work preferences might vary across individual physicians, reflecting their diverse wishes on the consecutive number of working days, the grouping of working shifts, the duration of the breaks after day shifts and night shifts, and work patterns on weekends and on the day before or after a weekend. We first gathered all the individual preferences and included them into our model. We found a feasible solution; however, the physicians and the administration felt that accommodating each individual work preference would not lead to sustainable schedules because it would raise questions of fairness and equity among the workers. Therefore, we worked with the physicians to construct general rules on work preferences that would reasonably capture each individual's preferences. Exceptions to these rules were made only to accommodate one physician who exclusively works *overnight* shifts.

Developing the complete set of general rules was an iterative process in which we would solve the corresponding integer program and propose a schedule to the physicians and the scheduler; they would determine what would work and what was not desirable. We would then formulate the changes, generate a new schedule, and submit it for feedback again until they felt the resulting schedule was appropriate. To communicate the results to the physicians and scheduler, we built VBA macros in Excel; they translated the AMPL solution into a visual representation of the physicians' workload on a yearly calendar and capture statistics. This effort was essential to verify that we had not omitted any requirement and to ensure that our proposed schedule could be implemented.

A critical aspect of cyclic scheduling is the choice of cycle length. Short cycles provide more repeatability and can be more easily remembered by the workers; however, they offer fewer possibilities for the inclusion of all the required attributes, in particular for workload and work-pattern necessities. For example, the physicians are mandated to work 11 PM or *overnight* shifts on Fridays throughout the year. This requirement cannot be fulfilled with a one-week cycle, because the work pattern would be repeated 52 times over the course of a year. In short, cycle length is largely driven by the nature of the constraints to be included. Because generating the final schedule was an iterative process, we solved models for 4-week, 8-week, and 13-week cycles. Once the cycle length has been chosen, yearly workload requirements can be scaled down to their corresponding figures over the cycle duration. We rounded up these figures because removing shifts is much easier than adding extra shifts.

In consideration of all the work requirements and preferences, we finally opted for an eight-week cycle, because the physicians felt that this cycle length was neither too short nor too long, and would permit a balanced distribution of type of weekend shifts. Without a doubt, devising the rules that would generate the work patterns during the weekend shifts was a crucial step for two reasons. First, work requirements that govern the number of working weekend shifts are stipulated in each physician's working contract, and each physician works the same number of weekend shifts. Second, working weekends is usually not a physician preference; therefore, to ensure fairness, the physicians agreed that they would work the same number of AM, PM, and *overnight* shifts on weekends. In addition, the number of weekends worked should be minimized, which means that weekend shifts should be clustered together, resulting in Saturdays and Sundays worked together, and possibly Friday PM and *overnight* shifts also.

In addition to the constraints development and choice of cycle length, recall that to operate 24/7, additional emergency physicians from the base emergency department must fill in the remaining unassigned shifts. This has two implications for the final schedule that we developed for the branch. First, the scheduler expressed the need for the new schedule to about evenly spread the unassigned shifts across the cycle because it makes it easier to find a physician from the base to cover these shifts. We did not specifically formulate constraints to honor this request because throughout this iterative process all the schedules showed the uncovered shifts to be spread out fairly evenly across the cycle. This is due to the nature and the number of constraints already included. Second, although one physician works only *overnight* shifts, the proportion of shifts for the other physicians still must be as close as possible to one-third of each shift type, ensuring that most of the total *overnight* load is covered. This requirement allows the physicians from the base who cover the remaining shifts at the branch to benefit from the work pattern of the physician who works *overnight* shifts only, especially given the fact that *overnight* shifts are the least desirable.

One last important detail is that some physicians who work at the branch also must work a fraction of their workload at the base. In scheduling the worked shifts, we did not consider this specific interaction with the base. This enables us to limit the scope and size of the problem formulation. It does not represent a strong limitation because the scheduler can decide, based on the generated schedule, which assigned shifts these physicians will work at the base, and thus best fit the schedule used for the group of physicians working there. It also allows us to relax the regulatory constraint that requires exactly one physician to be assigned to each shift, because we can assign two physicians to a particular shift to fulfill other constraints, provided one of the two physicians is part of the subset who can work at the base. In the future, the hospital administration plans to hire additional physicians so that the branch facility can operate independently of the base. Once this is achieved, the concerns mentioned above about the nature of the interactions between the two facilities will no longer exist.

Results and Implementation

Table 1 shows the final eight-week cycle we generated by solving the integer program, and the workloads and work patterns for the five physicians to be scheduled; number 1 is the physician who works *overnight* shifts only. Additional physicians from the base will fill the shifts that are left unassigned, and the hospital's scheduler will decide which physicians to assign to these shifts.

At each iteration, and for the final-schedule template implementation, we rolled out the cycle on the yearly calendar for 2009–2010, based on the hospital's fiscal year, which starts July 1. The next step was to include the work assignments on legal holidays, which the scheduler developed independently, following internal rules of fairness. To input these work assignments into the schedule, we manually replaced our work assignments during the legal holidays with those given by the scheduler and then removed the assigned shifts adjacent to these holiday periods that would create incompatibilities according to the list of rules. This was possible because we assigned more shifts to the physicians than they are required to work, permitting us to remove extra shifts based on particular requests or requirements that arise after the cyclic schedule has been established.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
8 AM–4 PM				2	2		
4 PM–12 AM	2		4	4	4		
12 AM–8 AM	1	1	1	1	5	5	5
			5	5	5		
		2			4	4	4
	3	3	2	2	1	1	1
	2	2		2	2	5	5
		5	5				
	1	4	3 or 4	3			
	4	4		4	4	3	3
		3	3		5	5	5
	1 or 5	1	1	1	2	2	2
			4	4	4		
	3	5	2	2			
	1	1	1 or 5	1 or 5	3	3	3
				3	3		
		4			2	2	2
	5	5	4	4	1	1	1
	5	5		5	5	4	4
			2		3	3	3
	1	4	4	2	1	1	1
				3	3	2	2
	4	5	5				
	1	2	2	5	4	4	4

Table 1: The eight-week cycle for the five physicians will be repeated throughout the year.

Once the legal holidays were arranged, we addressed the vacation requests, removing physicians' shifts assigned during their vacation time. Changes in vacation requests throughout the year can be accommodated, provided they are submitted with sufficient lead time. Because the physicians did not request as much vacation as they are legally allowed, the resulting schedules still displayed more assigned shifts than the physicians' yearly workload requirements. Therefore, we can remove additional shifts. In practice, some agreement could be made as to how many shifts a physician can request off as vacation time, and how many shifts the scheduler can remove based on the ease of finding another physician to cover those shifts. The statistics we provided on the number and types of shifts worked helped the scheduler and the physicians keep track of each element in the work requirements, and helped identify the shift assignments that can be removed without violating a physician's workload requirement (see Table 2). In particular, extra Fridays and weekend shifts could be considered first as candidates for removal. Likewise, single-shift assignments that occur as a result of holiday and vacation accommodations can be identified and possibly removed, depending on their effect on the other statistics.

Although the cyclic schedule automatically assigns physicians during the legal holiday periods, physicians know that the legal holiday schedule is built following specific rules, independent of the cyclic

Total worked shifts	Name	Color code	Day	Evening	Night	Single shifts
145	1	1	0	0	145	1
141	2	2	48	46	47	7
116	3	3	35	39	42	3
155	4	4	58	47	50	2
165	5	5	54	58	53	1
		Total	195	190	337	14
Total shifts to assign	1/3 overnights	Name	"Fridays" (11 or 15)	"Weekends" (28)	Holiday worked	Holiday/vacation taken
133	NA	1	18	36	9	9
132	44	2	12	32	8	11
107	36	3	12	38	6	7
144	48	4	16	32	8	6
142	47	5	13	41	8	7
		Total	71	179	39	40

Table 2: The table illustrates work-requirement statistics induced by the schedule.

Explain your satisfaction (dissatisfaction) with the implemented cyclic schedule (provide examples)	How do you feel about these adjustments you had to make?
How do you feel about the predictability of the implemented cyclic schedule?	Over the past quarter, how many times have you had to make trades?
How do you feel about your working patterns under the implemented cyclic schedule (e.g., distribution of shifts, time off between shifts, weekends work pattern, and distribution)?	What would make a cyclic schedule better for you?
What adjustments have you had to make to the implemented cyclic schedule?	What other comments, suggestions, and changes would you make to the cyclic schedule?

	Strongly disagree	Disagree	Indifferent	Agree	Strongly agree
I have had more difficulties making trades	1	2	3	4	5
I have had more difficulties planning my leisure time	1	2	3	4	5
I feel overall less fatigued	1	2	3	4	5
My quality of life has overall improved	1	2	3	4	5
I have had more difficulties planning my nonclinical work	1	2	3	4	5
My sleep quality has improved	1	2	3	4	5

Table 3: The table shows the list of emergency-physician survey questions.

schedule. As such, it preempts the regular schedule. Consequently physicians must candidly express their holiday preferences, and cannot request time off for only those holidays that they would be assigned according to the cyclic schedule.That is, a physician who is off at Christmas according to the cyclic schedule and therefore does not request the holiday off, could be assigned to work on Christmas. Assisting the scheduler in building the legal-holiday work schedule could be part of future work, in which we would maximize the accommodation of the physicians' preferences based on their rankings of legal holidays and account for other fairness rules in use at the hospital.

Three months after the cyclic schedule was implemented, we surveyed the physicians for feedback. We asked them eight open-ended questions about their satisfaction with the cyclic schedule and their suggestions for future improvements (see Table 3). We also included six questions using a Likert scale to assess improvements over the previous schedules on planning leisure time and nonclinical time, quality of life, sleep, and overall fatigue. Using this survey instrument, the hospital administration could record the physicians' feedback at multiple points in time, compare responses over time, and possibly make further adjustments to the scheduling rules.

In their responses to the questions, the physicians stated that the new scheduling method provided them with more predictability over their workweek, which in turn "relieves stress" and helps them "plan their activities outside of work." Having a calendar template for the entire year gives the physicians the ability to plan their vacation requests with the knowledge of their scheduled work, and enables them to make better use of their unassigned time. They also found the new work patterns to be "quite livable" and "well-balanced." Moreover, in developing the cyclic schedule we had anticipated that the resulting schedule would provide physicians with more consistent work patterns and an improved sense of fairness in the distribution of shifts, especially weekend shifts. The physician's feedback corroborated these qualitative benefits. Finally, the scheduler was also satisfied with the final schedule template, which she used to develop and publish the physician's work assignments using the hospital's internal software.

Conclusion

We provided the physicians with a new process that resulted in schedules that fulfilled their requirements, constraints, and requests. This process also helped them to identify some trade-offs in formulating their requests and classifying what was critical, as opposed to desirable. Feedback from the physicians and the scheduler showed that they are satisfied with the new schedule. We engage in a continuous dialogue to address new issues as the schedule unfolds, and to assist in developing the schedule for the next fiscal year. Future work could entail helping the administrators plan for growth as they hire additional physicians at the branch to cover all 21 shifts of the week, without having to use physicians from the base. Based on anticipated work requirements in the contract, adding three physicians to the group of five currently employed should be sufficient to run the branch independently. With eight physicians, the problem size will grow, resulting in an integer program of a much larger size, and possibly leading us to resort to using heuristics to find a solution.

Appendix

Let

$\mathcal{I} = \{1, \ldots, I\}$ represents the set of shifts in a day. For our environment, $I = 3$ with shift $1 \equiv$ AM (8 AM–4 PM), $2 \equiv$ PM (4 PM–12 AM), and $3 \equiv$ *overnight* (12 AM–8 AM).

$\mathcal{J} = \{1, \ldots, J\}$ represents the set of days within the cycle. $J = 56$ in our eight-week cycle.

$\mathcal{K} = \{1, \ldots, K\}$ represents the set of physicians to schedule. $K = 5$.

$\mathcal{L} = \{1, \ldots, L\}$; $L = 8$ for the eight-week cycle.

The following decision variable defines each physician's work schedule:

$x_{ijk} = 1$ if physician k works shift i on day j, zero otherwise, for $k = 1, \ldots, K$; $i = 1, \ldots, I$; $j = 1, \ldots, J$.

The following variables measure the degree of deviations from a desired goal:

d_{ij}^{0}, deviation from the goal of ensuring that only one physician is assigned to work shift i on day j.

d_{k}^{1l}, deviations from the goal of making weekend l a three-day weekend off by including the adjacent Friday or Monday for physician k.

d_{jk}^{2}, deviation from the goal of physician k working two or fewer *overnights* in a row in the Monday–Thursday j time window.

To achieve particular requirements in the problem formulation, we define variables that capture the occurrence of specific situations involving successive worked or nonworked days. These variables are defined as continuous, and based on the constraints in which they are employed, their values will be forced to either 0 or 1.

To ensure a succession of worked days:

$s_{jk}^{1} = 1$ if physician k does not work on day $(j-1)$ but works on day j, zero otherwise.

To ensure a succession of days off between groups of *overnights* worked:

$s_{jk}^{2} = 1$ if physician k works on day $(j-1)$ but not on day j, zero otherwise.

To give either Monday or Friday off when the weekend is off:

$s_{jk}^{3} = 1$ if physician k does not work on Saturday j, zero otherwise, for $j = 6, 13, 20, \ldots, 55$.

$s_{jk}^{4} = 1$ if physician k does not work on Sunday j, zero otherwise, for $j = 7, 14, 21, \ldots, 56$.

To ensure a limitation in the number of successive *overnights* worked:

$s_{jk}^{5} = 1$ if physician k does not work *overnight* on day j, zero otherwise.

The objective function aims at minimizing the deviations from the goals on the soft constraints. The larger coefficient on the deviation variables that relate to more than one physician assigned to a shift ensures that this situation is avoided with priority over the other deviations. Because of the cyclic nature of the schedule we are developing, for some constraints the indexing notation has to reflect that the beginning and the end of the cycle are connected to each other. This represents special cases, that we do not specifically write out in the model formulation here, but that we included in our implementation.

$$\min \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} 15 d_{ij}^{0} + \sum_{k \in \mathcal{K}} \sum_{l \in \mathcal{L}} d_{k}^{1l} + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} d_{jk}^{2}, \tag{1}$$

subject to the following constraints, which are labelled according to the description in the *Main Formulation* section of the paper.

R1, P5, P6: Constraints on time off between two worked shifts for physician k. Equations (2) to (7) apply $\forall j \in \mathcal{J}$, $\forall k \in \mathcal{K}$:

If physician k works PM on day j, then he/she cannot work AM on either day $(j+1)$ or $(j+2)$:

$$x_{2jk} + x_{1(j+1)k} \leq 1; \tag{2}$$

$$x_{2jk} + x_{1(j+2)k} \le 1. \tag{3}$$

If physician k works *overnight* on day j, then he/she cannot work AM and PM on either day $(j+1)$ or $(j+2)$:

$$x_{3jk} + x_{1(j+1)k} \le 1; \tag{4}$$

$$x_{3jk} + x_{2(j+1)k} \le 1; \tag{5}$$

$$x_{3jk} + x_{1(j+2)k} \le 1; \tag{6}$$

$$x_{3jk} + x_{2(j+2)k} \le 1. \tag{7}$$

R2: Assign at most two physicians per shift. Equations (8) and (9) apply $\forall\, i \in \mathcal{I}$, $\forall\, j \in \mathcal{J}$:

$$\sum_{k\in\mathcal{K}} x_{ijk} - d_{ij}^0 \le 1 \quad \text{and} \quad d_{ij}^0 \le 1. \tag{8}$$

When two physicians are assigned to a shift (i.e., $d_{ij}^0 = 1$), then one of them should be either MD3 or MD5, the two physicians who are required to work a fraction of their workload at the emergency department of the main hospital:

$$x_{ij3} + x_{ij5} \ge d_{ij}^0. \tag{9}$$

W1: Respect physician k's workload:

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} x_{ijk} = L_k \quad \forall\, k \in \mathcal{K}, \tag{10}$$

where L_k is the total workload during the eight-week cycle for physician k, per work contract. For example, for MD1, $L_1 = 24$ shifts, which translates into an average weekly workload of three shifts.

W2: Limit the proportion of AM, PM, and *overnight* shifts for each physician:

$$a_{ik} \le \sum_{j\in\mathcal{J}} x_{ijk} \le b_{ik} \quad \forall\, i \in \mathcal{I},\ \forall\, k \in \mathcal{K}, \tag{11}$$

where a_{ik} and b_{ik} are, respectively, the lower and upper bound on the number of shifts of type i for physician k, per work contract. For example, for MD1, $a_{11} = b_{11} = a_{21} = b_{21} = 0$ and $a_{31} = b_{31} = 24$ because all 24 shifts during the cycle have to be *overnights*. For the other physicians, the distribution of types of shifts should be as close as possible to a third each (i.e., $a_{ik} = \lfloor L_k/3 \rfloor$ and $b_{ik} = \lceil L_k/3 \rceil$).

W3: For each physician (except MD1), at least 20 percent of the AM shifts have to be assigned on Friday. Therefore, for physician k, the sum of all Friday AM shifts over the eight-week cycle has to equal 2:

$$\sum_{j=Fri} x_{1jk} = \begin{cases} 2 & \forall\, k \ne 1 \\ 0 & k = 1. \end{cases} \tag{12}$$

W4 and P1: Weekend assignments:
Once per cycle, physician k works Saturday and Sunday AM consecutively (except MD1 who works *overnights* only):

$$x_{1,\,6+(l-1)7,\,k} - x_{1,\,7+(l-1)7,\,k} = 0 \quad \forall\, k \ne 1,\ \forall\, l \in \mathcal{L}; \tag{13}$$

$$\sum_{l\in\mathcal{L}} (x_{1,\,6+(l-1)7,\,k} + x_{1,\,7+(l-1)7,\,k}) = 2 \quad \forall\, k \ne 1. \tag{14}$$

Once per cycle, physician k works Friday, Saturday, and Sunday PM consecutively (except MD1 who works *overnights* only, and MD4 who is assigned specific rules below to accommodate the fact that MD4 is required to work one more Friday PM or *overnight* than the other physicians):

$$x_{2,\,5+(l-1)7,\,k} - x_{2,\,6+(l-1)7,\,k} = 0 \quad \forall\, k \ne 1, 4,\ \forall\, l \in \mathcal{L}; \tag{15}$$

$$x_{2,\,6+(l-1)7,\,k} - x_{2,\,7+(l-1)7,\,k} = 0 \quad \forall\, k \ne 1, 4,\ \forall\, l \in \mathcal{L}; \tag{16}$$

$$\sum_{l\in\mathcal{L}} (x_{2,\,5+(l-1)7,\,k} + x_{2,\,6+(l-1)7,\,k} + x_{2,\,7+(l-1)7,\,k}) = 3 \quad \forall\, k \ne 1, 4. \tag{17}$$

Because MD4 is required to work three Fridays PM or *overnight* per cycle as opposed to two like the other physicians, we do not impose that MD4 only works Friday PM in combination with Saturday PM. As a result, the weekend PM assignments are slightly different than those for the other physicians because we allow the possibility of working Friday PM without Saturday PM and ensure that two Friday PM and one Friday *overnight* are assigned. Similar to the other physicians, the Friday, Saturday, Sunday consecutive PM will occur once per cycle:

$$x_{2,\,5+(l-1)7,\,k} - x_{2,\,6+(l-1)7,\,k} \ge 0 \quad k = 4,\ \forall\, l \in \mathcal{L}; \tag{18}$$

$$x_{2,\,6+(l-1)7,\,k} - x_{2,\,7+(l-1)7,\,k} = 0 \quad k = 4,\ \forall\, l \in \mathcal{L}; \tag{19}$$

$$\sum_{l\in\mathcal{L}} x_{2,\,5+(l-1)7,\,k} = 2 \quad k = 4; \tag{20}$$

$$\sum_{l\in\mathcal{L}} x_{3,\,5+(l-1)7,\,k} = 1 \quad k = 4; \tag{21}$$

$$\sum_{i=2}^{3}\sum_{l\in\mathcal{L}}(x_{i,\,5+(l-1)7,\,k}+x_{i,\,6+(l-1)7,\,k}+x_{i,\,7+(l-1)7,\,k})=7 \quad k=4. \tag{22}$$

Once per cycle (three times for MD1), physician k works Friday, Saturday, and Sunday *overnight* consecutively:

$$x_{3,\,5+(l-1)7,\,k}-x_{3,\,6+(l-1)7,\,k}=0 \quad \forall k\in\mathcal{K},\ \forall l\in\mathcal{L}; \tag{23}$$

$$x_{3,\,6+(l-1)7,\,k}-x_{3,\,7+(l-1)7,\,k}=0 \quad \forall k\in\mathcal{K},\ \forall l\in\mathcal{L}; \tag{24}$$

$$\sum_{l\in\mathcal{L}}(x_{3,\,5+(l-1)7,\,k}+x_{3,\,6+(l-1)7,\,k}+x_{3,\,7+(l-1)7,\,k})=\begin{cases}3 & \forall k\neq 1\\ 9 & k=1.\end{cases} \tag{25}$$

P2: Physician k never works more than two weekends in a row:

$$\sum_{i\in\mathcal{I}}\sum_{l=n}^{n+2}(x_{i,\,6+(l-1)7,\,k}+x_{i,\,7+(l-1)7,\,k})\le 4 \quad \forall n\in[1,6]. \tag{26}$$

P3: Assign physician k at most four shifts in any given week:

$$\sum_{i\in\mathcal{I}}\sum_{m=1}^{7}(x_{i,\,m+(l-1)7,\,k})\le 4 \quad \forall k\in\mathcal{K},\ \forall l\in\mathcal{L}. \tag{27}$$

For physician k, assign shifts to group at most three consecutive working days (except MD1):

$$\sum_{i\in\mathcal{I}}\sum_{j=n}^{n+3}x_{ijk}\le 3 \quad \forall k\neq 1,\ \forall n\in[1,53]. \tag{28}$$

Note that in each of the next four sets of constraints, (29)–(32), (33)–(36), (37)–(40), and (41)–(44), the stated goal could be achieved with a more concise formulation, involving fewer inequalities and avoiding the use of the s variables. However, our constraints (29)–(44) constitute a tighter problem formulation because each constraint comprises fewer binary variables, allowing the solver to reach a solution faster when branching from the LP relaxation.

For physician k, assign shifts to group at least two consecutive worked days (except MD1). Equations (29) to (31) define s^1_{jk}, whereas Equation (32) ensures that if physician k is not working on day $(j-1)$ but is working on day j, then he/she has to be assigned to work on day $(j+1)$. Equations (29) to (32) apply $\forall k\neq 1,\ \forall j\in\mathcal{J}$:

$$s^1_{jk}-\sum_{i\in\mathcal{I}}x_{ijk}\le 0; \tag{29}$$

$$s^1_{jk}+\sum_{i\in\mathcal{I}}x_{i(j-1)k}\le 1; \tag{30}$$

$$s^1_{jk}-\sum_{i\in\mathcal{I}}x_{ijk}+\sum_{i\in\mathcal{I}}x_{i(j-1)k}\ge 0; \tag{31}$$

$$s^1_{jk}-\sum_{i\in\mathcal{I}}x_{i(j+1)k}\le 0. \tag{32}$$

P4: For physician k, give at least two days off between groups of *overnight* shifts. Equations (33) to (35) define s^2_{jk}, whereas Equation (36) ensures that if physician k is working *overnight* on day $(j-1)$, but not on day j, then he/she cannot be assigned to work *overnight* on day $(j+1)$. Equations (33) to (36) apply $\forall j\in\mathcal{J}$, $\forall k\in\mathcal{K}$:

$$s^2_{jk}-x_{3(j-1)k}\le 0; \tag{33}$$

$$s^2_{jk}+x_{3jk}\le 1; \tag{34}$$

$$s^2_{jk}-x_{3(j-1)k}+x_{3jk}\ge 0; \tag{35}$$

$$s^2_{jk}+x_{3(j+1)k}\le 1. \tag{36}$$

P7: For physician k, if the weekend is off, then give Monday or Friday off (soft constraint). Equations (37) to (39) define s^3_{jk} and s^4_{jk}, whereas Equation (40) ensures that if physician k does not work Saturday and Sunday, then either Monday or Friday is off too; otherwise, the penalty kicks in Equations (37) to (40) apply $\forall k\in\mathcal{K}$, $\forall l\in\mathcal{L}$:

$$s^3_{6+(l-1)7,\,k}+\sum_{i\in\mathcal{I}}x_{i,\,6+(l-1)7,\,k}\le 1; \tag{37}$$

$$s^4_{7+(l-1)7,\,k}+\sum_{i\in\mathcal{I}}x_{i,\,7+(l-1)7,\,k}\le 1; \tag{38}$$

$$s^3_{6+(l-1)7,\,k}+s^4_{7+(l-1)7,\,k}+\sum_{i\in\mathcal{I}}x_{i,\,6+(l-1)7,\,k}+\sum_{i\in\mathcal{I}}x_{i,\,7+(l-1)7,\,k}\ge 2; \tag{39}$$

$$2s^3_{6+(l-1)7,\,k}+2s^4_{7+(l-1)7,\,k}+\sum_{i\in\mathcal{I}}\sum_{j=5+(l-1)7}^{1+7l}x_{ijk}-d^{1l}_k\le 5. \tag{40}$$

P8: For physician k, assign no more than two *overnights* in a row in the Monday–Thursday time window (except MD1; soft constraint). Equations (41) to (43) define s^5_{jk}, whereas Equation (44) ensures that if physician k is working *overnight* on day $(j-1)$ and on day j, then he/she cannot be assigned to work *overnight* on day $(j+1)$; otherwise, the penalty kicks in Equations (41) to (44) apply $\forall\, k \neq 1$, $\forall\, j \in [2+(l-1)7, 4+(l-1)7]$, $\forall\, l \in \mathscr{L}$:

$$s^5_{(j-1)k} + x_{3(j-1)k} \leq 1; \tag{41}$$

$$s^5_{jk} + x_{3jk} \leq 1; \tag{42}$$

$$s^5_{(j-1)k} + s^5_{jk} + x_{3(j-1)k} + x_{3jk} \geq 2; \tag{43}$$

$$s^5_{(j-1)k} + s^5_{jk} - x_{3(j+1)k} + d^2_{jk} \geq 0. \tag{44}$$

Constraints that apply to MD1 only:
Assign shifts to group at most four consecutive worked days:

$$\sum_{j=n}^{n+4} x_{3jk} \leq 4 \quad k=1,\ \forall\, n \in [1, 52]. \tag{45}$$

Assign shifts to group at least three consecutive worked days. Equations (46) to (50) apply for $k=1$, $\forall\, j \in \mathscr{J}$:

$$s^1_{jk} - x_{3jk} \leq 0; \tag{46}$$

$$s^1_{jk} + x_{3(j-1)k} \leq 1; \tag{47}$$

$$s^1_{jk} - x_{3jk} + x_{3(j-1)k} \geq 0; \tag{48}$$

$$s^1_{jk} - x_{3(j+1)k} \leq 0; \tag{49}$$

$$s^1_{jk} - x_{3(j+2)k} \leq 0. \tag{50}$$

Acknowledgments

We thank Cindy Cohen, the scheduler at Cincinnati Children's Hospital Medical Center, for all her insights and suggestions, which greatly helped to ensure the successful outcome of our schedule development. We also thank all the emergency physicians at the branch facility who assisted us throughout this project. Finally, we are thankful to the associate editor and two anonymous reviewers, whose comments and suggestions greatly helped to improve the flow of the paper.

References

Baker, K. R. 1976. Workforce allocation in cyclical scheduling problems: A survey. *Oper. Res. Quart.* **27**(1) 155–167.

Baker, K. R., M. J. Magazine. 1977. Workforce scheduling with cyclic demands and day-off constraints. *Management Sci.* **24**(2) 161–167.

Bartholdi, J. J., III, J. B. Orlin, H. D. Ratliff. 1980. Cyclic scheduling via integer programs with circular ones. *Oper. Res.* **28**(5) 1074–1085.

Beaulieu, H., J. A. Ferland, B. Gendron, P. Michelon. 2000. A mathematical programming approach for scheduling physicians in the emergency room. *Health Care Management Sci.* **3**(3) 193–200.

Bourdais, S., P. Galinier, G. Pesant. 2003. HIBISCUS: A constraint programming application to staff scheduling in health care. F. Rossi, ed. *Principles and Practice of Constraint Programming—CP 2003*. Springer-Verlag, Berlin, 153–167.

Brunner, J. O., J. F. Bard, R. Kolisch. 2009. Flexible shift scheduling of physicians. *Health Care Management Sci.* **12**(3) 285–305.

Buzon, I. E. 2001. La confection des horaires de travail des médecins d'urgence résolue à l'aide de la recherche avec tabous. Unpublished master's thesis, École Polytechnique, Montréal.

Carter, M., S. D. Lapierre. 2001. Scheduling emergency room physicians. *Health Care Management Sci.* **4**(4) 347–360.

Cohn, A., S. Root, C. Kymissis, J. Esses, N. Westmoreland. 2009. Scheduling medical residents at Boston University School of Medicine. *Interfaces* **39**(3) 186–195.

Dittus, R. S., R. W. Klein, D. J. DeBrota, M. A. Dame, J. F. Fitzgerald. 1996. Medical resident work schedules: Design and evaluation by simulation modeling. *Management Sci.* **42**(6) 891–906.

Ernst, A. T., H. Jiang, M. Krishnamoorthy, B. Owens, D. Sier. 2004. An annotated bibliography of personnel scheduling and rostering. *Ann. Oper. Res.* **127**(1–4) 21–144.

Franz, L., J. L. Miller. 1993. Scheduling medical residents to rotations: Solving the large-scale multiperiod staff assignment problem. *Oper. Res.* **41**(2) 269–279.

Gendreau, M., J. Ferland, B. Gendron, N. Hail, B. Jaumard, S. Lapierre, G. Pesant, P. Soriano. 2007. Physician scheduling in emergency rooms. E. K. Burke, H. Rudová, eds. *Physician Scheduling in Emergency Rooms*. Springer-Verlag, Berlin, 53–66.

Laporte, G., G. Pesant. 2004. A general multi-shift scheduling system. *J. Oper. Res. Soc.* **55**(11) 1208–1217.

Maier-Rothe, C., H. B. Wolfe. 1973. Cyclical scheduling and allocation of nursing staff. *Socio-Econom. Planning Sci.* **7**(5) 471–487.

Mueller, C. W., J. C. McCloskey. 1990. Nurses' job satisfaction: A proposed measure. *Nursing Res.* **39**(2) 113–117.

Nelson, D. 2008. Prevention and treatment of sleep deprivation among emergency physicians. *Pediatric Emergency Care* **23**(7) 498–503.

Rousseau, L. M., M. Gendreau, G. Pesant. 2002. A general approach to the physician rostering problem. *Ann. Oper. Res.* **115**(1–4) 193–205.

Shader, K., M. E. Broome, C. D. Broome, M. E. West, M. Nash. 2001. Factors influencing satisfaction and anticipated turnover for nurses in an academic medical center. *J. Nursing Admin.* **31**(4) 210–216.

Stachota, E., P. Normandin, N. O'Brien, M. Clary, B. Krukow. 2003. Reasons registered nurses leave or change employment status. *J. Nursing Admin.* **33**(2) 111–117.

Joseph W. Luria, MD, Clinical Medical Director, Cincinnati Children's Hospital Medical Center, writes: "I am writing to verify that the content of the paper 'Building Cyclic Schedules for Emergency Department Physicians,' written by Yann Ferrand, Michael

Magazine, Uday S. Rao, and Todd F. Glass is accurate. Through their interactions with the emergency physicians at one of our care sites and the scheduler for Cincinnati Children's Hospital Medical Center, the authors were able to develop schedules that would meet the necessary constraints and the desirable objectives of five emergency physicians' schedules using mathematical models.

"The researchers were attentive to our needs and our specific constraints, and communicated their proposed solutions to us effectively with the use of visual templates of the schedules and statistics that would keep track of the requirements for each individual physician. We implemented the new schedule starting July 2009 for these five physicians.

"Though the feedback we have received from the physicians impacted by the change has been mixed, most of the concerns are based on the mechanics of implementing the schedule as opposed to how well it addressed the constraints and desires of the group. The other physicians working at the main facility who have to fill in empty shifts at the branch site and who are scheduled under our original system have not reported concerns with the existence of the new schedule. This indicates that it is possible for us to operate with two distinct scheduling methods for our two sites.

"We plan to continue to use this scheduling strategy while working out some of the implementation issues that surfaced. We will also continue to work with the research team to refine the template, and will consider expanding the number of physicians who use this scheduling technique as we increase the number of physicians working at our branch facility."