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Media Company Uses Analytics to Schedule Radio Advertisement Spots

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In this paper, we describe the implementation of an optimization suite (OS) to facilitate the scheduling of radio advertisements for one of the largest media companies in the United States. Advertisements are scheduled adhering to complex criteria from the advertisers with the objective of maximizing the revenue for the company. Advertisers offer two types of flexibility for demand fulfillment: market flexibility provides an opportunity to shift the demand across demographics, and time flexibility allows the demand to be shifted across the broadcasting time horizon. The scale of inventories, fair and equitable distribution, flexibilities, and other complex criteria from the advertisers necessitated the development of a sophisticated OS to generate rosters for the placement of advertisements. The OS uses optimization models and four heuristics procedures to generate an advertisement placement roster for each station. The company has adapted the OS into its information systems to seamlessly incorporate optimization into its decision-making process.

Keywords: prescriptive analytics; decision support; radio broadcasting; advertisement placement; advertisement schedules; time and market flexibility.
History: This paper was refereed.

In the United States, radio broadcasts reach 90 percent of every age segment of the age 12-and-over population every week—243 million people; hence, it is one of the most effective media for advertising (Arbitron 2014). Radio reaches more people every day than the Web (Nielsen 2014); another study indicates that the return on investment for radio advertising improved 49 percent compared to television advertising during 2004–2005 (Radio Advertising Bureau 2014). Radio advertising reaches millions of listeners and can allow an advertiser to target many consumer markets because it can reach multiple radio stations and can also have various program formats. A significant challenge, however, is the fragmentation of the audience, because there are approximately 10,000 radio stations (Federal Communications Commission 2014). Each station represents a small piece of the entire network, and its audience demographics can change over time. Advertisers buy spots within different programs at the stations to target their desired customer demographic. Typically, the larger the audience reached for an advertiser's region, the more the advertiser will be willing to pay to air advertising spots. Given the diversity of the radio stations and customer demographics, advertisers like to place advertisements in programs that maximize their exposure to specific customer segments. Although radio programs and the associated demographics of the advertisers' targeted audience are often known, optimally allocating spots to an advertiser's order is challenging because of the high volume of advertising spots and the complex requirements from advertisers. Additionally, some advertisers allow flexibilities for placing their advertisements in any of the requested stations. The scheduling process should take advantage of these flexibilities, while ensuring that advertisers' orders are delivered according to their contracts.

iHeartMedia, Inc. (IHM), which owns over 850 stations in more than 150 cities, is one of the largest providers of network radio programming and traffic information in the United States. As part of its wide-ranging portfolio of products and services, IHM provides traffic reports, local news, sports, and weather information to more than 2,250 radio stations. IHM currently uses the optimization suite (OS) that we discuss in this paper. The OS is supplied with sales data from a contracting system, which we developed in-house; the sales data are allocated into a capacitated set of available advertising-spot inventory to be aired on the radio stations. The goal of the OS is to maximize the revenue generated from the inventory placed, minimize the number of contracted spots that are not placed, and adhere to the advertisers' contract specifications.

The spot selling process introduces additional complexities into the placement allocation process because of the mix of buyers. Local buyers tend to purchase radio advertising on a per-spot basis; that is, a one-to-one correspondence exists between the quantity purchased and the quantity delivered. National buyers may purchase their radio advertising on a gross rating point (GRP) basis; that is, a variable number of spots will be required to satisfy the GRP requirement. A GRP is a measure that quantifies the number of people who hear the advertising spot as a percentage of the population reached rather than as an absolute number of people. This measure introduces uncertainty about the number of spots potentially required to satisfy the GRP target, because the GRPs delivered can vary from station to station, and by day of the week, time of the day, and demographic group targeted. In addition to the contract type and decision to purchase either by spot counts or GRPs, advertisers' contract specifications contain a number of standard components. These include airing dates, airing times, and restrictions, such as program mix, spot lengths, program or station exclusions and inclusions, and category-conflict avoidance. Additionally, advertisers expect a fair and equitable distribution of spots, that is, a proportional assignment of spots across the times of the day, stations, program types, and days of the week. Thus, advertisers expect IHM to distribute their spots evenly across the available inventory according to their order specifications.

Literature Review

To the best of the authors' knowledge, the literature contains no references to scheduling commercials on radio stations. The problem is analogous to scheduling commercials on television programs; such commercials have generated reasonable research interest. Goodhardt et al. (1975), Headen et al. (1979), Henry and Rinne (1984), Webster (1985), and Rust and Eechambadi (1989) have extensively studied the scheduling of programs for television. The majority of these studies focus on scheduling television programs, rather than commercials, to maximize a specific criterion such as audience size. In Simon (1982), the author devised a model to demonstrate the slowdown of sales even with higher level of advertising and showed that a pulsation or cyclic strategy in advertising was optimal for both constrained and unconstrained budgets. Other papers on advertisement scheduling include the works of Mahajan and Muller (1986), and Lilien et al. (1992). These works, however, focus on the effectiveness of advertisement campaigns rather than on their scheduling.

Bollapragada et al. (2002) presented the pioneering work for scheduling advertisements on television programs. The authors developed a mathematical model to generate near-optimal solutions for sales plans based on advertiser requirements. The model works at an aggregated level based on the available inventory from the programs in a week, and then allots the sales orders to spots, subject to constraints. The model minimizes both the amount of premium inventory assigned to a sales plan and the penalties for not meeting client requirements. It also includes constraints on supply, such as air time and product conflict, and client requirements, such as budget, show mix, weekly weights, and unit mix.

In a subsequent work, Bollapragada et al. (2004) presented algorithms for scheduling commercials and focused on the even spread of given advertisement spots. The authors used an integer program and solved it sequentially for each advertiser with an objective of minimizing the use of premium inventory. The authors propose several integer programs and heuristics to efficiently solve the problem. The models emphasize sequence and equal separation of the commercials, and report computational results for problem sizes of approximately 26–40 advertisement spots.

Jones (2000) introduced the advertising allocation problem as an example of combinatorial auctions for which hundreds of potential advertisers can submit bids for airing their commercials in advertising slots. The author proposed a mixed-integer programming (MIP) model, which considers the locations of the time slots. The model is computationally prohibitive and uses a constraint-programming heuristic to find feasible solutions. The campaigns are assumed to be continuous; breaks are not allowed between the weeks of the advertisement schedule (i.e., the advertisements must run each week of the schedule).

Zhang (2006) used a two-step hierarchical approach to allocate television advertisements. The author proposes a winner-determination problem to select advertisers; subsequently, an assignment program schedules the selected advertisers' commercials. The winner-determination problem uses a column-generation algorithm, and the assignment program is an MIP model. The author reported computational results for a maximum of 32 programs and 200 advertisers and presented an approximation algorithm to solve the model.

Market and time flexibilities are unique aspects of the radio advertising market and are typically not found in the television advertising market. In exchange for lower rates, some advertisers allow a percentage of their demand to shift between defined markets and time periods. As a result, we need to consider all the markets together in making decisions. Our approach also allows discontinuous campaigns (i.e., breaks between the weeks of the advertisement schedule).

Background

A market refers to a large demographic representation (e.g., Houston market, New York market); a station is an entity that broadcasts advertisements within a given market. Stations are (1) rated with GRP, (2) nonrated without GRP, or (3) noncommercial. Nonrated stations are new or have low demographic coverage. Noncommercial stations are typically religious channels. Approximately 70 percent of the stations are rated; therefore, only these stations can be used for GRP orders. GRPs are collected by marketing agencies and used to quantify the order fulfillments for the placement of advertisements. IHM has approximately 200 markets, and each market typically has 10–20 stations. It divides a day into eight parts, which we refer to as day parts, of three hours each.

IHM has three departments that work closely together to sell and schedule advertisement spots: the sales department sells advertisements to be aired during program breaks; the traffic department schedules advertisements on programs that meet the advertisers' requirements; and the billing department bills advertisers for the advertisements that they have scheduled and have been aired. The schedulers within the traffic department are responsible for preparing rosters for the inventory of advertisement slots. This team prepares rosters for all the markets for the following week. Prior to implementing the OS, this was a weeklong process. Each week, the team finalizes the rosters for the following week by Thursday night, and sends the rosters to the broadcasting team in each individual market on Friday. Inevitably, last-minute orders will need to be included with minimal disruptions to the existing roster, and unsold spots will be replaced by the paid spots. The objective of the work we present in this paper is to assist the traffic department in preparing the rosters with fair and equitable distribution and with all other restrictions that are included in the advertiser orders. All of the remaining unsold inventory, which we call overdeliveries, may be allotted to the advertisers; overdeliveries for an advertiser are typically used for public service announcements or allocated for other internal purposes (e.g., promotions).

Current Practice and the Need for Analytics

An advertiser's order typically has multiple order lines. Each order line specifies the details of the demand, including market, broadcasting granularity (e.g., daily, weekly, quarterly, monthly, or yearly), inclusion and exclusion rules, preferences, days in a week, air services (i.e., types of programs), rate, broadcast type, and type of order (i.e., spot or GRP), for airing that advertiser's advertisements. Figure 1 shows a sample order form. The form captures the details for an order line, such as quantity, market, inventory group, broadcast times, weeks on and (or)

Advertiser:	Agency:	Proposal ID:		
Billing code:	Flight desk:			
Line item no.:				
Quantity:	Standard:		Co Op Code:	
Market:	Co-Op:		Revision id:	
Inventory groups:	AR ready to schedule:			
Flight times:	Gross amt:			
Weeks on:	Net amt:			
Flight dates:	Proposal line id:			
Rate:				
Spot type:				
GRPs				

Days/Services/Formats/Stations/Controversial Titles

Air days:	Air services:	Air formats:	Exclude titles:	For stations:
Disp:	All services:	All formats:		Market:
Monday:	News wire:	Country:		Rated:
Tuesday:	AP-15s:	Oldies:		Nonrated:
Wednesday	Beach:	Jazz hits:		Noncommercial:
Thursday:	Weather channel:	O.D.:		
Friday:	Real traffic:	Classical:		
Saturday:	Web traffic:			
Sunday:				

Figure 1: IHM uses a mock customer order screen to capture various requirements and restrictions for an advertiser's order line. The inclusion and exclusion selections for an order line define its inventory.

off, rate, spot type, and spot distribution. The form includes fields that capture days, service, formats, and stations, and define the inventory to be used for the placement of an advertisement of an order line. We refer to these as inclusion-exclusion rules for the order lines. Order lines typically specify the proportion of spots to be placed in the rated and nonrated stations in a spot order; GRP order lines can be satisfied only by rated stations.

The advertisers' order lines can be for discontinuous campaigns; in these campaigns, the advertiser selects the broadcasting weeks and (or) days from the entire order's broadcasting horizon. Advertisers give their preferences for stations, days of the week, and day parts. Advertiser orders also specify the period within which a predefined number of advertisements is to be broadcast. This period can be daily, weekly, monthly, or quarterly. Prior to implementing the OS, IHM used an in-house process to prepare the rosters. This process used a rule-based approach by which it ranked the order lines based on a series of criteria and scheduled the order lines based on their rankings. The rosters were applicable to only the first week in the planning horizon. Table 1 shows some challenges that this practice presented.

Feature	Inefficiency
Scheduling in a sequential manner	Low-quality schedules for orders with fewer restrictions and priorities
Scheduling only one market or week at a time	Suboptimal inventory use
Fairness and equitable distribution	Not modeled
Solution requires manual updates	Inefficient and error prone
Scheduling only one week in the future	Short-sighted planning process
Time and market flexibility	Not modeled

Table 1: The list of features depicts the practice at IHM prior to implementing the OS. Some key features, such as fairness, equitable distribution, time, and market flexibility, are desirable characteristics for advertisement rosters; however, IHM did not model them because of their complexities.

The following flexibilities, which we build into the OS, provide advertisers with better opportunities for placing advertisements. Market flexibility (MF) offers an advertiser the opportunity to move the demand in an order line from the specified market to another desirable market. As an incentive, based on the advertiser's flexibility, additional advertisement spots are provided in the market to which the advertisements

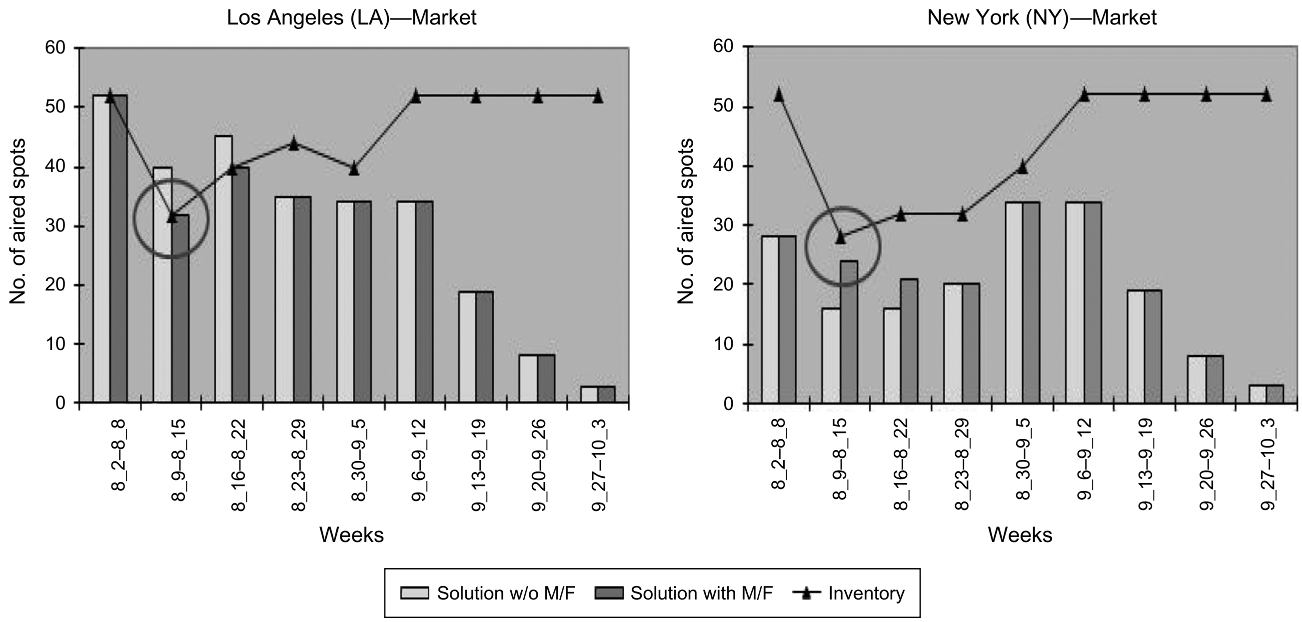

Figure 2: The graphs illustrate the movement of an order line's demand from the Los Angeles to New York market as a result of market flexibility.

are moved. To illustrate, suppose an advertiser's order line specifies 100 spots for the Los Angeles market in the next five weeks. If the order line also includes the New York market as acceptable with a limit of 20 percent, then the order line allows broadcasting 80 advertisement spots in the Los Angeles market and the remaining 20 in the New York market; however, because of the 20 percent incentive provided, the advertiser will receive 24 broadcast spots in the New York market. MF is applicable only in markets acceptable to the advertiser. This flexibility allows IHM to compensate an advertiser who wishes to advertise in a high-demand market by using inventories from other markets. The example in Figure 2 shows the net effect of moving excess demand from the Los Angeles market to the New York market. This shift in demand is significant in terms of revenue generation because it provides a means to move an advertiser's demand across markets, thereby increasing the fill rate for the order lines.

Time flexibility (TF) permits uneven distribution of placements for an advertiser's order line across placement weeks to accommodate the varying amounts of overall market demand for inventory in the placement weeks. These flexibilities provide better inventory utilization, and help IHM to place the advertisements based on advertisers' preferences. A time-flexible order line offers the flexibility to schedule the advertisement spots within a range of the planning horizon without strictly adhering to an equal distribution of an order line. To illustrate, suppose an advertiser's order line requests 100 spots in four weeks with a TF factor of 20 percent. The number of advertisement spots to be broadcast based on the order line ranges from 20 to 30 each week. TF is illustrated in Figure 3, where the pressure on inventory in week 8_16 to 8_22 is eased by spreading order lines across weeks 8_9–8_15 and 8_23–8_29. TF also requires the advertiser's consent. MF and TF are unique features in radio advertisements, which are not available for scheduling television advertising. Another consideration in placing advertisements is that advertisers prefer to avoid repeat broadcasts, that is, broadcasting the same order line from an advertiser at same time and same station for two or more consecutive days or multiple times within a same day part. In addition, order lines with spot requirements have limitations on the number of spots that can be broadcast on a nonrated or noncommercial station.

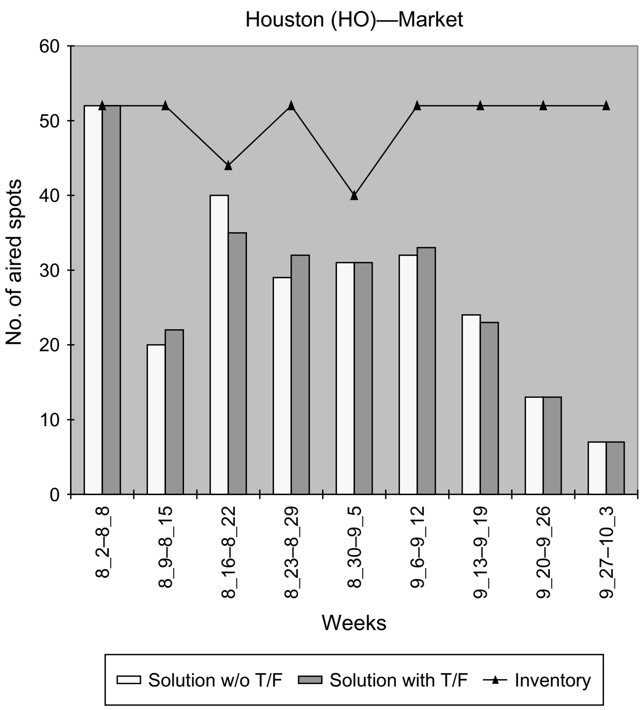

Figure 3: The graph illustrates the movement of an order line's demand from one week to another within the planning horizon as a result of time flexibility.

Because of these challenges, IHM's management sought assistance from external firms who specialize in modeling and developing decision support systems to determine if better ways were available to address its rostering process. All these challenges necessitated a sophisticated OS to maximize the revenue from the advertisers' orders. Given the magnitude of orders and advertisers, manually incorporating market and time flexibilities in IHM's current planning process was not trivial. The OS maximizes profit, and minimizes the bumping of advertiser order lines and penalties for deviating from fair and equitable distribution. A feasible solution should include all the complex criteria from advertisers' order lines. The OS uses two optimization models and four heuristics procedures to generate a roster for all advertisers' order lines across all markets.

Developing Optimization Models

Figure 4 depicts the process within the OS. We use two optimization models. The market week (MW) model addresses strategic planning using a linear programming (LP) model, which finds the target for the number of placements at each advertiser's order line. The station day-part (SD) model addresses operational planning using an MIP model, which assigns the placement of individual advertisement spots to the advertiser's order lines based on the target from the MW model. The OS also uses four heuristics, which are summarized in Table 2.

Mathematical Models

MW model: The MW model considers all the markets, stations, and day parts for a given planning horizon and determines which order lines of an advertiser's order should be accepted. The model generates targets at the market-week level; these targets will be used in the subsequent SD model. The inventory is consolidated for each day part within a station, and advertisement spots are placed using the consolidated inventory. The MW model seeks to obtain suitable targets for the SD models. It considers all markets simultaneously for the usage of time and market flexibilities, both of which the SD model excludes. The MW model also maximizes the revenue from all advertisers' order lines minus the penalties that result from deviating from fair and equitable distribution of inventories across stations, days of week, or day parts. The model includes four constraints: (1) time and market flexibility: limit the amount of demand that can be shifted to other periods and markets; (2) fairness and equitability: measure the deviations from the target for a perfect distribution based on station, day-part, program-type, and day-of-the-week inventory available for an order line; (3) time separation: limit the number of assignments to the same advertiser within a day part; and (4) inclusions and exclusions: define the available inventory for an order line.

Other features, such as simulcast (simultaneous broadcasting of advertisements across different stations), linked feed (broadcasting on the set of requested stations), and minimum time separation (minimum amount of time between two consecutive advertisements of the same type), are specific to placing order lines with an inventory spot; therefore, they are not in the MW model, but are addressed in the SD model. Because the MW model is an LP

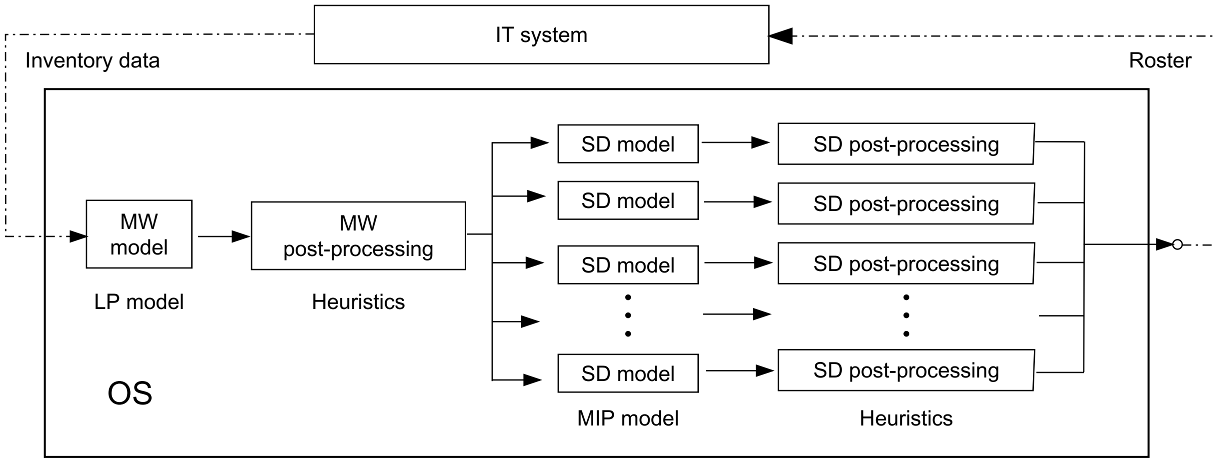

Figure 4: The optimization suite consists of a linear programming model to determine the targets for the markets, a mixed-integer programming model for each market to assign individual inventory spot to an advertiser's order line, and a series of heuristics to improve the quality of the solution.

model, the solution provides fractional targets, which are rounded off to integers using a rounding heuristic (RoH) that reflects business rules. For nonflexible and (or) flexible orders, the rounded target values are compared against the weekly demand and (or) maximum allowable shift in demand because of the flexibilities, and the targets are adjusted accordingly. The targets are then used in the SD MIP models.

SD Model: The SD models, which use the targets from the MW model, are independent of each other and provide the optimal placements for advertisers' order lines. Each SD model represents a market and is created at a smaller granular level of inventory spots within a station and day part. SD Models also have fairness and equitability constraints and time-separation constraints; however, they are defined based on inventory spots. Other constraints include back-to-back-advertiser constraints, which restrict assigning two consecutive inventory spots to the same advertiser, and group-link constraints for simulcast and linked-feed functionality. The SD model is an MIP model and runs for a stipulated time. If the model does not find an optimal solution, then the solution is improved using the heuristics. We created penalty profiles for the optimization runs based on sample runs and business needs, and created different profiles by prioritizing fair and equitable distributions for station, day of week, and day part. The planner selects a profile for the optimization run based on his (her) needs.

Heuristics	Purpose
Rounding heuristic (RoH)	Round the advertisers' order line targets from the MW model
Recovery heuristic (RuH)	Improve the integer solution from the SD model
Overdelivery heuristic (OdH)	Deplete the unsold inventory for the upcoming scheduling week considering fair and equitable distribution
Sequence heuristic (SeH)	Remove the violations in minimum time separation between similar types of advertisement spots in the SD model

Table 2: Heuristics are used after executing the MW LP and SD MIP models; these heuristics help improve the quality of the solution.

Heuristics

We used heuristics to improve the solution from the perspective of profit maximization and fair and equitable distribution. Additionally, heuristics address some qualities for the roster, such as allocating a fixed percentage of overdelivery for national accounts, substituting unrated spots for rated spots to fulfill an order line, and allocating inventory spots for the unfilled orders based on the time and market flexibilities of an order line. The recovery heuristic (RuH) is used to improve the solution of an SD model

	Data		Characteristics	
Weeks	No. of markets	No. of stations	No. of day parts	No. of inventory
6	50	570	191,520	114,397
	100	967	324,912	227,955
	150	1,522	511,392	392,239
	216	2,277	765,072	581,842
12	50	570	383,040	229,456
	100	967	649,824	457,982
	150	1,522	1,022,784	788,314
	216	2,277	1,530,144	1,168,161
18	50	570	574,560	343,768
	100	967	974,736	686,356
	150	1,522	1,534,176	1,181,302
	216	2,277	2,295,216	1,749,552
24	50	570	766,080	458,244
	100	967	1,299,648	914,958
	150	1,522	2,045,568	1,573,118
	216	2,277	3,060,288	2,329,829

Table 3: The number of inventory spots indicates the magnitude of the planning horizon. A higher number of day parts compared to inventory spots indicates that some day parts have no inventory to schedule.

after a stipulated run time. The SD model uses a prescribed MIP gap percentage to keep the overall run time tractable; however, this may cause unfulfilled order lines, even in the case of unsold inventories. This heuristic allocates the unsold inventory to the unfulfilled order lines. The overdelivery heuristic (OdH) assigns the unsold advertisement spots to the advertisers based on their revenue or demand. It provides a targeted overdelivery for each order line. Based on each order line's proportion of demand or revenue, the heuristic allocates the unsold inventories to the advertisers, while considering the inclusion and exclusion restrictions. The minimum time separation between similar advertisement types is modeled only as a bound for each day part in the SD model. Currently, there are approximately 79 types of advertisements, and the minimum time separation is 10 minutes for 80 percent of the orders; the remainder have a requirement of 45 or 60 minutes. For each day, approximately 1,000 inventory spots are available on average. The MIP model cannot explicitly handle this feature, because doing so would add an exponential number of knapsack constraints in the model. Hence, any violations in the output from a SD model are handled in the sequence heuristic (SeH). We explain the details of the heuristics in the appendix.

Implementation

The typical planning horizon is 10–24 weeks, and the number of markets is approximately 200. The planning horizon is determined based on advertisers' demand. In addition, most order lines have requests for the initial six to eight weeks of the planning horizon. Running the OS beyond 24 weeks would not be useful because the placements will eventually be changed in subsequent runs. Table 3 provides details regarding the characteristics of the data used in a typical planning process. We used the same data for several optimization runs for various numbers of weeks, and performed optimization runs for subsets of markets. The column headings represent the following: No of. markets—the total number of markets; No. of stations—the total number of stations; No. of day parts—the total number of day parts; and No. of inventory—the total number of available advertisement spots. The inventory count is less than the number of day parts, indicating that many day parts within the stations have no inventory.

The largest instance has a 24-week planning horizon, 2.3 million advertisement spots, 2,277 stations, and three million day parts. Based on the feedback from IHM's traffic department about the quality of the solution, we calibrated the objective coefficients of

	Model characteristics				
Weeks	No. of markets	No. of constraints	No. of variables	No. of nonzeros	No. of SD models
6	50	272,680	949,271	3,898,498	47
	100	527,107	1,845,252	7,870,534	93
	150	897,560	3,158,754	13,884,904	139
	216	1,357,735	4,787,945	21,515,376	200
12	50	408,268	1,401,284	5,733,659	47
	100	788,424	2,703,616	11,427,934	94
	150	1,357,331	4,716,650	20,409,688	142
	216	2,064,182	7,193,587	31,734,440	203
18	50	469,898	1,564,070	6,320,084	47
	100	906,648	3,002,257	12,567,416	94
	150	1,564,072	5,263,427	22,500,979	142
	216	2,386,161	8,054,460	35,000,594	203
24	50	517,190	1,678,192	6,747,135	48
	100	997,751	3,215,710	13,402,445	95
	150	1,724,151	5,633,993	23,994,261	143
	216	2,635,219	8,657,920	37,364,503	204

Table 4: The number of constraints, variables, and nonzeros indicate the characteristics of the market-week model; the number of SD models represents the number of station-day mixed-integer programming models solved during each run.

the MW and SD models to meet IHM's needs. We also created various combinations of penalty profiles. A user from the traffic department selects a suitable profile based on his or her requirements. In Table 4, No. of markets represents the total number of markets; No. of constraints, No. of variables, and No. of nonzeros represent the total number of constraints, variables, and nonzeros, respectively, for the MW model, and No. of SD models is the total number of SD models created. We developed the models using Java with CPLEX 12.1 as the optimization engine. For the implementation, we used Dell T710 (two quad core processors) hardware with 64GB main memory running on RedHat Enterprise Linux 5.5 (x86-64). Barrier optimizer in CPLEX was used to solve the MW model. For a 10-week planning horizon, it takes approximately 15 minutes on average to reach optimality. We used different levels of optimality gaps for the SD models, a 0.5 percent MIP gap for the first week of the planning horizon, and one percent for the other weeks. Because the OS is run every week, we used a lower percentage for an MIP gap for the first week in the planning horizon. The overall run time for a 10-week planning horizon is approximately five hours using the hardware listed above. Because the SD models are independent of each other, we solve them in parallel. The overall optimality was compromised because of using sequential optimization, an LP model for MW, and a rounding heuristic to determine the targets for the SD models.

During the implementation of the OS, IHM's responsibilities included defining business requirements, design review, data collection and formatting, testing and validation of placements, and system integration. The duties of the OS provider included gathering business requirements, data validation, development and tuning of mathematical models and heuristics, results review, and project management. IHM and the OS provider staff members together expended approximately 6,500 person-hours.

Benefits Summary

IHM's initial interest in the development of an OS was driven by customer service issues and a desire to have better visibility of its future inventory availability. Using the previous spot-placement process, IHM could not accurately deliver GRP orders; it focused only on creating the placements for the next broadcast week. The OS benefits IHM in the following ways:

1. Enhanced inventory usage: Market and time flexibilities help to shift the demand, and the OS

improves inventory usage. The previous placement process recognized only spot orders; therefore, IHM had to translate GRP orders into an estimated number of spots. An order line would specify the required GRPs in a market, and IHM would estimate the required number of spots by assuming a fair and equitable distribution of spots across the specified stations based on the station's available inventory. However, if other spot-based orders requested only specific stations (cherry-picking), which are commonly the higher-rated stations, then the estimated number of spots would be insufficient to meet the targeted GRP demand. In some cases using the former process, the GRP delivery shortage for an order could be as high as 25 percent. Over the course of a year, the revenue impact could total $1,000,000 (or more) and could damage customers' goodwill. IHM has been able to provide GRP estimates to the OS, thus allowing the OS to accurately assess the order's GRP targets and alleviating these problems.

2. Enhanced customer service: The OS also provides fairer and more equitable distribution of the spots across the days of week and day parts. A good distribution keeps the allocated placements in closer alignment with customer expectations. Although the improved distribution provides no direct revenue impact, it improves customer goodwill, which typically translates into future revenue from the advertisers.

3. Streamlined allocation of overdelivery: The OS also streamlines the allocation of overdelivery spots. In any available commercial spot that is not sold, something must be aired during its specified time or be assimilated back into the radio program (e.g., the on-air personalities must fill the additional time). The available commercial time may be filled with items such as additional free spots to the advertiser (overdelivery), public service announcements, or promotional items. The previous placement process would typically allocate any unsold spots to those advertisers who had purchased the most number of spots. This process tended to reward the volume purchasers of spots rather than the advertisers who bought smaller quantities, but paid higher rates. The OS allows a more targeted delivery of the excess inventory to those customers who are deemed more valuable, thereby assigning overdelivery in a more structured fashion. In IHM's previous process, no bound was placed on the number of overdelivery spots allocated to an advertiser. For example, an advertiser who purchased 100 spots might have received 200 spots in overdelivery. This practice on a potentially recurring pattern could cause the advertisers to expect large quantities of overdelivery spots and to therefore reduce their paid spots based on this expectation. Furthermore, in a competitive space, advertisers who may have paid higher rates and who are more valuable to the organization are overshadowed with the sheer magnitude of spots (both paid and overdelivered). Although this does not have a direct revenue impact, it contributes to the advertiser's goodwill, which helps IHM to maintain a better client relationship and maintain revenues.

4. Enhanced business processes: Accurate inventory visibility for the future weeks allows the sales teams to know which inventory is low so that they can try to get premium pricing for any additional advertisers who want to purchase the inventory. Conversely, if the sales teams know which inventory is plentiful, they can be more aggressive with their pricing to sell it. The OS has also increased the efficiency of placement operations. Compared to the volume of spots handled previously, the IHM operations staff can now handle twice the volume of spots it handled previously.

5. Monetary benefits: Considering the spot orders only, the OS improved revenue by one percent over the previous placement process, which translates into multiple millions of dollars on an annual recurring basis. Furthermore, in a typical week, the increase in inventory utilization for spot orders is approximately two percent. The difference in the revenue and spot placement percentages is attributable to the focus on placing higher-revenue spots first so that the additional spots are of lower revenue value than those that were placed initially, and provide a better overall utilization of inventory. Based on gross profit, IHM conservatively estimates the monetary benefit it receives as a result of enhanced customer service and better business processes to be over $500,000 per year.

We can summarize the key benefits of the OS as follows:

- An increase of one percent in invoiced revenue on the placement of spot orders;

- Additional revenue of more than $1,000,000 each year from the GRP orders as a result of better fulfillment of GRP targets;
- Decrease in customer complaints as a result of the reduction in underdeliveries of GRP orders;
- Reduction in the inventory used for overdelivery, which increases IHM's revenue and provides a more targeted overdelivery approach;
- Increase in visibility to supply and demand for future weeks, thus aiding senior management in making better sales and promotional decisions;
- Reassignment of some employees previously required for scheduling to other tasks.

Conclusions and Future Directions

Advertisement placement is a complex tactical problem. The models in the OS allow for the best use of the available inventory, provide market and time flexibilities to satisfy demand, maximize revenue, and enhance customer satisfaction. Additionally, the OS provides strategic decision support for the sales department by helping it target future business. One enhancement could be to use the OS to check for the feasibility or availability of inventory for new proposals from the advertisers, thereby minimizing the unsatisfied orders in the future. Another possibility is to add a pricing module that could recommend pricing adjustments for potential orders based on the current inventory utilization.

Appendix

Sets

$\mathcal{W}$ Set of all weeks in the planning horizon, $w \in \mathcal{W}$.
$\mathcal{A}$ Set of all advertisers, $a \in \mathcal{A}$.
$\mathcal{M}$ Set of all markets, $m \in \mathcal{M}$.
$\mathcal{L}$ Set of all order lines, $l \in \mathcal{L}$.
$\mathcal{S}$ Set of all stations, $s \in \mathcal{S}$.
$\mathcal{I}$ Set of all inventory, $i \in \mathcal{I}$.
$\mathcal{T}$ Set of all periods (weeks and day parts) in the planning horizon, $t \in \mathcal{T}$.
$\mathcal{I}_{mw} \subseteq \mathcal{I}$ Set of all rated inventory in market m during week w.
$\mathcal{I}_{sw} \subseteq \mathcal{I}$ Set of all rated inventory in station s during week w.
$\mathcal{I}_{st} \subseteq \mathcal{I}$ Set of all rated inventory in station $s \in S$ for period $t \in T$.
$\mathcal{N}\mathcal{I}_{st} \subseteq \mathcal{I}$ Set of all nonrated inventory in station $s \in S$ for period $t \in T$.
$\mathcal{L}_a \subseteq \mathcal{L}$ Set of all order lines for an advertiser $a \in A$.
$\mathcal{R}_a \subseteq \mathcal{I}$ Set of all rated inventory (market, week, station, period) belonging to advertiser $a \in A$ from the inventory set I.
$\mathcal{N}_a \subseteq \mathcal{I}$ Set of all nonrated inventory (market, week, station, period) belonging to advertiser $a \in A$ from the inventory set I.

Parameters

i^r_{st} Total available rated inventory for a station s for period t.
i^n_{st} Total available nonrated inventory for a station s for period t.
d_{al} Demand of an advertiser a for order line l.
r_a Revenue rate of an advertiser a.
u_{amw} Maximum quantity that can be assigned to advertiser a in market m during week w.
l_{amw} Minimum quantity that can be assigned to advertiser a in market m during week w.
θ_{asw} Proportion of quantity to be assigned to advertiser a on station s that would give a perfect fairness and equitability (F&E) rotation in week w.
μ_{ast} Maximum allowable quantity to be assigned to advertiser a on station s for period t.
ρ_{ail} Preferences score of inventory i with respect to an advertiser a for order line l.
τ_{ail} Maximum limit on the number of nonrated spots for advertiser a for order line l.

Penalties

λ^f Penalty for each unit of unsatisfied demand below a minimum threshold.
λ^e Penalty for each unit of assignment that is above or below the F&E target.
λ^s Penalty for each unit of assignment exceeding the allowable amount.

Decision Variables

X_{ail} Allotted rated inventory for advertiser a to order line l in inventory set $\mathcal{R}_a$.
Y_{ail} Allotted nonrated inventory for advertiser a to order line l in inventory set $\mathcal{N}_a$.
U_{asw} Surplus in assignments to station s for advertiser a for week w based on the F&E target.
V_{asw} Slack in assignments to station s for advertiser a for week w based on the F&E target.
U'_{amw} Slack in assignments to advertiser a for market m for week w based on the time or market flexibility lower bound for station s.
V'_{ast} Surplus in assignments to advertiser a for station s for period t based on the bound for minimum separation placements for station s.

Model Formulation

$$\max\Big\{\sum_{a\in\mathcal{A}}\sum_{i\in\mathcal{R}_a\cup\mathcal{N}_a}\sum_{l\in\mathcal{L}_a}(r_a+\rho_{ail})(X_{ail}+Y_{ail}) - \lambda^f\sum_{a\in\mathcal{A}}\sum_{m\in\mathcal{M}}\sum_{w\in\mathcal{W}}U'_{amw} - \lambda^e\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{w\in\mathcal{W}}(U_{asw}+V_{asw}) - \lambda^s\sum_{a\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}V'_{ast}\Big\} \tag{1}$$

Inventory constraints

$$\sum_{i\in\mathcal{R}_a}X_{ail}+\sum_{i\in\mathcal{N}_a}Y_{ail}\le d_{al}\quad \forall\, a\in\mathcal{A},\ l\in\mathcal{L}_a, \tag{2}$$

$$\sum_{a\in\mathcal{A}}\sum_{l\in\mathcal{L}_a}\sum_{i\in\mathcal{R}_a\cap\mathcal{I}_{st}}X_{ail}\le i^r_{st}\quad \forall\, s,\ t\in\mathcal{I}_{st}, \tag{3}$$

$$\sum_{a\in\mathcal{A}}\sum_{l\in\mathcal{L}_a}\sum_{i\in\mathcal{N}_a\cap\mathcal{I}_{st}}Y_{ail}\le i^n_{st}\quad \forall\, s,\ t\in\mathcal{I}_{st}. \tag{4}$$

Time and market flexibility constraints

$$\sum_{i\in\mathcal{I}_{mw}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}\le u_{amw}\quad \forall\, a\in\mathcal{A},\ m\in\mathcal{M},\ w\in\mathcal{W}, \tag{5}$$

$$\sum_{i\in\mathcal{I}_{mw}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}+U'_{amw}\ge l_{amw}\quad \forall\, a\in\mathcal{A},\ m\in\mathcal{M},\ w\in\mathcal{W}, \tag{6}$$

Affiliate F&E constraints

$$\sum_{i\in\mathcal{I}_{s'w}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}-U_{as'w}\le\theta_{as'w}\sum_{i\in\mathcal{I}_{sw}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}\quad \forall\, a\in\mathcal{A},\ s'\in\mathcal{S},\ w\in\mathcal{W}, \tag{7}$$

$$\sum_{i\in\mathcal{I}_{s'w}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}+V_{as'w}\ge\theta_{as'w}\sum_{i\in\mathcal{I}_{sw}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}\quad \forall\, a\in\mathcal{A},\ s'\in\mathcal{S},\ w\in\mathcal{W}, \tag{8}$$

Time-separation constraints

$$\sum_{i\in\mathcal{I}_{st}\cap\mathcal{R}_a}\sum_{l\in\mathcal{L}_a}X_{ail}-V'_{ast}\le\mu_{ast}\quad \forall\, a\in\mathcal{A},\ s\in\mathcal{S},\ t\in\mathcal{T}, \tag{9}$$

Select and exclude constraints

$$\begin{gathered} X_{ail}\ge 0\quad \forall\, a\in\mathcal{A},\ i\in\mathcal{R}_a,\ l\in\mathcal{L}_a,\\ 0\le Y_{ail}\le\tau_{ail}\quad \forall\, a\in\mathcal{A},\ i\in\mathcal{N}_a,\ l\in\mathcal{L}_a,\\ X_{ail},\ Y_{ail}\ge 0. \end{gathered} \tag{10}$$

In the objective function, we maximize the potential revenue from the advertisers less the penalties resulting from MF and TF, deviation from F&E, and exceeding the targets based on minimum time separation. Constraint (2) states that the assignment for any advertiser should not exceed its order-line demand, and constraints (3) and (4) state that the same inventory cannot be assigned to more than one advertiser. The derived parameter u_{amw} gives the hard upper bound on the number of inventory spots to be allowed for the market and week for an advertiser based on its adoption toward MF. Constraint (5) defines the hard upper bound, whereas constraint (6) defines the soft lower bound for MF, because the penalty helps to provide a minimum guarantee to the advertiser's order to be broadcast in the requested market. For constraints (7) and (8), the parameter θ_{asw} is calculated as the ratio of the number of potential inventory spots for the advertiser in a particular station s, to the sum of potential inventory spots for the advertiser from all potential stations. The variables U_{asw} and V_{asw} get the surplus and slack from attaining the target for the advertiser a for station s to maintain the fair and equitable distribution. Constraint (9) limits the number of spots allotted to advertiser a for a particular period t; hence, it helps the postprocessing process to efficiently minimum separation requirements. Finally, constraint (10) gives the inclusion and (or) exclusion for the variables based on the requirements of an advertiser's order. For conciseness, we have defined only the variables and constraints for a rated station. Similar to constraints (5)–(9), constraints and variables should also be defined for nonrated and commercial stations. Similarly, F&E constraints are defined only at a station level, and similar type of variables and constraints should be constructed to assure F&E at days of week, program types, and day parts. We solve the model as an LP model; it gives the consolidated target for each advertiser's order line in each market and week. However, continuous targets from the LP model will be incorrect for the subsequent SD model; hence, we use a rounding heuristic, which we explain in the *MW Postprocessing* section.

MW Postprocessing

The following rounding heuristic is used to round off the MW model targets to be used in the SD model. For each advertiser a, order line $\mathcal{L}_a$, and market for order-line m, let x_w and y_w be the targets from the MW model for the rated, nonrated, and noncommercial allocation in week w. Let x'_w, y'_w be their rounded values, respectively.

1. *Spot orders (not time flexible)*: For each week w, round x_w and y_w target values such that the total delivery in each week is equal to the closest integer value of the original sum, and thus does not exceed the weekly demand; that is, $x'_w+y'_w\le\lfloor x_w+y_w+0.5\rfloor$, $\forall\, w$. The change in the values is limited to rounding up or down; that is, $\lfloor x_w\rfloor\le x'_w\le\lfloor x_w\rfloor+1$, $\forall\, w$, and similarly, $\lfloor y_w\rfloor\le y'_w\le\lfloor y_w\rfloor+1$, $\forall\, w$.

2. *Time flexibility*: The goal is to round x_w, y_w for all $w=1,\ldots,W$ by shifting fractional values among them such that:

(a) Total delivery over all weeks should be equal to the closest integer value of the original sum, and thus does not exceed the total demand, which we achieve by

$$\sum_w(x'_w+y'_w)\le\Big\lfloor\sum_w(x_w+y_w)+0.5\Big\rfloor,\quad \forall\, w.$$

(b) Total delivery in each week does not exceed the TF percentage constraint, which we achieve by

$$x'_w+y'_w\le\lfloor x_w+y_w\rfloor+1,\quad \forall\, w.$$

(c) Total nonrated delivery over all weeks should not exceed the total nonrated demand, which we achieve by

$$\sum_w y'_w \le \left\lfloor \sum_w y_w \right\rfloor + 1, \quad \forall w.$$

(d) The change in the values is limited to rounding up or down, which we achieve by

$$\lfloor x_w \rfloor \le x'_w \le \lfloor x_w \rfloor + 1, \quad \forall w \quad \text{and}$$

$$\lfloor y_w \rfloor \le y'_w \le \lfloor y_w \rfloor + 1, \ \forall w.$$

5. *GRP Orders (market and nonmarket flexibility)*: For GRP orders, we round x_w for all $w = 1, \ldots, W$ by shifting fractional values among them such that:

(a) Total GRP delivery from rated spots over all weeks should equal the closest rounded value (two decimal places) of the original sum, and thus does not exceed GRP demand. We achieve this by

$$\sum_w x'_w \le \left\lfloor 10^2 * \sum_w x_w + 0.5 \right\rfloor \cdot 10^{-2}.$$

(b) The change in the value is limited to rounding up or down; that is,

$$\lfloor 10^2 x_w \rfloor \cdot 10^{-2} \le x'_w \le \lfloor 10^2 x_w + 1 \rfloor \cdot 10^{-2} \quad \forall w.$$

Station Day-Part Model Formulation

The SD model assigns the inventories of advertisement spots to order lines based on the targets from the MW model for each market and week. Some constraints from the MW model are excluded and some new constraints are added. The following constraints are retained from the MW model: time-separation targets, inclusion-exclusion, total number of assigned spots or total GRP delivery, fairness and equitability constraints with targets from the MW model, total number of assigned spots, and total GRP delivery constraints. The SD model has a much smaller scope, but operates at a smaller granularity level than the MW model. The SD model includes additional placement constraints that the MW model cannot enforce. During execution, the markets with the largest sell-out ratio (demand and (or) inventory) are run first, because if some target placements cannot be placed successfully as a result of additional constraints introduced in the SD model, then MF and TF can be invoked again to shift those spots into markets with smaller sell-out ratios. The SD model may be run for any number of weeks into the future; however, running the SD for all the future weeks in the planning horizon is unnecessary. For conciseness in the model description below, we have included only the market flexibility and fairness and equitability constraints for rated stations. Similar type of variables and constraints can be constructed for nonrated and commercial stations. The objective function is similar to the MW model.

Sets

$\mathcal{F}_t \subseteq \mathcal{I}$ Set of all individual inventory spots (feeds) for period $t \in T$.

$\mathcal{R}'_a \subseteq \mathcal{I}$ Set of all rated individual inventory feeds (feed, market, week, station, period) belonging to advertiser $a \in A$ from inventory set I.

$\mathcal{N}'_a \subseteq \mathcal{I}$ Set of all nonrated individual inventory feeds (feed, market, week, station, period) belonging to advertiser $a \in A$ from inventory set I.

$\mathcal{G}_t$ Set of all inventory feeds that are to be broadcast together at period t.

Parameters

θ'_{asw} Target quantity from the MW model to be assigned to advertiser a for station s for F&E rotation.

Penalties

λ^e Penalty for each unit of assignment that is above or below the F&E target.

λ^s Penalty for each unit of assignment exceeding the allowable amount in a period.

λ^b Penalty for back-to-back placement.

Decision Variables

$$X_{afl} = \begin{cases} 1, & \text{if rated inventory feed } f \in \mathcal{R}'_a \text{ is allotted for a} \in \mathcal{A} \text{ and } l \in \mathcal{L}_a; \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_{afl} = \begin{cases} 1, & \text{if nonrated inventory feed } f \in \mathcal{N}'_a \text{ is allotted for a} \in \mathcal{A} \text{ and } l \in \mathcal{L}_a; \\ 0, & \text{otherwise.} \end{cases}$$

U_{asw} Surplus in assignments to station s for advertiser a for week w based on the F&E target.

V_{asw} Slack in assignments to station s for advertiser a for week w based on the F&E target.

U'_{amw} Slack in assignments to advertiser a for market m for week w based on the time-market flexibility lower bound for station s.

V'_{ast} Surplus in assignments to advertiser a for station s for period t based on the bound for minimum separation placements for the station s.

P_{al} Unallocated demand for advertiser a for order line l.

B_{afl} Surplus for violation of back-to-back placement for advertiser a for order line l.

Model Formulation

$f(s, m, w)$:

$$\max \Big\{ \sum_{a\in\mathcal{A}} \sum_{f\in\mathcal{R}'_a \cup \mathcal{N}R'_a} \sum_{l\in\mathcal{L}_a} (r_a + \rho_{afl})(X_{afl} + Y_{afl}) - \lambda^b \sum_{a\in\mathcal{A}} \sum_{f\in\mathcal{R}'_a \cup \mathcal{N}R'_a} \sum_{l\in\mathcal{L}_a} B_{afl} - \lambda^e \sum_{a\in\mathcal{A}} (U_{asw} + V_{asw}) - \lambda^s \sum_{a\in\mathcal{A}} \sum_{t\in\mathcal{T}} V'_{ast} \Big\} \tag{11}$$

Inventory constraints

$$\sum_{a\in\mathcal{A}}\sum_{l\in\mathcal{L}_a}\sum_{f\in\mathcal{R}'_a} X_{afl} \le 1 \quad \forall f\in\mathcal{F}_t,\, a\in\mathcal{A},\, l\in\mathcal{L}_a, \tag{12}$$

$$\sum_{l\in\mathcal{L}_a}\sum_{f\in\mathcal{R}'_a} X_{afl} + \sum_{l\in\mathcal{L}_a}\sum_{i\in\mathcal{N}'_a} Y_{afl} + P_{al} = \theta'_{asw} \quad \forall a\in\mathcal{A},\, l\in\mathcal{L}_a, \tag{13}$$

Back-to-back advertiser constraints

$$X_{afl} + X_{af'l} - B_{afl} \le 1$$
$$\forall a\in\mathcal{A},\, l\in\mathcal{L}_a,\, f, f'\in\mathcal{R}'_a,\, |t(f)-t(f')| = 1, \tag{14}$$

Affiliate F&E constraints

$$\sum_{f\in\mathcal{R}'_a\cap\mathcal{I}_{s'w}}\sum_{l\in\mathcal{L}_a} X_{afl} - U_{as'w} + V_{as'w} = \theta'_{as'w} \quad \forall a\in\mathcal{A}, \tag{15}$$

Group-link constraints

$$X_{afl} - X_{af'l} = 0 \quad \forall a\in\mathcal{A},\, l\in\mathcal{L}_a,\, f, f'\in\mathcal{R}'_a\cap\mathcal{G}_t, \tag{16}$$

Time-separation constraints

$$\sum_{f\in\mathcal{I}_{st}\cap\mathcal{R}'_a}\sum_{l\in\mathcal{L}_a} X_{afl} - V'_{ast} \le \mu_{ast} \quad \forall a\in\mathcal{A},\, t\in\mathcal{T}, \tag{17}$$

Select and exclude constraints

$$\begin{aligned} X_{afl}&\in\{0,1\} \quad \forall a\in\mathcal{A},\, i\in\mathcal{R}'_a,\, l\in\mathcal{L}_a,\\ Y_{afl}&\in\{0,1\} \quad \forall a\in\mathcal{A},\, i\in\mathcal{N}'_a,\, l\in\mathcal{L}_a. \end{aligned} \tag{18}$$

Constraint (12) allocates each inventory feed to only one advertiser a for an order line l. Constraint (13) deduces the unallocated demand for each customer's order line based on the target from the MW model. For advertiser a's order line l, X_{afl} and Y_{afl} give the allotted rated and nonrated inventory feeds, respectively. Constraint (14) captures the value for B_{ail} to be penalized in the objective function for the allotment of back-to-back inventory feeds, that is, inventory feeds that are broadcast exactly at the same time on two consecutive days for the same advertiser a. Constraint (15) is similar to the F&E constraint from the MW model; it captures the slack and surplus assignments based on the MW model target, and the slack and surplus are penalized in the objective function. If any inventory feed in a simulcast or linked group is assigned to an order line on a given broadcast date, then all feeds in that simulcast or linked group must be assigned to the same order line on that broadcast date on different stations. Constraint (16) assures the allotment of group-link inventories to the same advertiser a and order line l. Constraint (17) is similar to constraint (9) in the MW model, which gives the bound on the number of spots to be broadcast for period t, so the time-separation constraint can be handled efficiently in the postprocessing of the SD model. Constraint (18) gives the binary restrictions for X_{afl} and Y_{afl} variables.

SD Postprocessing

The following heuristics are used to improve the final assignments of advertisers order lines in the SD model. The algorithms are designed to compensate for the MIP gap in solving the SD model, overdelivery of unallocated inventory spots, and violation of minimum time separation between similar types of advertisements.

1. *Recovery heuristic*: The recovery heuristic schedules order lines to inventory feeds that are within specifications, but are unscheduled so far. Here, we show the order of execution:

(a) Close the MIP optimality gap from the SD model. For any order line with an unmet MW target, and if spots are unscheduled after solving the SD model, we assign the spots to the order line to minimize bumping.

(b) Overdelivery for compliance.

The algorithm works as follows: Let L be the set of order lines to be scheduled, and D be the set of broadcast dates. Perform the following for each order line l in L:

Recovery heuristic:

Step 1: Initialization: Let q be the target quantity for this algorithm to schedule an order line l on broadcast dates in D. Let S be the set of all unscheduled spots for l with broadcast dates in D.
Step 2: If $q \le 0$ or $S = \emptyset$, STOP.
Step 3: Let s be the first element in S. Let x be the number of demand units that s can satisfy for an order line l (x may be greater than 1 for simulcast or linked groups, or fractional for GRP). Schedule s to l. Set $q \leftarrow q - x$, and $S \leftarrow S\setminus\{s\}$. Go to Step 1.

4. *Overdelivery heuristic*: The overdelivery heuristic is used to deplete the unsold inventory for the upcoming scheduling week in an F&E manner. The quantity allotted to each order line is proportional to the demand quantity of the order line. The heuristic works as follows: Let w be the first week of the planning horizon. For each market m, let u be the total unscheduled inventory of market m in week w. Let S be the set of all stations in the market. Let L be the set of order lines in market m that accept overdelivery and have demand in week w. Let q be the total ordered quantity in week w. Perform the following process for each order line l in L:

Overdelivery heuristic:

Step 1: Initialize $j = 1$. Let q^l be the demand of order line l in week w. Calculate the target total overdelivery quantity for l as $t^l = u * q^l/q$. Let u^l be the number of spots (scheduled or unscheduled) that are within specifications for an order line l in market m in week w.
Step 2: If $j > |S|$, STOP.
Step 3: Let u^{s_j} be the number of spots (scheduled or unscheduled) for an order line l in station s_j in week w. Calculate the station-wise target for order line l as $t^{s_j} = t^l * u^{s_j}/u^l$.

Step 4: Let U^{s_j} be the set of all unscheduled spots that can be scheduled to l as overdelivery with broadcast date in w.
Step 5: If $t^{s_j} \leq 0$ or $U^{s_j} = \emptyset$, set $j \leftarrow j+1$, and go to Step 1.
Step 6: Let u be the first element in U^{s_j}. Schedule u to l. Set $t^{s_j} \leftarrow t^{s_j} - 1$, and $U^{s_j} \leftarrow U^{s_j} \setminus \{u\}$. Go to Step 4.

7. *Sequence-constraint heuristic*: Sequential constraints to address the minimum separation among similar types of advertisement spots are difficult to include in the MW or SD optimization models, because including them would exponentially increase the number of constraints in the model. Hence, in both the MW and the SD models, we address sequential constraints as upper limits for each advertiser type for any day part. The upper limit on the allowed number of assignments for an advertiser type is expected to be helpful in the postprocessing. We set the following heuristic in the postprocessing to remove any violations of sequence constraints. These models are very small; they typically have 20–30 variables and 10–15 constraints, and are solved in parallel.

Sequence-constraint heuristic procedure:

Step 1: Based on the SD solution, let t be the day part at which the sequence constraint is violated. Let the timeline T be set as $T = \{t, t+1\}$. Initialize $k = 1$.
Step 2: Construct a SD model only for the set T with all the assignments from the solution as targets for the order lines. If feasible, then use the new solution and go to Step 2.
Step 3: On infeasibility, add the day part k if $|(t-(k-1))| < |(k+1)-t|$ and $k <$ maxLimit, then add $(k-1)$ to the set T; otherwise add $(k+1)$ to T. Increment k and go to Step 2.

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Verification Letter

John Kaufman, President, Business Operations, iHeart-Media, Inc., 125 West 55th St., New York, NY 10019, writes:

"To whomsoever it may concern:

"The purpose of this letter is to verify the paper titled "Media Company Using Analytics to Schedule Radio Advertisement Spots." The optimization suite (OS) described in this paper is currently being used within the sales planning/placement processes on a daily basis by multiple of our businesses. We have achieved the quantitative and qualitative benefits described in the Benefits section of this paper from using the OS in our Total Traffic and Weather Network (TTWN) business unit."

Saravanan Venkatachalam is an assistant professor in the Industrial and Systems engineering department at Wayne State University. He has nine years of work experience in the design and development of decision support systems in the domains of supply chain management, revenue management, and contact center operations. He has a PhD in Industrial Engineering from Texas A&M University.

Fion Wong is a pricing and revenue management practitioner who has solved real-world revenue optimization problems in the media, airline, and hospitality industries. Techniques that she has applied include mathematical programming, time-series forecasting, and consumer choice modeling. Fion has a Master of Science degree in industrial engineering from Georgia Institute of Technology. She currently works at the Revenue Management department of Delta Air Lines in Atlanta.

Emrah Uyar is a senior operations research consultant at JDA International Ltd, Bracknell, UK. He has seven years of experience in design and implementation of pricing/revenue management solutions in various service industries including media, leisure, transportation and hospitality. He has a PhD in industrial engineering from Georgia Institute of Technology and is a former Thomas

Johnson Fellow. He also holds Richard E. Rosenthal Young Researcher Award 2010 from INFORMS.

Stan Ward is a senior practice director for the JDA Software Group, Inc. He has over 23 years of experience in pricing/revenue management and large scale data analytics as a consultant, implementer and innovator across industries, including television broadcasting, radio, package delivery, hotels, cruise lines, and airlines. He earned an MBA from the Fuqua School of Business at Duke University and a BEE from the Georgia Institute of Technology.

Amit Aggarwal is currently executive vice president of Revenue Management at iHeartMedia, Inc. He has more than 15 years of experience in revenue strategy, pricing, systems and data across several industries including airlines (Priceline.com), hotels (Starwood Hotels and Resorts), and media (iHeartMedia, Inc.).