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THE FRANZ EDELMAN AWARD
Achievement in Operations Research

Operations Research Enables Better Planning of Natural Gas Pipelines

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Abstract. China’s natural gas consumption has nearly doubled over the last five years. To better meet demand, the China National Petroleum Corporation (CNPC), China’s largest oil and natural gas producer and supplier, partnered with researchers from the University of California, Berkeley, and Tsinghua University in Beijing to apply innovative operations research to develop and implement new software that helps CNPC improve the management of its gas pipeline network. Previously, all pipeline production and construction planning for CNPC, which controls 72% of the country’s natural gas resources and 70% of its pipeline network, was conducted by traditional methods using spreadsheets. However, because of the network’s increasing size and complexity, using the traditional method resulted in excess costs and wasted resources. Since the implementation of the new software, which uses a three-stage convex relaxation method and iterative piecewise linear approximation methods, at the end of 2014, CNPC has realized approximately \$530 million in increased profits. Moreover, the resulting increased efficiency of the existing pipeline network allowed the company to postpone adding new pipelines, leading to an official budget reduction of over \$20 billion in construction costs for the subsequent five years.

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Keywords: natural gas pipeline transmission • CNPC • convex relaxation

Introduction

As the world’s second largest economy, China has transitioned into a stage of high-quality development from the stage of high-speed growth. At the Davos World Economic Forum in 2018, He Liu, one of China’s top economists, stressed that fighting pollution is one of China’s three critical battles: “green and low-carbon development is what the Chinese people want the most in a break with the traditional growth model” (He 2018).

To ensure the sustainable development of its economy, China’s 12th Five-Year National Plan (2011–2015) considered natural gas to be an important source of green energy and encouraged its use over coal in winter residential heating. As a result, from 2010 to 2015, China’s annual natural gas consumption increased from 107

billion to 193 billion cubic meters, and the total length of the gas pipeline network expanded from 32,800 kilometers to 45,000 kilometers. By transitioning from coal to natural gas for winter residential heating, Beijing, the capital of China, experienced better air quality in 2017 than it had in the previous five years (Zhuang 2017).

One major challenge for increasing natural gas usage is the limitation of transmission capacity. The majority of China’s natural gas users are located far from its sources; therefore, natural gas must be transported by pipelines. Because the demand for natural gas is increasing, China hopes to more efficiently utilize its current pipeline network capacity, while planning its pipeline network construction in advance. The transmission capacity changes with the gas flow dynamics; therefore, it is hard to directly evaluate the

capacity without considering the physical characteristics (e.g., pressure and temperature) of natural gas. A logical question is whether the current pipelines' transmission capacity satisfies the demands of customers.

In 2014, it was unclear whether China's pipeline network had sufficient capacity to meet the nation's increasing demand. Answering such a question is complex. First, the network includes thousands of demand nodes. The pipelines leading to the demand nodes are interconnected and form numerous cycles; therefore, the demand at one node could be satisfied by multiple pipelines. Second, the flow of natural gas is determined by the pressure and temperature of each node. The physical laws governing the relationship between temperature, pressure, and the resulting flow of natural gas is nonlinear and nonconvex. Flows also depend upon the varying elevations through the network.

To assist the decision-making process for new pipeline construction, this project aimed to answer the following questions: (1) How much natural gas can be transported by a given network? (2) What is the optimal transmission plan, including the decisions on gas production, imports, transportation, storage, and sales, based on the locations of available natural gas resources, locations of demand, and prices for natural gas at different locations in the network. We address these questions primarily by integrating natural gas fluid dynamics into a mathematical optimization framework.

The China National Petroleum Corporation (CNPC) is China's largest oil and natural gas producer and supplier. From 2014 to 2017, CNPC ranked in the top four of the Fortune Global 500. CNPC operates a network of more than 50,000 kilometers of natural gas pipelines, which is the second largest natural gas pipeline network owned by a single corporation worldwide. CNPC now holds approximately 72% of China's natural gas resources and 70% of its pipeline network. Within the corporation, the China Petroleum Planning and Engineering Institute (CPPEI) is the research and consulting branch for the technological and economic analyses of petroleum-related and petrochemical-related construction projects.

In 2012, CNPC initiated a key research project, "Optimization of Natural Gas Pipeline Transmission Network," which covered planning, operations, and scheduling for natural gas pipeline transmission in response to China's 12th Five-Year National Plan (2011–2015). The planning of natural gas transmission often considers the annual and monthly plans in an interval of three to five years. The operations problem considers the amount of monthly and weekly natural gas flow and pressure, and the scheduling problem often focuses on the hourly gas flow and pressure within a day.

CPPEI, the University of California, Berkeley, and Tsinghua University in Beijing jointly worked on the

dynamic planning of natural gas transmission. In 2013, they completed the mathematical modeling and algorithm development. They developed and implemented solution algorithms in a software package, which CNPC has applied to its natural gas pipeline transmission planning problems since December 2014. The algorithms utilize a three-stage convex relaxation (3SCR) method and an iterative piecewise linear approximation method. Our resulting decision support system, which combined the optimization model, geographic information system, and a database system, can be applied to networks containing more than 1,000 nodes and having any topological structure. In 2017, our project won the first prize for progress in science and technology within CNPC because of its contributions to increasing revenue in natural gas pipeline transmission planning.

The software developed in this project has been broadly applied at CNPC's headquarters, research institutes, and field stations. In 2015–2017, our project team used the decision support system to update plans two to three times per month. Each time, it provided more than 1,000 sets of optimization results under different parameter settings. The results generated by the software helped CNPC to evaluate transmission capacity, make annual and monthly transmission plans, and provide instructions for weekly and daily natural gas transmission scheduling.

Literature Review

A vast amount of literature on natural gas network design and operations is available. In this section, we provide a review of the models, methods, and applications of natural gas pipeline transmission problems.

Most research problems in natural gas pipeline transmission can be categorized as either operations problems or investment problems based on the objective functions and the time horizons. Operations problems consider the optimization of natural gas flow rates, pressure, or other physical parameters, given a fixed network infrastructure. Investment problems consider the optimization of the network topology and the decisions of building new infrastructure. Operations problems usually consider the problem within three to five years, while the investment problems optimize the network for over 10 years.

Traditional models of natural gas pipeline transmission problems are summarized in Table 1, where we list whether the problem is an operations one or an investment one, the goal of optimization, whether the problem is a multiperiod problem, whether the problem considers steady or transient state of natural gas, and whether the problem assumes fixed flow directions. In this study, we focus on the operations problems because we assume a fixed network topology and consider optimizing the natural gas flow rates, pressure, and temperature.

Table 1. Major Models in the Literature Can Be Categorized as Operations Models or Investment Models

Research	Operations	Objective	Multiperiod	State	Fix direction
Zhang and Zhu (1996)	No	Pipeline diameter	No	Steady	Yes
André et al. (2013)	No	Topology, pipeline diameter	No	Steady	Yes
André et al. (2009)	No	Pipeline diameter	No	Steady	No
Mahlke et al. (2010)	Yes	Compressor fuel	Yes	Transient	Yes
Bonnans et al. (2011)	Yes	Compressor fuel Purchasing and sales	No	Steady	No
Domschke et al. (2011)	Yes	Compressor fuel	Yes	Transient	Yes
De Wolf and Smeers (1996)	No	Pipeline diameter	No	Steady	No
De Wolf and Smeers (2000)	Yes	Revenue and purchasing	No	Steady	Yes

Operations problems with an objective of minimizing the operating cost (De Wolf and Smeers 2000), maximizing the revenue of the operator (Bonnans et al. 2011), or minimizing the fuel cost (Mahlke et al. 2010, Bonnans et al. 2011, Domschke et al. 2011) have been studied in the literature. In this project, our objective is maximizing the profit influenced by the central planner, who decides how much natural gas to purchase or produce at each location.

Most natural gas pipeline transmission optimization problems are complex because the associated models are mixed-integer nonconvex optimization models. The nonconvexity implies that we may not be able to find a global optimal solution. For nonconvex problems, a good initial solution point can significantly simplify the solution process. For example, De Wolf and Smeers (2000) and Babonneau et al. (2012) use energy minimization methods to find good initial points for the piecewise linear approximation of the nonlinear constraint. Convex relaxation can be another effective approach to deal with nonconvex problems (see Borraz-Sánchez et al. 2016 and Xue et al. 2016).

Table 2 summarizes recent applications of natural gas pipeline transmission optimization models on real networks and their network sizes and features. The only publicly available large networks investigated previously come from the open data library Gaslib (Humpola et al. 2015b), which we numerically tested in this study.

Many commercial software packages can simulate a network’s steady or transient performances (e.g., CNPC uses Transient Gas Network [TGNET], a module

of Pipeline Studio). However, this type of software cannot optimize flow and pressure. We also evaluated other software packages, including Synergie Gas (American), IHS Energy GULP (British), and OLGA (Brazilian), but found none to be capable of handling all the necessary tasks. Furthermore, CNPC prefers to develop its own software rather than relying on licensed software.

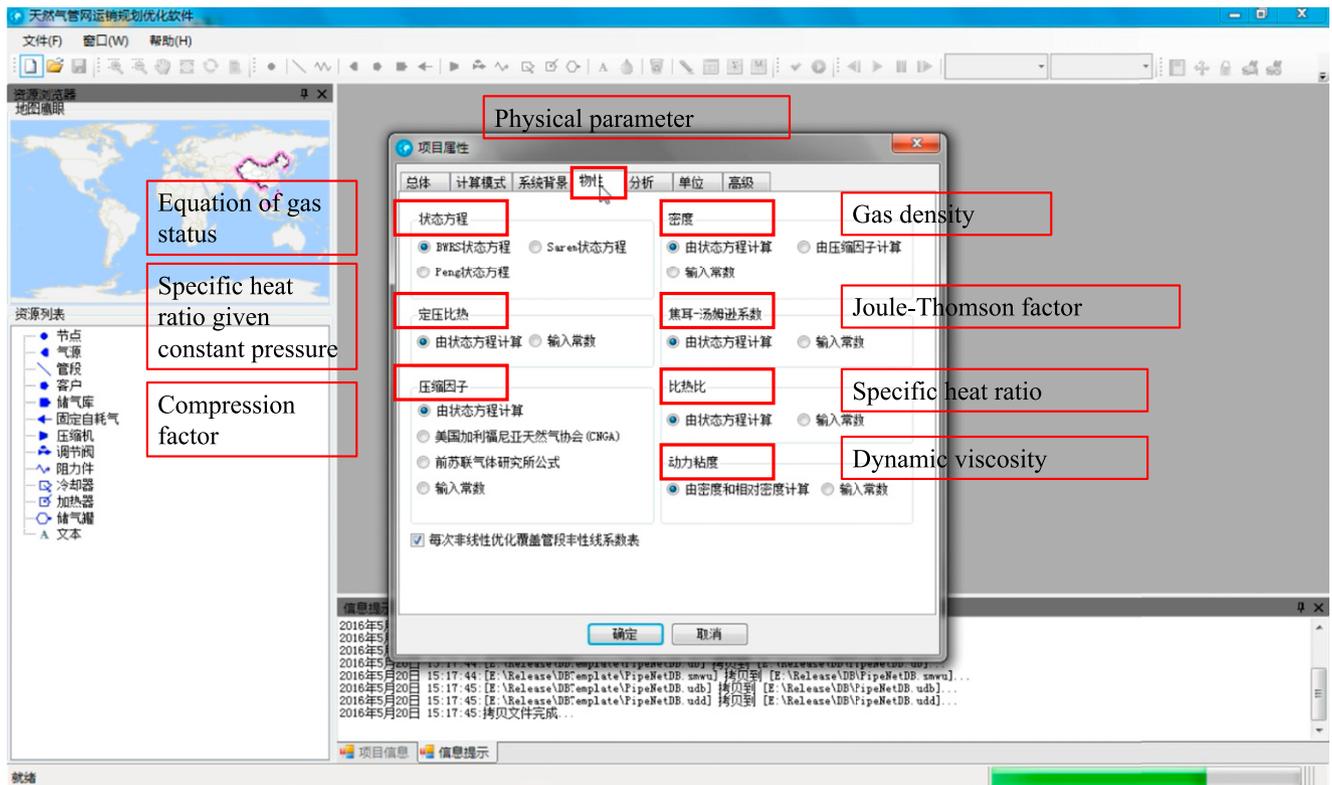
Technical Challenges and How We Approached Them

The research team strove to develop and implement novel analytical models that complete in seconds, generate solutions with satisfactory accuracy, and are robust, and user friendly. However, given the complex nature of the problem, the process presented numerous challenges, which we discuss in this section.

A precise description of the pipeline model is challenging for two reasons. First, the equations describing the relationship between natural gas flow, pressure, and temperature involve highly complex nonlinear partial differential equations (e.g., the Benedict-Webb-Rubin equation for computing the compression factor and the Joule-Thomson equation for describing temperature change; see Benedict et al. 1942 and Perry 1950). Second, the equations describing the flow dynamics are only applicable when the pipelines are no more than three miles long; however, a major pipeline can often be as long as 100 miles. It is necessary to divide long pipelines into multiple small ones so that the equations can be applied directly. Consequently, the complexity increases significantly.

Table 2. Researchers Have Optimized Pipeline Transmission Problems of Varying Sizes and Network Features

Research	Country	No. of nodes	No. of arcs	Network feature
André et al. (2013)	France	81	None	Tree structure
Chebouba et al. (2009)	Algeria	7	6	Gun-barrel structure
Mahlke et al. (2010)	Germany	30	29	None
Danilovic et al. (2011)	Serbia	5	4	Gun-barrel structure
De Wolf and Smeers (2000)	Belgium	20	24	None
Pfetsch et al. (2015)	Germany	600	562	None
Borraz-Sánchez et al. (2016)	Germany	582	609	None
Humpola et al. (2015a)	Germany	32	27	None
Xue et al. (2016)	China	26	40	Cyclic

Figure 1. (Color online) A User Has Multiple Ways to Compute Parameters in the Software

Many possible models describing pipeline flow dynamics are available because no standard pipeline environment exists. CNPC preferred the optimization toolbox to enable it to handle all the possible models in a unified manner. Figure 1 lists a parameter panel in the software; for each parameter, the user has multiple ways to compute its value.

In a decision support tool, computation time is an important criterion to ensure a positive user experience. Preferably, the core computation should complete in seconds. This requirement is extremely demanding, particularly in the case of a large-scale, nonconvex model. One direct approach is to approximate the model with a linear programming model. However, this approach may lead to significant inaccuracies. The estimated cost savings and revenue increase may be inaccurate if the model itself has significant errors.

Because of uncertainty in future costs and prices, the software should be able to compare a large set of future scenarios. Considering numerous combinations of cost and price, the software being designed should handle the uncertainty by evaluating multiple scenarios in an efficient manner and store all computed results in the database for future reference.

We faced many infeasible cases while testing the analytical model and methods. To deal with this problem, we took several approaches. First, we tested repeatedly on networks with specific topologies and

parameter settings to find the infeasible cases and then refined the algorithm to avoid them. Second, we embedded into the software different algorithms, including those based on piecewise linear approximation and convex relaxation, so that users could select the most appropriate methods for their needs. Third, we designed nine levels of data checking to avoid the data input errors, which could cause the software to generate invalid outputs.

Traditional Method Using Spreadsheets

Before 2012, engineers at CNPC manually generated the natural gas transmission plans by balancing the amount of natural gas in the network using spreadsheets in the following five steps: (1) divide the entire system into more than a dozen subsystems; (2) establish the internal balance of production, imports, transmission, storage, and sales amounts within each subsystem using Microsoft Excel; (3) adjust the decisions made in the second step based on the management team's experience and feedback; (4) seek a preliminary balance among the production, imports, transmission, storage, and sales amounts in the entire system by adjusting the interflows between each subsystem; and (5) check whether the amount of natural gas in each subsystem is balanced according to the preliminary balance of the entire system. If not, go back to step 2 and iterate.

The manual method could not guarantee that the transmission capacity is utilized efficiently. The basic principle of the manual method is to balance the natural gas flow rates, without using any optimization method. Engineers used their personal experiences to set the flow directions, and then separately generated the flow rate in each pipeline. The logic of the process is to try to make full use of the transmission capacity and satisfy customer demands as much as possible. Engineers modified the flow rates repeatedly when they found unbalanced nodes.

Plans generated by the manual method were not optimal in most cases and could result in wasting natural gas resources or investing unnecessarily in pipeline construction. When the customer demands could not be satisfied, engineers again balanced the gas flow by adjusting the amount of natural gas to be in storage, to be imported, and to be produced. If demand was still unsatisfied, they would propose the construction of new pipelines.

Since 2013, with the expansion of the pipeline network, the planning problem could no longer be handled manually. The size of the network increased dramatically, and the network topology became increasingly complex. Engineers could no longer generate plans by separately analyzing each pipeline. Furthermore, in addition to satisfying demand, CNPC was interested in maximizing its profits. Thus, to pursue efficiency and address complexity, CNPC turned to operations research.

New Methodology

CNPC faced an important trade-off between model accuracy and computational efficiency. If we built a complex model that closely matched the real network, then it would be impossible to find optimal solutions; conversely, if we used an approximate model, the inaccuracy might be such that the computed optimal solution would be practically infeasible. Thus, the operations research team designed two approaches. Although both satisfy the accuracy and computational efficiency requirements, one provides highly accurate solutions for small-scale problems and the other offers more efficient computations for large-scale problems, such as planning the entire pipeline.

The mathematical problem of natural gas pipeline transmission can be stated as follows. A natural gas pipeline transmission network is frequently represented by a directed network graph that consists of nodes and arcs. The decision variables on nodes include net supply (total inflow minus total outflow), pressure, and temperature. The decision variables on each arc are the flow rates. The objective is to maximize the total profit or minimize the total cost, equivalently, where the total profit equals revenue minus purchasing cost. Constraints include the flow balance, flow range

(min and max), pressure range, and equations describing the physical laws in different components (e.g., compressor, regulator valve, and pipeline.) A simplified model of the problem is provided in Appendix A. The original model for the natural gas pipeline transmission problem is nonlinear and nonconvex, and we tried two approaches to solve the model.

In the first approach, we model the natural gas transmission problem as a mixed-integer linear programming problem (MILP) by iteratively approximating all nonlinear relationships as piecewise linear. This yields satisfactory results for small-sized problems. In the real project, the piecewise linear approximation method is used for solving problems in small-size networks, particularly when analyzing a proper subset of the network. For large-scale problems, however, we encountered several issues that theoretically should not have occurred when solving the problem using iterative piecewise linear approximation. For example, we found that although all algorithms in IBM ILOG CPLEX should hypothetically provide the same optimal value for the same problem, this is not the case in practice. For example, for some numerical instances, one algorithm found an optimal solution, but another algorithm returned an infeasible or unbounded error. Furthermore, the computation time is always above the maximum computation time criterion set by CPPEI.

In the second approach, we first developed an iterative two-stage framework to separately obtain solution optimality and solution accuracy (Xue et al. 2016). During each iteration, we relax the original nonconvex natural gas transmission problem as a convex optimization problem. The convex relaxation allows each iteration to be solved extremely fast, thus making possible the iteration and attainment of a highly accurate solution. In numerical tests using real data from a Chinese network with more than 20 nodes, the method can generate solutions with a convergence error within 2% in one second, where the convergence error has been defined as the percentage difference in the pipeline resistance coefficient between two adjacent iterations (Xue et al. 2016). The iterative two-stage framework has been verified to obtain optimal solutions in small-scale networks with no more than 100 nodes. However, in networks with many cycles, the iterative two-stage method may not generate feasible solutions that converge within a reasonable time (i.e., it may run for several hours).

We then updated the second approach and proposed a 3SCR algorithm to replace the iterative approach. Appendix B provides details. The convex relaxation on a single pipeline's flow-pressure constraint was extended to constraints on a series of connected pipelines. We identified the conditions under which the convex relaxation does not compromise optimality (Appendix C). In the first stage, we solve a convex relaxed model.

If the resulting solution is feasible, the run concludes; otherwise, a second-stage model minimizes energy in pipelines and updates the flow rates. A third stage determines the pressure and temperature of each node, given the flows established in the second stage. The proposed algorithm can obtain a solution with satisfactory accuracy for the largest Chinese network (i.e., 1,285 nodes) when other methods cannot even generate a feasible solution.

Implementation

The software has five components: data input, data check, unit conversion, optimization, and data output. Data are first imported from the database into the optimization system and stored in memory. Second, the arbitrary numerical data are converted into data with standard units. Third, the system checks the data for correctness. The natural gas pipeline planning problem is then optimized by the algorithm the user selected at the beginning. The user can select the 3SCR method or the piecewise linear approximation method based on the requirements for efficiency and accuracy. After the calculation has completed, the results are converted to required formats and generated as output.

During software development, we worked to ensure that the software interface is user friendly. First, the main interface displays the pipeline network loaded from the database. The geographical information

of each node is stored in the database. The design is similar to that of commercial map software; the user can move, enlarge, and shrink the network similar to using a Google map (Figure 2). A user can also click on each node and arc to view its information.

Second, when the software is initialized, a user can select the objective functions and other system settings. For example, the objective function can be either linear or piecewise linear, and the physical parameters can be computed in various ways.

Third, the team designed nine levels of data checking, including magnitude, conflicts, size relationship, overlaps, network topology, and check module required by other software (e.g., TGNET). The software is designed such that in the event of an error, the user receives an error message and can decide whether to continue the calculation.

Fourth, users often do not want to wait for an extended time without receiving any information. We therefore designed the system such that it provides users with the calculation schedule.

Validation and Verification

We verified the results from the proposed 3SCR algorithm using data from real networks, including previous studies, open data sets, and CNPC’s network. We compared computation time, optimality, and solution accuracy with the results from piecewise linear

Figure 2. (Color online) The Design of the Software Is Similar to That of Commercial Map Software

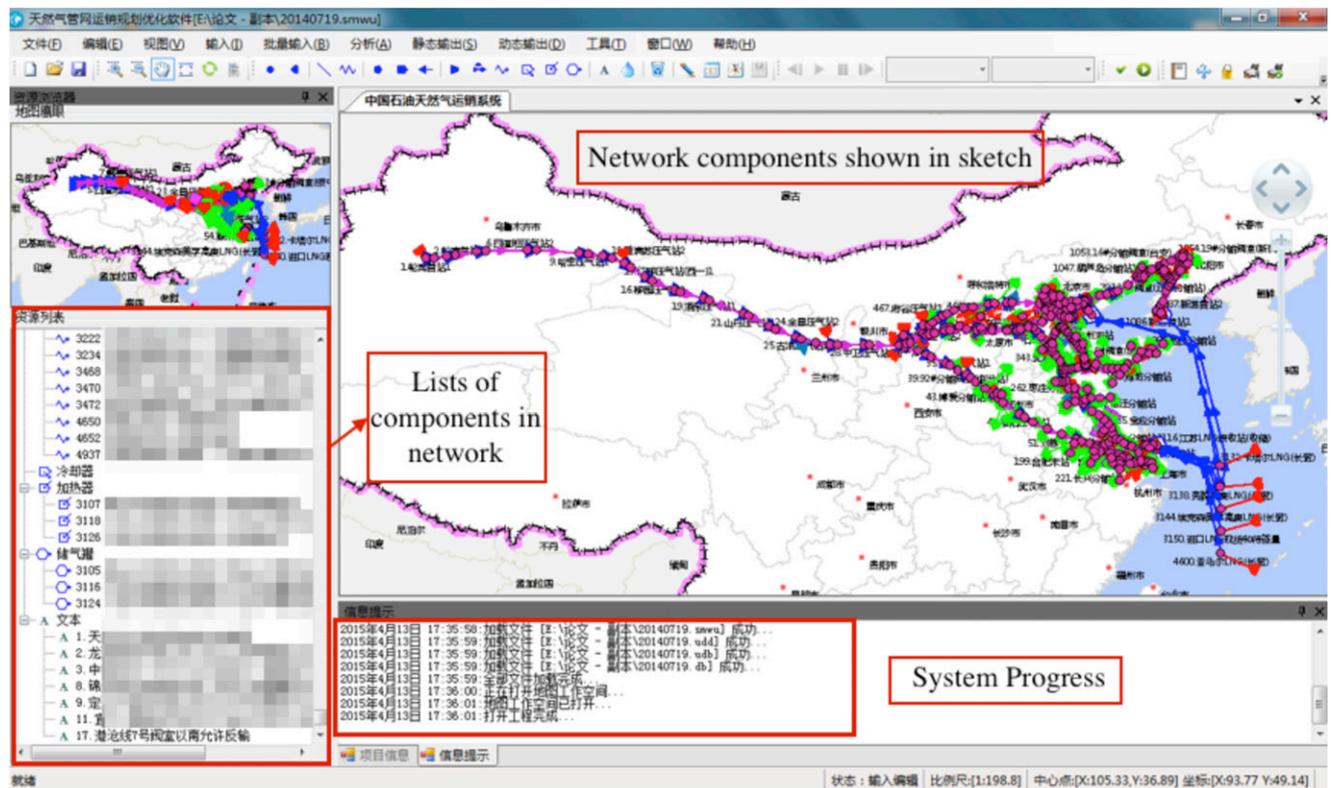


Table 3. We Use Networks from Open Data Source GasLib and CNPC for Our Numerical Tests

Network	Nodes	Arcs	Country	Cyclic	Elevation
GasLib 135	135	170	Germany	Cyclic	Uniform
GasLib 582	582	609	Germany	Cyclic	Nonuniform
CNPC	1,285	1,475	China	Cyclic	Nonuniform

approximation methods reported in the literature (De Wolf and Smeers 2000, Babonneau et al. 2012). We observed that in large-scale networks, our proposed algorithm was the only one that generated feasible solutions.

Table 3 provides summary statistics for the networks we tested. We tested German networks from the open data set GasLib (Humpola et al. 2015b) and Chinese networks provided by CNPC. The tested networks varied in size and elevation configuration. All the numerical tests were conducted using a computer with one Intel i7-5500U CPU, 8 GB RAM, and a Windows 10 operating system. We solved all the problems using C++ in Visual Studio 2010 and the IBM ILOG CPLEX 12.5 optimizer, which is the production computation environment for the software.

GasLib 135 and GasLib 582 are German networks published in the open data source. GasLib 135 contains 135 nodes, 141 pipelines, and 29 compressor stations, and GasLib 582 has 582 nodes, 278 pipelines, 5 compressor stations, 23 control valves, 8 resistors, 26 valves, and 269 short pipes. We reported the computational time, number of iterations, minimum total cost, and maximum and average relative difference of the pipeline’s optimized discharge pressure and the simulated value of the discharge pressure, where we used simulation to calculate the simulated value, given the suction pressure and flow rate. The relative difference of discharge pressure with its simulation value is an important index that engineers in CNPC use to evaluate the solution accuracy. Table 4 indicates that all three methods successfully obtained a solution in the GasLib 135 network, while only the 3SCR method obtained a feasible solution in the GasLib 582 network. We also found that 3SCR

required the least computation time, and its advantage in terms of the computation time is significant compared with the other two methods.

We tested 3SCR using the CNPC network (Figure 3), which has 1,285 nodes, 882 pipelines, 264 compressors, 185 regulator valves, and other components, such as resistors, heaters, and coolers. In Figure 3, a dot represents a client node, a demand node, or a connection node in the network, and a line represents a component, such as a pipeline, a compressor, or a regulator valve. The CNPC network represented CNPC’s complete natural gas pipeline transmission system. The iterative piecewise linear approximation approach for the MILP could not generate feasible solutions in the CNPC network after several hours of computation time.

Table 5 shows that 3SCR could obtain optimal solutions for the CNPC network within 17 seconds. Most importantly, the maximum differences in the discharge pressures were smaller than 7.5% and the average differences were smaller than 0.3%. This result is impressive for a network with more than 1,000 nodes. The maximum relative difference was slightly larger than 5%, but the results were still acceptable. Thus, we concluded that 3SCR could efficiently and effectively solve the natural gas pipeline transmission problem for large-scale networks, whereas the iterative piecewise linear method could not obtain solutions for the GasLib 582 and CNPC networks.

Impact

The benefits of our operations research effort have been substantial. The main economic impact consisted of direct revenue increases and indirect budget savings for CNPC. Furthermore, our project generated noneconomic impacts.

Direct Profit Increase

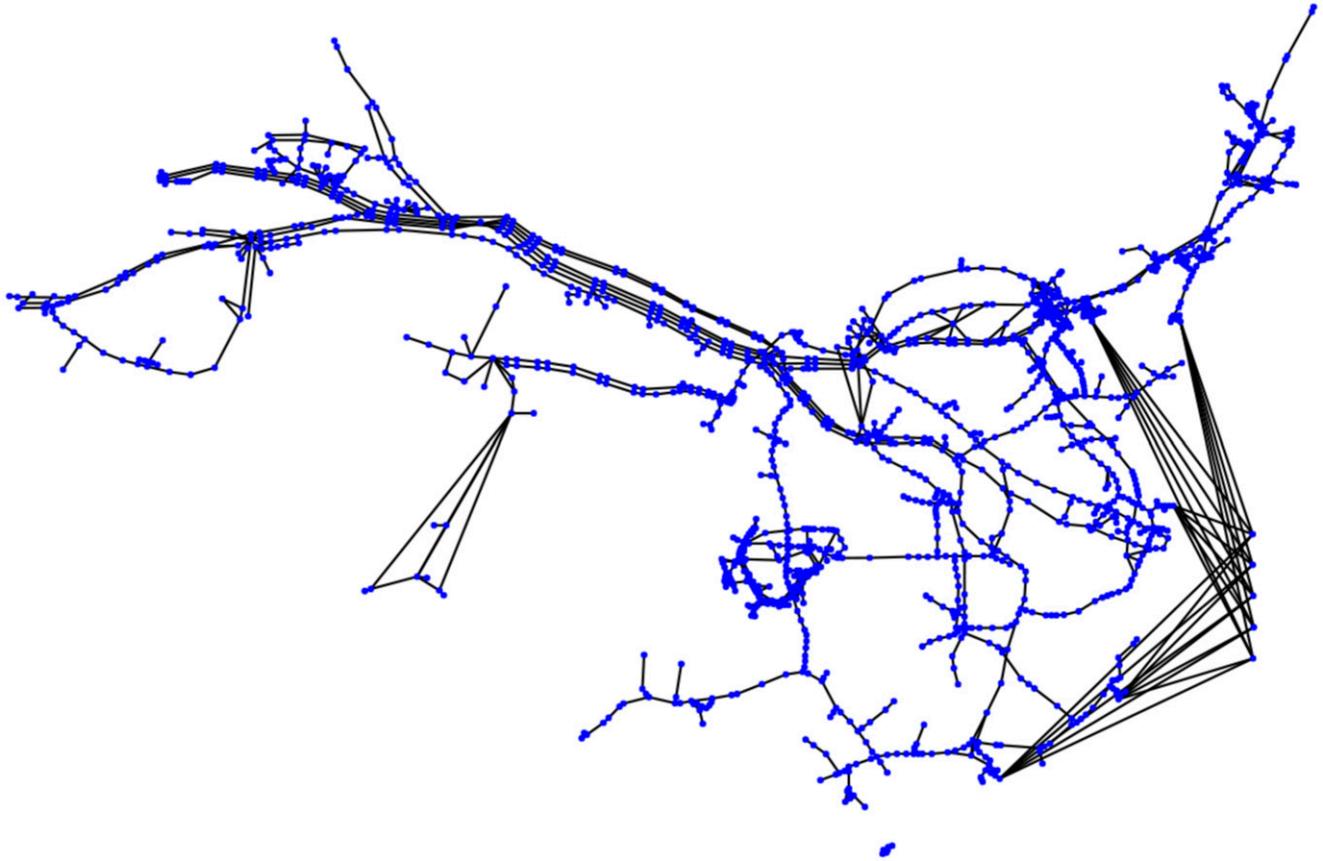
Implementation of the software generated more than \$530 million in additional profits for CNPC’s natural gas transmission in the years following the implementation (2015–2017), which we categorize into three profit components. The first comes from the increased revenue from optimizing the allocation of natural gas

Table 4. Three-Stage Convex Relaxation Is the Only Method That Can Generate Feasible Solutions Within the GasLib 582 Network

Network	Method	Time (s)	No. of iterations	Minimum cost	Max gap	Average gap
GasLib 135	IPLA-De	1109.57	40	-299.27	1979.30%	26.88%
	IPLA-Ba	90.57	40	-273.98	1676.67%	13.35%
	3SCR	2.73*	NA	-344.43*	24.17%*	11.36%*
GasLib 582	IPLA-De	NA	NA	Infeasible	NA	NA
	IPLA-Ba	NA	NA	Infeasible	NA	NA
	3SCR	4.12*	NA	-84.43*	0.50%*	0.21%*

Note. IPLA-De, piecewise linear approximation extended from De Wolf and Smeers (2000); IPLA-Ba, piecewise linear approximation extended from Babonneau et al. (2012); 3SCR, three-stage convex relaxation; * indicates the best performance in a group; NA indicates no value.

Figure 3. (Color online) We Used a Representation of CNPC’s Network with 1,285 Nodes to Test the Three-Stage Convex Relaxation Approach



sales to different regions. Figure 4 illustrates the difference in allocation of sales between the manual results and the optimization results. The decision support system provides plans that allow CNPC to sell more natural gas to customers who are willing to pay higher prices. In the 2015–2017 period, the optimized plan resulted in increased sales of \$1.07 billion cubic meters of gas over the manual method and increased profits of \$340.3 million.

The second component of profit increase is achieved by reducing the natural gas production and import costs. For example, the optimization results (Table 6) showed that in 2015–2017, the amount of imported gas from lines A and B of the Central Asia-China gas pipeline decreased by 540 million cubic meters in 2015 and 2017 and 770 million cubic meters in 2016 and 2017. Meanwhile, the amount imported from line C of the Central Asia-China pipeline increased by 450 million cubic meters in 2016 and the imported liquefied natural gas increased by 540 million cubic meters and 320 million cubic meters in 2015 and 2016, respectively. These changes resulted in a decrease of \$125.5 million in natural gas purchasing costs and a decrease of \$62.2 million in transportation costs. Thus, profits increased by \$187.7 million.

Finally, CNPC estimates that the new decision support system saved approximately \$2 million in labor costs over the previous manual approach. Furthermore, the new database system associated with the decision support system significantly increases the efficiency of employees whose jobs involve file maintenance and data maintenance.

Capital Expenditure Savings

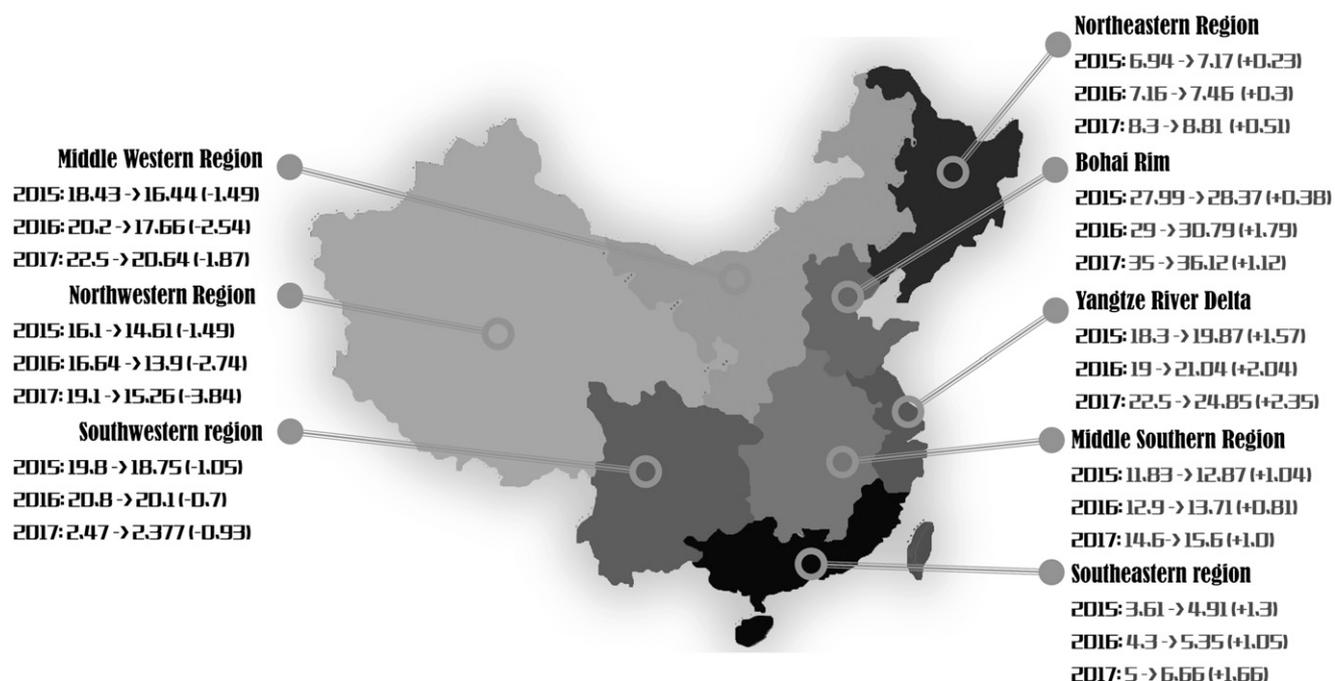
Capital expenditure savings of more than \$20 billion were made possible because the software helped

Table 5. The Three-Stage Convex Relaxation Method Obtained Good Performance Results in CNPC’s Network

Data set	Time (s)	Obj value (CNY)	Max gap	Average gap
1	15.92	−42,528,600	6.71%	0.11%
2	14.98	−31,788,800	5.80%	0.11%
3	16.53	−32,931,700	5.80%	0.12%
4	12.68	−48,223,600	5.80%	0.28%
5	13.60	−54,867,100	5.80%	0.13%
6	13.74	−61,435,600	7.49%	0.23%
7	14.42	−42,846,300	7.49%	0.11%
8	14.57	−82,902,200	7.49%	0.12%
9	15.09	−62,926,900	7.49%	0.21%

Note. Obj, objective function.

Figure 4. The Results of Our Optimization Method in Comparison with the Results of the Manual Method Suggest That CNPC Should Sell More Gas in Its Eastern Regions (Unit: Billion Cubic Meters; Manual Method → Optimal Method)



CNPC optimize the sequences of constructing several main pipelines, including line D of the Central Asia-China gas pipeline, and the third, fourth, and fifth west-to-east pipelines. Figure 5 shows the postponed pipelines. On average, the construction projects were postponed for at least five years.

The optimization tool increases the accuracy of decisions and helps CNPC make better decisions on investments. Before implementing our software, the transmission capacity of the current network was utilized inefficiently. As a result, decisions to construct new pipelines were made. The estimated demands on gas in 2016 and 2017 were 130 billion cubic meters and 151.7 billion cubic meters, respectively. CNPC’s original plan to satisfy demand used both the transmission capacity of the current pipelines plus the construction of new pipelines. As a result of our project, CNPC can now satisfy the same demand with the current pipelines. It has been able to postpone plans for constructing new pipelines, including line D of the Central Asia-China pipeline, the central and western parts of the third west-to-east pipeline, the fourth west-to-east pipeline, and

the fifth west-to-east pipeline, and remove \$20 billion from its capital expenditures budget.

Increased Work Efficiency

The successful implementation also significantly increased CNPC’s employee efficiency. For example, using the manual method required three days to finish an annual plan and two weeks to finish a monthly plan. After implementing our software, completing an annual plan required only 40 minutes and finishing the monthly plan took 10 hours. Moreover, this efficiency improvement allowed us to consider more scenarios.

Environmental Impact

The project also provided environmental benefits. Before implementing our software, CNPC could make only a conservative estimate of customer demand based on the limitations of its production and transmission capacities. Following the implementation, it was able to make more precise evaluations of the transmission capacity and found that its existing capacity, which can be up to 178 billion cubic meters, is more than sufficient

Table 6. Our Software Provides CNPC with a Plan to Optimize the Amounts and Sources of Natural Gas Imports

Gas import method	2015			2016			2017		
	Manual	Software	Difference	Manual	Software	Difference	Manual	Software	Difference
Pipeline	35.7	35.16	-0.54	39.84	39.52	-0.32	43.77	42.45	-1.32
Liquefied	1.52	2.06	+0.54	0	0.32	+0.32	16.23	17.55	+1.32

Notes. The unit of natural gas amount is billions of cubic meters. We compare the natural gas import plans obtained by manual method and software from 2015 to 2017.

to meet demands. Therefore, CNPC now has more confidence about selling natural gas to customers.

The increased natural gas sales facilitated China's coal-to-gas switch in winter 2017. In 2017, the planned amount of gas sales in Beijing was estimated to be 16.6 billion cubic meters; in contrast, planned sales in 2013 were 9.95 cubic meters. Natural gas has gradually replaced coal and become the main energy source for residential heating. The increasing usage of natural gas led to the decreasing demand on coal, which largely contributed to the improvement in air quality. For example, the planned gas sales in Beijing increased by one billion cubic meters from 2016 to 2017, while natural gas was estimated to account for over 97% of energy for residential heating in Beijing (Qi 2017). With the increased usage of natural gas, the amount of coal used decreased by 13 million tons compared with the usage in 2012 (Beijing Bureau of Statistics 2017), which contributed substantially to improving air quality in Beijing. The air quality was rated excellent or good for 32 of

the 36 days between November 15, 2017 (when Beijing turned on its heating plants for gas) and December 20, 2017. The average PM_{2.5} level (a commonly used index measuring the air pollution) was 38 micrograms per cubic meter. For the same period in the previous four years, the average PM_{2.5} level was 93 micrograms per cubic meter.

Operations Research in CNPC

In addition to the foregoing, the importance of optimized decision support has been recognized throughout the corporation, and the modeling methods and the new software system have become core techniques for reducing operational costs. Indeed, CNPC has introduced fundamental changes in natural gas pipeline activities. The software is now used at different levels within CNPC, including corporate headquarters, regional offices, and subdivisions, where large-scale natural gas transmission planning and precise evaluation of transmission capacities are no longer undertaken without the software.

Figure 5. After Implementing Our Software, CNPC Decided to Postpone the Construction of Several Main Pipelines in China



Note. The dotted lines in the figure represent the postponed pipelines.

Portability

Within CNPC, in addition to addressing natural gas dynamic planning, our approach to handling pipeline and compressor dynamics can also be applied directly to transmission fuel cost minimization and to problems of transient transportation. In other countries, where the sizes of natural gas networks are typically smaller and the topologies are simpler, the algorithm and software we developed is also applicable.

Furthermore, we believe that the innovative methods we developed in this project can be applied to continuous-process industries with sophisticated technologies, such as metal smelting, chemicals, oil refineries, and waste water treatment.

Persuading CNPC to Accept and Apply Algorithms

Initially, it was difficult for our research team to persuade CNPC to accept and apply the algorithms because of its high science and technology standards. We conducted numerical tests using networks from our previous work, the open data source (GasLib), and CNPC, and then compared the results obtained from the different algorithms. In particular, we carried out extensive numerical tests on CNPC's networks of different sizes and with different network configurations and parameter settings as we tried to test all the situations that CNPC would face.

We communicated constantly with our CNPC clients. We spent a significant amount of time understanding the requirements of CNPC's users, and comparing our results with results from CNPC's previous work. As operations researchers, we have been involved with software development and testing. We wrote 230,000 lines of code for the project, as we tested algorithms, data processing, and user interactions. Meanwhile, our team cooperated with experts from IBM to take better advantage of IBM ILOG CPLEX, which is embedded as the optimization solver in our software. We believe all the work we discuss in this paper eventually led to CNPC's acceptance and implementation of the software, helping us confirm that the models described the application work well enough to be used, and helping us judge that the software contained no significant bugs.

Timeliness

Since 2011, the natural gas industry in China has experienced rapid growth. The world's gas market is also set to boom (Paraskova 2017). Specifically, global natural gas demand is expected to grow by 1.6% annually over the next five years, with China accounting for 40% of this growth (International Energy Agency 2016). This anticipated increase in the amount of natural gas consumed and the need for larger network

pipelines forced CNPC to upgrade its natural gas pipeline network. The introduction of operations research in gas transmission planning helped CNPC handle its large-scale and complex planning problems, thereby ensuring that the company is able to complete its annual and monthly planning accurately and on time. By introducing operations research when it did, CNPC could find opportunities to postpone new pipeline construction early enough to generate billions in cost savings.

In 2017, the Chinese authorities announced a plan to reform the oil and gas industry. In the future, the role of markets will be increasingly decisive in resource allocation, which changes the government's role in energy. The increasingly dynamic and competitive natural gas markets will make revenue and transmission efficiency more significant at CNPC. The well-timed application of operations research in gas transmission planning is giving CNPC an opportunity to win the future battle for natural gas markets.

Future Plans of CNPC

After realizing the power of operations research, CNPC decided to establish a lab on natural gas supply chain optimization. One of the key missions of this lab is to apply operations research to every aspect of its natural gas supply chain, ranging from upstream wellheads to downstream end users. Meanwhile, operations research at CNPC is being broadly considered as an improvement process. With this broader view of operations research, CNPC plans to develop systems that integrate market forecasting, economic evaluation, and artificial intelligence to form a data-driven supply chain optimization platform for natural gas. Our team plans to continue its research with the lab, further expand the impact of operations research, and make operations research an integral part of CNPC's future business development.

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Appendix A. Natural Gas Pipeline Transmission Model

In this appendix, we provide a simplified model for the natural gas pipeline transmission problems at CNPC. The simplification in this description is to assume the system is isothermal (and thus has an input of the ambient temperature). In the real project, we assume the system is nonisothermal (and thus the temperature of natural gas at each node is a decision variable).

A natural gas pipeline transmission network is frequently represented by a directed graph $G(\mathcal{N}, \mathcal{A})$, where \mathcal{N} is the set of nodes and \mathcal{A} is the set of arcs. For each node $i \in \mathcal{N}$, let s_i be its

net supply, which is unrestricted in sign, where it is positive when node i is a supply node and negative when node i is a demand node. Let p_i be node i 's natural gas pressure and let π_i denote the square of p_i . For each arc $(i, j) \in \mathcal{A}$, let $f_{i,j}$ denote the natural gas volumetric flow per unit of time. We focus on a directed graph; therefore, the natural gas must flow from i to j on arc (i, j) and $f_{i,j}$ cannot be negative. By definition, the triplet (f, s, π) uniquely describes a feasible solution to the natural gas pipeline transmission problem.

Two constraints must be satisfied. First, each node's total outflow minus its total inflow must equal its net supply. Node i 's total inflow can be expressed as $\sum_{j:(j,i) \in \mathcal{A}} f_{j,i}$. Similarly, node i 's total outflow is $\sum_{j:(i,j) \in \mathcal{A}} f_{i,j}$. Therefore,

$$\sum_{j:(i,j) \in \mathcal{A}} f_{i,j} - \sum_{j:(j,i) \in \mathcal{A}} f_{j,i} = s_i, \forall i \in \mathcal{I}. \quad (\text{A.1})$$

Equation (A.1) denotes "flow-balance." Second, each node's net supply and pressure should be within a range. Let \underline{s}_i and \bar{s}_i denote node i 's minimum and maximum net supply, respectively. Further, let \underline{p}_i and \bar{p}_i be node i 's minimum and maximum gas pressure, respectively. Then, we require

$$\underline{s}_i \leq s_i \leq \bar{s}_i, \quad \forall i \in \mathcal{I}; \quad (\text{A.2})$$

$$(\underline{p}_i)^2 \leq \pi_i \leq (\bar{p}_i)^2, \quad \forall i \in \mathcal{I}. \quad (\text{A.3})$$

Equations (A.2) and (A.3) denote "flow-range" and "pressure-range," respectively. For example, we can impose $\underline{s}_i = \bar{s}_i = 0$ for a node that is not a gas source or a gas client.

Third, the gas pressures at the two ends of an arc must satisfy certain physical laws. In our graph representation, an arc (i, j) can represent a pipeline, a compressor, or a regulator valve. Let \mathcal{A}_p denote the set of arcs representing pipelines. Similarly, let \mathcal{A}_c and \mathcal{A}_r denote the sets of arcs representing compressors and regulator valves, respectively. For each regular pipeline $(i, j) \in \mathcal{A}_p$, we define $\beta_{i,j}$ as its resistance coefficient, which depends on the pipeline length, pipeline diameter, pipeline ambient temperature, and many other physical parameters. We assume that given a pipeline $(i, j) \in \mathcal{A}_p$, the suction pressure is not smaller than the discharge pressure (i.e., $\pi_i \geq \pi_j$) and the resistance coefficient $\beta_{i,j}$ is positive. As described in Xue et al. (2016), if we consider the effects of nonuniform network elevation, given a pipeline (i, j) with resistance coefficient $\beta_{i,j}$, the flow rate $f_{i,j}$ and gas pressure squares π_i and π_j satisfy

$$\beta_{i,j} f_{i,j}^2 = \pi_i - \alpha_{i,j} \pi_j, \quad \forall (i, j) \in \mathcal{A}_p. \quad (\text{A.4})$$

In Constraint (A.4), $\alpha_{i,j}$ is defined as the elevation coefficient for pipeline $(i, j) \in \mathcal{A}_p$, which considers the effect of nonuniform network elevation on a pipeline's flow-pressure relationship. Pipeline Constraint (A.4) can consider the variation in elevation for pipeline (i, j) because we introduce the elevation coefficient $\alpha_{i,j}$.

We prefer to employ π_i instead of p_i in Constraint (A.4) because the optimization problem is linear with respect to π_i but nonlinear in p_i . Unlike the linear Constraints (A.1)–(A.3), the nonlinear and nonconvex flow-pressure relationship in Constraint (A.4) represents significant challenges to the natural gas pipeline transmission problem.

In natural gas pipeline transmission networks, compressors are used to increase gas pressure and enhance the transmission capacity. By contrast, regulator valves are used to reduce the gas pressure. For a compressor $(i, j) \in \mathcal{A}_c$, it is often assumed that

$$\pi_i \leq \pi_j, \quad \forall (i, j) \in \mathcal{A}_c, \quad (\text{A.5})$$

because a compressor increases the gas pressure. Furthermore, we do not consider the nonlinear relationship between the flow and the pressure of compressors in the current model. By contrast, for a regulator valve $(i, j) \in \mathcal{A}_r$,

$$\pi_i \geq \pi_j, \quad \forall (i, j) \in \mathcal{A}_r, \quad (\text{A.6})$$

because a regulator valve reduces the gas pressure.

We consider the natural gas pipeline transmission network from a central planner's perspective. The planner decides how much gas to purchase or produce at each location. In the graph representation, node i can represent a supply node, demand node, or connection node. Let \mathcal{I}_s denote the set of supply nodes. Similarly, let \mathcal{I}_d denote the set of demand nodes. The cost of purchasing and selling natural gas is modeled as a linear function of the gas supplies. Let c_i^s be the unit gas procurement price at node $i \in \mathcal{I}_s$, which is the cost of producing natural gas or purchasing natural gas from foreign gas suppliers. The total purchasing cost is $\sum_{i \in \mathcal{I}_s} c_i^s \cdot s_i$ (see De Wolf and Smeers 2000). Then, the planner decides how much gas to sell at each location. Let c_i^d be the unit gas selling price at demand node $i \in \mathcal{I}_d$. The total revenue is $\sum_{i \in \mathcal{I}_d} c_i^d \cdot (-s_i)$. Therefore, the central planner would like to optimize

$$\min \left\{ \sum_{i \in \mathcal{I}_s} c_i^s \cdot s_i - \sum_{i \in \mathcal{I}_d} c_i^d \cdot (-s_i) \right\}.$$

In summary, we have considered all the constraints on natural gas pipeline transmission that are mainly studied in the field of operations research. The objective function and Constraints (A.1)–(A.3), (A.5), and (A.6) have all been studied extensively (e.g., see the models in De Wolf and Smeers 1996, 2000; Rømo et al. 2009; and Babonneau et al. 2012). In general, due to the existence of Constraint (A.4), the problem of natural gas pipeline transmission is nonconvex. Therefore, an efficient solution approach is needed for large-scale pipeline transmission networks. We summarize the objective function and Constraints (A.1)–(A.6) in one model as (A.7), which is the basic model that we aim to solve:

$$\begin{aligned} \min \quad & \left\{ \sum_{i \in \mathcal{I}_s} c_i^s \cdot s_i - \sum_{i \in \mathcal{I}_d} c_i^d \cdot (-s_i) \right\} \\ \text{s.t.} \quad & \sum_{j:(i,j) \in \mathcal{A}} f_{i,j} = \sum_{j:(j,i) \in \mathcal{A}} f_{j,i} + s_i, \quad \forall i \in \mathcal{I}; \\ & \beta_{i,j} f_{i,j}^2 = \pi_i - \alpha_{i,j} \pi_j, \quad \forall (i, j) \in \mathcal{A}_p; \\ & \pi_i \leq \pi_j, \quad \forall (i, j) \in \mathcal{A}_c; \\ & \pi_i \geq \pi_j, \quad \forall (i, j) \in \mathcal{A}_r; \\ & \underline{s}_i \leq s_i \leq \bar{s}_i, \quad \forall i \in \mathcal{I}; \\ & (\underline{p}_i)^2 \leq \pi_i \leq (\bar{p}_i)^2, \quad \forall i \in \mathcal{I}; \\ & f_{i,j} \geq 0, \quad \forall (i, j) \in \mathcal{A}. \end{aligned} \quad (\text{A.7})$$

Appendix B. Three-Stage Convex Relaxation Approach

In Appendix A, we introduce the basic model we built for CNPC's natural gas pipeline transmission problem, which is nonlinear and nonconvex. In the literature, the piecewise linear approximation methods (see De Wolf and Smeers 2000 and Babonneau et al. 2012) are used to linearize and approximate the original problem. However, in practice, the piecewise linear approximation methods are unable to meet requirements on computation time for large-scale pipeline networks. Thus, in this section, we propose a convex relaxation-based solution approach, which we show is significantly more efficient computationally. In particular, the proposed approach comprises three stages: we relax Model (A.7) to a convex problem and solve the relaxed problem in the first stage; if the solution to the original problem is feasible, we end the algorithm; otherwise, we proceed to the second and third stages to address the feasibility issue by further solving a modified energy minimization problem and a pressure feasibility problem, respectively.

First Stage: Relaxing the Flow-Pressure Equation

In Model (A.7), Constraint (A.4) is the key nonlinear constraint. In the first stage, we relax this constraint in the following manner. The flow directions are fixed; therefore, we handle the flow direction and nonlinear constraint separately. For any pipeline $(i, j) \in \mathcal{A}_p$, Constraint (A.4) and the node pressure bounds together imply that

$$\begin{cases} \beta_{i,j} \cdot f_{i,j}^2 = \pi_i - \alpha_{i,j} \pi_j \\ \pi_i \leq (\bar{p}_i)^2, \pi_j \geq (\underline{p}_j)^2 \end{cases} \Rightarrow \beta_{i,j} f_{i,j}^2 \leq (\bar{p}_i)^2 - \alpha_{i,j} (\underline{p}_j)^2. \quad (\text{B.1})$$

Constraint (B.1) can be further generalized to a series of connected pipelines. We illustrate this idea using the network in Figure B.1, where three pipelines (1, 2), (2, 3), and (3, 6) form a tandem line. We have

$$\begin{aligned} & \beta_{1,2} f_{1,2}^2 + \alpha_{1,2} \beta_{2,3} f_{2,3}^2 + \alpha_{1,2} \alpha_{2,3} \beta_{3,6} f_{3,6}^2 \\ &= \pi_1 - \alpha_{1,2} \pi_2 + \alpha_{1,2} (\pi_2 - \alpha_{2,3} \pi_3) + \alpha_{1,2} \alpha_{2,3} (\pi_3 - \alpha_{3,6} \pi_6) \\ &= \pi_1 - \alpha_{1,2} \alpha_{2,3} \alpha_{3,6} \cdot \pi_6 \\ &\leq (\bar{p}_1)^2 - \alpha_{1,2} \alpha_{2,3} \alpha_{3,6} (\underline{p}_6)^2. \end{aligned} \quad (\text{B.2})$$

We provide several remarks on Constraints (B.1) and (B.2). First, Constraint (B.2) is stricter than Constraint (B.1). For example, we assume that the network in Figure B.1 has uniform elevation (i.e., $\alpha_{i,j} = 1$), and that all nodes have identical pressure bounds \underline{p} and \bar{p} . Then, Constraint (B.1) implies that

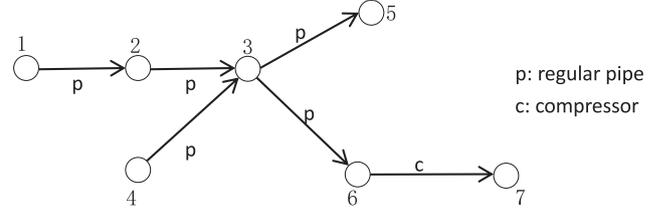
$$\beta_{1,2} f_{1,2}^2 \leq (\bar{p})^2 - (\underline{p})^2,$$

whereas Constraint (B.2) implies

$$\beta_{1,2} f_{1,2}^2 + \beta_{2,3} f_{2,3}^2 + \beta_{3,6} f_{3,6}^2 \leq (\bar{p})^2 - (\underline{p})^2.$$

Apparently, Constraint (B.2) is much tighter than Constraint (B.1). Second, Constraint (B.2) provides an upper bound on the amount of natural gas that can be transported via pipelines (1, 2), (2, 3), and (3, 6). The goal is often to transport as much natural gas as possible, and thus Constraint (B.2) is

Figure B.1. We Illustrate the Convex Relaxation Method Using an Example of a Network with Pipelines and a Compressor



likely to hold at equality at optimal solution. If Constraint (B.2) is binding the optimal solution, then using Constraint (B.2) instead of Constraint (B.1) does not compromise solution accuracy. Third, both Constraints (B.1) and (B.2) are convex constraints, and the relaxed problem is a convex optimization problem.

This idea is simple because it replaces a nonconvex equality constraint with a relaxed convex inequality. However, the procedure for constructing the convex relaxed model is not straightforward because we need to enumerate all the pipeline-only paths that link a supply node and a demand node. We introduce some new notation and describe the procedure as follows.

We construct a collection of M vectors, which are sets of pipelines in the natural gas pipeline transmission network. Let $Y(m)$ and $n(m)$ denote the m th vector and its length, respectively, where $Y(m)$ is the m th set of pipelines. For $1 \leq m \leq M$ and $1 \leq i \leq n(m)$, we denote $Y(m, i)$ as the i th element of vector $Y(m)$, which means that $Y(m, i)$ is the i th element in $Y(m)$. The following algorithm outlines the construction of M , $Y(m)$, and $n(m)$ for each $1 \leq m \leq M$.

Algorithm B.1 (Enumerating all the pipeline-only paths)

1. **(Initialization)**
 - a. Build an empty stack s . Set $m = 0$.
 - b. Push each $i \in \mathcal{I}_s$ into s .
 - c. For each $(i, j) \in \mathcal{A}_c \cup \mathcal{A}_r$, push the end node j into s . Go to Step 2.
2. **(Termination)** Check if $|s| = 0$.
 - a. If true, set $M := m$, terminate.
 - b. Else, go to Step 3.
3. **(Iteration)**
 - a. Pop one vector Y' out of s , where i is the last element of Y' .
 - b. Search $j \in \mathcal{I}$, for j that satisfy the following conditions: $(i, j) \in \mathcal{A}_p$ and $(i, j) \notin Y'$.
 - i. Search j , check if $j \in Y'$.
 1. If true. Set $m := m + 1$, $Y(m) = Y'$, $n(m) := |Y'|$. Go to b in Step 3.
 2. Else. Push (Y', j) back to s . Go to b in Step 3.
 - ii. Else. Set $m := m + 1$, $Y(m) = Y'$, $n(m) := |Y'|$. Go to Step 2.

We illustrate the construction of $Y(m)$ using the network in Figure B.1. In the initialization stage, we build an empty stack s and set $m = 0$. Nodes 1 and 4 are source nodes; therefore, we push them into stack s (i.e., $s = \{(1), (4)\}$). Furthermore, (6, 7) is the only compressor; therefore, we add Node 7 to s . In the termination stage, s is not empty and we proceed to the

positive parameters that satisfy $r_j = r_i \cdot \alpha_{i,j}, \forall (i,j) \in \mathcal{A}_p$. We solve the following Problem (B.6):

$$\begin{aligned} \min \quad & \sum_{(i,j) \in \mathcal{A}_p} \frac{\beta_{i,j}}{3} r_i f_{i,j}^3 \\ \text{s.t.} \quad & \sum_{j:(i,j) \in \mathcal{A}} f_{i,j} = \sum_{j:(j,i) \in \mathcal{A}} f_{j,i} + s_i, \quad \forall i \in \mathcal{F}; \\ & f_{i,j} \geq 0, \quad \forall (i,j) \in \mathcal{A}; \\ & \underline{s}_i \leq s_i \leq \bar{s}_i, \quad \forall i \in \mathcal{F}; \\ & \sum_{i \in \mathcal{F}_s} c_i^s \cdot s_i - \sum_{i \in \mathcal{F}_d} c_i^d \cdot (-s_i) \leq z^* \cdot (1 + \gamma \cdot \text{sign}(z^*)). \end{aligned} \quad (\text{B.6})$$

By solving Problem (B.6), we obtain the optimal gas flow rates $f_{i,j}^{2*}, \forall (i,j) \in \mathcal{A}$ and the optimal gas supplies at each node $s_i^{2*}, \forall i \in \mathcal{F}$. In the third stage, we find the corresponding optimal gas pressures.

Third Stage: Solving for Gas Pressures

In the third stage, we calculate the natural gas pressure at each location given the gas flow rates $f_{i,j}^{2*}$ and gas supplies s_i^{2*} . The problem of optimizing the gas pressures is formulated as Problem (B.7). In this problem, the gas pressures need to satisfy Constraints (A.4)–(A.6), and Constraint (A.3). We set the objective function as $\max \sum_{i \in \mathcal{F}} \pi_i$ because it is often preferable to transport natural gas in high-pressure environments in practice.

$$\begin{aligned} \max \quad & \sum_{i \in \mathcal{F}} \pi_i \\ \text{s.t.} \quad & \pi_i - \alpha_{i,j} \pi_j = \beta_{i,j} (f_{i,j}^{2*})^2, \quad \forall (i,j) \in \mathcal{A}_p; \\ & \pi_i \leq \pi_j, \quad \forall (i,j) \in \mathcal{A}_c; \\ & \pi_i \geq \pi_j, \quad \forall (i,j) \in \mathcal{A}_r; \\ & (\underline{p}_i)^2 \leq \pi_i \leq (\bar{p}_i)^2, \quad \forall i \in \mathcal{F}. \end{aligned} \quad (\text{B.7})$$

The first equation in Problem (B.7) is exactly Constraint (A.4). The second constraint is valid for compressors. The third is valid for regulator valves. The fourth constraint gives the pressure bound on each node. Let π_i^{3*} be the optimal solution to the square of the gas pressure at node i . If Model (B.6) has an optimal solution, $(f_{i,j}^{2*}, s_i^{2*}, \pi_i^{3*})$ comprises a feasible solution to Problem (A.7). Problem (B.7) is a linear programming problem with decision variable π_i , so it can be solved in a straightforward manner.

Appendix C. Proof of Theorem

Theorem 1. Under Assumptions B.1–B.3 Problem (S1-R) has the same optimal value as Problem (S0).

Proof of Theorem 1. Recall that Problem (B.4) is a relaxation problem of Problem (A.7). If we are able to show that the optimal solution to Problem (B.4) is feasible to Problem (A.7), then Problems (B.4) and (A.7) must have the same optimal values.

Let $f_{i,j}^*$ and s_i^* be the optimal solutions of Problem (B.4). Under Assumption B.1, each node has the same pressure bounds. For each node $i \in \mathcal{F}$, set

$$\begin{aligned} \pi_i^* &= (\bar{p})^2 / \prod_{l=1}^{i'-1} \alpha_{Y(m',l), Y(m',l+1)} \\ &- \sum_{k=1}^{i'-1} \beta_{Y(m',k), Y(m',k+1)} (f_{Y(m',k), Y(m',k+1)}^*)^2 / \prod_{l=k}^{i'-1} \alpha_{Y(m',l), Y(m',l+1)} \end{aligned}$$

for m' and i' that satisfy $i = Y(m', i')$. Equivalently speaking, node i is at the i' th position of vector $Y(m')$. For this definition to be valid, we have to verify two results. First, there are m' and i' that satisfy $i = Y(m', i')$. Second, π_i is constant for all m' and i' that satisfy $i = Y(m', i')$.

The first result follows from the construction of $Y(m)$. In Algorithm B.1, the construction of $Y(m)$ is such that $Y(m)$, $1 \leq m \leq M$ contains all nodes $i \in \mathcal{F}$. Therefore, m' and i' must exist such that $i = Y(m', i')$.

The second result holds as well. If node i is a root, the result holds trivially as $i' = 1$ and $\pi_i^* = (\bar{p})^2$. If not, then node i has a predecessor. Suppose there exist $m_1, m_2, i_1 > 1$, and $i_2 > 1$, such that

$$Y(m_1, i_1) = Y(m_2, i_2) = i,$$

but

$$\pi_{Y(m_1, i_1)} \neq \pi_{Y(m_2, i_2)}.$$

Because the network is a distribution network, each node has one predecessor at most. If node i is not a root, then both $Y(m_1, i_1 - 1)$ and $Y(m_2, i_2 - 1)$ must be node i 's only predecessor, which means

$$Y(m_1, i_1 - 1) = Y(m_2, i_2 - 1).$$

Continually applying this logic, we infer that

$$Y(m_1, j) = Y(m_2, j)$$

for all $1 \leq j \leq i_1$ and $i_1 = i_2$. Then

$$\begin{aligned} \pi_{Y(m_1, i_1)} &= (\bar{p})^2 / \prod_{l=1}^{i_1-1} \alpha_{Y(m_1, l), Y(m_1, l+1)} \\ &- \sum_{k=1}^{i_1-1} \beta_{Y(m_1, k), Y(m_1, k+1)} (f_{Y(m_1, k), Y(m_1, k+1)}^*)^2 / \\ &\quad \prod_{l=k}^{i_1-1} \alpha_{Y(m_1, l), Y(m_1, l+1)} \\ &= (\bar{p})^2 / \prod_{l=1}^{i_2-1} \alpha_{Y(m_2, l), Y(m_2, l+1)} \\ &- \sum_{k=1}^{i_2-1} \beta_{Y(m_2, k), Y(m_2, k+1)} (f_{Y(m_2, k), Y(m_2, k+1)}^*)^2 / \\ &\quad \prod_{l=k}^{i_2-1} \alpha_{Y(m_2, l), Y(m_2, l+1)} \\ &= \pi_{Y(m_2, i_2)}, \end{aligned}$$

which contradicts the assumption that $\pi_{Y(m_1, i_1)} \neq \pi_{Y(m_2, i_2)}$.

Finally, we argue that $f_{i,j}^*, s_i^*$, and π_i^* constitute a feasible solution to Problem (A.7). Apparently, this optimal solution satisfies the constraints in Problem (B.4) because an optimal solution must be feasible. We have to prove that the constructed solution satisfies the Constraints (A.4) and (A.3) that are in Problem (A.7) but not in Problem (B.4):

$$\begin{aligned} \beta_{i,j} f_{i,j}^2 &= \pi_i - \alpha_{i,j} \pi_j, \quad \forall (i,j) \in \mathcal{A}_p; \\ (\underline{p})^2 &\leq \pi_i \leq (\bar{p})^2, \quad \forall (i,j) \in \mathcal{A}. \end{aligned}$$

First, we prove that $f_{i,j}^*, s_i^*$, and π_i^* satisfy Constraint (A.4). For pipeline $(i,j) \in \mathcal{A}_p$, m' must exist such that $i, j \in Y(m')$ and node i is the only predecessor of node j , and i' must exist

such that $i = Y(m', i')$ and $j = Y(m', i' + 1)$. By definition of π_i^* and π_j^* , we have

$$\begin{aligned}\pi_i^* &= (\bar{p})^2 / \prod_{l=1}^{i'-1} \alpha_{Y(m', l), Y(m', l+1)} \\ &\quad - \sum_{k=1}^{i'-1} \beta_{Y(m', k), Y(m', k+1)} (f_{Y(m', k), Y(m', k+1)}^*)^2 / \\ &\quad \prod_{l=k}^{i'-1} \alpha_{Y(m', l), Y(m', l+1)}, \\ \pi_j^* &= (\bar{p})^2 / \prod_{l=1}^{i'} \alpha_{Y(m', l), Y(m', l+1)} \\ &\quad - \sum_{k=1}^{i'} \beta_{Y(m', k), Y(m', k+1)} (f_{Y(m', k), Y(m', k+1)}^*)^2 / \\ &\quad \prod_{l=k}^{i'} \alpha_{Y(m', l), Y(m', l+1)}.\end{aligned}$$

Thus, we can have

$$\begin{aligned}\pi_i^* - \alpha_{ij} \pi_j^* &= \pi_{Y(m', i')}^* - \alpha_{Y(m', i'), Y(m', i'+1)} \pi_{Y(m', i'+1)}^* \\ &= (\bar{p})^2 / \prod_{l=1}^{i'-1} \alpha_{Y(m', l), Y(m', l+1)} \\ &\quad - \sum_{k=1}^{i'-1} \beta_{Y(m', k), Y(m', k+1)} (f_{Y(m', k), Y(m', k+1)}^*)^2 / \\ &\quad \prod_{l=k}^{i'-1} \alpha_{Y(m', l), Y(m', l+1)} - (\bar{p})^2 / \prod_{l=1}^{i'-1} \alpha_{Y(m', l), Y(m', l+1)} \\ &\quad + \sum_{k=1}^{i'-1} \beta_{Y(m', k), Y(m', k+1)} (f_{Y(m', k), Y(m', k+1)}^*)^2 / \\ &\quad \prod_{l=k}^{i'-1} \alpha_{Y(m', l), Y(m', l+1)} \\ &\quad + \beta_{Y(m', i'), Y(m', i'+1)} (f_{Y(m', i'), Y(m', i'+1)}^*)^2 \\ &= \beta_{Y(m', i'), Y(m', i'+1)} (f_{Y(m', i'), Y(m', i'+1)}^*)^2 = \beta_{ij} \cdot (f_{ij}^*)^2.\end{aligned}$$

Next, we have to prove that π_i^* satisfies Constraint (A.3). Given m that satisfies $1 \leq m \leq M$, set $i_0 = Y(m, 1)$. Under Assumption B.2, for any $j = Y(m, j')$ where $2 \leq j' \leq n(m)$, we have

$$\pi_j^* \leq \pi_{i_0}^* = (\bar{p})^2.$$

Because $\pi_{i_0}^*$ and f_{ij}^* satisfy constraints in Problem (B.4),

$$\begin{aligned}\pi_{i_0}^* - \pi_j^* &\cdot \prod_{l=1}^{j'-1} \alpha_{Y(m, l), Y(m, l+1)} = \sum_{k=1}^{j'-1} \beta_{Y(m, k), Y(m, k+1)} (f_{Y(m, k), Y(m, k+1)}^*)^2 \\ &\quad \cdot \prod_{l=1}^{k-1} \alpha_{Y(m, l), Y(m, l+1)} \\ &\leq (\bar{p})^2 - (p)^2 \cdot \prod_{l=1}^{j'-1} \alpha_{Y(m, l), Y(m, l+1)}.\end{aligned}$$

Because $\pi_{i_0}^* = (\bar{p})^2$, we have

$$\pi_j^* \geq (p)^2.$$

Thus, $(p)^2 \leq \pi_i^* \leq (\bar{p})^2$ holds for any $i \in \mathcal{I}$. Because Problem (B.4) is a relaxation of Problem (A.7), if the optimal solution to

Problem (B.4) satisfies constraints in Problem (A.7), Problems (A.7) and (B.4) must have the same optimal value. \square

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