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Medium-Term Production Scheduling of the Lumwana Mining Complex

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In this paper, we discuss our development of a life-of-mine production plan for Barrick Gold Corporation's Lumwana operation, a large copper mining complex based on the Chimiwungo and Malundwe reserves. In our production plan, we maximize recovered copper metal based on a mixed-integer program (MIP) formulation with reserve aggregations that approximate those used in operational mine planning. We discuss the application of a MIP to medium-term planning based on a 60-month production schedule. Constraints on shovel placement, uranium levels in mill feed, stockpiling and mining, and processing capacity ensure that the resulting production schedules and resource allocation are operationally feasible. At Lumwana, our MIP solution strategies optimize the scheduling problem at two levels of time and production-volume granularity. A coarse solution based on annual production from aggregated reserve blocks sets the overall production strategy. This strategy is then imposed on a schedule of monthly production for the scheduling volumes used in actual production planning. This problem was not solvable as a single multiperiod monolith. Instead, we solved a sequence of overlapping multiperiod problems in which each subproblem advances the schedule horizon a given number of periods while fixing the solution to the initial periods of the previous subproblem. We solve multiple options relating to production capacity with a life-of-business optimization system (LOBOS) that we developed.

Keywords: strategic planning; OR; natural resources; open pit mining; surface mining; optimization; integer programming; applications.
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The Lumwana mine is located in Zambia, 220 kilometers (km) west of the Copperbelt and 65 km west of Solwezi, the provincial capital. Commissioned in December 2008 (Mining Technology 2012), it is now Africa's largest copper mine. Barrick Gold Corporation purchased the Lumwana reserve (Lumwana) from Equinox Minerals on April 25, 2011. The reserve has 5.97 billion pounds of copper and 7.8 billion pounds of inferred (i.e., probable) copper, giving it a 37-year mine life (Barrick 2012). Lumwana is the largest capital investment in Zambia's history; at full capacity, Zambia expects it to produce 20 percent of the total national copper output (Zambia Advisor 2012).

In June 2011, Barrick's engineers were coming to grips with this new acquisition. Barrick contacted the authors at MineSmith (www.minesmith.com.au) to ask them to develop production plans for maximizing copper metal production over planning horizons of 20 and 60 months. MineSmith optimized Lumwana's production schedule to maximize copper metal production, and continued in this role into 2013. Lumwana management had previously collaborated with the authors on a number of mine optimization projects at OTML's Ok Tedi mine in Papua New Guinea (Smith et al. 2007) and were familiar with the level of detail and quality of plans they could obtain using MineSmith's line-of-business optimization system (LOBOS). LOBOS is a platform for generating and solving linear program (LP) and MIP representations of mine planning problems. It includes utilities that expedite importing and converting mine designs and resource models into schedule-ready reserve units that have the material classifications and attributes associated with production scheduling. LOBOS supports the inclusion of operation-specific variables and constraints in a graphical environment. Models are generated, passed to an LP-MIP solver and, once solved, can be viewed in the LOBOS user interface as three-dimensional (3D) schedule animations and various plots and graphs of production and

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financial outcomes. Solutions are exported to Excel for in-depth analysis. LOBOS includes a number of solution strategies and heuristics for solving large-scale planning problems.

The resulting schedule of mining and processing activity and resulting metal production provided Barrick with a means of estimating the value of the Lumwana project based on an optimized mine plan. LOBOS provided the sequence of extraction of reserves from multiple open pits, rates of depletion of the reserves, and likely sequence of reserve extraction both within and between pits—across numerous production options associated with alternative production fleet sizes and allocations. Previous to this study, Lumwana engineers implemented production scheduling in Excel using assumed sequences and rates of reserve extraction without consideration of complications associated with blending, stockpile usage, or contaminants in concentrate. Lumwana management recognized that the accuracy of the previous Excel-based work was questionable in terms of the granularity and feasibility of the resulting production schedule. Lumwana requires monthly schedules of sufficient level of detail for medium-term production planning. These schedules account for a variety of constraints encompassing two major deposits, five ore classifications, blending and production constraints. In contrast to the limitations of Excel, MineSmith's schedules, formulated in LOBOS as MIPs, provide Lumwana mine planners a level of detail and inclusion of operational constraints not possible using a spreadsheet approach.

Lumwana includes significant uranium resources. Therefore, we include uranium oxide production in the optimization model as a constraint on the maximum uranium level in the copper concentrate; beyond this maximum level, smelting the concentrates is unsafe. Copper ore, which is high in uranium, is either stockpiled or blended at the mill to avoid uranium concentrate penalties by the smelter, thus keeping uranium below the maximum allowable concentration in the copper concentrate.

Lumwana's operations encompass two major mineral resources: Malundwe and Chimiwungo. Barrick engineers provided pit designs and resource models for each reserve; their designs provided input for our mine plan optimization. A pit is generally considered as a single, continuous surface excavation. At Lumwana, pits are excavated in benches (i.e., horizontal intervals of 10–12 meters (m) in height). For simple pit geometries having spatial homogeneous concentrations of mineral, benches can extend continuously across the pit. This is common for quarries, which are distinguished from metal mines in that the rock itself is the final product, not the contained mineral, which is processed to recover metal. However, most pit geometries are more complex and mineralization is heterogeneous; within the pit, concentrations of mineralized rock are classified as either waste or high-, medium-, or low-grade ore. Therefore, pits are mined not only in benches but in stages, where a stage is a vertically contiguous interval of benches that are expected to be depleted as a group. Each bench is divided into smaller volumes or mining blocks, which roughly correspond to a zone of production over a span of time in which a single shovel could dig the contained volume. We schedule mining blocks, rather than the entire bench. At the finest level of detail are the resource blocks. Also referred to as a block model, the resource model provides an estimate of the mineral contained in a 3D matrix of rectangular volumes called blocks. We do not schedule blocks because the mining process is not sufficiently selective to work at this level. The Maludwe reserve consists of seven stages (see Figure 1). The Chimiwungo reserve includes three pits, each with a concentric configuration of expanding pit stages (for those unfamiliar with mining terminology, see the *Mining Block Definition* section).

Applying Mixed-Integer Programs to Mine Scheduling

The first author's experience with applying MIPs to active mining operations started in the 1990s (Smith and Tao 1994). At that time, few academics or mining companies believed that a MIP could solve complex mine planning problems. Although this opinion persists, our experiences (this paper discusses one example of many) have convinced us that MIP technology has undergone tremendous advances in recent years; used intelligently, it can be valuable in very large-scale mining applications. Bixby and Rothberg (2007)

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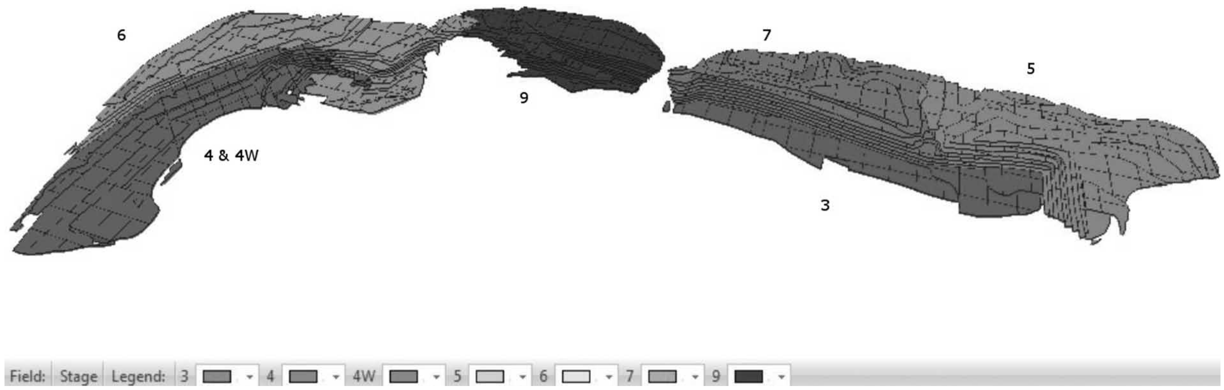

Figure 1: The seven production stages of the Malundwe deposit use mining blocks, rather than resource model blocks. Mining blocks approximate the volume required for shovel operations during a production period (here, one month). Malundwe, the focus of medium-term (20-month) production, is one of three major deposits planned for production at Lumwana.

report solution time improvements for difficult MIPs; as the CPLEX versions have changed (e.g., CPLEX 5 in 1998 to CPLEX 9 in 2003), solution times have improved between 16- and 344-fold. These improvements in algorithmic development, affordable computing power, and formulations have made possible the work we describe herein. For example, February 2013 production scenarios for Lumwana resulted in MIP models with 923,628 rows, 357,678 columns, and 3,708 binary variables. CPLEX's presolve, in combination with LOBOS modeling and solution utilities, resulted in significant problem-size reductions to 48,834 rows, 30,875 columns, and 2,512 binary variables. A Lumwana MIP problem of this size solves to a 1.26 percent tolerance of optimality in 720 seconds using a first-generation Intel Core i7 Processor.

Mine plan optimization is a recurrent theme in the literature (Newman et al. 2010). Topics in mine planning include scheduling ore and waste production from benches and stages, sequencing and scheduling underground mine production, managing production resources (e.g., major equipment) to generate a production schedule, and optimizing strategic decisions (e.g., mining and milling capacity or cutoff grade, which we define as the measure of value in the mineralization used to separate ore from waste). Since the first application of graph theory to finding the ultimate pit extent (Lerchs and Grossman 1965), research has focused on pit optimization. The number of reported applications of MIPs to mine planning is relatively limited. These applications generally address scheduling; for example, Graham-Taylor (1992) discusses the application of a MIP to open-cut production scheduling in single-production periods with binary variables, which relate to shovel activity at the production face. Applications to underground production scheduling include stoping (i.e., underground production excavation) and backfilling the resulting voids with cemented waste fill at Mount Isa Mines in Queensland (Trout 1995), and strategic planning at Mount Isa, a large multimine, multiproduct mining and processing complex (Smith et al. 2003). Other frequently cited applications include scheduling of sublevel cave drill and blast rings at Kiruna in northern Sweden (Kuchta et al. 2004) and underground production at the Stillwater complex (Carlyle and Eaves 2001). Open-pit scheduling provides a challenging example of applying a MIP to very large-scale problems (Gershon 1983), with recent examples of improved branch-and-cut strategies (Bley et al. 2010, Caccetta and Hill 2003) that seek a direct means of scheduling resource blocks. With constraints and variables potentially being generated for each block in the block model, the size of the resulting MIP can be immense.

The MIP application we present in this paper uses mining blocks to avoid the very large MIP that results from direct block model optimization; the actual

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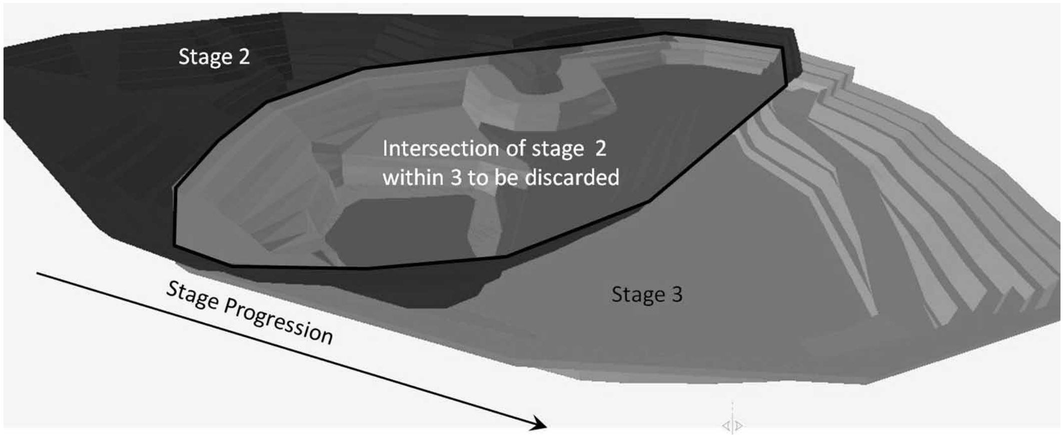

Figure 2: Chimiwungo West Stages 2 and 3 illustrate the overlap in design surfaces with the deletion of intersecting pits. The resulting design surfaces show the expansion of the final pit with increasing copper value.

scheduling units used in operating mines are commonly based on the volume of rock mined in a drill-blast-load production cycle or the practical working area of shovel for the selected scheduling interval (Smith et al. 2007) (see Figure 2). MIP technology is being applied to combined open-cut and underground mining at Codelco's Chiquicamata complex (Epstein et al. 2012).

The idealized perception of mine planning is that life of mine (LoM) annual production schedules are available and current, and that the LoM plan can be used to set medium-term monthly or quarterly production targets, which can then be resolved into weekly plans. The reality for most operations is probably more similar to that of Lumwana—at best, the immediate need for production drives planning into a medium-term cycle.

Medium-term planning is often all that is necessary for active operations, which have an established program of overlying waste removal (i.e., prestripping) to ensure sufficient ore feed to the mill. Commercial mine scheduling packages support these operations with a focus on short- to medium-term scheduling. Optimization technology is commonly absent from these commercial planning systems, which produce schedules based on fixed production targets and activity rates, user-defined ridged sequencing of activities, and predetermined assignment of shovels (Runge Pincock Minarco 2012).

Optimization technology is more commonly found in commercial applications aimed at strategic planning for greenfield projects (preproduction projects in the financial evaluation stage). These packages seek an optimal solution to issues such as cutoffs, stockpiling (i.e., providing an inventory of lower-grade ore), final pit dimensions, and stages of the pit's evolution. Commercial mine optimization solutions include COMET (Strategy Optimization Systems 2012), Compass (MineSight 2013), Whittle (Dassault Systemes 2012), and Chronos (Maptek 2013). Mine planners for operating mines, such as Lumwana, are challenged to obtain an operationally feasible solution from commercial scheduling packages with a long-term greenfields focus. For a schedule to be operationally feasible, it must provide a level of detail sufficient for allocation of major production resources, such as shovels, to a reserve unit having a volume that when mined (by that shovel) consumes a significant portion of the shovel's capacity over the production planning interval. Commercial systems in active mining operations fail for two primary reasons. First, the granularity of the optimization inputs and outputs is too coarse, typically annualized schedules of production from entire benches in a pit stage. Second, they provide no access to a model of production or the underlying data structure. Without the flexibility required to implement operational model components, which are often site specific, the solutions from

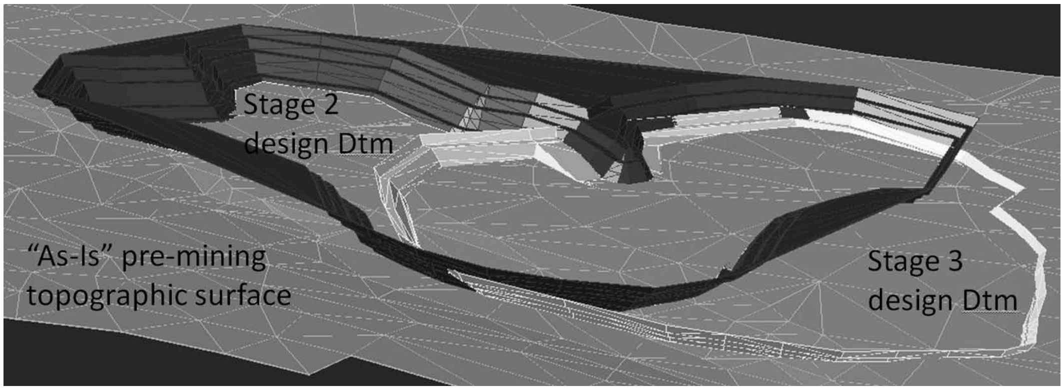

Figure 3: The design pit surfaces, modeled as digital terrain surfaces, also intersect against the current mining surface. The intersection of two or more surfaces results in a closed solid.

these commercial packages cannot be turned into feasible medium-term schedules, and issues relating to changes in solution granularity cannot be resolved. This study was implemented using LOBOS, a platform that is aimed at formulating and solving MIP models for mine-planning applications and allows alternative data inputs and solution outputs. LOBOS facilitates the implementation of alternative variables and constraints at the level of granularity needed in active mining operations.

Mining Block Definition

We take a sequence of fixed mine designs, and then optimize schedules to these designs. These designs optimize an extractable volume at a given level of profitability (i.e., a pit stage) (Lerchs and Grossman 1965). Initially, surfaces represent existing topography and mining face positions (i.e., excavations). Additional design surfaces represent mining excavations, which will contain the extracted volumes. We convert these surfaces into closed solid volumes before generating mining blocks suitable for production schedule optimization.

Figure 3 presents two stages in the mining of Lumwana's Chimiwungo West pit, which plans for extraction in seven stages. Pit optimization results in Stages 2 and 3, where Stage 3 is an expansion of Stage 2. Here, the pit algorithm that Barrick's engineers use produces overlapping surfaces. Therefore, we convert the current topography and design surfaces into design solids as follows.

1. Discard any portion of the design surfaces not corresponding a stage in the evolution of the pit. In this example, discard the Stage 3 surface within Stage 2 along with any surface extending above the current topography, as Figures 3–5 illustrate.

2. The outputs of Step (1) are surfaces enclosing the volume of a mining stage, as defined using the Whittle implementation of the Lerchs-Grossman algorithm (Dassault Systemes 2012). In our example, from Stages 2 and 3 of the Chimiwungo West pit, we trim the Stage 2 surface to the surface topography, the surface topography to the interior of the Stage 2 design surface, and the Stage 3 surface by both the topography and the Stage 2 surface (see Figure 4). When we combine these surfaces, the result is a set of continuous surfaces enclosing a volume (see Figure 5). A solid results from combining these surfaces into a single surface with no exposed edges.

3. The resulting solids have a volume; however, they lack density, mineralization, or any of the other properties required for production planning. The resource model (often referred to as the block model) provides information about mineralization and material properties. Figure 6 shows a portion of the Chimiwungo resource model that encompasses Stages 2 and 3.

Using both solid geometries of the stage design volume and the resource model, we generate mining blocks with reserves that are geometric and can be scheduled.

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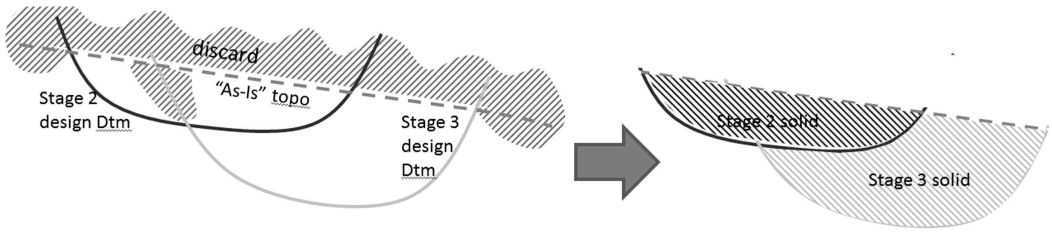

Figure 4: To prepare surfaces for the generation of closed solids, we clip the surfaces above the mining topography following the order of stage expansion to produce nonoverlapping, contiguous surfaces. When combined, the surfaces convert the Stage 2 and 3 solids into a progressive expansion of the pit by stages.

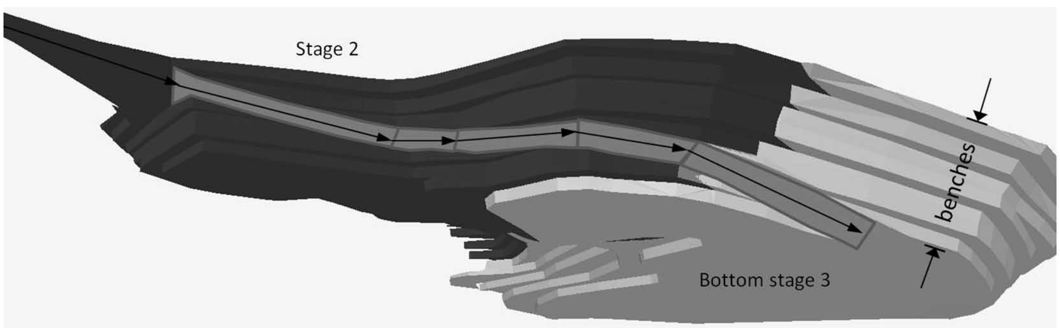

Figure 5: The Chimiwungo West pit, Stages 2 and 3 as closed solids, as seen from below, show access ramps and benches. The final Stages 2 and 3 solids include ramp access across both stages.

Step 1: Subdivide the Design Geometry Into Production Volumes. Production scheduling is based on mining blocks, a subdivision of the bench into volumes suitable for the mining cycle of drilling a pattern of blastholes to the depth of the bench across the extent of the mining block, loading these with explosives, blasting and loading the fragmented material into haulage trucks using shovels. These volumes are often classified in terms of development (ramps) and production (drill-blast units). Depending on the operation's size, a drill-blast unit may only take a few weeks to excavate. At Lumwana, the mining blocks should not be much smaller than the working area of a shovel in a month, the duration of the shortest scheduling interval. The set of mining blocks on a bench is a blast master, the nonintersecting set of bench-specific volumes used in mine planning. Figure 6 shows the Stage 2 solid, sliced into a blast master of $100 \times 100 \times 12$ cu m (cubic meter) mining blocks. We use LOBOS utilities to progressively merge $100 \times 100 \times 12$ cu m reserve geometries into larger volumes, while tracking cumulative tonnes. The final merged volume is approximately equal to the shovel's capacity for a month.

Step 2: Associate the Mining Block with Its Contained Mineralization. As Figure 6 shows, mining block geometries intersect with the resource model (in which the mineralization is discretized into rectangular volumes of $15 \times 15 \times 12$ cu m). Resource block properties include the block centroid and (x, y, z) dimensions. Comparing the mining block geometry with the resource block centroid and dimensions, LOBOS calculates the percentage of the resource block contained within the mining block. These volume-adjusted resource blocks are now available as parcels of material of a given volume, tonnage, and mineralization associated with the mining block.

Step 3: Merge Resource Blocks Into Material Parcels. In addition to tonnes and grades, each block in the resource model has classification fields that relate to production and processing performance; the

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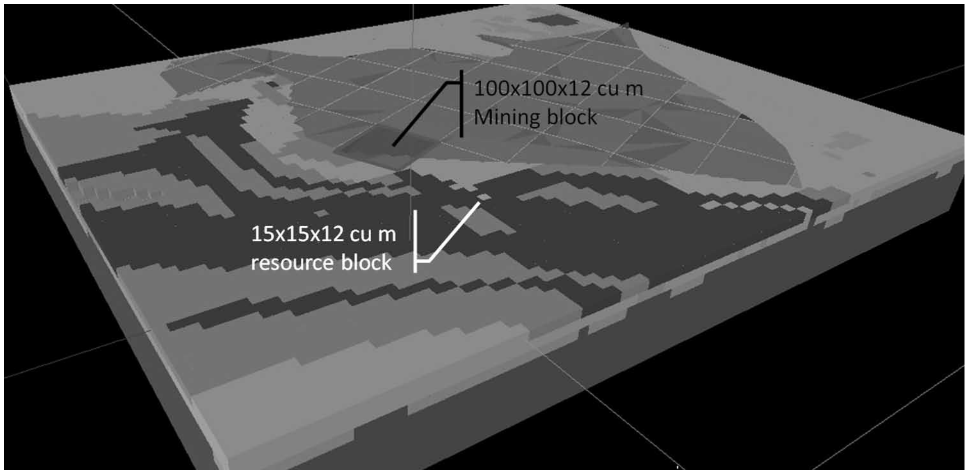

Figure 6: This Chimiwungo block model intersection shows Stage 2 mining blocks with intersecting resource model blocks.

most important are density, hardness (associated with milling rate), and mineral recovery. Resource blocks contained in the mining block are merged into bins as a function of these classifications: mineralogy (e.g., sulphide or oxide ore), metallurgical performance (e.g., recovery of copper and uranium in concentrate), and copper and uranium grades. The resulting reserve-related scheduling units consist of solids that are associated with aggregations of resource model blocks into material parcels with similar production and processing characteristics.

Step 4: Assign Destinations to Material Parcels. We further classify material parcels by destination (e.g., dump, stockpile, concentrator). Parcels fall into material bins according to a cutoff, a measure of value based on mineral grade or dollars-per-tonne value. Lumwana cutoffs combine copper and uranium grades. Figure 7 shows a mining block with three material bins on Bench 1,328: medium-grade ore (MGS), low-grade ore (LGS), and waste; the result will be three corresponding continuous production variables in the MIP. A given material bin can have multiple destinations; alternative destinations allow the optimization process to determine a destination, even splitting a bin's tonnage between destinations, if necessary, to meet constraints or improve total copper metal recovery (see Figure 8). In this figure, the mining block, $i \in \mathcal{I}$, acts as a container for the intersected resource blocks. According to material and cutoff classifications, these resource blocks are grouped into material bins $p \in \mathcal{P}_i$, having $j \in \mathcal{J}$ alternative destinations, which are shown as a waste dump, low-grade (LG) stockpile, or concentrator. For high-grade ore and waste destinations, material tonnage X_{ijpt} is sent directly to the dump or concentrator in the current scheduling period t. Low-grade ore can be sent to a stockpile and then rehandled, if and when needed, as S_{ipv} from the stockpile in period $t < v \leq \mathcal{T}$.

After reducing the resource model and mine design into a schedule-ready blast master of mining blocks, we can define the order in which we access and deplete mining blocks.

Sequencing Mining Blocks

The mining inventory consists of 2,509 mining blocks in Malundwe and 1,178 in Chimiwungo. Malundwe and Chimiwungo also have six and seven material classes, respectively; these account for copper recovery and stockpiling of high uranium level ore. Sequencing follows from pit designs, which include ramp locations for bench access, with blocks sequenced from ramp entry points to the bench (see Figure 9).

Figure 9 illustrates the relationship between a design and the mining blocks used for production

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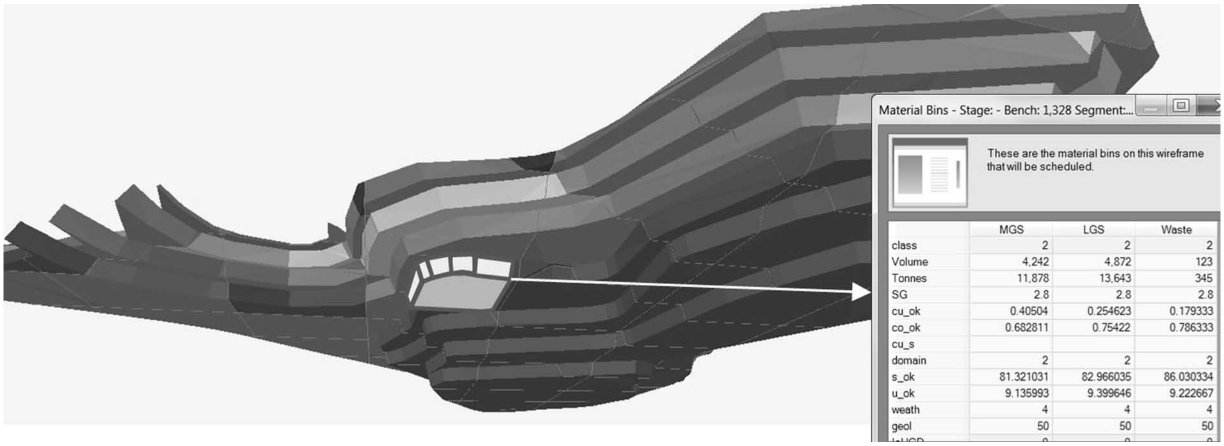

Figure 7: With the intersection of the mining block geometries on the resource model, each mining block includes the all-contained resource blocks. Material bins are the aggregation of resource blocks according to a reserve classification scheme.

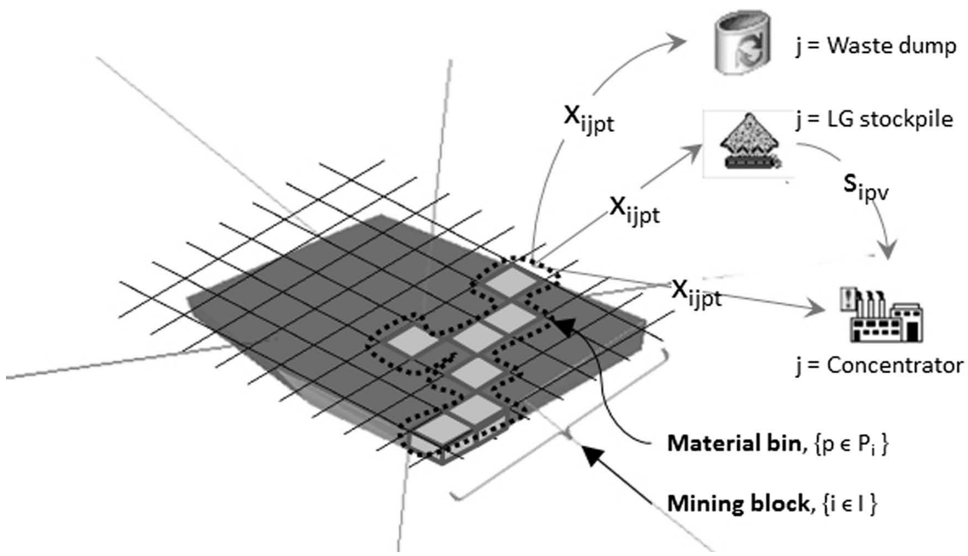

Figure 8: A many-to-one relationship exists between the resource blocks used for mineral resource estimation and the mining block used for production planning.

scheduling. The designs for two pit stages (shown as design strings) are superimposed on mining blocks (solid fill) for a bench that spans the width of both stages. Ramps access the bench in both stages from the left and right; different ramps access one or more benches. The ramp placement determines the initial mining block in production on the bench. We base our MIP formulation on general precedence relationships of predecessor and successor blocks. Commercial systems rely on predetermined sequences in which the path through the bench (the order in which the blocks are mined) is predetermined (MineRP 2012). The Lumwana MIP defines only operationally feasible sequences of blocks, as the mine planner defines. LOBOS provides semi-automated utilities to generate block-to-block sequencing constraints. Figure 10

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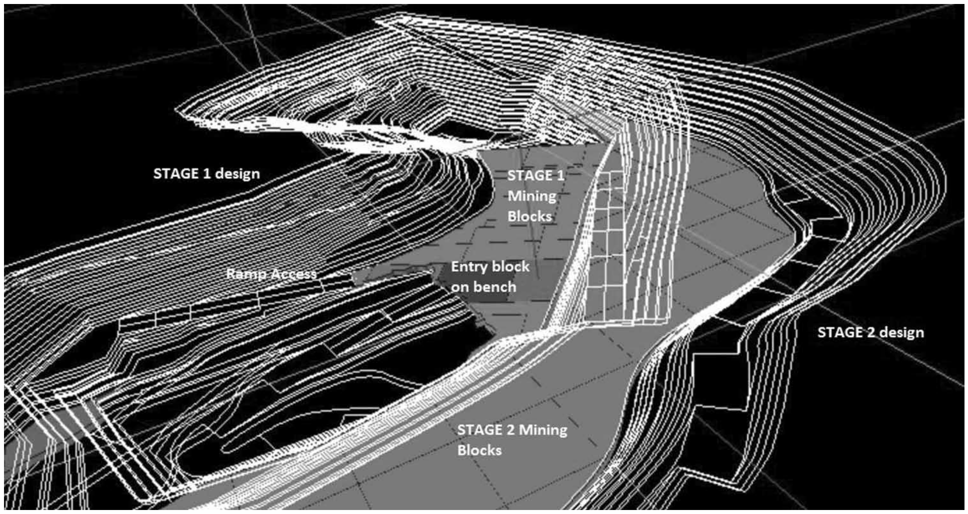

Figure 9: This design of two stages in Malundwe shows a single bench of mining blocks spanning both stages. Mid-bench limits of the stages and ramp locations are the basis for the layout of mining blocks. The ramp access shows the transition from the previous bench to the entry block on the next bench down. Bench sequencing is initiated from this block.

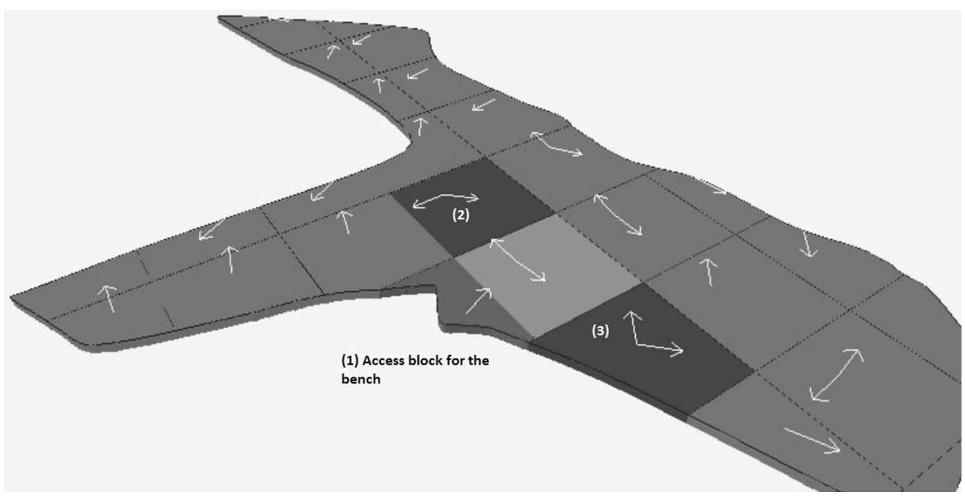

Figure 10: This example shows several alternative sequences: developing the bench to the left (2), or right (3), and in any of the sequences shown (as arrows).

shows an example of sequencing a single bench. The benches have one entry point (1), which constrains only a single block. From this position, mining progresses along alternative paths—to the left (2), the right (3), or both directions simultaneously. This allows the shovel to work through the bench on alternate paths. Vertical sequencing constraints ensure a minimum working platform around mining blocks, allowing the construction of temporary ramps within benches. A minimum working space criterion results in a working slope limit. This is shallower than that of the final design pit slopes, resulting in an operationally feasible progression through the benches. In general, the sequencing constraints allow maximum flexibility in meeting production constraints.

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Lumwana's Material Bin Definitions

Reserve classification is by destination: dump, stockpile, or mill. This allocation presumes that material classes are allocated to destinations according to some measure of value. Lumwana's 12 material types are classified according to weathering, copper grade, uranium grade, and copper-to-sulfur ratio (Cu:S). The ore classification is assigned to all parcels above a copper cutoff and below a uranium cutoff. Cutoffs segregate material into ore, waste, and material to be stockpiled. All sulphide ore is sent directly to the concentrator. Copper ore exceeding the uranium cutoff or with poor metallurgical recovery is stockpiled; thus, it does not contribute to maximizing copper metal. Uranium ores with insignificant copper are also stockpiled. Thus, we can reduce 12 geologic classifications to five material bins:

1. High-grade copper, low-level uranium, low Cu:S ore is sent directly to the concentrator.
2. High-grade copper, high-level uranium, high Cu:S ore is sent to a stockpile.
3. High-grade copper, low-level uranium, low-recovery ore is sent to a stockpile.
4. Uranium ore goes to low- and high-grade stockpiles.
5. Waste material below both copper and uranium cutoffs goes to waste dumps.

We make no special provisions for the contents, location, or capacity of dumps and stockpiles. In the model, we define separate stockpiles in terms of accepted material (e.g., high- and low-level uranium). Our initial scenarios did not allow for low-grade copper stockpile concentrator feed (i.e., stockpile rehandling) or for cutoff optimization (i.e., optimizing ore feed properties to maximize copper production over the life of the operation). However, later in the Lumwana project, we used stockpiles with predefined, low-grade copper cutoffs. Although we do not report on either cutoff or stockpiling optimization, stockpiling increased and sustained copper production.

Problem Description

First, we introduce the basic scheduling problem; we use a horizon of 20 months, primarily address the active Malundwe deposit, and illustrate the challenges of medium-term scheduling of a large multipit complex in the absence of a LoM plan. We then describe the solution strategies we applied to determine more challenging medium- and long-term schedules.

Medium-Term Schedule

We optimize Lumwana's production schedule using various mining capacities (i.e., the mix of contractor-operated earth-moving fleets and a Lumwana-operated fleet of trucks and shovels) and scheduling horizons; we do not consider supporting equipment such as dozers. Given the high per-tonne operating cost of using contractors, our production scenarios focus on the impact of reducing the use of contractors over time. Our first scenario, a 20-month production schedule, encompasses the seven Malundwe stages currently in production (see Figure 1). This schedule confirms the feasibility of achieving management's copper production target of 120 to 130 Kta (kilo tonne per annum). Except for those stages already producing ore, all remaining Malundwe stages require stripping of waste prior to ore production. Given the long project life and the absence of a LoM plan, a major challenge in generating a 20-month schedule is the placement of stripping capacity in stages unable to produce significant copper within the first 20 months. In some scenarios, we assign contractor truck-shovel fleets to alternate pits to distribute excess shovel capacity (i.e., capacity higher than that required to maintain maximum ore production in currently active stages). Our primary problem in the medium term is how to allocate mining capacity to achieve production targets. In the longer term, reducing reliance on contractors should decrease mining costs. This means reducing contracted capacity over time, while seeking an allocation of owner-operated shovels to pits that maintain copper production.

Figure 11 shows a 20-month Malundwe ore production schedule. This area chart displays the contribution to ore production by each pit stage during this period. Because Malundwe is already in production, prestripping of waste is complete and Stages 3, 4E, and 4W produce ore and will continue to produce it for nine months. Even after prestripping the overlying cover of waste, waste associated with mineralized

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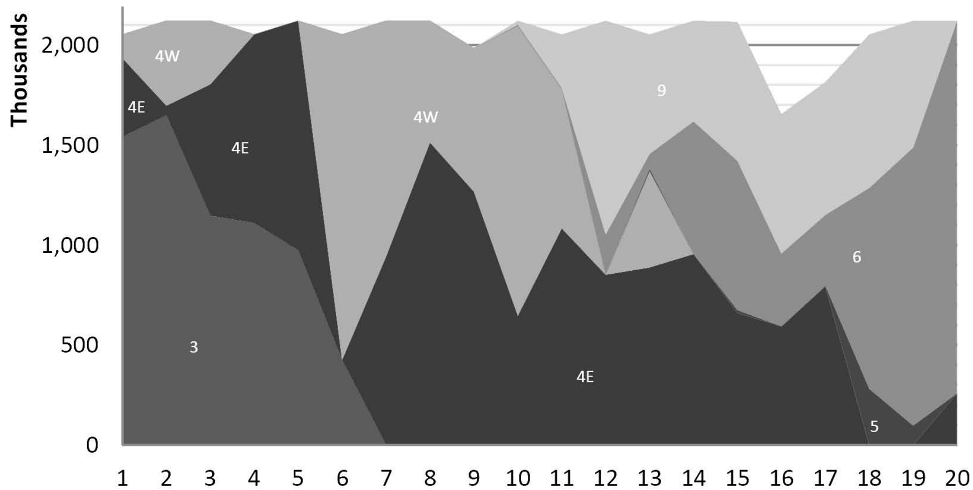

Figure 11: In a typical Malundwe 20-month ore production schedule (Kt), some stages (e.g., 3, 4E, and 4W) produce ore at the start of the schedule; others require many months of stripping prior to ore production (e.g., six and nine, as Figure 12 illustrates). In these examples, multiperiod scheduling allows us to look ahead to see the metal production that will result from waste stripping.

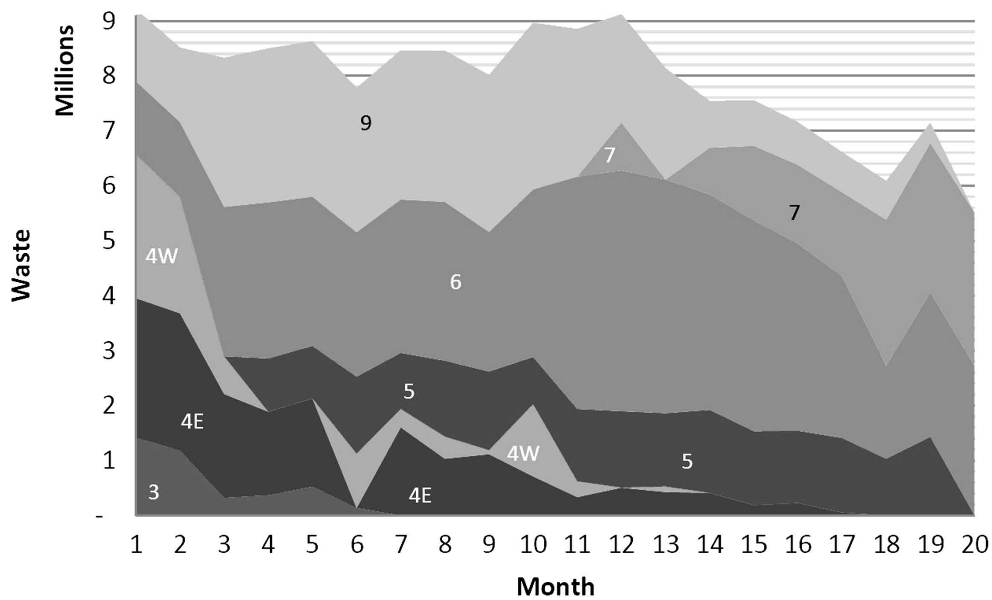

Figure 12: This typical Malundwe 20-month waste production schedule (Mt) shows that within the confines of a limited horizon, the time to start waste stripping may not be obvious; for example, compare the limited ore production of Stages 5 and 7 in Figure 11 to the waste schedule for the same stages in this figure.

material below cutoff and sterile material that must be removed from overlying benches to maintain stable pit slopes are still present. Examining the waste production schedule for Malundwe (see Figure 12) shows that Stages 3, 4E, and 4W produce both waste and ore. Most waste production occurs as prestripping in an earlier period (see Figure 12). For example, Stages 5–7, and 9 each produce waste before they produce ore. Only Stages 5, 6, and 9 produce any ore in this period—and only after almost a year of

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prestripping. These area charts are a common means of illustrating a production schedule from active production areas (i.e., pit stages). The area under each curve is proportional to the production tonnage of the stage, which varies by month. In a stage's initial development, waste is removed to uncover ore. For Lumwana, the production schedule provides monthly targets for ore production and waste removal in each mining stage. Given that the tonnages shown in Figures 11 and 12 are based on a schedule of production volumes (mining blocks) that relate to operational practice for shovels, we can resolve this medium-term schedule into a feasible plan for monthly shovel placement and production.

The production schedule in Figures 11 and 12 illustrates the difficulty of scheduling waste stripping in the absence of either predetermined LoM production targets or of an optimization methodology that allows a horizon of sufficient length to determine the copper production that follows the waste removal. The waste removal schedule (see Figure 12) shows that prestripping (in Stages 9 and 6) must commence immediately and prestripping in Stage 5 must start by month 4 if ore production is to be maintained in the second year, assuming that (1) ore production in Stage 9 starts by month 11, (2) Stage 6 does not reach significant production before month 14, and (3) Stage 5 does not start before month 18. Stage 7 cannot produce ore within a horizon limited to 20 months; however, production in Stage 7 is necessary in the subsequent 60-month schedule. With no clear evidence of the value of the deeply buried ore in stages such as 7, we must extend the scheduling horizon to determine whether to include Stage 7 in the reserve. Knowing the value of prestripping, we assign shovels to Stage 7, which operates at its waste removal capacity. Shovel allocation should (1) maintain ore production, (2) prestrip waste, thereby uncovering sufficient ore later in the schedule, and (3) provide the best utilization of the shovel fleet. Given the Stage 7 example, we see that we must extend the horizon beyond 20 months to understand where to place shovels to maintain future ore production. With the feasibility of copper metal production targets confirmed to 20 months, we extend the scheduling horizon to 60 months. The 60-month scenario is challenging to solve as a multiperiod MIP at the level of detail required to support production planning. We discuss solution strategies for this extended schedule in the next section.

Medium-Term Solution Strategies

We initiated the Lumwana scheduling project to support the next 20 months of production planning—not as a LoM study. The mine was in production and contractors were in place. However, a LoM plan to provide production targets for the 20-month schedule was not available. Our initial solutions were largely exploratory; our objectives were to (1) determine if mine production capacity was sufficient to maintain copper production, and (2) determine a production capacity allocation that would provide the best shovel fleet utilization.

We completed the 20-month schedule, assuming the pits that would receive excess shovel capacity. This solution was not optimal for production beyond this time frame; however, it was optimal for those reserves that would be depleted over 20 months (i.e., Stages 3, 4E, 4W, 9, and the significant reserves in Stage 6, as Figures 2 and 3 illustrate). This allowed us to fix production from these stages for the first 20 months in a second phase of schedule optimization, which encompassed 60 months and the Chimiwungo pits.

The requirements for the 60-month schedule were largely the same as those for 20 months; however, the resulting MIP's size was much greater because we added the Chimiwungo pits. With this addition, 60 months of production is insufficient to deplete 37 years of reserves. Therefore, we must still address where to place waste-stripping capacity. We use alternative scenarios to evaluate placing varying numbers of shovels in different pits at different times. Given the problem's difficulty, a rapid and effective solution that reduces the problem size and improves convergence is essential. We summarize the successful solution strategy in the following.

1. An initial pass with increased granularity of the scheduling unit (i.e., aggregation of mining blocks) and production interval (i.e., years).

2. Multiperiod sequential optimization using a rolling horizon.

3. Revision of the initial coarse LoM solution to a finer granularity by including:

- Earliest and latest start dates loosely based on the coarser solution.

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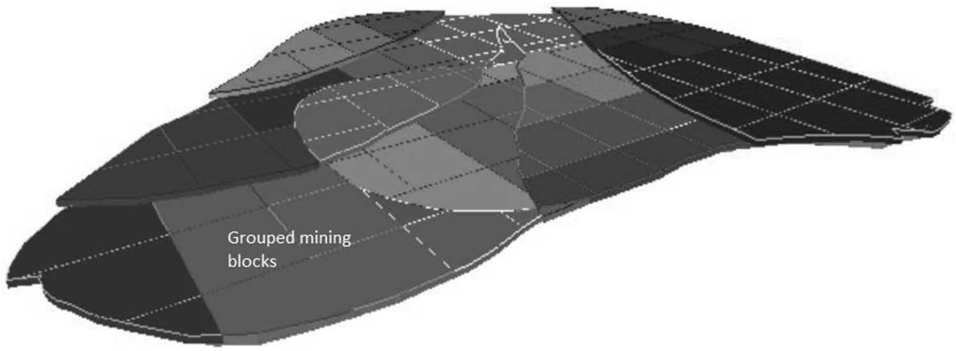

Figure 13: Scheduling occurs at successively finer levels of granularity. To work from an overall production strategy down to a feasible solution, we initially optimize longer scheduling horizons with groupings of mining blocks. The coarse LoM solution establishes earliest and latest start dates and solution bounds when optimizing the component blocks on a monthly interval. This example shows both large scheduling blocks used in LoM strategy planning and finer component mining blocks used for scheduling in one of the Chimiwungo pits.

- Upper and lower bounds on total mining and processing capacity spanning the coarser solution.
- Bounds on mining capacity in individual stages corresponding to alternate contractor and shovel allocations.

4. A final pass of multiperiod optimization using a rolling horizon to generate a monthly schedule based on mining blocks.

Overall, we sought to generate a schedule based on the optimization of the longest possible monthly horizon using the original mining blocks.

We began by increasing the size of the scheduling units, thereby reducing the size of the problem and the complexity of the resulting sequences. Figure 13 shows a solution for groupings of merged mining blocks used in the initial pass of optimization at a coarse granularity, with an overlay of the component mining blocks used in the final pass at the required level of granularity. Ultimately, the size of these merged blocks in the initial pass could be an entire bench in a stage, as is commonly the case in LoM studies; however, Lumwana engineers require a level of detail to support planning over 60 months. At Lumwana, retaining a level of scheduling-unit granularity much smaller than an entire bench is essential. This is partly an operational requirement to allow mine planning engineers to work directly with the optimized solution. Sub-bench granularity also improves ore-waste selectivity. With a total shovel capacity barely sufficient to sustain ore production, flexibility within the bench is essential. Mining entire benches requires that we assume that ore and waste parcels deplete at equal rates. For benches with a high proportion of waste and insufficient mining capacity, scheduling entire benches removes any opportunity to find a production sequence within and between benches that maintains ore feed to the mill. Effectively, using entire benches overestimates the required shovel capacity.

We were unable to solve the Lumwana MIP using one 60-month horizon. Instead, we solved the problem in a series of overlapping horizons. In the rolling horizon approach, we build the MIP for an initial interval (e.g., months 1–10 of a 60-month schedule). If this interval (1–10) is feasible and within an acceptable tolerance of optimality, then we deplete the mining blocks active in period 1 according to the solution obtained for these blocks in period 1, and we advance the scheduling horizon to months 2–11, 3–12, etc., to the full scheduling horizon. This approach allows us to review the solution, make necessary adjustments, and produce an operationally acceptable mine plan. For example, examining the solution may show that shovel capacity is not fully utilized following month 20; in this case; we restart the optimization process from month 20 and reallocate the shovels. Solving multiperiod problems by a rolling horizon demands close monitoring. At Lumwana, Barrick seeks a production strategy that both minimizes mine

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production capacity and meets copper production targets. As we note previously, shovel capacity should be sufficient to ensure that the rate of waste stripping uncovers sufficient ore to keep the mill full and meet minimum copper production targets. Therefore, we optimize production scenarios over a series of successively reduced shovel-fleet configurations. Eventually, as stripping capacity is restricted, the problem becomes infeasible with respect to minimum copper production targets. A time lag always exists between insufficient stripping capacity and ore production; therefore, when we encounter an infeasibility, we must restart the rolling horizon some periods earlier than the period in which the copper metal target was violated. With the source infeasibility resolved, we continue the rolling horizon until we find a solution to the entire scheduling horizon.

We take the solution from the coarse scenario to set "postsolve" earliest start (ES) and latest start (LS) periods. In this context we refer to a postsolve ES that is derivative of the earliest period of mining activity as found in the solution to the coarse scenario. In the initial pass of optimization using the coarse scenario, we solve merged mining blocks at a coarser time interval (quarters instead of months followed by years instead of quarters). The tonnage in the merged mining blocks follows the principle of scheduling reserve units, which approximate the capacity of a shovel in the smallest scheduled time interval. We assign a conventional presolve ES value to merged mining blocks based on precedence relationships within the pit (for an applied example in underground mining see Kuchta et al. 2004). In this approach, for each mining block in the pit, the set of mining blocks that must be depleted before that mining block is active is determined and cumulative tonnage of this precedence set is compared against the mine capacity to obtain the mining block's (presolve) ES value (for algorithmic details see Lambert et al. 2014). Note that processing capacity in a multipit application has little application in setting the ES value unless the order of the pits is already known, which it is not when solving the coarse scenario. Therefore, the methods reported elsewhere are not fully applicable at Lumwana.

In the second pass of schedule optimization, we reduce the granularity of the mining blocks and period durations, unmerge grouped mining blocks, and redefine the period horizon. Postsolve ES and LS values are now taken from the solution to the coarse scenario, with the period of the solution mapped from the coarse mining block to its component blocks and the coarse scenario's periods mapped to the new scheduling horizon. We constrain the production from the material parcels in the less-granular mining blocks to ensure that it does not occur before the postsolve ES period and commences by the LS period. We build some flexibility into these postsolve ES and LS values by relaxing the ES and LS periods, dependent on their position within the scheduling horizon. ES dates in mining blocks included in aggregations mined early in the coarse schedule have little, if any, flexibility; mining blocks solved later in the scheduling horizon have greater flexibility. Relaxation of the LS reverses that of the ES, with more flexibility provided later in the schedule. This strategy reflects the increased uncertainty of the production schedule over time when going from a coarse to a finer level of solution granularity, and helps avoid infeasibility when imposing an annualized schedule on a monthly model. Constraining the second, finer-granularity pass of optimization with the solution to the initial coarse model using the postsolve ES and LS periods may result in the loss of some value, especially if there is insufficient relaxation of the ES and LS values. However, we have found that this approach consistently results in a second pass of optimization that yields a higher value than the initial coarse pass.

Model sizes and solution times required for this study varied greatly; we solved a large number of scenarios, each of which uses a rolling horizon. In most cases, we achieved a gap of two to five percent of optimality. A 12-month horizon using agglomerated blocks of a size sufficient to accommodate a shovel for a month would result in roughly 100,000 rows, 30,000 columns, and 660,000 nonzeros following probing and aggregation using CPLEX 12.1. The solution of such a model out to 30 months usually requires four hours on an i7 CPU running eight threads. Solution time depends on the problem's difficulty as much as its size. For example, as we reduce the mining capacity, finding a mining sequence that meets minimum copper production targets becomes more difficult as we further reduce mining capacity. As the difficulty increases, the solution time also increases and

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the number of feasible solutions found and convergence to optimality decreases.

Conclusions

The final result is a schedule of production for 60 months, defining production in terms of ore and waste sources, total copper production, uranium levels in ore, stockpile inventories, and shovel assignments at a level of mining block activity that could be implemented at a feasible operational level. As of this writing, we have completed a full LoM study and will follow it with an analysis of mining capacity and cutoffs.

A major lesson we learned is the need for a method to reduce problem size. The primary solution strategy increases granularity in time and geometry. LOBOS utilities expedite optimization at increasing levels of granularity through a process of mining block generation by intersecting designs with resource models and by aggregating mining blocks to volumes, which correspond to the scheduling interval durations. Using a rolling horizon is also critical, because it enables a solution of relatively long time-dynamic MIPs, while avoiding a massive multiperiod LoM MIP. However, in a large multipit complex such as Lumwana, even when we used a rolling horizon, we found that determining the value of waste prestripping for pits in which ore was overlaid by multiple years worth of waste production was difficult. Allocation of excess mining capacity to prestripping required a level of advanced knowledge of the relative ranking of pits based on comparative value and the time needed to access ore. For this, we relied on trial and error and guidance from site planning engineers.

Additional developments have come out of this work. Prestripping requirements are a major impediment to multiperiod optimization using a rolling horizon. Stripping costs do not result in revenue and mining will never occur in cases in which significant value from ore production occurs later than the interval being used. Overlying waste blocks (i.e., overburdening) also presents an opportunity to reduce model size by reducing waste benches to a single mining block. Merging resource blocks involves averaging and artificially reduces the variance of grade distributions, giving an incorrect impression of homogeneous mill feed grade. Therefore, we avoided aggregating mining blocks with a high proportion of ore. In the Chimiwungo pits, the upper benches are all waste, and merging these benches into a single mining block per bench had no negative impact on the accuracy of the copper grade delivered to the mill. This substantially reduced the model's size. Additionally, once the coarse solution has identified the production sequence of the pits, prestripping can be implemented prior to optimization of the subsequent finer granularity scenario with a corresponding loading on all related resource constraints. We could potentially represent all overburden benches as a single mining block of one parcel, substantially reducing the model's size.

ES and LS dates calculated a priori for single pits are ineffective for multipit problems; all pits do not start at the same time, and multiple pits can be in production at the same time. Consequentially, an ES date on a mining block only provides an approximation of the actual timing of production for whatever pit is the first in production. Additionally, when scheduling single pits, ES values are based on mining and milling capacities; however, when many mines supply one concentrator, only the mining capacity relates to the ES date. We have partially overcome these limitations by solving the scheduling problem at a coarse granularity followed by a finer level of granularity. In the coarse scenario, we apply, a priori, an ES based on precedence relationships and the pit's mining capacity. In the subsequent fine-granularity scenario, ES and LS periods are determined ex posteriori based on the solution to the parent mining block and a relaxation of the ES and LS dates when going from the coarser to the finer scheduling horizon (e.g., from years to quarters). The use of the ex posteriori ES and LS periods greatly reduces the size of the MIP, allowing optimization of monthly schedules over a much longer interval.

Successive solutions at increasing levels of detail also allow the inclusion of additional constraints on production capacities in the pit stages. This allows the solution to a coarser LoM model to be imposed as bounds on a model with an operational level of detail. In an initial optimization, we apply minimum constraints and bounds, and then refine the solution by adding operational constraints, such as leveling of production and the placement of shovels.

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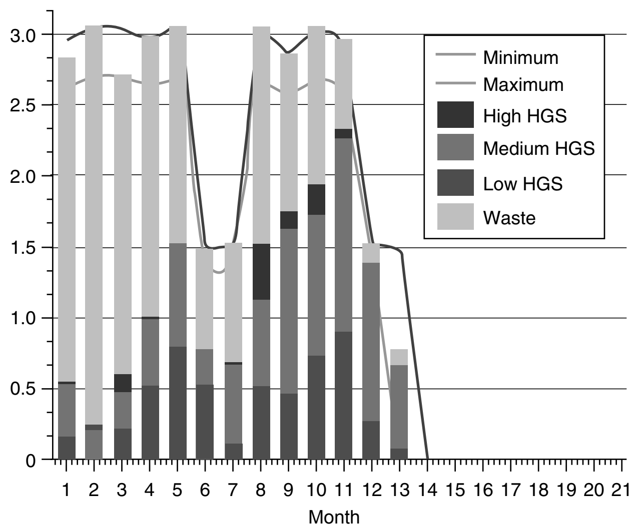

Figure 14: This example of a production solution from a pit stage shows lower and upper bounds on that solution (Mt) by period (month). These bounds are based on the solution to a more granular model using larger production blocks, and then imposed on a production scenario using a finer level of mining block granularity.

These constraints limit the search space and provide an operationally feasible schedule (see Figure 14). Setting lower and upper bounds on pits, stages, or activity level of production resources is necessary for operational mine planning in which the utilization of production resources must be high and sharp fluctuations in production are not tolerated. However, we have found that when targets are based on a coarser scenario whose solution is then applied to a finer level of time granularity, infeasibilities commonly result. This arises from the inability of production resources to meet monthly targets because of spikes in waste movement, which were hidden in an annualized schedule. Therefore, we have had greater success with relaxed ES and LS values; they place no hard lower bounds on production, but still enforce the same cumulative production levels as the solution to the coarser scenario.

Scheduling production for an operation of Lumwana's size is challenging; however, mining operations that are as large or larger than Lumwana are becoming increasingly common. The research in Lambert et al. (2014) is based on the direct scheduling of blocks in a block model that can apply to the production scheduling of mining blocks as implemented in this paper. However, a major difference exists in application and model structures between the reported advances in direct block scheduling and what we report in this paper. First, we schedule the aggregation of (resource) blocks into material parcels in a mining block, thereby reducing the model size, while using a similar volume and geometry of reserve as mine planners use in active operations. Second, whereas direct block scheduling is only concerned with labeling (resource) blocks as ore or waste, we expand on the destination to include the stockpiling of low-grade ore for potential processing at a later date. We also address the issue of routing different materials within the larger mining complex. Since we initially developed our model, we have added modeling components associated with the mining complex's material handling network; these include point of origin (mining block), destination, routing, and the consumption of haulage resources as a function of haulage productivity, which can vary by origin, destination, routing, season (e.g., rainy or dry), and haulage-unit properties. The resulting model has a substantial network component associated with not only material origins and destinations, but also with the routing of that material from the bench to its final destination.

We continue to improve model formulations, the generation of efficient mining block inputs, and solution heuristics with a focus on the following.

1. An improved MIP model formulation whose LP relaxations provide near integrality on binary depletion and activity variable, as Lambert et al. (2014) discuss. As of this writing we have completed such a formulation and reapplied it to the Lumwana problem with a several-fold improvement in solution speed and convergence.

2. With improvements in solution performance of the MIP formulation, we now find that although far fewer subproblems are being solved to find a good integer-feasible solution, the time spent solving the LP relaxation is far greater. This is clearly a consequence of adding side constraints on production and network flow variables. These material flow variables are cast in terms of tonnes while mining block sequencing constraints use fractional 0-1 continuous and binary variables, as seen with the direct block sequencing

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models. There may be opportunities to significantly improve LP performance with changes in solver settings, model formulations, and improvements in scaling and block aggregation techniques.

3. We have yet to prove the degree of optimality of the rolling horizon solution to the complete MIP. One suggestion originating from the review of this paper is to apply the rolling horizon solution as a MIP starting solution. Unfortunately, the resulting MIP starting solution tends to be infeasible with respect to the complete multiperiod LoM MIP. Although a relaxation of bounds associated with tight constraints may resolve this, indications are that solving the resulting LP-relaxation will prove challenging.

It is not possible to quantify an improvement in value resulting from the change in scheduling methods; spreadsheet solutions for complex operations such as Lumwana are generally not implemented at a comparable level of detail nor do they account for all sequencing, capacity, and blending constraints. The solutions generated are simply not comparable, because the schedule generated in Excel is unlikely to even be feasible with respect to the constraints implemented in the MIP.

This project, as with most, was a learning experience. With decades of experience in the mining industry, we relied heavily on a common sense approach to solving the client's operational requirements so that the results obtained could be put into practice.

As noted, our emphasis in this project is providing operationally feasible solutions for active operations. Although we may assume that we have increased project value, the infeasibility of prior solution methods invalidates any meaningful comparison. Primarily, we demonstrate in this paper how MIP technology can be rapidly applied to active operations to optimize the life of complex mine plans while producing operationally feasible solutions.

Appendix. Model Formulation

We present a MIP model of production scheduling for one mine in one production stage. Multiple mines, with multiple stages delivering material to alternative destinations, can be modeled with additional indices, which we omit in this treatment for the sake of clarity and brevity.

Notation

1. Indices and sets
   - $i \in \mathcal{I}$: Set of mining blocks representing the basic geometry used in production scheduling.
   - $p \in \mathcal{P}_i$: Set of material parcels contained in mining block i.
   - $j \in \mathcal{J}$: Set of destinations available for parcel j, including parcels reporting to the set of stockpiles, $j' \in \mathcal{S}$, and, $j'' = \mathcal{C}$, reporting to the mill.
   - $t \in \mathcal{T}$: Set of production scheduling periods in production horizon $\mathcal{T}$, including t', the period of stockpile rehandle to the concentrator.
   - $b \in \mathcal{B}$: Set of benches in a mining stage.
   - $i \in \mathcal{I}_b$: Set of mining blocks in bench b.
   - $d \in \mathcal{D}_t$: Set of diggers (main production shovels) available in period t.
   - $k \in \mathcal{K}_i$: Set of mining blocks that must be mined out to provide access to mining block i.
2. Parameters
   - T: The final period in the production horizon.
   - c_{ip}: Copper grade (percent) in mining block i of parcel p.
   - r_p: Metallurgical recovery (percent) of parcel p.
   - f_t: Discount factor ($\in [0, 1]$) for copper recovery in period t.
   - R_{ip}: Reserve (tonnage) available of mining block i, parcel p.
   - $\bar{B}_b$: Tonnage production capacity in bench b, resulting from shovel placement capacity in the bench.
   - $\bar{D}_{dt}$: Global tonnage production capacity over all locations of digger fleet d in period t.
   - u_{ip}: Mining block i uranium grade (ppm) of parcel p.
   - $\bar{U}$: Uranium grade limit (ppm) in the mill feed.
   - $\bar{N}_t$: Number of production shovels available in period t.
   - $\bar{C}_{ipt}$: Tonnage capacity of stockpile i of parcel p per period t.
3. Variables
   - x_{ijpt}: The tonnage from mining block i delivering material from parcel p at reserve level R_{ip} to destination j of $\mathcal{J}$ alternative destinations in scheduling period t.
   - y_{ipt}: The stockpile inventory tonnage of mining block i, parcel p in period t.
   - $s_{ipt'}$: Tonnage of stockpile rehandle source from mining block i, parcel p in period t'.
   - δ_{it}: 1 if mining block i is fully extracted in period t, 0 otherwise.
   - η_t: Number (integer or continuous) of shovels required in period t for production.

Formulation

The objective is to maximize copper metal production (1) subject to constraints on production (2)–(4), blending (5), stockpiling (6) and (7), and mining block sequencing (8) and (9):

$$\text{maximize } z = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_i} \sum_{t \in \mathcal{T}} c_{ip} \cdot r_p \cdot f_t \cdot (x_{ijpt} + s_{ipt}), \qquad (1)$$

$$\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} x_{ijpt} \leq R_{ip} \quad \forall\, i \in \mathcal{I}, \forall p \in \mathcal{P}_i, \qquad (2)$$

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$$\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_i} x_{ijpt} \le \bar{B}_b \quad \forall b\in\mathcal{B}, \forall i\in\mathcal{I}_b, \forall t\in\mathcal{T}, \tag{3}$$

$$\eta_t = \sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_i}\frac{x_{ijpt}}{\bar{D}_d} \quad \forall b\in\mathcal{B}, \forall i\in\mathcal{I}_b, \forall t\in\mathcal{T}, \forall d\in\mathcal{D}_t, \tag{4}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_i} u_{ip}\cdot(x_{ijpt}+s_{ipt}) \le \bar{U}\cdot\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_i}(x_{ijpt}+s_{ipt}) \quad \forall t\in\mathcal{T}, \tag{5}$$

$$y_{ipt} = \sum_{t=1}^{t'-1} x_{ij''pt} \quad \forall i\in\mathcal{I}, \forall j''\in\mathcal{S}, \forall p\in\mathcal{P}_i, \forall t\in\mathcal{T}, \forall t'\in\mathcal{T}\mid t<t', \tag{6}$$

$$y_{ipt'} = y_{ipt'-1} + x_{ij'pt'} - s_{ipt} \quad \forall i\in\mathcal{I}, j'=\mathcal{C}, \forall p\in\mathcal{P}_i, \forall t\in\mathcal{T}\mid t\neq t' \forall t'\in\mathcal{T}\mid t<t', \tag{7}$$

$$\sum_{p\in\mathcal{P}_i} R_{ip}\cdot\delta_{it} \le \sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_i}\sum_{u=1}^{t} x_{ijpu} \quad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}, \tag{8}$$

$$\sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_k} x_{kjpt} \le \sum_{p\in\mathcal{P}_k} R_{kp}\cdot\delta_{it} \quad \forall i\in\mathcal{I}, \forall k\in\mathcal{I}\mid k\in\mathcal{K}_i, \forall t\in\mathcal{T}, \tag{9}$$

$$0\le x_{ijpt}\le R_{ip} \quad \forall i\in\mathcal{I}, \forall p\in\mathcal{P}_i, \forall j\in\mathcal{J}, \forall t\in\mathcal{T}, \tag{10}$$

$$0\le s_{ipt}\le R_{ip} \quad \forall i\in\mathcal{I}, \forall p\in\mathcal{P}_i, \forall t\in\mathcal{T}\mid t\neq 1, \tag{11}$$

$$0\le y_{ipt}\le \bar{C}_{ipt} \quad \forall i\in\mathcal{I}, \forall p\in\mathcal{P}_i, \forall t\in\mathcal{T}, \tag{12}$$

$$0\le \eta_t\le \bar{N}_t \quad \forall t\in\mathcal{T}, \tag{13}$$

$$\delta_{it}\in\{0,1\} \quad \forall i\in\mathcal{I}, \forall t\in\mathcal{T}. \tag{14}$$

The objective (1) maximizes recovered copper metal. Constraints on reserve parcels (2) limit production from the material parcel to the available resource. Bench capacity constraints (3) limit production from all mining blocks on the same bench. The global shovel capacity (4) is limited in the period to the capacity of the available production fleet. Although the fleet size η_t can be defined as integer, a solution as a continuous variable is often preferred by operators who would like some indication of the degree of integer violation in a given period. It is important to remember that the model does not produce the final definitive result. Rather, the engineer interprets the solution to make an informed decision. The solution might result in 6.1 shovels for Q1 2014, and the operator would undoubtedly interpret this as 6. However, if the solution is 6.3 shovels for Q1 to Q3, an engineer who believes that productivity is already overstated may well interpret it as 7 for 2014. The formulation does allow for the solution of η_t as integer; however, doing so results in loss of this information. We assume that required shovel capacity for stockpile rehandle is separate from the shovel resource available for mine production. A uranium blending constraint (5) limits the average uranium grade of the ore feed to a maximum feed limit. A pre-rehandle stockpile inventory constraint (6) equates the size of the stockpile prior to any rehandle to all mine production inflows. Generally, the earliest date of rehandle is defined a priori by the period-varying stockpile capacity $\bar{N}_t$. A standard inventory constraint (7) balances stockpile input and output with starting inventory. Sequencing constraints (8) in combination with (9) controls the order of depletion of the mining blocks, such that any block k restricting access to block i must be fully depleted ($\delta_{kt}=1$) prior to production from block i.

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Verification Letter

Graham Lindsay, Senior Planning Engineer, Lumwana Mining Company Limited, P.O. Box 110199, Solwezi, Zambia, writes:

"I can verify that the scheduling work undertaken by MineSmith for the 2–5 year planning horizon proved extremely useful to us and enabled the technical services team to look a number of different options/scenarios in a relatively short space of time. The work proved valuable as it gave direction as to the next area/stage that should be mined in order to continue with a sustainable production plan over the medium term.

"The schedules that MineSmith put together were both sensible and practical in terms of machine allocation and bench progression.

"They are very thorough in their drive to understand the workings of the site and the unique nature of the operation. This enabled them to put together a plan that we could use for both operational purposes and management reporting."

Martin L. Smith is managing director and principal mining engineer for MineSmith Pty. Ltd. He has 30 years of experience in the minerals industry working as a practicing mining engineer, researcher, consultant, and educator. He is also the creator of the Life of Business Optimisation System (LOBOS).

Stewart J. Wicks is director and technology officer for MineSmith. Stewart has 18 years of experience in software engineering, with the last seven years focused on schedule optimisation solutions for the mining industry and the development and application of LOBOS.

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