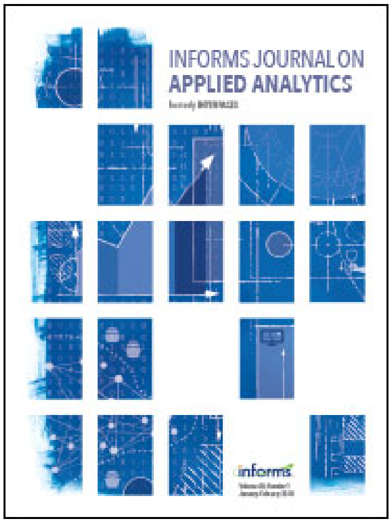

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Empowering Denver Public Schools to Optimize School Bus Operations

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Abstract. Denver Public Schools (DPS) serves roughly 90,000 K–12 students using a mixed bus fleet. Developing and reviewing bus-route assignments manually has been challenging and time consuming for DPS. During 2017–2018, DPS analysts reviewed and adjusted over 700 routes assigned to approximately 200 buses, considering time and capacity feasibility. We developed a decision support tool (DST) to generate feasible bus-route assignments and help inform DPS's decisions. The DST employs optimization models to solve the bus-route assignment problem using distance data from Google Maps Application Programming Interface and various interroute reposition-time scenarios to account for the impact of potential traffic delays. The model incorporates multiple objectives related to minimizing cost, meeting demand, and maximizing "consistency"—that is, the difference between a newly created and previously implemented solution The solutions generated by the DST for the 2017–2018 school year utilized significantly fewer buses and lower reposition mileage compared with the DPS solution. Considering the convenience, efficiency, and flexibility of generating high-quality bus-route assignments using the DST, the DPS transportation team has used the DST in the route planning process since 2018.

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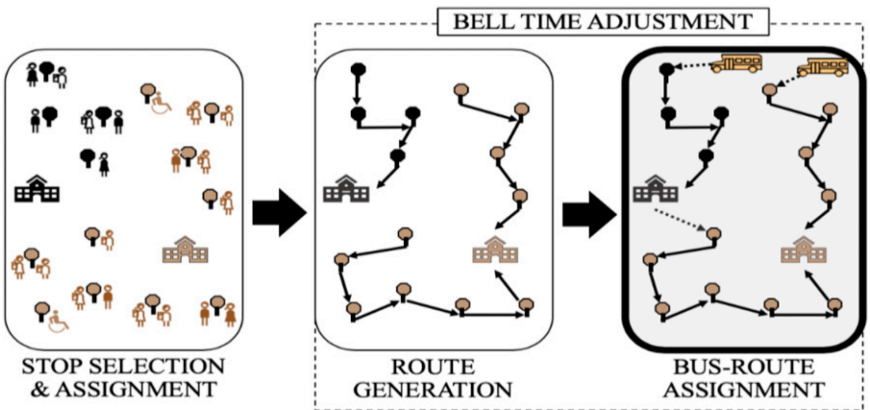
Keywords: decision support system • school bus routing • optimization • assignment • consistency • transportation

Transporting K–12 students in the United States is a massive endeavor involving more than 25 million students annually and costing more than $23 billion (Snyder et al. 2016). The provision of school bus transportation is an essential service to students and families, especially for those with little to no access to modes of safe transportation. Many states require by law that school districts provide transportation to all eligible students. Student eligibility depends on criteria such as walking distance from school, disability status, type of school, and whether a student attends a school outside of his or her zoning district. Although school systems strive to provide service to all eligible students, many challenges exist, including operational costs, bus-stop and -route assignments, ridership uncertainty, and rider and driver equity. Efficient school-bus operations could help reduce chronic absenteeism, a "hidden educational crisis" (U.S. Department of Education 2019), as well as the overall cost of transportation incurred by the school systems.

The school bus-routing problem (SBRP) (Newton and Thomas 1969, Park and Kim 2010) consists of interconnected subproblems: selection of bus stops to ensure safe and easy access by students, choosing routes covering all the bus stops, setting school bell times, and bus-route assignment (Figure 1). A route (also referred to as a segment by Denver Public Schools (DPS)) consists of a set of bus stops with students assigned to those stops to be picked up in the morning and dropped off in the afternoon. Prior to software-based solutions, school districts solved SBRPs manually. In the late 1970s, researchers examined specific school districts and developed algorithms for solving SBRPs, considering the objectives of minimizing transportation costs and average student ride time (Bodin and Berman 1979). Various solution methods exist addressing one or more subproblems, including special-education routing (Russell and Morrel 1986, Braca et al. 1997); route generation and bus-route assignment (Spada et al. 2005, Simchi-Levi et al. 2014); route generation and bus-route assignment with school-bell time adjustment (Fugenschuh 2009, Bertsimas et al. 2019); and mixed-load route generation (Park et al. 2012, Lima et al. 2016).

Our work focuses on DPS's bus-route assignment problem, which is challenging for various reasons, including the large number of routes and buses, various start and end times for the routes, reposition times and distances between the routes, and uncertainty in

Figure 1. (Color online) SBRP Has Several Components, Including Stop Selection and Assignment of Students to Stops, Route Generation, and Bus-Route Assignment



Notes. Bell-time adjustment can be considered when generating routes or assigning buses to routes. The dashed lines in the bus-route assignment diagram represent the repositioning of buses from the terminal to the first stop of a route or between consecutively served routes. Our work focuses on the bus-route assignment subproblem.

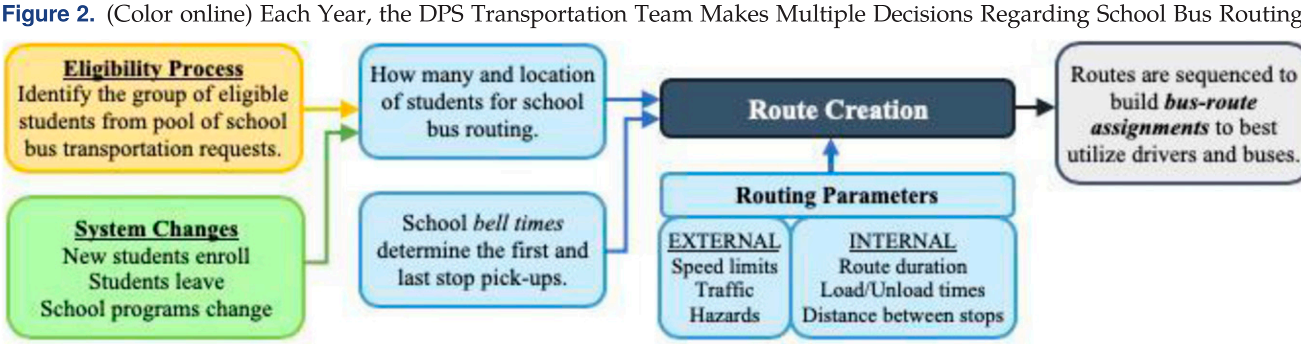
reposition times. While generating bus-route assignments, we consider a heterogenous bus fleet, multiple bus-terminal locations, and capacity and time constraints. We consider multiple objectives, such as minimizing the reposition mileage, the number of buses utilized (with the goal of reducing operational costs), delays (due to uncertainty in reposition times), using slack (or idle time) between routes, and "inconsistency"—that is, reducing the difference between a newly created and current (or previous) solution. The solution methods—namely, optimization models that are embedded into the decision support tool (DST)—are the results of a collaboration between a group of researchers from the Georgia Institute of Technology and data analysts and transportation supervisors from DPS. This work differs from other work in the literature because it considers multiple objectives (as well as numerous constraints), evaluates trade-offs between consistency and other objectives, and embeds the developed solution approaches into a fully functional decision-support tool, which has been successfully used in practice by DPS and can be adapted to other school systems.

Denver Public Schools System

DPS transportation route analysts develop bus-route assignments for a fleet of around 400 buses, transporting close to 40,000 K–12 students across 207 schools (Denver Public Schools Transportation 2018). The team determines bus-stop locations, assigns students to stops, creates a set of routes, and assigns buses to routes each academic year starting in April and ending in July. For the 2017–2018 academic year, DPS route analysts created 700–800 routes and assigned buses to routes to maximize the number of routes served per bus, while also picking up and dropping off students on time (Figure 2).

Each bus starts and ends its day at one of two terminals. This is a single-load problem because each route involves transporting students going to or departing from one school. We categorize the routes into two types: morning or afternoon. In the morning, each bus repositions from its terminal to the first stop of its first assigned route. After visiting all the stops and picking up students on its route, the bus takes the students to their schools by a planned drop-off time because students cannot be dropped off too early at

Figure 2. (Color online) Each Year, the DPS Transportation Team Makes Multiple Decisions Regarding School Bus Routing



their schools due to safety reasons. Then, the bus repositions to the first stop of the next route (if any) assigned to it. In the afternoon, each bus starts at a school to pick up students. After visiting all the stops on the route and dropping off students, it repositions to the school of the next route (if any) assigned to it. Finally, after serving all its assigned routes, the bus returns to its terminal. In our deterministic and stochastic models, morning and afternoon routes are considered separately due to the sufficiently long break between the routes; this helps to decrease the model size and run time.

While planning the routes and estimating their durations, DPS analysts utilize a conservative estimate of bus speed and account for potential travel-time uncertainties within each route. DPS shares information with families about the estimated arrival time of the bus at each stop. Drivers adjust their speed and try to arrive at each stop "on time" so that students are not accidentally missed because the bus is too early or late. Because the estimated and actual route times are typically very close to each other, we assume that route durations are known/deterministic.

There is often some uncertainty in the repositioning times between routes. In the deterministic model, we use average repositioning times as input and utilize the concept of slack to address this uncertainty. Slack time between two consecutive routes assigned to the same bus is equal to the second route's start time minus the first route's end time minus the (average) reposition time between those routes. That is, the slack is the extra time available while a bus repositions from one route to another. A slack that is too small may increase the chances of route delays. Hence, it is generally undesirable for the slack time to be too small. In the stochastic model, we explicitly model the uncertainty by considering various reposition-time scenarios.

Challenges

Many factors impacting school bus routes are outside the control of the transportation team. These factors include school closures and openings, new riders, families leaving or moving into the school district, students changing schools, road closures, rezoning, and city development. Even with minimal changes to such factors within the school system, DPS route analysts struggle to evaluate trade-offs and the feasibility of adjustments to bus-route assignments before implementation.

A major consideration in school bus-routing decisions is cost, including costs of bus procurement and maintenance, driver salaries, fuel (particularly due to repositioning between routes), and utilization of commercial school-bus transportation software. Prior to this collaborative project and the development of the DST, DPS analysts used a software package to generate routes and then manually created bus-route assignments. This tedious and time-consuming adjustment process left little time for review before the start of the new academic year. Manually adjusting the bus-route assignments often led to a cascading impact of potential infeasibilities, where one change could affect tens or hundreds of other assignments. The manual process also made what-if analyses of the bus-route assignments nearly impossible. For example, prior to this work, when a school wanted to change its bell time, there was no simple method to evaluate the overall impact on the bus-route assignments, feasibility, and cost.

Another key consideration in bus-route assignment decisions is scarce resources, such as buses and drivers, capacity constraints on the number of students each bus can accommodate, and accessible buses that must be assigned to routes with students using wheelchairs. If there are not enough buses or drivers, DPS has to rely on expensive third-party busing services to cover any routes not able to be served by the DPS bus fleet. Hence, scarce resources add to the challenge in the manual generation of feasible and low-cost bus-route assignments. The added expense of third-party services might be unnecessary or excessive, but, prior to this work, DPS analysts did not have enough time or bandwidth to determine whether the use of third-party services could be decreased or not.

Finally, DPS route analysts prefer "consistency" in bus-route assignments; that is, whenever possible, a route assigned to a particular bus/driver in the current solution should be assigned to the same bus/driver in a newly generated solution. Consistency helps minimize the impact on students, families, and drivers; makes implementation easier; and helps DPS meet its contractual obligations to drivers by limiting the changes to bus-route assignments. There are challenges in achieving consistency due to the various objectives and constraints that need to be considered, and there is often a trade-off between consistency and cost. We focus on overall cost minimization as a primary objective and include consistency as a secondary objective in our models by using adjustable parameters to help DPS develop and evaluate consistent bus-route assignments. Better route assignments result in more efficient use of the current bus fleet and available drivers without compromising service.

Solution Approach

We developed deterministic integer programming (D-IP) and stochastic integer programming (S-IP) models, which are embedded into the DST for bus-route assignment generation and for sensitivity analysis considering uncertainty in reposition times, respectively. The objective function has five main components—namely, the number of buses used, number of unserved routes, reposition miles, inconsistency (deviation from the current solution), and slack penalty (applied only if the slack time is below a specified threshold).

Model inputs include the following:

- Route data: For each route, ridership demand, route end time and duration, and estimated reposition time and miles between every pair of routes.
- Fleet data: For each bus, capacity, terminal location, and other specifications (e.g., wheelchair accessibility).
- Current DPS solution (i.e., current bus-route assignments).
- User-assigned parameters, including the following:
  - For the objective function components (the number of buses used, the number of unserved routes, reposition miles, inconsistency, and slack), the weights/penalties are represented by scalars (c, e, r, v, and s), respectively.
  - Lower and upper bounds on slack time.
  - Parameters defining the magnitude of the buffer added to repositioning times.

The models generate new bus-route assignments, considering time and capacity constraints, while aiming to minimize (expected) operational costs and penalties due to inconsistency (i.e., differences between the current and the newly generated assignments).

Key constraints are based on time and capacity, where each route is assigned to a bus that has the capability to serve it (e.g., enough capacity to accommodate all students on the route and wheelchair accessibility if students using wheelchairs are on the route), and assignments of consecutive routes to a bus must be serviced on time. Optional constraints include a lower bound on how early buses can leave the terminal and upper and lower bounds for the slack time. DPS analysts have the flexibility to turn these constraints on or off in the DST. We discuss further details, such as definitions and explanations of decision variables, input parameters, objectives, and constraints of the D-IP model, in Appendix A.

The S-IP model builds upon the D-IP model by considering uncertainties in reposition times and potential delays—that is, the possibility of a bus arriving at its destination after the scheduled arrival time. Although the D-IP model takes reposition times between routes as a deterministic input, the S-IP model considers multiple scenarios, each with an assigned probability, for reposition times between each pair of routes in the system and incorporates a penalty for the expected delay in the objective function. For a feasible solution, the model calculates the delay under each scenario and the total expected delay across all scenarios. For a given problem instance, we expect that the S-IP solution is likely to result in lower overall expected delay, on average, compared with the D-IP solution. We discuss further details, such as definitions and explanations of decision variables, input parameters, objectives, and constraints of the S-IP model, in Appendix B.

Bus-Route Assignment Decision Support Tool

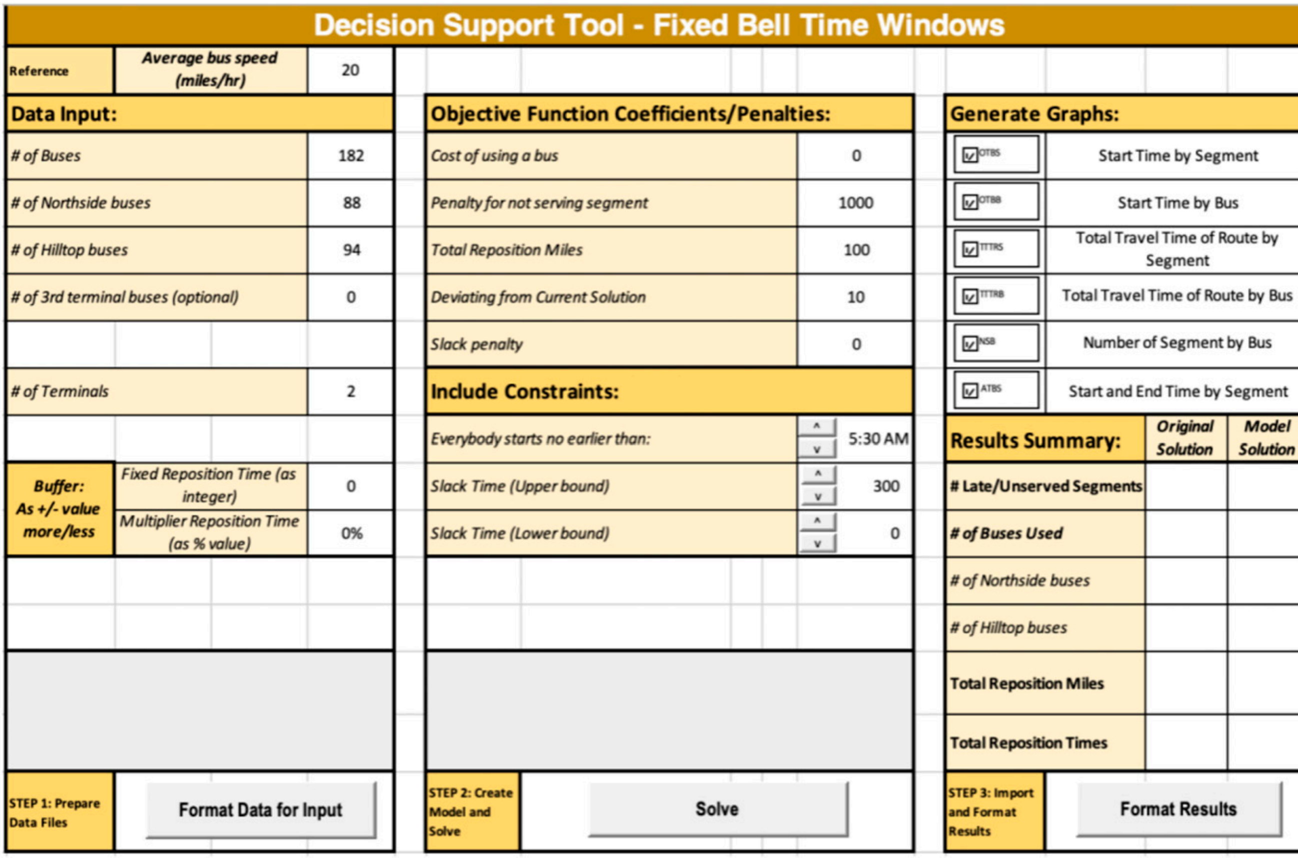
To aid DPS analysts in decisions related to school-bus routing, we developed an Excel-based DST with macros embedded using Microsoft Visual Basic for Applications. Populated with data from DPS, the DST employs the models coded by using a mathematical programming language, AMPL, enabling the DST to call any integer programming (IP) solver. The DST currently uses the GNU Linear Programming Kit, a solver freely available at https://www.gnu.org/software/glpk/, to keep the implementation and usage cost low; in the future, DPS could utilize other solvers if desired. The DST contains all functionality needed by the user to create and analyze bus-route assignments. Figure 3 provides a view of the main menu of the DST.

Decision Support Tool Inputs

The data input, collected via the main menu of the DST tool (Figure 3), includes bus-fleet mix (e.g., capacity and terminal location of each bus), route information (e.g., planned load, wheelchair load, start time, and duration), reposition times and miles between routes, number of terminals, number of buses by terminal, and information about the current bus-route assignment (i.e., current DPS solution). The user also inputs additional parameters, such as the penalties for the objective function components, which allow that user to consider trade-offs between cost and consistency while maintaining time and capacity feasibility.

The user can enter some or all of the reposition-time and mile values. If the reposition-time and mile information are not readily available for all route pairs, the DST provides two different distance functions for calculating reposition miles and times for a subset or all route pairs: the Google Maps Distance Matrix Application Programming Interface (API) and the Euclidean distance formula. The user also has the option to add "buffers" to the reposition times. The buffer can be added as an absolute value or as a fraction of the current reposition time, or by adjusting the bus speed. The bus speed is used to calculate the reposition time based on the reposition distance. This functionality of the DST enables DPS to adjust reposition times as needed (e.g., based on past experience or changes in traffic patterns or road conditions)

Figure 3. (Color online) The Main Menu Tab in the DST Enables the User to Prepare Model Input Files, Specify Model Parameters, Run the User-Defined Model, and Read in the Model Solution



Note. Other supplementary functions (not shown in the figure) include reposition time and distance matrix calculations using the Google Map Distance Matrix API or a Euclidean distance formula.

while creating new bus-route assignments or evaluating alternative solutions. In the future, DPS could utilize different time estimates considering the time of day.

Decision Support Tool Outputs

The DST output, in a format similar to what DPS analysts have historically used, includes the bus-route assignment solution generated by the model (Figure 4(a)) highlighting consistency, the reposition times and miles between routes (Figure 4(b)), and lists of buses utilized and routes (not) served. The DST allows the user to output select histogram plots and view summary statistics of the model solution. Appendix C shows example outputs in Figures C.1 and C.2.

Note that each route has a planned load, which can be greater than the maximum available bus capacity, because, typically, some eligible students choose not to use the bus. Therefore, the route-load input to the model is the minimum of the planned route load and the DPS-assigned bus capacity to this route.

Computational Study

To understand the impact and performance of the developed models in comparison with the DPS bus-route assignments, we completed various studies detailed below. DPS analysts provided the bus-route assignments implemented during the 2017–2018 academic year, which consisted of 358 morning and 350 afternoon routes. First, using the D-IP model, we created bus-route assignments with and without considering consistency. Then, we analyzed different combinations of weights (c, e, r, v, and s) for the five penalty components in the objective function (i.e., number of buses used, number of unserved routes, total reposition miles, inconsistency, and slack) to examine their impact. In the computational study, we coded the D-IP and S-IP models in Python and solved them with Gurobi Optimizer 8.1.

For each route pair (j, k), we first calculated the estimated reposition time t_{jk} (the reposition time from the end of route j to the beginning of route k) using the Google Maps Distance Matrix API and an estimated bus speed (Duran and Walkowicz 2013);

Figure 4. (Color online) The Output Solution Tab in the DST Displays the Bus-Route Assignment Generated by the D-IP Model

(a)

DPS Bus Number	DPS Bus Capacity	DPS Bus WC Capacity	Model Bus Number	Model Bus Capacity	Model Bus WC Capacity	Segment Number	Segment Load	Segment WC Load	Terminal
1026-N	72	0	1026-N	72	0	1	120	0	1
1035-N	72	0	1026-N	72	0	13	72	0	1
1028-N	72	0	1028-N	72	0	3	140	0	1
1028-N	72	0	1028-N	72	0	4	72	0	1
1029-N	72	0	1029-N	72	0	5	120	0	1

(b)

Start Time	Duration	End Time	Slack Time	Reposition Time (t → s)	Reposition Time (seg → seg)	Reposition Time (a → t)	Reposition Miles (t → s)	Reposition Miles (seg → seg)	Reposition Miles (a → t)
6:46 AM	59.00	7:45 AM	0.00	17.18712186			5.72904062		
7:52 AM	38.00	8:30 AM	3.00		3.649933254	26.90847116		1.216644418	8.969490385
7:27 AM	33.00	8:00 AM	0.00	13.65835595			4.552785317		
8:28 AM	17.00	8:45 AM	13.00		14.75631851	15.62126694		4.918772836	5.20708898
7:03 AM	37.00	7:40 AM	0.00	14.1952205			4.731740165		
7:57 AM	28.00	8:25 AM	1.00		15.35842701	26.83949897		5.119475669	8.946499658

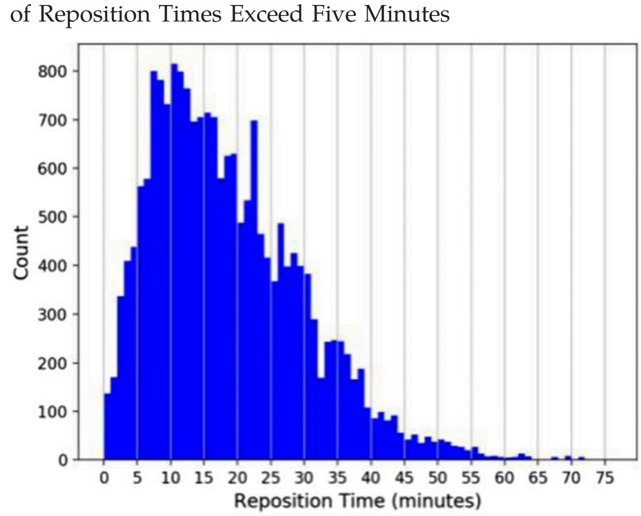
Notes. In panel (a), the output solution tab shows the current DPS solution and the solution generated by the model and highlights the differences (inconsistency) between them; the shaded cells indicate when the bus-route assignment is the same in both solutions. In panel (b), the time stamps and reposition information for the routes assigned to each bus.

see Appendix D for details in Figure D.1. We stored these values in matrix R_0, where each cell (j, k) in the matrix corresponds to t_{jk}. We ran the D-IP model using R_0 as input, with and without slack penalties. We also ran the D-IP model using different reposition-time matrices as input, which we created by modifying R_0 with the addition of buffers (an absolute value, α, and a fraction of the current reposition time, β) to each t_{jk}.

To understand the impact of reposition-mile uncertainty on the quality and robustness of the solutions, we generated multiple reposition-time scenarios (where each scenario corresponds to a particular set of realizations of the reposition times). First, note that 92.5% of reposition times are larger than five minutes (see Figure 5). For $t_{jk} > 5$, we modeled the percentage difference between the realized and expected reposition times using a triangular distribution (minimum = −10, mode = 0, and maximum = 200). This ensures that the realized reposition times are more likely to be longer (versus shorter) than the average, and the absolute deviations are higher when the expected reposition time is longer. To generate each scenario u, for each pair (j, k) we drew a number d from the distribution and set $t^u_{jk} = (1 + d/100)t_{jk}$. For very small repositioning times—that is, $t_{jk} \leq 5$—we added a delay of $(x \in 1,\ldots,5)$ minutes following a triangular distribution (minimum = 1, mode = 1, and maximum = 5). We generated 15 reposition-time scenarios with scenario u stored in matrix R^u, $u = 1,\ldots 15$.

For each D-IP solution, we computed the expected delay across these 15 reposition-time scenarios. Note that route delay is the positive difference between the actual and scheduled arrival time of the bus at the route's destination (if the bus arrives earlier than scheduled, the delay is zero). The S-IP model incorporates an expected delay penalty (ℓ) in the objective function to favor bus-route assignments that minimize expected delay across all scenarios.

Figure 5. (Color online) The Distribution of Estimated Reposition Times from Matrix R_0 Shows that More than 90% of Reposition Times Exceed Five Minutes



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Results and Discussion

The D-IP model focuses on the assignment of buses to routes with the goal of minimizing reposition mileage between routes and reducing the number of buses used, while ensuring timely pick-up and delivery of students. In the objective function, we set $c = 100$ (the penalty for the number of buses used), $e = 1{,}000$ (the penalty for not serving a route), and $r = 1$ (the penalty for total reposition miles) to ensure that the model always favors solutions that serve as many routes as possible.

Trade-Off Between Cost and Consistency

To understand the trade-off between cost and consistency, we ran the D-IP model with various penalties for deviating from the current solution (i.e., by setting the inconsistency penalty in the objective function to v = 0, 10, 25, 50, 100). In this part of the computational study, we set $s = 0$ (i.e., no penalty for slack) and used the average reposition-time estimates (R_0) as input (i.e., $\alpha = \beta = 0$).

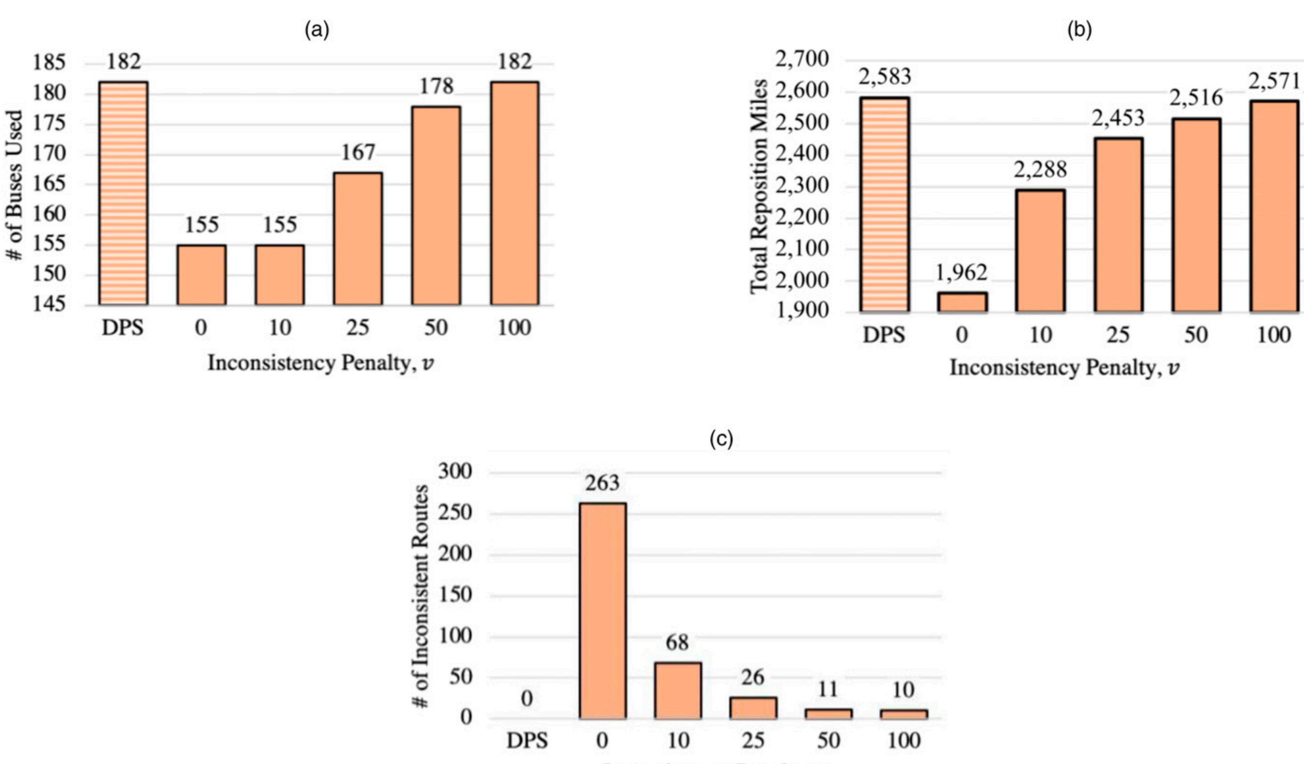
When we set $v = 0$, the D-IP solution for morning routes utilized 15% fewer buses and 24% fewer reposition miles compared with the DPS solution as shown in Figure 6. For the afternoon routes (see Appendix E for a summary of the results in Figure E.1), the D-IP solution utilized 19% fewer buses and 27% fewer reposition miles compared with the DPS solution. As we increased v, the consistency (similarity) between the D-IP solution and the DPS solution increased; the D-IP solution results in no more buses used and a smaller number of reposition miles compared with the DPS solution, while serving all the routes.

D-IP Solution Quality and Robustness Under Reposition-Time Uncertainty

In the D-IP model, we utilized two approaches to hedge against reposition-time uncertainty: (1) We used different reposition-time matrices as input, which we created by modifying R_0 with the addition of buffers to each t_{jk}; and (2) we ran the D-IP model using R_0 as input, with slack penalties.

We modified the reposition times by adding a "buffer" with the goal of reducing delays. The buffer is a combination (α, β), including a fixed value ($\alpha \geq 0$) and a fraction ($\beta < 1$) of the estimated reposition time. For example, suppose the reposition time between two routes is 20 minutes; (0, 0) represents no buffer, whereas (5, 0.2) represents a buffer of $5 + (0.2)(20) = 9$

Figure 6. (Color online) The Graphs Show the D-IP Model Solution for the Morning Routes with Objective Function Penalty Parameters $(c; e; r; s) = (100; 1{,}000; 1; 0)$



Notes. As v increases, the number of buses used (a) and the total reposition miles (b) increase, while the number of inconsistent routes decreases (c). Even a small v (i.e., $v \geq 25$) results in a solution with high consistency (over 90%) and utilizing significantly fewer buses compared with the DPS solution.

minutes added to the reposition time. The set of (α, β) combinations used were: $\{(0, 0), (0, 0.1), (0, 0.2), (5, 0), (5, 0.1), (5, 0.2), (10, 0), (10, 0.1), (10, 0.2)\}$.

To understand the impact of using buffers, we created eight different reposition-time matrices, $\bar{R}_1, \ldots, \bar{R}_8$, by adding different buffer values to the estimated reposition times stored in matrix R_0.

We solved the D-IP model by inputting each of these matrices; as before, we set the objective function penalties as $(c = 100, e = 1{,}000, r = 1)$. For each of the D-IP solutions, we computed the route delays under the 15 reposition-time scenarios $(R^u, u = 1, \ldots, 15)$ and the corresponding expected delay across all scenarios. Generally, as the buffer increased, (1) the total expected and maximum delay decreased, and (2) all routes could be served with fewer buses (except when $\alpha = 10$ in the morning results) than the DPS solution. Note that although the mean expected delay was quite low in the D-IP solution, the maximum expected delay could be higher compared with S-IP, but significantly lower than the maximum expected delay in the DPS solution. The morning and afternoon results with the D-IP using buffers are presented in Tables E.1 and E.2 in Appendix E.

In addition, the D-IP model includes a slack-time penalty in the objective function to influence the model to prefer bus-route assignments with a certain amount of slack. As we mention above, the slack time is equal to the planned start time of a route minus the reposition time minus the planned end time of the previous route assigned to the bus. If the slack is positive, it helps reduce delays in case of reposition times that are longer than expected.

To understand the impact of using the slack penalty, we prepared the following input for the D-IP model via preprocessing: For each pair of routes that can be feasibly served consecutively by the same bus, we set the slack penalty equal to $s \times \max\{0, \textit{threshold} - \textit{slack}\}$. Hence, if the slack between a pair of routes assigned to the same bus is less than a threshold, the slack penalty increases as the slack decreases. We set the *threshold* = 15, which incentivizes the model to prefer bus-route assignments with slack close to and above 15 minutes. We tested the D-IP model using R_0 as input with $s = 0, 10, 25, 50, 100$, where $s = 0$ corresponds to the base D-IP solution. For each of the D-IP solutions (found using one of the slack penalties), we computed the route delays under the 15 reposition-time scenarios $(R^u, u = 1, \ldots, 15)$ and the corresponding expected delay across all scenarios. In general, as the slack penalty increased, (1) the total expected and maximum delay decreased, and (2) all routes could be served (except when $s = 100$ in the morning routes), while the total number of reposition miles increased. Tables E.3 and E.4 in Appendix E display the morning and afternoon results for the D-IP solutions under different slack penalties. Note that DPS previously could not quantify the impact of including slack penalties in generating bus-route assignments.

Recall that the S-IP model incorporates an expected delay penalty (ℓ) in the objective function to favor bus-route assignments to minimize the expected delay across all scenarios. The total expected delays resulting from the S-IP and D-IP solutions (with buffers or slack penalties) are similar. Hence, in this application, D-IP solutions are generally robust under reposition-time uncertainty. Furthermore, the impact of buffers and slack penalties are generally similar; hence, depending on their knowledge and experience, DPS analysts can benefit from using slack penalties and buffers interchangeably, or in combination.

Implementation and Benefits

The process of developing the models and the DST sparked a robust data-validation process led by the district routing department and supported by information from DPS bus terminals. The DPS route analysts collaborated on updating the route data sets with the latest information from the bus terminals and departments across the district. The DST helped DPS quickly identify potential issues with bus-route assignments, given the strategic goal of minimizing reposition time and miles between routes to address changing traffic conditions across Denver.

The pilot testing on a subset of the bus routes during the 2017–2018 school year indicated that the DST solution would lead to an 8% decrease in buses used and a 20% decrease in reposition miles compared with the bus-route assignment implemented by DPS during that time frame. The DST enabled the transportation team to create high-quality bus-route assignments, as well as efficiently and effectively evaluate them (e.g., identify bus routes at risk for being late); doing these activities manually previously took several weeks or was not time-feasible at this scale and scope. DPS analysts used the DST for the planning of the 2018–2019 school year and the bus-route assignments outputted from the DST with minor modifications, which helped DPS start the new school year with a realistic and robust school bus-route assignment plan and reduced the need for contracted third-party services to cover bus-driver vacancies (considering the nationwide bus-driver shortage). The time savings in the planning process enabled the DPS transportation team to focus on other critical tasks, while helping to enable DPS to better prepare and transport students each academic year.

Lessons Learned

The school bus-routing problem is difficult to solve, especially for large school systems such as DPS. The solution approach presented in this paper provides a data-driven optimization approach to help DPS in creating high-quality school bus-route assignments, as well as utilizing the tool to evaluate alternatives—for example, alternative route options, adding buffers to repositioning times to reduce potential delays, changes to bell times, etc. The model employed in the DST is especially useful in evaluating the trade-offs between cost and "consistency" in bus-route assignments. The unique perspective and insight of the DPS analysts and transportation team helped with the implementation and integration of the DST into the route-planning process. The DST has a general design and could be easily adapted for use by other school systems. We hope that this successful implementation of operations research and analytics in the public sector and the benefits realized by DPS encourage more collaborations between the public-sector organizations and academia in applying analytics-based tools and approaches.

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Appendix A. D-IP Model Formulation

In this section, we describe the details of the D-IP model formulation that determines the optimal bus-route assignment, given a set of routes and the bus fleet. We present the definitions of the sets, inputs, parameters, preprocessing, and decisions variables first and then present and explain the model objectives and constraints. All time values are in minutes.

Sets

- B: Set of buses ($i = 1, \ldots, n$)
- S: Set of routes ($j = 1, \ldots, m$)
- S_0: Set of routes including 0 to represent the bus's terminal ($j = 0, 1, \ldots, m$)

Input Data

Route Data.

- l_j: Load of route $j \in S$ is the minimum of the current DPS assigned bus capacity and the planned route load
- l_j^w: Wheelchair load of route $j \in S$
- d_j: Duration of route $j \in S$ from first to last stop
- $\bar{e}_j$: End time of route $j \in S$ at its last stop
- $\overrightarrow{m}_{ij}$: Reposition miles from bus i's terminal to the start of route j, $i \in B$, $j \in S$
- m_{jk}: Reposition miles from the end of route j to the start of route k, $j \in S$, $k \in S$
- $\overleftarrow{m}_{ij}$: Reposition miles from end of route j to the terminal of bus i, $i \in B$, $j \in S$
- $\vec{t}_{ij}$: Reposition time from bus i's terminal to the start of route j, $i \in B$, $j \in S$
- t_{jk}: Reposition time from the end of route j to the start of route k, $j \in S$, $k \in S$
- $\overleftarrow{t}_{ij}$: Reposition time from end of route j to the terminal of bus i, $i \in B$, $j \in S$
- a_{jk}: Slack time if route k is served immediately after route j by the same bus

Fleet Data.

- c_i: Capacity of bus $i \in B$
- c_i^w: Wheelchair capacity of bus $i \in B$

User-Assigned Parameters.

- b: Value under which slack time is penalized in the objective function
- $\underline{w}$, $\bar{w}$: Lower and upper bound on slack time used in optional constraints
- p: Earliest departure time from all terminals
- $\alpha \geq 0$: Buffer value added to reposition times
- $0 \leq \beta < 1$: Buffer value as a fraction of reposition times
- c: Penalty for using a bus
- e: Penalty for not serving a route
- r: Penalty for reposition miles
- v: Penalty for deviating from the current bus-route assignments
- s: Penalty for small slack time between two consecutive routes

Consistency Parameters

The consistency parameters represent the current bus-route assignments in the model and are listed below. The lists, G_{i0j}, G_{ijk}, and G_{ij0}, represent the current bus-route assignment solution.

- $\hat{y}_{i0j} = 1$ for bus $i \in B$ that serves route $j \in S$ first in the current bus-route assignment solution, G_{i0j}.
- $\hat{y}_{ijk} = 1$ for bus $i \in B$ that serves routes $j, k \in S$, $j \neq k$ sequentially in the current bus-route assignment solution, G_{ijk}.
- $\hat{y}_{ij0} = 1$ for bus $i \in B$ that serves route $j \in S$ last in the current bus-route assignment solution, G_{ij0}.

Preprocessing

To improve the model run time and reduce complexity, we perform preprocessing to determine the capacity and time-feasible bus-route assignment pairs based on the set of routes, S, and buses, B, as two 0–1 matrices called F_{ij} (which routes can be served by each bus) and F_{ijk} (which pairs of routes can be consecutively served by the same bus).

Bus $i \in B$ could potentially serve route $j \in S$ if the bus has sufficient capacity to serve the route and meets other restrictions of the route (e.g., wheelchair-accessible); namely, $F_{ij} = 1$ if $c_i \geq l_j$ and $c_i^w \geq l_j^w$.

Bus $i \in B$ can serve route $j \in S$ then route $k \in S$ if the bus-route assignment is both capacity and time feasible, i.e., $F_{ijk} = 1$ if $F_{ij} = F_{ik} = 1$ and if there is sufficient time for the bus to travel from the end of route j to the start location of route k in time to begin serving route k (i.e., $\bar{e}_j + ((1+\beta)^* t_{jk} + \alpha) + d_k \leq \bar{e}_k$).

Decision Variables

- $y_{i0j} = 1$, bus $I \in B$ serves route j first; 0, otherwise, for $i \in B$, $j \in S$, if $F_{ij} = 1$.

- $y_{ijk} = 1$, bus i serves routes j and then route k; 0, otherwise, for $i \in B$, $j \in S$, $k \in S$ if $F_{ij} = 1$.
- $y_{ij0} = 1$, bus $i \in B$ serves route j last; 0, otherwise, for $i \in B$, $j \in S$, if $F_{ij} = 1$.
- $x_j = 1$, route j is not served by any buses; 0, otherwise, for $j \in S$.

Objective Function and Constraints

The D-IP is a multiobjective model. We list and explain each component of the objective function below.

$$\min c\left[\sum_{i\in B,\, j\in S: F_{ij}=1} y_{i0j}\right] + , \tag{A.1}$$

$$e\left[\sum_{j\in S} x_j\right] + , \tag{A.2}$$

$$r\left[\sum_{i\in B,\, j\in S: F_{ij}=1}\left(\vec{m}_{ij}\, y_{i0j} + \overleftarrow{m}_{ij}\, y_{ij0} + \sum_{k\in S: F_{ijk}=1} m_{jk} y_{ijk}\right)\right] + , \tag{A.3}$$

$$v\left[\sum_{\substack{i\in B,\, j\in S:\\ (i,j)\in G_{i0j}}} (\hat{y}_{i0j} - y_{i0j}) + \sum_{\substack{i\in B,\, j\in S:\\ (i,j)\in G_{ij0}}} (\hat{y}_{ij0} - y_{ij0}) + \sum_{\substack{i\in B, j\in S,\, k\in S:\\ (i,j,k)\in G_{ijk}}} (\hat{y}_{ijk} - y_{ijk})\right] + , \tag{A.4}$$

$$s\left[\sum_{i\in B,\, j\in S,\, k\in S: F_{ij}=F_{ik}=F_{ijk}=1} \max\{0, (b - a_{ij})\} y_{ijk}\right]. \tag{A.5}$$

Component (A.1) captures the penalty for using a bus. Component (A.2) represents the penalty for not serving a route. Component (A.3) is the penalty for repositioning in miles, including miles from the bus's assigned terminal to the start of its first route, between routes, and from the end of the last route returning to the bus's assigned terminal. Component (A.4) represents the penalty for deviating from the current bus-route assignments. This penalty enables minimizing the changes between the new bus-route assignments and the current solution. Component (A.5) penalizes positive slack time below a specified threshold with the purpose of penalizing lower slack-time values. Any objective can be removed from consideration by the model by setting the corresponding penalty to zero.

$$\text{s.t.} \sum_{j\in S: F_{ij}=1} y_{i0j} \le 1,\ \forall i \in B, \tag{A.6}$$

$$\sum_{l\in S_0: F_{il}=F_{ilj}=1} y_{ilj} = \sum_{k\in S_0: F_{ik}=F_{ijk}=1} y_{ijk},\ \forall i\in B,\ j\in S,\ \textit{if}\ F_{ij} = 1, \tag{A.7}$$

$$x_j + \sum_{k\in S_0: F_{ij}=F_{ik}=F_{ikj}=1} y_{ikj} = 1,\ \forall j \in S. \tag{A.8}$$

Constraint (A.6) specifies that each bus can only serve one route first. Then, Constraint (A.7) serves as the "flow-in flow-out" constraint—that is, if bus i serves route j, then it needs to next serve another route k or return to the terminal. Finally, Constraint (A.8) requires that either a bus or the dummy bus serves each route.

Optional constraints, listed below, that the user can enable in the DST include the following: a lower bound on how early buses can leave the terminal (A.9) and upper and lower bounds for the slack time ((A.10) and (A.11)). The departure-time constraint helps the model consider solutions that align with driver work schedules. In regard to the slack-time constraints, if the slack is too small, this may increase the chances of route delays; hence, it could be desirable to put a lower bound on slack to minimize potential delays. If the slack is large and there are no delays, the bus waits until it needs to move to the start of the next route; hence, it is possible to put an upper bound on slack to limit wait times. Note that Constraints (A.10) and (A.11) utilize an indicator function, denoted as $\hat{A}(A)$, where if the condition A is true then $\hat{A}(A) = 1$; otherwise, $\hat{A}(A) = 0$.

$$(p + \vec{t}_{ij})\, y_{i0j} \le \bar{e}_j - d_j,\ \forall i\in B,\ j\in S,\ \text{if}\ F_{ij} = 1, \tag{A.9}$$

$$y_{ijk} \le \hat{A}(a_{ij} \ge \underline{w}),\ \forall i\in B,\ j\in S,\ k\in S,\ \text{if}\ F_{ijk} = 1, \tag{A.10}$$

$$y_{ijk} \le \hat{A}(a_{ij} \le \bar{w}),\ \forall i\in B,\ j\in S,\ k\in S,\ \text{if}\ F_{ijk} = 1. \tag{A.11}$$

Appendix B. S-IP Model Formulation Details

In the S-IP model, we incorporate uncertainty by using multiple reposition-time matrices as scenarios. Let U define the set of reposition-time scenarios where each scenario, $u \in$ U, corresponds to a random reposition-time matrix, R^u. The remaining sections describe the updated notation used in the S-IP formulation based on the D-IP model formulation. Some of the D-IP model parameters and decision variables are used in the S-IP model formulation and not reiterated here.

Updated Input Data

Route Data.

- $\vec{t}^{\,u}_{ij}$: Reposition time from bus i's terminal to start of route j for scenario u, $i \in B$, $j \in S$, $u \in$ U.
- t^u_{jk}: Reposition time from the end of route j to start of k for scenario u, $i \in B$, $j \in S$, $u \in$ U.
- $\vec{t}^{\,u}_{ij}$: Reposition time from end of route j to terminal of bus i for scenario u, $i \in B$, $j \in S$, $u \in$ U.
- M: Big-M value for the time tracking constraints equal to $\max_{j\in S} \bar{e}_j$.

User-Assigned Parameters.

- ℓ: Penalty for delay.

Preprocessing

As outlined in the D-IP preprocessing section in Appendix A, for S-IP, we update the two 0–1 matrices, F_{ij} and F_{ijk}, with capacity and time-feasible bus-route assignment pairs based on the set of routes, S, buses, B, and scenarios, U.

Bus $i \in B$ can serve route $j \in S$, then route $k \in S$ if the bus-route assignment is both capacity- and time-feasible—i.e., $F_{ijk} = 1$ if $F_{ij} = F_{ik} = 1$—and if there is sufficient time for the bus to travel from the end of route j to the start location of route k in time to begin serving route k for at least one scenario u (i.e., $\bar{e}_j + ((1+\beta)^* t^u_{jk} + \alpha) + d_k \le \bar{e}_k$).

Auxiliary Variables

- T^u_j = start time of route $j \in S$ based on scenario $u \in$ U.

- $T_j^{u'}$ = end time of route $j \in S$ based on scenario $u \in$ U.
- δ_j^u = nonnegative delay at the end of route $j \in S$ based on scenario $u \in$ U.
- ε^u = expected delay across scenario $u \in$ U.

Objective Function and Constraints

In the S-IP model, each reposition-time scenario applies to feasible bus-route assignment pairs (i.e., the bus can serve the route pair on time with the reposition time from at least one of the scenarios). Thus, there can be bus-route assignment pairs that are not time-feasible for some scenarios. The S-IP model determines the set of bus-route assignment pairs (i.e., the bus-route assignment solution) that minimizes the expected delay across all scenarios. We calculate the expected delay by using the auxiliary variables, which keep track of delay for each feasible bus-route assignment pair for all scenarios. The weighted cost for expected delay is shown below.

$$\ell \sum_{u \in \mathrm{U}} \varepsilon^u = \ell \sum_{u \in \mathrm{U}} p^u \left(\sum_{j \in S} \delta_j^u \right). \tag{B.1}$$

The probabilities for each scenario $u \in$ U are p^u. The formulation of the S-IP model is an extension of the D-IP model formulation to include the auxiliary variable calculations, as shown below.

$$\text{s.t.} \sum_{j \in S: F_{ij}=1} y_{i0j} \leq 1, \ \forall i \in B, \tag{B.2}$$

$$\sum_{l \in S_0: F_{il}=F_{ilj}=1} y_{ilj} = \sum_{k \in S_0: F_{ik}=F_{ijk}=1} y_{ijk}, \ \forall i \in B, \ j \in S, \ if \ F_{ij} = 1, \tag{B.3}$$

$$x_j + \sum_{k \in S_0: F_{ij}=F_{ik}=F_{ikj}=1} y_{ikj} = 1, \ \forall j \in S, \tag{B.4}$$

$$T_j^u \geq (\bar{e}_j - d_j) \sum_{i \in B: F_{ij}=1} y_{i0j} - M\left(1 - \sum_{i \in B: F_{ij}=1} y_{i0j}\right), \ \forall u \in \mathrm{U}, \ j \in S, \tag{B.5}$$

$$T_k^u \geq T_j^{u'} + t_{jk}^u \sum_{i \in B: F_{ij}=F_{ik}=F_{ijk}=1} y_{ijk} - M\left(1 - \sum_{i \in B: F_{ij}=F_{ik}=F_{ijk}=1} y_{ijk}\right), \ \forall u \in \mathrm{U}, \ j \in S, \ k \in S, \tag{B.6}$$

$$T_j^{u'} \geq \bar{e}_j, \ \forall u \in \mathrm{U}, \ j \in S, \tag{B.7}$$

$$T_j^{u'} \geq T_j^u + d_j, \ \forall u \in \mathrm{U}, \ j \in S, \tag{B.8}$$

$$\delta_j^u \geq T_j^u - \bar{e}_j + d_j, \ \forall u \in \mathrm{U}, \ j \in S, \tag{B.9}$$

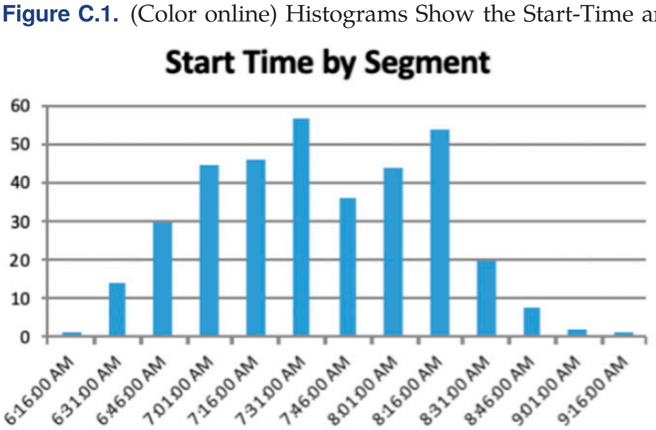
$$\delta_j^u \geq 0, \ \forall u \in \mathrm{U}, \ j \in S. \tag{B.10}$$

Constraints (B.2)–(B.4) are the same as constraints (A.6)–(A.8) from the D-IP model formulation. Constraints (B.5)–(B.9) track the start and departure times of each route $j \in S$ in the model solution for each scenario. Constraint (B.5) sets the start-time auxiliary variable equal to the planned start time of the route if a bus $i \in B$ serves route $j \in S$ first. Otherwise, Constraint (B.6) sets the start-time auxiliary variable equal to a calculated start time plus reposition if bus i serves route j then route k. In the case where a route j is not served by any bus, the big-M value makes Constraints (B.5) and (B.6) nonbinding. Constraints (B.7) and (B.8) represent the end time of route $j \in S$ as the maximum between Constraint (B.7) (the lower bound based on the route end time) and Constraint (B.8) (the calculated end time using the start-time auxiliary variable). Finally, Constraint (B.9) tracks the delay of route $j \in S$ based on any delays of routes (if any) served before route j.

Appendix C. Additional Model Outputs

Each illustration below shows an example of additional outputs from a model solution. Figures C.1 and C.2 provide more details about the solution that is beneficial to DPS analysts for comparing different solutions. Note that the term segment and route are synonymous.

Figure C.1. (Color online) Histograms Show the Start-Time and End-Time Distributions of the Routes (i.e., Segments)



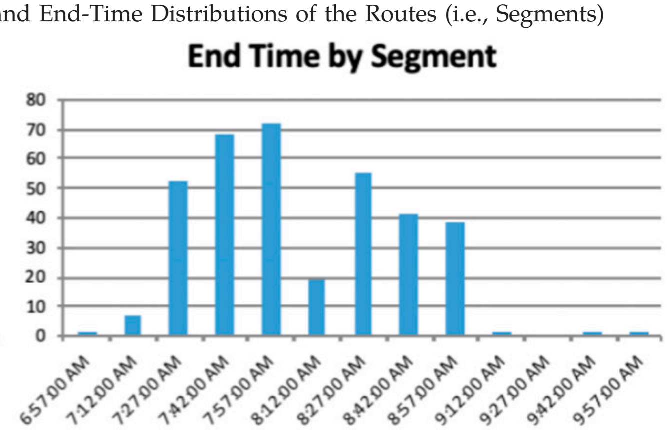
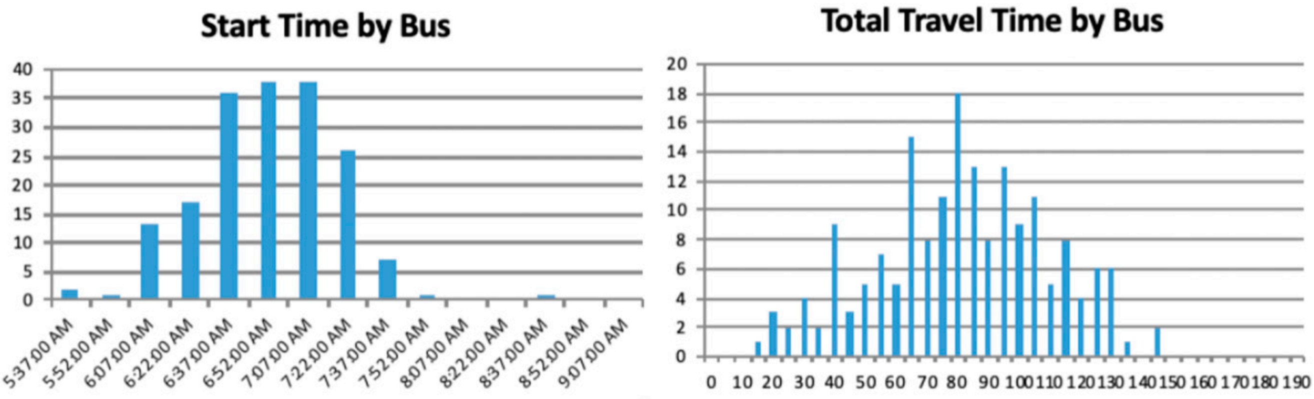


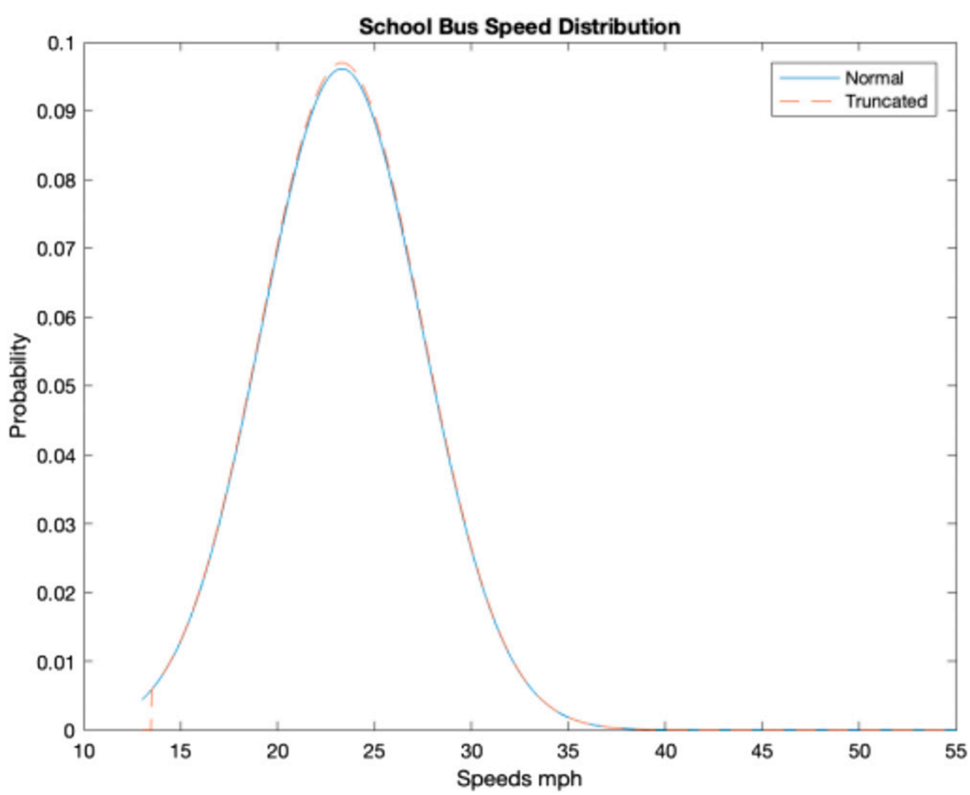
Figure C.2. (Color online) Histograms Show the Start-Time and Total Travel-Time Distribution of the Buses—The Length of the Bus-Route Assignments for Each Bus from Departure to Arrival at Its Terminal



Appendix D. Bus-Speed Distribution

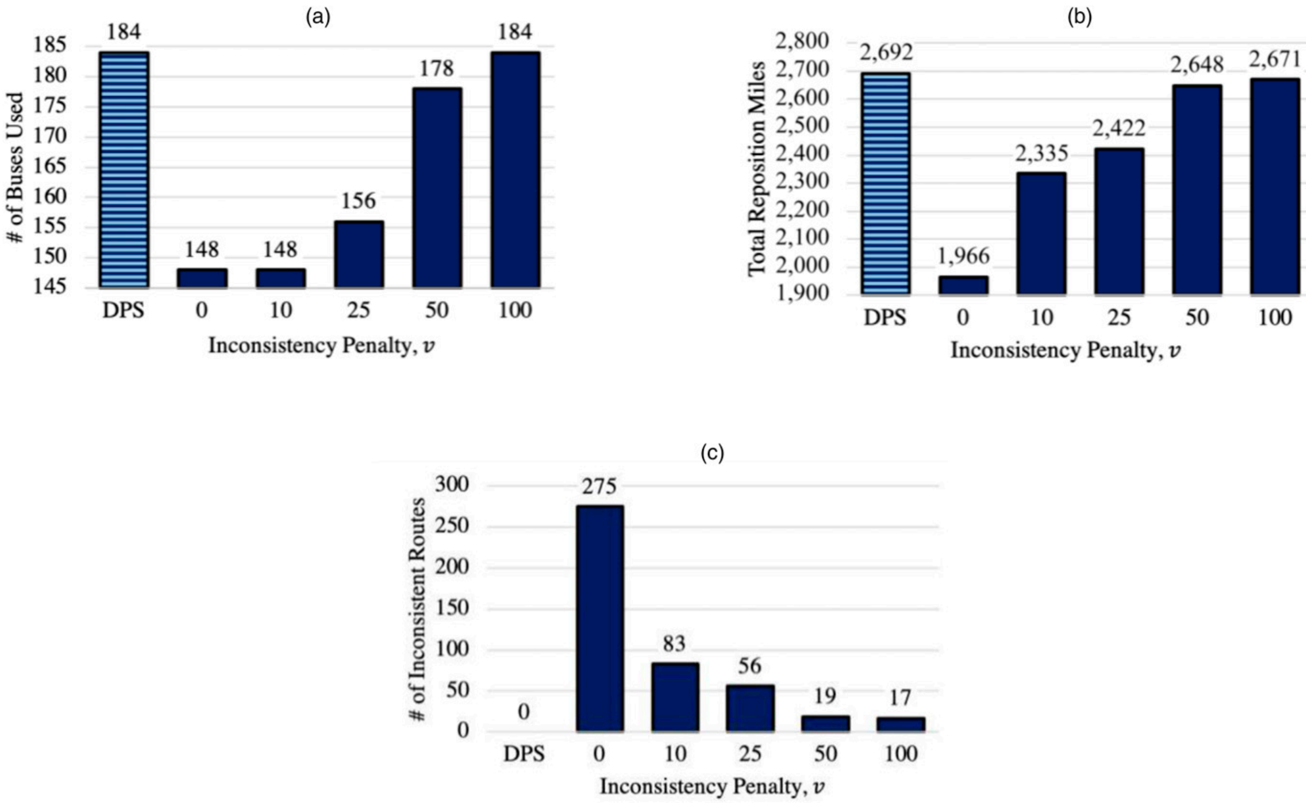
To estimate reposition times, we utilized travel-distance estimates and bus speed, as Duran and Walkowicz (2013) report for various states, including Colorado. We modeled the bus speed as a truncated normal distribution with minimum, mean, and maximum of 13.5, 23.31, and 53.77, respectively, and standard deviation of 4.15 in miles per hour (mph). We use the average bus speed of 23.31 mph for the reposition-time calculations for the D-IP model runs.

Figure D.1. (Color online) A Truncated Normal Distribution for the Expected Bus-Speed Distribution Is Used to Generate Reposition-Time Matrices



Appendix E. Computational Study: Supplemental Results

Figure E.1. (Color online) The Graphs Show the D-IP Model Solution for the Afternoon Routes with Objective Function Penalty Parameters (c; e; r; s) = (100; 1,000; 1; 0)



Notes. As v increases, the number of buses used (a) and the total reposition miles increase (b), while the number of inconsistent routes decreases (c). Even a small (i.e., $v \geq 25$) results in a solution with high consistency (over 80%) and utilizing significantly fewer buses compared with the DPS solution.

Table E.1. The Table Summarizes the Model Results for the 2017–2018 Morning Routes: D-IP Solutions (= 0; = 50), and DPS Solution

	Morning summarized results of D-IP solutions using buffers				
Solution	Buffer values (α, β)	Number of buses	Number of unserved routes	Total reposition miles	Expected delay: minimum, mean, standard deviation, maximum
D-IP with R_0	(0,0)	155	0	1,962	0, 2.1, 4, 29
D-IP with R_1	(0,0.1)	159	0	1,970	0, 1.6, 2.2, 10
D-IP with R_2	(0,0.2)	161	0	1,981	0, 1.3, 2.1, 14
D-IP with R_3	(5,0)	171	0	2,164	0, 0.7, 1.6, 9.5
D-IP with R_4	(5,0.1)	174	0	2,169	0, 0.5, 1.3, 7.5
D-IP with R_5	(5,0.2)	176	0	2,178	0, 0.3, 0.9, 6.3
D-IP with R_6	(10,0)	182	3	2,247	0, 0.2, 0.8, 5.3
D-IP with R_7	(10,0.1)	182	5	2,207	0, 0.09, 0.5, 5.3
D-IP with R_8	(10,0.2)	182	8	2,146	0, 0.07, 0.5, 5.3
S-IP	—	172	0	2,129	0, 0.06, 0.2, 1.3
DPS	—	182	0	2,583	0, 4.4, 12.4, 89

Note. For the morning routes, as the buffer increases, some routes become time-infeasible to serve, and all the buses are needed to serve the remaining time-feasible routes, leading to a slight decrease in the number of reposition miles.

Table E.2. The Table Summarizes Model Results for the 2017–2018 Afternoon Routes: D-IP Solutions Based on Different Inputs for Reposition Times, S-IP Solution (s = 0; = 50), and DPS Solution

	Afternoon summarized results of D-IP solutions using buffers				
Solution	Buffer values (α, β)	Number of buses	Number of unserved routes	Total reposition miles	Expected delay: minimum, mean, standard deviation, maximum
D-IP with R_0	(0,0)	148	0	1,966	0, 2, 3.5, 17
D-IP with R_1	(0,0.1)	149	0	1,983	0, 1.6, 3.1, 17
D-IP with R_2	(0,0.2)	150	0	1,993	0, 1.1, 2, 10
D-IP with R_3	(5,0)	157	0	2,050	0, 0.5, 1.5, 9
D-IP with R_4	(5,0.1)	160	0	2,075	0, 0.4, 1.3, 8.6
D-IP with R_5	(5,0.2)	160	0	2,080	0, 0.4, 1.2, 7.6
D-IP with R_6	(10,0)	166	0	2,192	0, 0.4, 1.3, 10.1
D-IP with R_7	(10,0.1)	167	0	2,205	0, 0.25, 1, 9.2
D-IP with R_8	(10,0.2)	170	0	2,223	0, 0.16, 0.6, 3.7
S-IP	—	155	0	2,096	0, 0.03, 0.1, 0.8
DPS	—	184	0	2,693	0.06, 20, 41, 189

Table E.3. The Table Summarizes Model Results for the 2017–2018 Morning Routes: D-IP Solutions Based on Different Slack Penalties, S-IP solution (s = 0; = 50), and DPS Solution

	Morning summarized results of D-IP solutions using slack penalty				
Solution	Number of buses	Number of unserved routes	Total reposition miles	Expected delay: minimum, mean, standard deviation, maximum	Expected slack: minimum, mean, standard deviation, maximum
D-IP with $s = 0$	155	0	1,962	0, 2.1, 4, 29	0, 12, 11, 58
D-IP with $s = 10$	179	0	2,323	0, 0.15, 0.6, 5.1	3.5, 19, 11, 64
D-IP with $s = 25$	182	0	2,372	0, 0.2, 0.7, 5.1	3.1, 19, 11, 64
D-IP with $s = 50$	182	0	2,381	0, 0.2, 0.8, 5.1	3.1, 19, 10.5, 54
D-IP with $s = 100$	182	1	2,399	0, 0.15, 0.6, 4.8	3.2, 19.2, 11, 64
S-IP	172	0	2,129	0, 0.06, 0.2, 1.3	0.08, 18, 11, 59
DPS	182	0	2,583	0, 4.4, 12.4, 89	0.1, 16, 13, 78

Table E.4. The Table Summarizes Model Results for the 2017–2018 Afternoon Routes: D-IP Solutions Based on Different Slack Penalties, S-IP Solution (s = 0; = 50), and DPS Solution

	Afternoon summarized results of D-IP solutions using slack penalty				
Solution	Number of buses	Number of unserved routes	Total reposition miles	Expected delay: minimum, mean, standard deviation, maximum	Expected slack: minimum, mean, standard deviation, maximum
D-IP with $s = 0$	148	0	1,966	0, 2, 3.5, 17	0, 19, 23, 141
D-IP with $s = 10$	167	0	2,296	0, 0.3, 1.2, 9	3, 22, 20, 133
D-IP with $s = 25$	174	0	2,333	0, 0.2, 0.8, 6	5, 23, 21, 133
D-IP with $s = 50$	177	0	2,363	0, 0.2, 0.7, 6	5, 23.5, 20, 133
D-IP with $s = 100$	178	0	2,366	0, 0.2, 1, 9	4, 24, 20, 133
S-IP	155	0	2,096	0, 0.03, 0.1, 0.8	1.1, 21, 21, 138
DPS	184	0	2,693	0.06, 20, 41, 189	0, 25, 24, 157

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