POLYTECHNIQUE MONTRÉAL

Département de mathématiques et de génie industriel
MTH1102(D) - Calcul II
Exercices supplémentaires - Réponses

Question 1

a) $\vec{r}(t) = 2\cos(t)\vec{i} + 2\sin(t)\vec{j} + 4[\sin^2(t) - \cos^2(t)]\vec{k}$, $0 \leq t \leq 2\pi$. Puisque $\vec{r}(0) = 2\vec{i} - 4\vec{k} = \vec{r}(2\pi)$, cette courbe est fermée.

b) $\displaystyle\oint_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{rot}\vec{F}\cdot d\vec{S} = \iint_D \left(y\vec{j} + (x^2 - y^2)\vec{k}\right)\cdot\left(2x\vec{i} - 2y\vec{j} + \vec{k}\right) dA = -8\pi.$

Question 2

a) $\displaystyle\Phi = \iint_{S_1} \operatorname{rot}\vec{F}\cdot d\vec{S} + \iint_{S_2} \operatorname{rot}\vec{F}\cdot d\vec{S}.$

b) $\displaystyle\iint_S \operatorname{rot}\vec{F}\cdot d\vec{S} = \oint_C \vec{F}\cdot d\vec{r} = \iint_D \operatorname{rot}\vec{F}\cdot d\vec{S}.$

c) $\displaystyle\Phi = \iint_D \operatorname{rot}\vec{F}\cdot d\vec{S} = \iint_D (-2ye^2\,\vec{k})\cdot\vec{k}\,dA = 0.$

Question 3

$$\Phi = \iint_S \vec{F}\cdot dS = -\iiint_E \operatorname{div}\vec{F}\,dV = -3\int_0^{2\pi}\int_0^{\pi/2}\int_0^3 \rho^4\sin(\phi)\,d\rho d\phi d\theta = -\frac{1458}{5}\pi.$$

Question 4

a) $\displaystyle\operatorname{vol}(B) = \iiint_B dV = \int_0^{2\pi}\int_0^6\int_r^{42-r^2} r\,dzdrd\theta = 720\pi.$

b) $\displaystyle\Phi = -(\Phi_S - \Phi_P) = -\left(\iiint_B (1+2+5)\,dV - \Phi_P\right) = -(\operatorname{vol}(B) - \Phi_P) = -(5760\pi - 6264\pi) = 504\pi.$

Question 5

a) $\displaystyle m = \iiint_E \alpha\,dV = \frac{\alpha}{3}\iiint_E \operatorname{div}(x\,\vec{i} + y\,\vec{j} + z\,\vec{k})\,dV = \frac{\alpha}{3}\iint_S (x\,\vec{i} + y\,\vec{j} + z\,\vec{k})\cdot d\vec{S}.$

b) $m = \frac{8\pi^2}{3}$.

Question 6

a) Vrai
b) Faux
c) Faux
d) Faux
e) Vrai
