Statistical Distributions of Traffic Characteristics

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Introduction

- There is considerable analytic value in modeling the time between the arrivals of successive vehicles
- The most simplistic approach to vehicle arrival modeling is to assume that all vehicles are equally or uniformly spaced.
- This results in what is termed a deterministic, uniform arrival pattern.
- Under this assumption, if the traffic flow is $360\ veh/h$, the number of vehicles arriving in any 5-minute time interval is 30 and the headway between all vehicles is 10 seconds (because h will equal $3600/q$).
- However, actual observations show that such uniformity of traffic flow is not always realistic because some 5-minute intervals are likely to have more or less traffic flow than other 5-minute intervals.
- This is due to the fact that the passing of vehicles at a cross-section is to a certain extent a matter of chance.
- Thus a representation of vehicle arrivals that goes beyond the deterministic, uniform assumption is often preferred.

Poisson Model

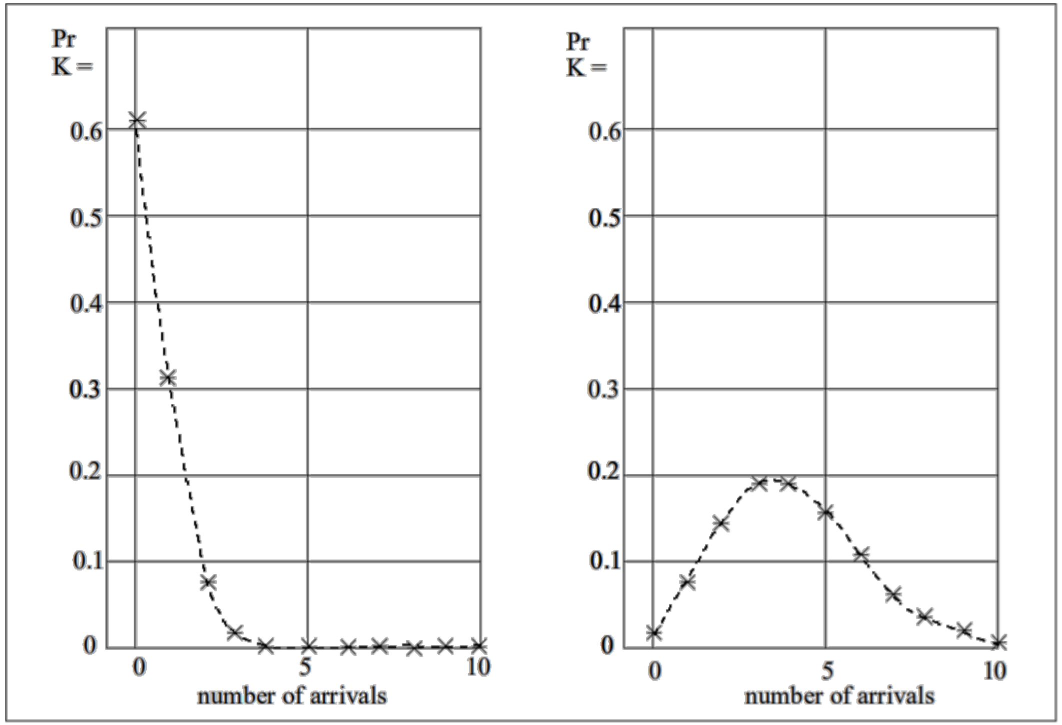
- If drivers are considered free (as opposed to constrained) in their choices, they will behave independently from each other.
- This implies that the passing of a cross-section becomes a pure random phenomenon.
- In general, this will be the case if there is ***relatively little traffic*** present (a small volume and density) and if there are no upstream disturbances, such as signalized intersections, that result in a special ordering of the vehicles in the stream.
- The problem then becomes one of selecting a probability distribution that is a reasonable representation of observed traffic arrival patterns.
- An example of such a distribution is the Poisson distribution, which is expressed as: where:

$$P(n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

  - $P(n)$ is the probability of having n vehicles arriving in time t,
  - λ is the average vehicle flow or arrival rate in vehicles per unit time,
  - t represents the duration of the time interval over which vehicles are counted,
  - e is the base of the natural logarithm ($e = 2.718$).

Poisson Model

- A special property of the Poisson distribution is that the variance equals the mean.
- This property can be used to test in a simple way if the Poisson process is a suitable model: from a series of observations, one can estimate the mean and the variance. If the variance over the mean does not differ too much, then it is likely that Poisson is an adequate model.



Probability function of Poisson with $\Delta t = 20s$; left $q = 90veh/h$; right $q = 720veh/h$.

Poisson Model

Example 1: Vehicle arrivals as a poisson process

An observer counts 360 veh/h at a specific highway location. Assuming that the arrival of vehicles at this highway location is Poisson distributed, estimate the probabilities of having 0, 1, 2, 3, 4, and 5 or more vehicles arriving over a 20-second time interval.

Poisson Model

Example 1: Vehicle arrivals as a poisson process

An observer counts 360 veh/h at a specific highway location. Assuming that the arrival of vehicles at this highway location is Poisson distributed, estimate the probabilities of having 0, 1, 2, 3, 4, and 5 or more vehicles arriving over a 20-second time interval.

Solution 1

λ=360 veh/h, or 0.1 (veh/s).

Using the Poisson equation with t = 20 s, the probabilities of having exactly 0, 1, 2, 3, and 4 vehicles arrive

$$P(n)=\frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

$$P(0)=\frac{(0.1\times 20)^0 e^{-0.1(20)}}{0!}=\underline{\underline{0.135}}$$

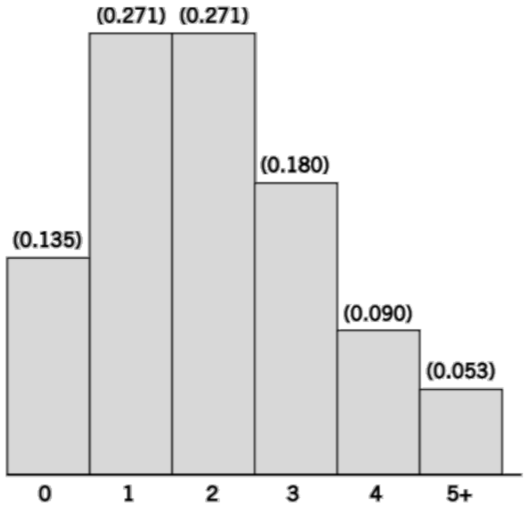
$$P(1)=\frac{(0.1\times 20)^1 e^{-0.1(20)}}{1!}=\underline{\underline{0.271}}$$

$$P(2)=\frac{(0.1\times 20)^2 e^{-0.1(20)}}{2!}=\underline{\underline{0.271}}$$

$$P(3)=\frac{(0.1\times 20)^3 e^{-0.1(20)}}{3!}=\underline{\underline{0.180}}$$

$$P(4)=\frac{(0.1\times 20)^4 e^{-0.1(20)}}{4!}=\underline{\underline{0.090}}$$

$$\begin{aligned}P(n\geq 5)&=1-P(n<5)\\&=1-0.135-0.271-0.271-0.180-0.090\\&=\underline{\underline{0.053}}\end{aligned}$$



Histogram of the Poisson distribution for λ = 0.1 vehicles per second.

Poisson Model

Example 2: Vehicle arrivals as a poisson process

Traffic data are collected in 60-second intervals at a specific highway location as shown in Table.

Assuming the traffic arrivals are Poisson distributed and continue at the same rate as that observed in the 15 time periods shown, what is the probability that six or more vehicles will arrive in each of the next three 60-second time intervals (12:15 P.M. to 12:16 P.M., 12:16 P.M. to 12:17 P.M., and 12:17 P.M. to 12:18 P.M.)?

Time period	Observed number of vehicles
12:00 P.M. to 12:01 P.M.	3
12:01 P.M. to 12:02 P.M.	5
12:02 P.M. to 12:03 P.M.	4
12:03 P.M. to 12:04 P.M.	10
12:04 P.M. to 12:05 P.M.	7
12:05 P.M. to 12:06 P.M.	4
12:06 P.M. to 12:07 P.M.	8
12:07 P.M. to 12:08 P.M.	11
12:08 P.M. to 12:09 P.M.	9
12:09 P.M. to 12:10 P.M.	5
12:10 P.M. to 12:11 P.M.	3
12:11 P.M. to 12:12 P.M.	10
12:12 P.M. to 12:13 P.M.	9
12:13 P.M. to 12:14 P.M.	7
12:14 P.M. to 12:15 P.M.	6

Observed Traffic Data

Poisson Model

Solution 2

- According to the Table, a total of 101 vehicles arrive in the 15-minute period from 12:00 P.M. to 12:15 P.M.
- Thus the average arrival rate, λ, is 0.112 veh/s. (101/(15*60)=0.112)
- To find the probabilities of exactly 0, 1, 2, 3, 4, and 5 vehicles arriving.

$$P(0)=\frac{(6.733)^0 e^{-6.733}}{0!}=\underline{\underline{0.0012}}$$

$$P(1)=\frac{(6.733)^1 e^{-6.733}}{1!}=\underline{\underline{0.008}}$$

$$P(2)=\frac{(6.733)^2 e^{-6.733}}{2!}=\underline{\underline{0.027}}$$

$$P(3)=\frac{(6.733)^3 e^{-6.733}}{3!}=\underline{\underline{0.0606}}$$

$$P(4)=\frac{(6.733)^4 e^{-6.733}}{4!}=\underline{\underline{0.102}}$$

$$P(5)=\frac{(6.733)^5 e^{-6.733}}{5!}=\underline{\underline{0.137}}$$

$$\rightarrow P(n\le 5)=\sum_{i=0}^{5}P(n)$$
$$=0.0012+0.008+0.027+0.0606+0.102+0.137$$
$$=0.3358$$

$$\downarrow$$

$$P(n\ge 6)=1-P(n\le 5)$$
$$=1-0.3358$$
$$=0.6642$$

$$\rightarrow P(n\ge 6)\text{ for three successive time periods}=\prod_{t_i=1}^{3}P(n\ge 6)$$
$$=(0.6642)^3$$
$$=\underline{\underline{0.293}}$$

Limitations of the Poisson Model

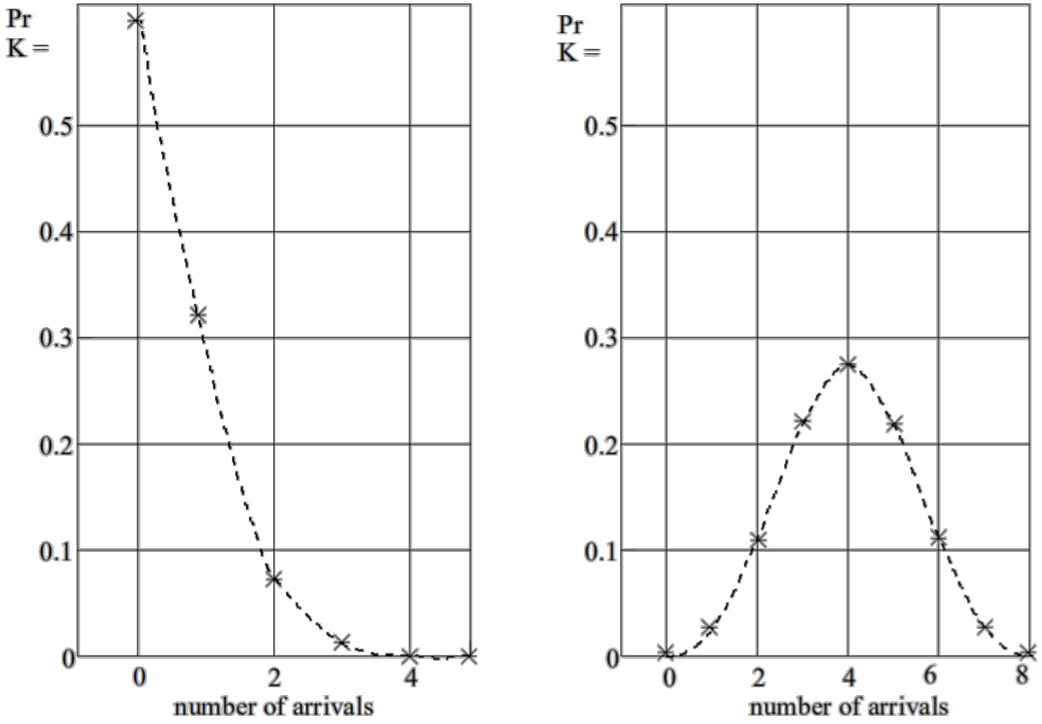
- Empirical observations have shown that the assumption of Poisson-distributed traffic arrivals is most realistic in lightly congested traffic conditions.
- As traffic flows become heavily congested or when traffic signals cause cyclical traffic stream disturbances, other distributions of traffic flow become more appropriate.
- The primary limitation of the Poisson model of vehicle arrivals is the constraint imposed by the Poisson distribution that the mean of period observations equals the variance.
- For example, the mean of period-observed traffic in Example 2 is 6.733 and the corresponding variance, σ^2 , is 7.210. Because these two values are close, the Poisson model was appropriate for this example.
- If the variance is significantly greater than the mean, the data are said to be over-dispersed.
- If the variance is significantly less than the mean, the data are said to be under-dispersed.
- In either case the Poisson distribution is no longer appropriate, and another distribution should be used

Binomial distribution

- When the flow of a traffic stream increases, more and more vehicles form platoons (clusters, groups), and the Poisson distribution is no longer valid.
- A model that is suitable for this situation is the model of a so-called Binomial-process, with probability function:

$$\Pr\{K=k\} = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for} \quad k = 0, 1, ..., n$$

- The binomial distribution describes the number of 'successes' k in n independent trials, at which the probability of success per trial equals p.
- Unfortunately, this background does not help to understand why it fits the arrival process considered here.
- In the case of a binomial distribution, the variance over the mean is smaller than 1.



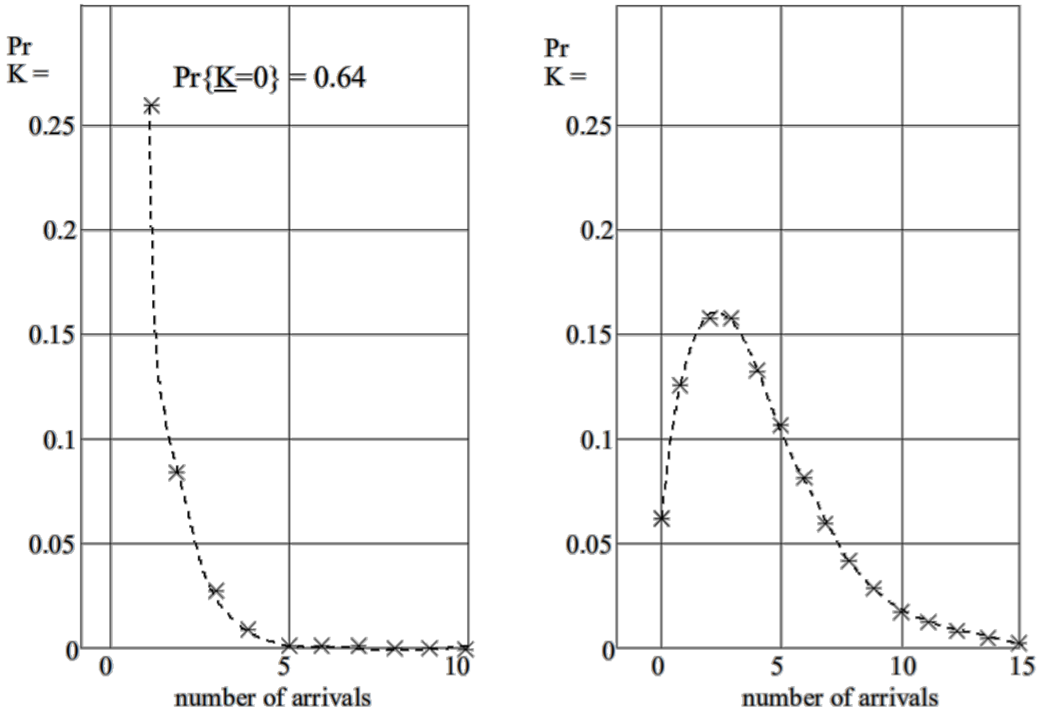
Binomial probability function with $\Delta t = 20s$; left $q = 90veh/h$; right $q = 720veh/h$.

Negative Binomial Distribution

- It has been mentioned that the Poisson distribution does not fit the traffic conditions downstream of a signalized intersection.
- When this is the case, one can state that high and low intensities follow each other.
- The variance of the number of arrivals then becomes relatively large, leading to variance over mean being larger than 1.
- In that case the model of a Negative Binomial distribution is adequate :

$$\Pr\{K = k\} = \binom{k+n-1}{k} p^n (1-p)^k \quad \text{for} \quad k = 0, 1, 2, \ldots$$

- As with the Binomial-process a traffic flow interpretation of the Negative-Binomial-process is lacking.



Negative binomial probability function with $\Delta t = 20s$; left $q = 90veh/h$; right $q = 720veh/h$.

Parameter Estimation

- From observations one calculates the sample mean m and the sample variance s^2.
- From these two parameters follow the estimations of the parameters of the three probability functions:
  - Poisson: $\hat{\mu} = m$
  - Binomial: $\hat{p} = 1 - s^2/m$ and $\hat{n} = m^2/(m - s^2)$
  - Negative binomial: $\hat{p} = m/s^2$ and $\hat{n} = m^2/(s^2 - m)$

Distribution	**Mean**	**Variance**	**St.dev**	**Relative St.dev.**
Poisson	μ	μ	$\sqrt{\mu}$	$1/\sqrt{\mu}$
Binomial	np	$np\,(1-p)$	$\sqrt{np\,(1-p)}$	$\sqrt{(1-p)\,/np}$
Negative binomial	$n\,(1-p)\,/p$	$n\,(1-p)\,/p^2$	$p\sqrt{n\,(1-p)}$	$1/\sqrt{n\,(1-p)}$

Mean, variance, standard deviation and, relative standard deviation of the three arrival distributions.

Applications

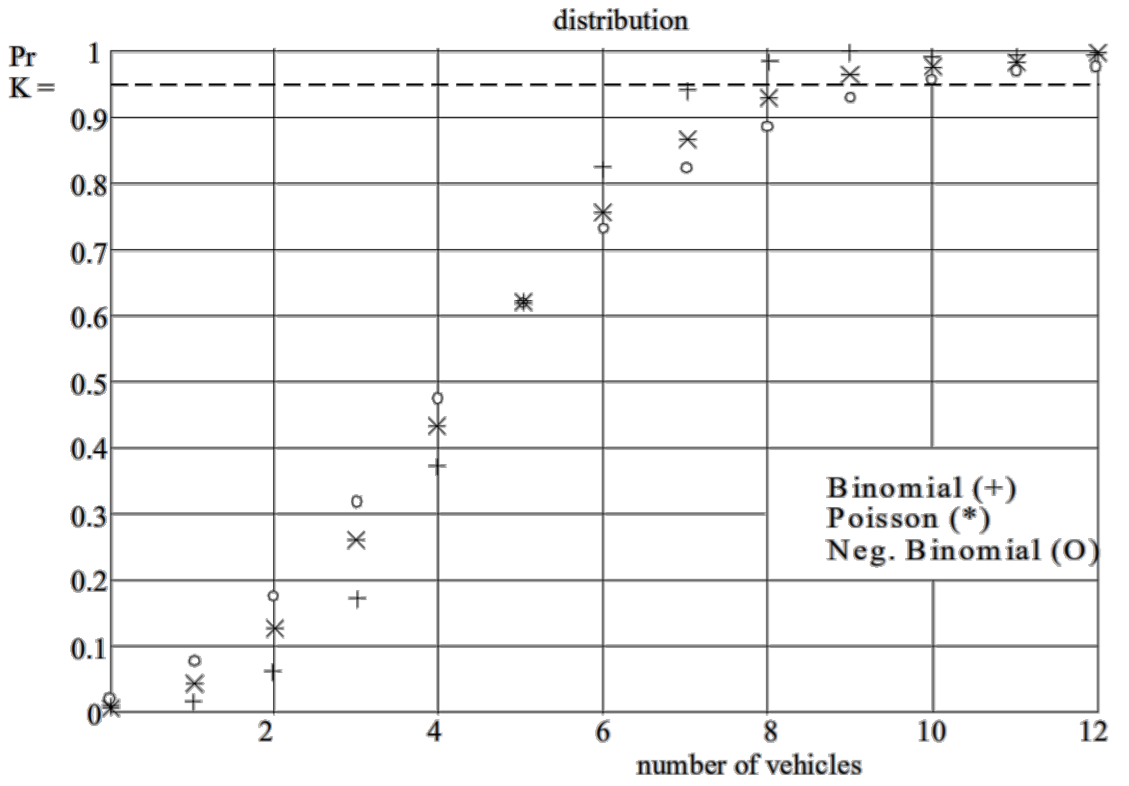
Example 3: Length of left-turn lane

- One has to determine the length of a lane for left turning vehicles.
- In the peak hour the traffic flow of the left turning vehicles equals 360 veh/h and the period they are confronted with red light is $50\ s$.
- Suppose the goal is to guarantee that in 95% of the cycles the length of the lane is sufficient.

Applications

Solution 3: Length of left-turn lane

- In 50 s will arrive on the average (50/3600) x 360 = 5 veh.
- From these two parameters follow the estimations of the parameters of the three probability functions:
  - Poisson: $\hat{\mu} = 5$
  - Binomial with $s^2 = 2.5$: $\hat{p} = 1 - 2.5/5 = 0.5$ and $\hat{n} = 25/(5 - 2.5) = 10$
  - Negative binomial with $s^2 = 7.5$: $\hat{p} = 5/7.5 = 0.667$ and $\hat{n} = 25/(7.5 - 5) = 10$
- With these parameter values the probability functions, and the distributions can be calculated.
- From the graph can be read that the 95-percentile of:
  - Binomial = 7,
  - Poisson = 9,
  - Negative Binomial = 10.
- These differences are not large but on the other hand one extra car requires 7 to 8 extra m of space.



Results of the estimated Binomial, Poisson and Negative-Binomial distributions

Applications

Example 4: Probe-vehicles

- Suppose one has to deduce the state of the traffic stream at a $2\ km$ long road section from probe-vehicles that broadcast their position and mean speed over the last km.

  It is known that on the average 10 probe vehicles pass per hour over the considered section. The aim is to have fresh information about the traffic flow state every 6 minutes. The question is whether this is possible.

Applications

Solution 4: Probe-vehicles

- It is likely that probe vehicles behave independently, which implies the validity of the Poisson-distribution.
- Per 6 minutes the average number of probes equals: $(6/60) \cdot 10 = 1$ probe.
- The probability of 0 probes in 6 minutes then equals: $e^{-1} = 0.37$
- This seems to be much too large for a reliable system.
- If one decides to update the traffic flow information every 20 minutes

$$\hat{\mu} = \left(\frac{20}{60}\right) * 10 = 3.33$$

- Therefore, the probability of zero probes equals: $e^{-3.33} = 0.036$
- This might be an acceptable probability of failure.

Exponential Distribution

- The assumption of Poisson vehicle arrivals also implies a distribution of the time intervals between the arrivals of successive vehicles (time headway).
- To show this, note that the average arrival rate is:

$$\lambda = \frac{q}{3600}$$

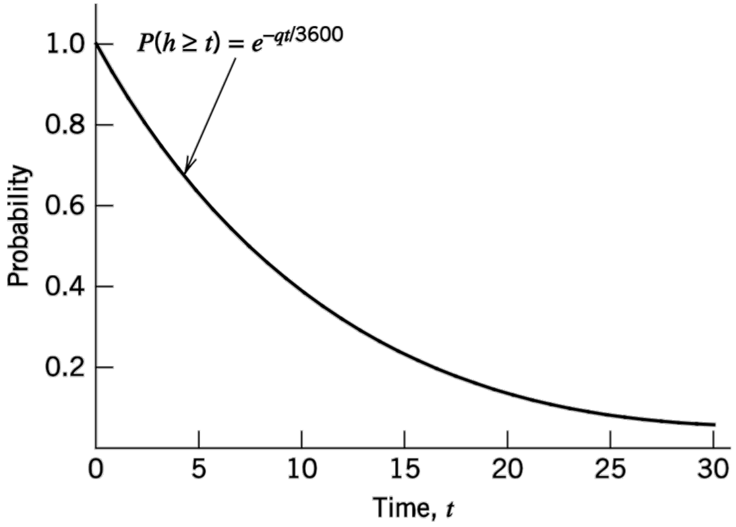
- Where
  - λ is the average vehicle arrival rate in veh/s,
  - q is flow in veh/h,
  - Number of seconds per hour is equal to 3600.
- Substituting this into the Poisson Model gives:

$$P(n) = \frac{(qt/3600)^n \, e^{-qt/3600}}{n!}$$

- Note that the probability of having no vehicles arrive in a time interval of length t, $P(0)$, is equivalent to the probability of a vehicle headway, h, being greater than or equal to the time interval t.

$$P(0) = P(h \geq t) = e^{-qt/3600}$$

- This distribution of vehicle headways is known as the negative exponential distribution and is often simply referred to as the *exponential distribution*.



Exponentially distributed probabilities of headways greater than or equal to t, with q = 360 veh/h.

Exponential Distribution

- In more general terms:

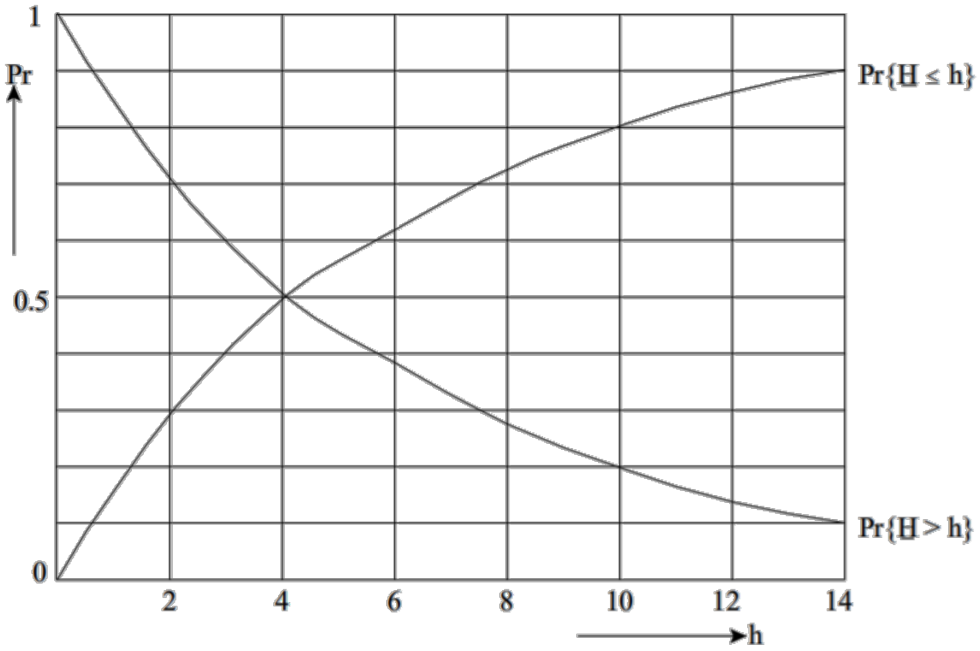
$$\Pr\{H > h\} = S(h) = e^{-qh}$$

The probability that a stochastic variable H is larger than a given value h.

- This is called the survival probability or the survival function $S(h)$.
- On the other hand, the complement is the probability that a stochastic variable is smaller than a given value ➔ the distribution function.
- Consequently, the distribution function of headways corresponding to the Poisson-process is:

$$\Pr\{H \leq h\} = P(h) = 1 - e^{-qh}$$

- This is called the *exponential distribution function*.



Survival function S (h) and distribution function P (h) of an exponential distribution, $q = 600\ veh/s$

Exponential Distribution

- Note that, for the exponential distribution the standard deviation divided by the mean, equals 1.

- The probability density function (p.d.f.) of the distribution function $P(h) = \Pr\{H \leq h\}$ is calculated by differentiation:

$$p(h) = \frac{d}{dh} P(h) = qe^{-qh}$$

The mean value becomes ➔

$$\mu = \int_0^\infty hp(h)\,dh = \int_0^\infty hqe^{-qh} dh = \frac{1}{q}$$

➔ The mean headway μ equals the inverse of the intensity q

The variance of the headways becomes ➔

$$\sigma^2 = \int_0^\infty (h-\mu)^2\, p(h)\,dh = \frac{1}{q^2}$$

➔ The variation coefficient (the standard deviation divided by the mean) = 1

Exponential Distribution

Example 5: Headways and the exponential distribution

Consider the traffic situation in Example 1 (360 veh/h). Again, assume that the vehicle arrivals are Poisson distributed. What is the probability that the headway between successive vehicles will be less than 8 seconds, and what is the probability that the headway between successive vehicles will be between 8 and 10 seconds?

Exponential Distribution

Solution 5

- This expression gives the probability that the headway will be less than 8 seconds as:

$$P(h < t) = 1 - e^{-qt/3600}$$
$$= 1 - e^{-360(8)/3600}$$
$$= \underline{\underline{0.551}}$$

- Compute the probability that the headway will be greater than or equal to 10 seconds:

$$P(h \geq t) = e^{-qt/3600}$$
$$= e^{-360(10)/3600}$$
$$= \underline{\underline{0.368}}$$

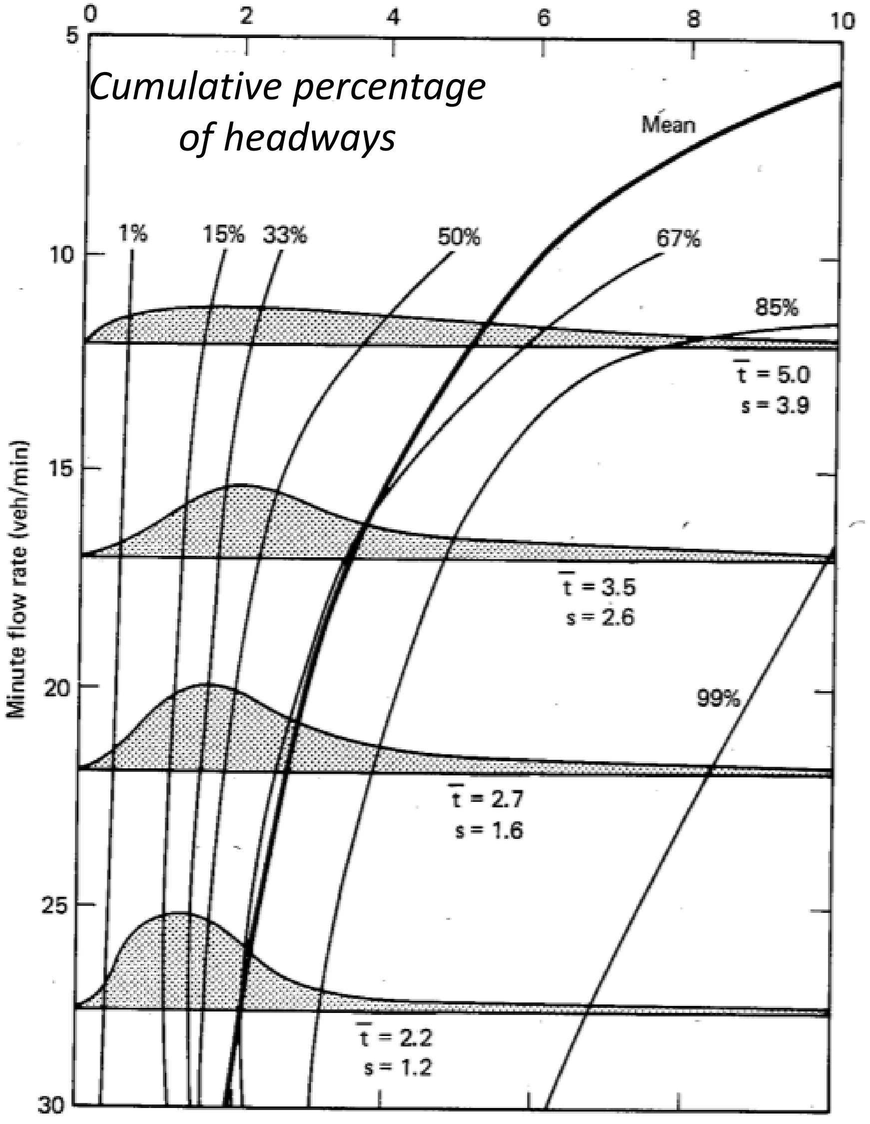
- So the probability that the headway will be between 8 and 10 seconds is:

$$1 - 0.551 - 0.368 = 0.081$$

Real Data and Headway Distribution

- Figure shows headways collected at a specific location for periods of time representing different flow situations.
- The four traffic flow levels depicted are for 10-14, 15-19, 20-24, and 25-29 vehicles per minute.
- Individual headways are rarely less than 0.5 seconds (1-2%).
- Individual headways are rarely over 10 s, unless the minute flow rate is below 15 veh/min.
- The median headway is always less than the mean. They converge as the flow rate increases.
- The mean headway tracks the 67 cumulative percentile curve.
- The ratio of standard deviation to the mean time headway approaches 1 under low flow rate conditions but decreases continuously as the flow rate increases.

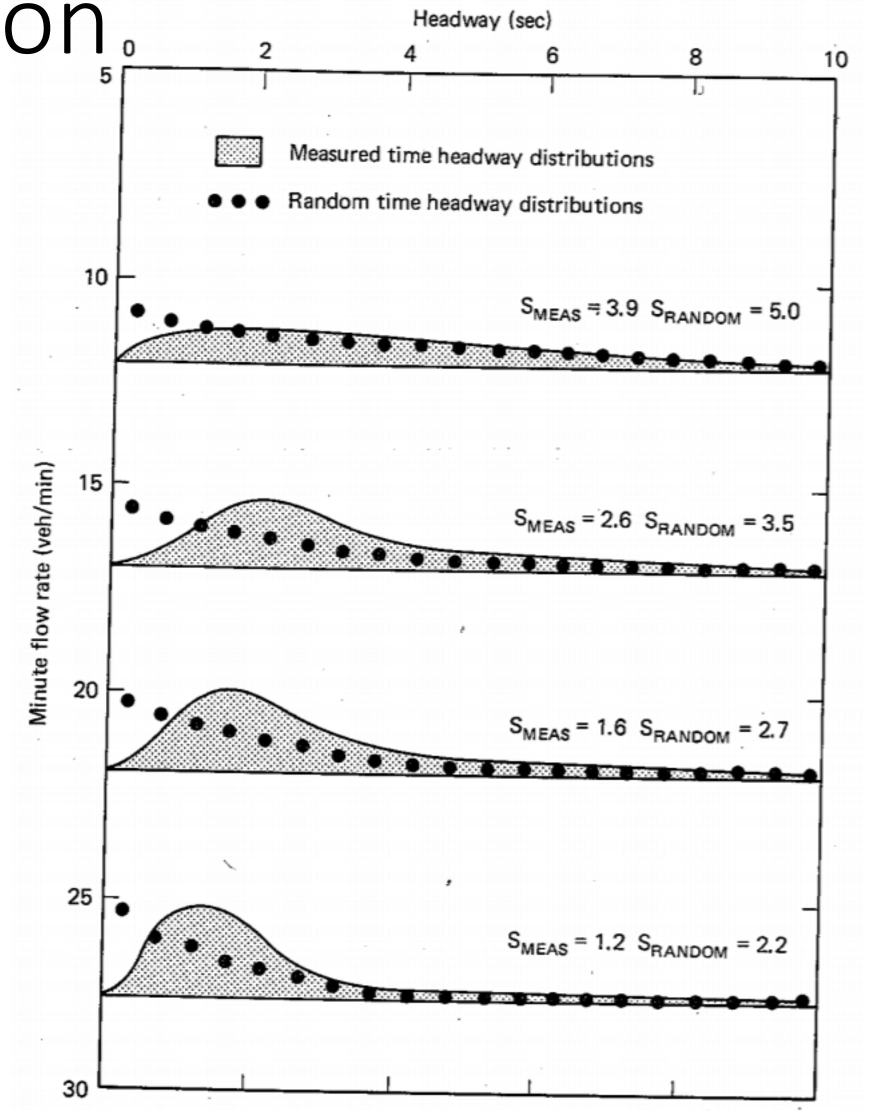
Mean time headways



Observed time headway distributions

Real Data and Exponential Distribution

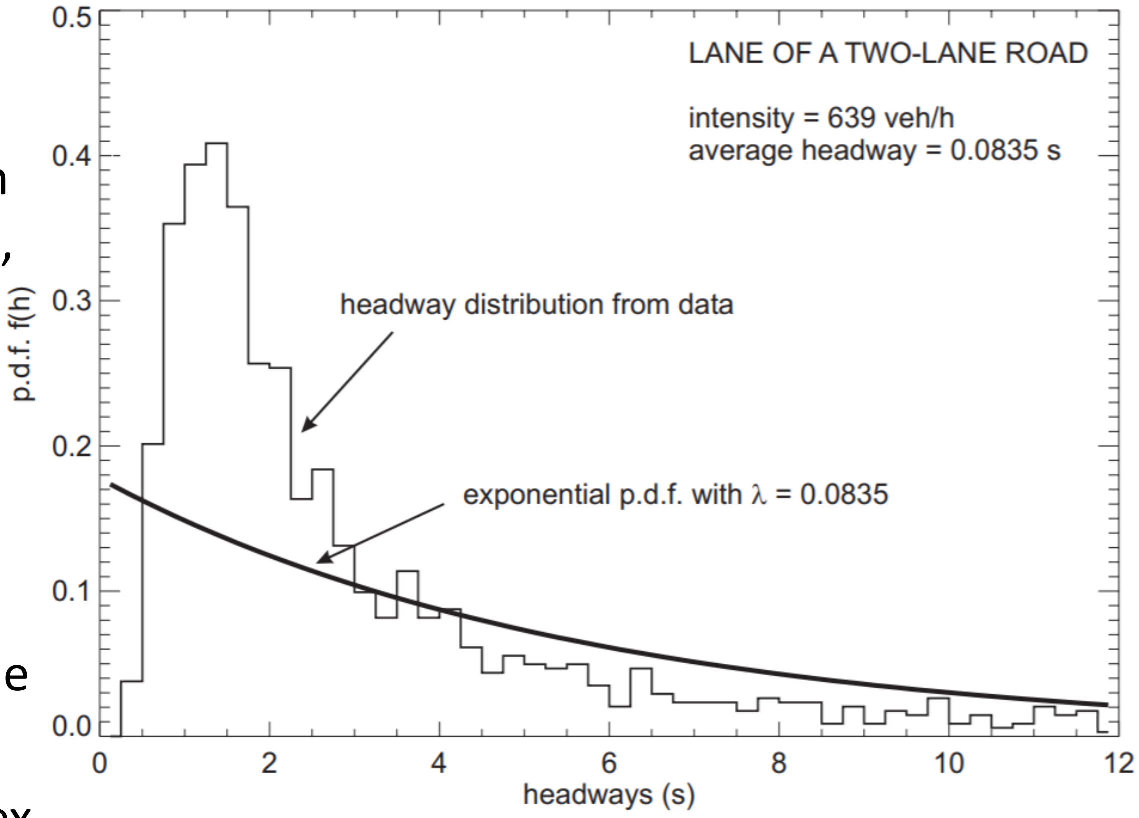
- The smallest headways are the most likely to occur and probabilities consistently decrease as the headway increases.
- The two distributions are very different, particularly under higher flow conditions.
- The theoretical probabilities are higher for intervals less than 1 s and lower for headways between 1.5 s to 4.5 s.
- The standard deviation for the measured distribution is always less than the standard deviation of the corresponding random distribution but appears to be converging at lower flow rate levels.



Observed time headway distributions and the exponential headway distributions

Real Data and the Exponential Distribution

- In general, the exponential distribution (ED) of the headways is a good description of reality at low flow rate and unlimited overtaking possibilities.
- If both above-mentioned conditions are not fulfilled, then there are interactions between the vehicles in the stream, leading to driving in platoons.
- In that case the ED is fitting reality badly.
- The minimum headways in a platoon are clearly larger than zero, whereas according to the ED the probability of extreme small headways is relatively large.
- The differences between the ED and reality have led to the use of other headway models at higher intensities
- Next, we will discuss some simple and a few more complex alternatives for the ED model.



Histogram of observed headways compared to the exponential probability density function

Alternatives for the Exponential Distribution

Shifted Exponential Distribution

- The shifted exponential distribution is characterized by a minimum headway h_m, leading to the distribution function:

$$\Pr\{H \le h\} = 1 - e^{-\lambda(h-h_m)}$$

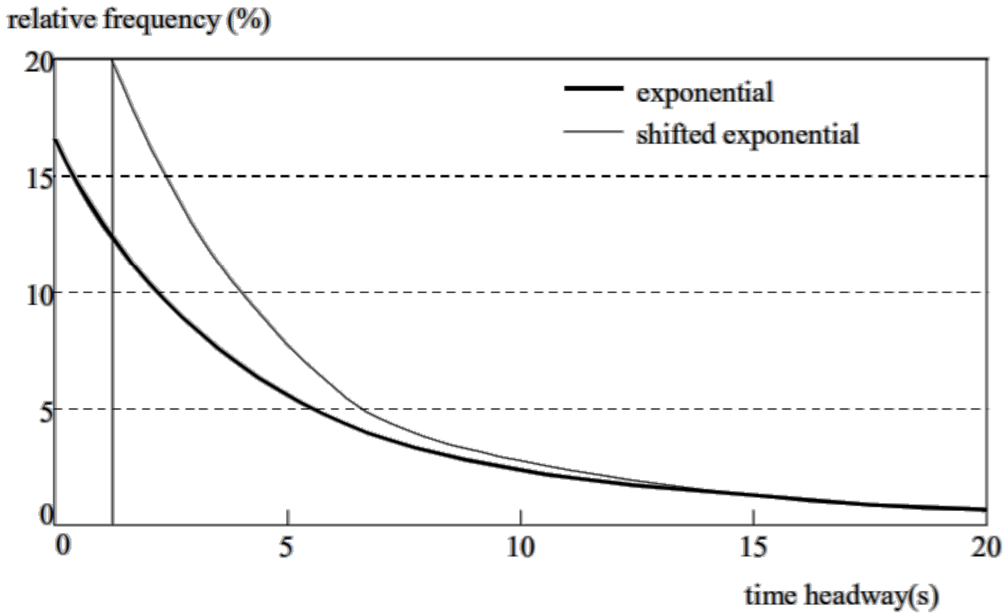
$$\text{with} \quad \lambda = \frac{q}{1 - q h_m} \quad \text{for} \quad h \ge h_m$$

- The probability density function:

$$p(h) = 0 \quad \text{for} \quad h < h_m$$
$$p(h) = \lambda e^{-\lambda(h-h_m)} \quad \text{for} \quad h \ge h_m$$

- The mean value is: $\mu = h_m + \frac{1}{\lambda}$
- The variance equals: $\sigma^2 = \left(\frac{1}{\lambda}\right)^2$
- The variation coefficient: $\frac{\sigma}{\mu} = \frac{1}{1 + \lambda h_m}$

  which is always smaller than 1 as long as $h_m > 0$.
- In practice it is difficult to find a representative value for h_m.
- The abrupt transition at h_m does not fit reality very well.



non-shifted and a shifted probability density with the same mean value (μ = 6 s) and a minimum headway hm= 1 s.

Alternatives for the Exponential Distribution

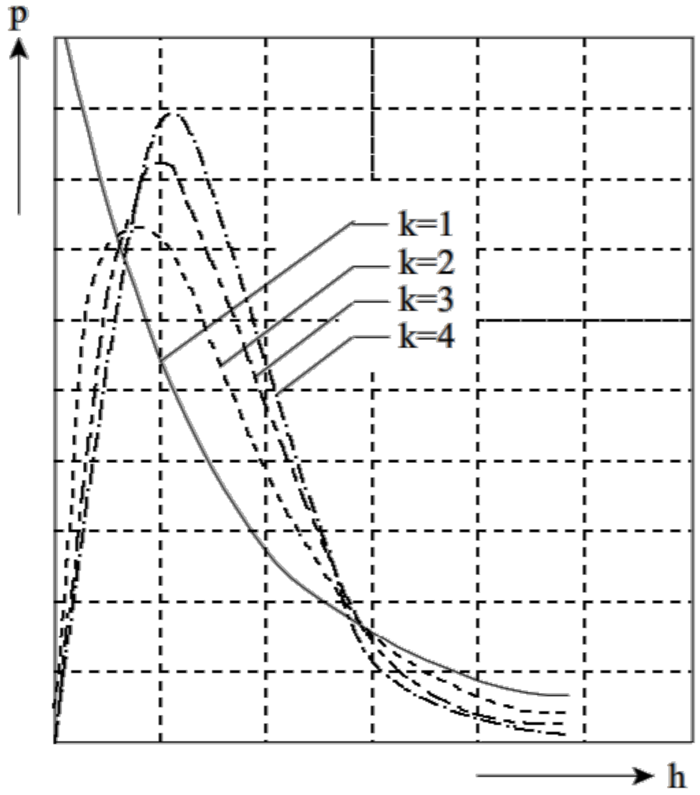
Erlang distribution

- A second alternative is the Erlang distribution with a less abrupt function for small headways.
- The Erlang distribution function is defined by

$$\Pr\{H \leq h\} = 1 - e^{-kh/\mu} \sum_{i=0}^{k} \left(\frac{kh}{\mu}\right)^{i} \left(\frac{1}{i!}\right)$$

- and the corresponding probability density:

$$p(h) = \frac{h^{k-1}}{(k-1)!} \left(\frac{k}{\mu}\right)^{k} e^{-kh/\mu}$$



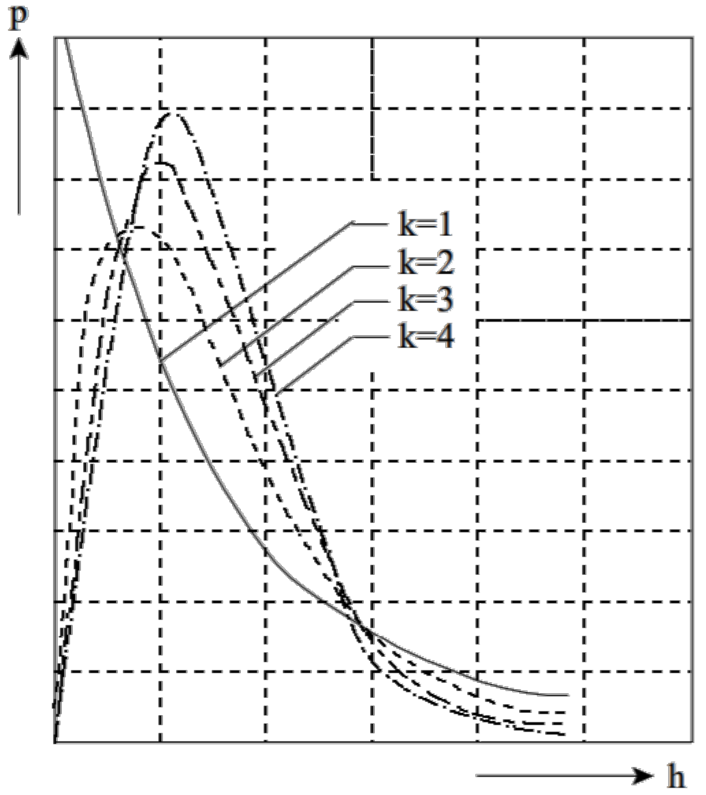
Erlang probability densities

Alternatives for the Exponential Distribution

Erlang distribution

- Note that for $k = 1$ we have $p(h) = (1/\mu)e^{-h/\mu}$.
- Consequently, the exponential distribution is a special Erlang distribution with parameter $k = 1$.
- For values of k larger than 1 the Erlang p.d.f. has a form that better suits histograms based on observed headways.
- The mean = μ
- The variance: μ^2/k
- The coefficient of variation: $1/\sqrt{k}$ which is smaller or equal to 1.

- Figure shows densities for $\mu = 6\ s$ and $k = 1, 2, 3$ and 4.



Erlang probability densities

Alternatives for the Exponential Distribution

Lognormal distribution

- If one has a set of observed headways h_i, then one can investigate whether $x_i = \log h_i$ has a normal distribution.
- If this is the case, then the headways themselves have a lognormal distribution. The p.d.f. of a lognormal distribution is:

$$p(h) = \frac{1}{h\sigma\sqrt{2\pi}} e^{-\frac{\ln^2\left(\frac{h}{\mu}\right)}{2\sigma^2}}$$

- The mean μ*:

$$E(H) = \mu^* = \mu e^{\frac{1}{2}\sigma^2}$$

- The variance:

$$var(H) = (\sigma^*)^2 = \mu^2 e^{\sigma^2}\left(e^{\sigma^2} - 1\right)$$

- The coefficient of variation:

$$C_v = \frac{\sigma^*}{\mu^*} = \frac{\mu e^{\frac{1}{2}\sigma^2}\sqrt{(e^{\sigma^2} - 1)}}{\mu e^{\frac{1}{2}\sigma^2}} = \sqrt{e^{\sigma^2} - 1}$$

- In contrast to the previously discussed distributions, the coefficient of variation of the lognormal distribution, can be smaller as well as larger than 1.

The parameters of the p.d.f., μ and σ are not the mean and st. dev. of the lognormal variate but of the corresponding normal variate. If $\mu*$ and ${\sigma^*}^2$ are given, μ and σ^2 follow from:

$$\mu = \frac{\mu^*}{\sqrt{1 + C_v^2}}$$

$$\sigma^2 = 2\ln\left(\sqrt{1 + C_v^2}\right)$$

Composite Headway Models

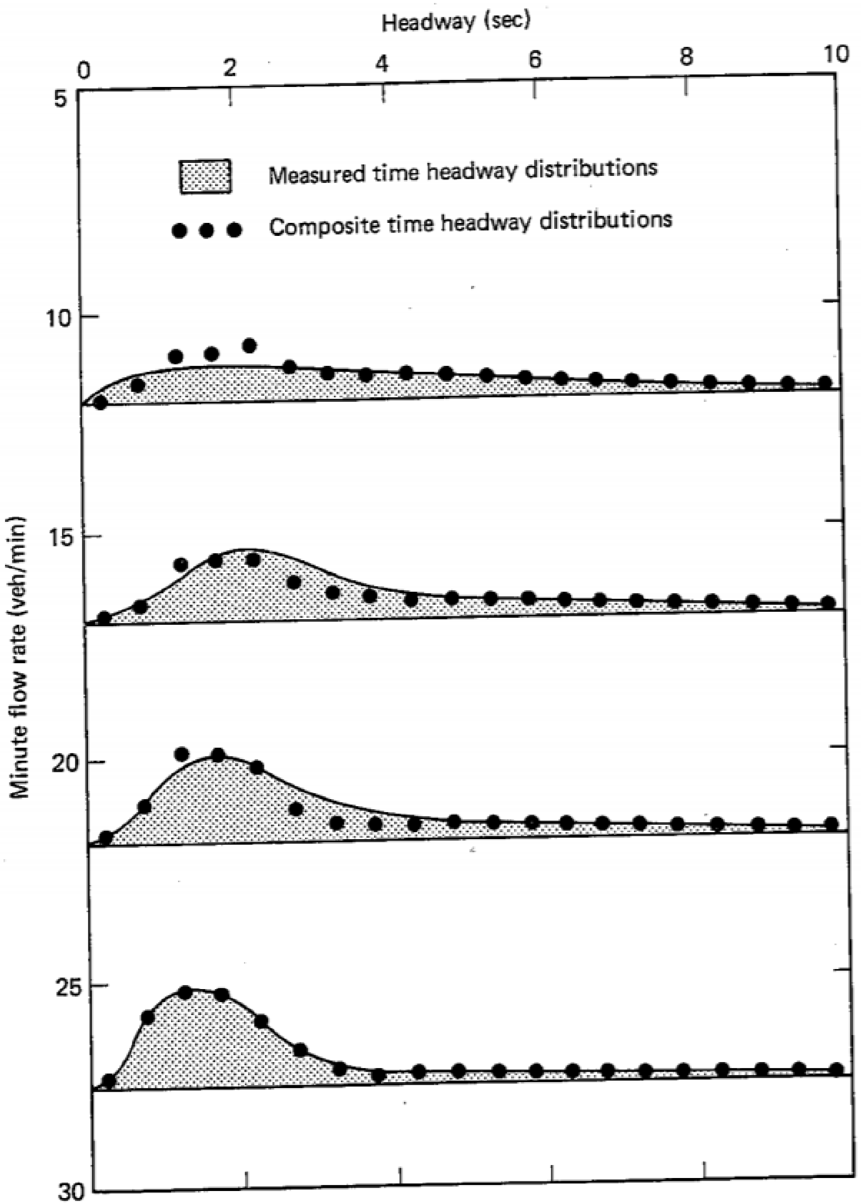
- Comparisons of observed histograms of headways and the simple models discussed earlier have often led to models badly fitting data.
- This has been an inspiration to develop models that have a stronger traffic behavioristic background than the ones discussed.
- In so-called composite headway models, it is assumed that drivers that are obliged to follow the vehicle in front (because they cannot make an overtaking or a lane change), maintain a certain minimum headway (the so-called empty zone or following headway). They are in a constrained or following state.
- If they have a headway which is larger than their minimum, they are called free drivers.
- Driver-vehicle combinations are thus in either of two states: free or constrained.
- As a result, the p.d.f. $p(h)$ of the headways has two components: a fraction φ of constrained drivers with p.d.f. $p_{fol}(h)$ and a fraction $(1-\varphi)$ with $p_{free}(h)$

$$p\left(h\right)=\phi p_{fol}\left(h\right)+\left(1-\phi\right)p_{free}\left(h\right)$$

- The remaining problem is how to specify the p.d.f. of both free drivers and constrained drivers.

Composite Headway Models

- The two distributions appear to have the same general shape
- The two distributions are most different under low flow rate conditions but become more similar as the flow level increases.
- Large differences occur when headways lie between 1 s and 2.5 s.



Observed time headway distributions and the composite headway distributions

Distance Headway Distributions

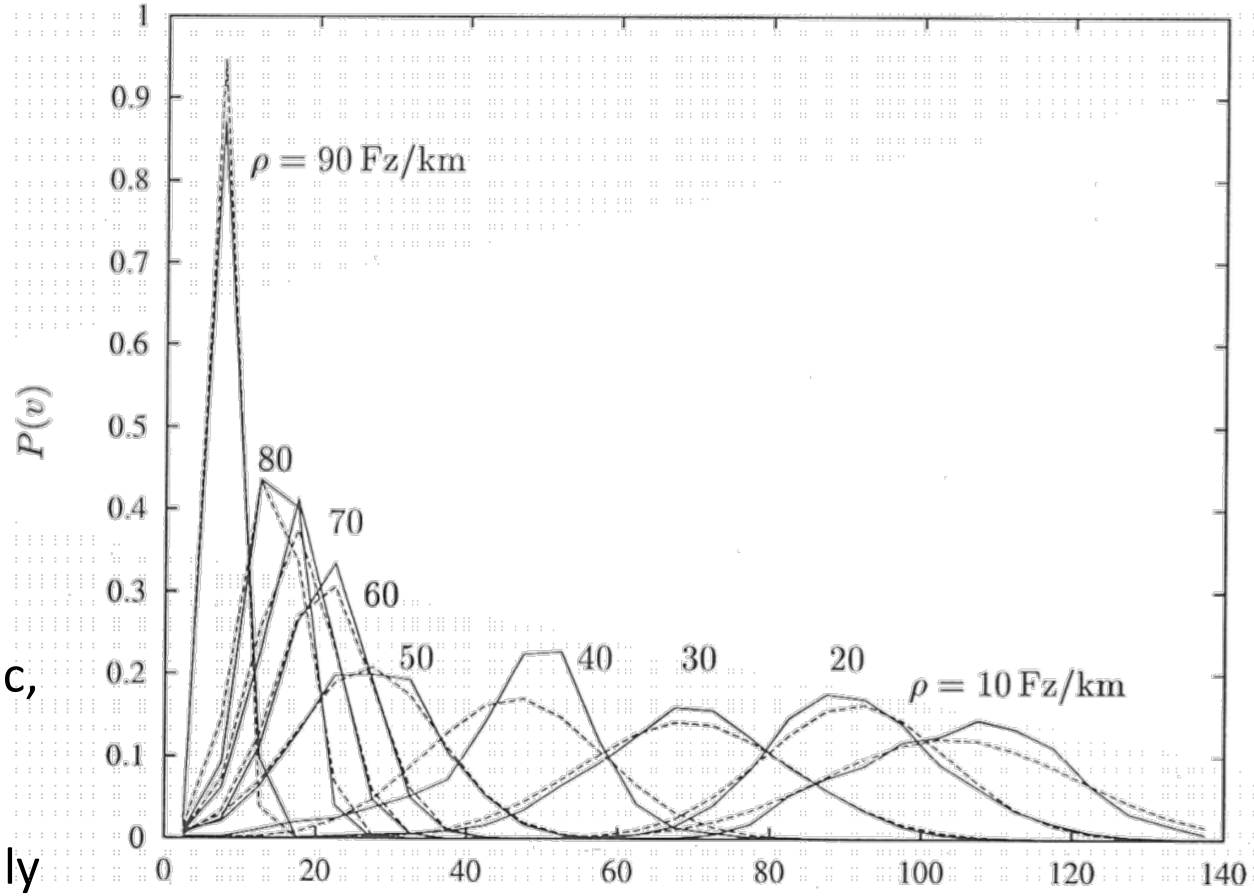
- Most results discussed for time headways are also valid (with occasional modifications) for distance headways.
- It is easier to observe time headways than distance headways, for the same reason as it is easier to observe intensity than density.

Individual Vehicle Speeds

- Just as time and distance headways, speeds have a continuous distribution function.
- It is observed that speeds usually have a Normal (or Gaussian) distribution.
- That means the p.d.f., with parameters mean μ and standard deviation σ is:

$$p\left(v\right)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(v-\mu)^2}{2\sigma^2}}$$

- If the histogram of observed speeds is not symmetric, then the Lognormal distribution is usually a good alternative as a model for the speed distribution.
- As with the mean speed, that could be defined locally and instantaneously, one can consider the distribution of speeds locally and instantaneously.



Probability density functions of local speeds collected at the A9 two-lane motorway in the Netherlands, for different density values. The distributions are compared to Normal probability density functions (dotted lines)

Number of Overtakings

- Vehicles on a road section usually have different speeds, which leads to faster ones catching up the slower ones, and a desire to carry out overtakings.
- One can calculate the number of desired overtakings from the quantity of traffic and the speed distribution.
- Consider a road section of length X during a period of length T.
- Assume the state of the traffic is homogeneous and stationary.
- Assume further that the instantaneous speed distribution is Normal with mean μ and standard deviation σ.
- Then the number of desired overtakings is: $$n = XTk^2\sigma/\sqrt{\pi}$$
- Note that the number of overtakings is:
  - Linear dependent on the time-distance one considers;
  - Increases with the squared of the quantity of traffic (term k^2);
  - Is larger if the speeds are more different (term σ).

Number of Overtakings

Example 6: Overtakings in a bike lane

Consider a one-way path of $1\ km$ length, for 1 hour.

The bicycle (bike) intensity = $600\ bic/h$

The moped (mop) intensity = $150\ mop/h$.

From observations it is known that speeds of bikes and mops have, with a good approximation, a Normal distribution:

- $u_{bic} = 19\ km/h$;
- $\sigma_{bic} = 3\ km/h$;
- $u_{mop} = 38\ km/h$;
- $\sigma_{mop} = 5\ km/h$.

Assess the operational quality of the bike lane.

It is assumed that the quality of operation is negatively influenced by the number of overtakings a cyclist has to carry out (active overtakings) or has to undergo (passive overtakings).

Number of Overtakings

Solution 6: Overtakings in a bike lane

$$OT[bic - bic] = (600/19)^2\, 3/\sqrt{\pi} = 1688$$
$$OT[mop - mop] = (150/38)^2\, 5/\sqrt{\pi} = 44$$

The number of calculated $OT[mop - bic]$ is valid when both speed distribution do not have overlap and in practice that is the case, and is calculated using:

$$n_2 = XTk_1k_2(v_1 - v_2)$$
$$OT[mop - bic] = (600/19)\,(150/38)\,(38—19) = 2341$$

The outcomes of the calculation show clearly that the mopeds are responsible for an enormous share in the OT’s.

The operational quality for the cyclists will increase much if mopeds do not use cycle paths.

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Thank
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