Microscopic Models of Traffic Flow – B

Hamzeh Alizadeh, Ph.D.

Director – Research and Data Valorization
ARTM

Single-regime models / Multiregime models

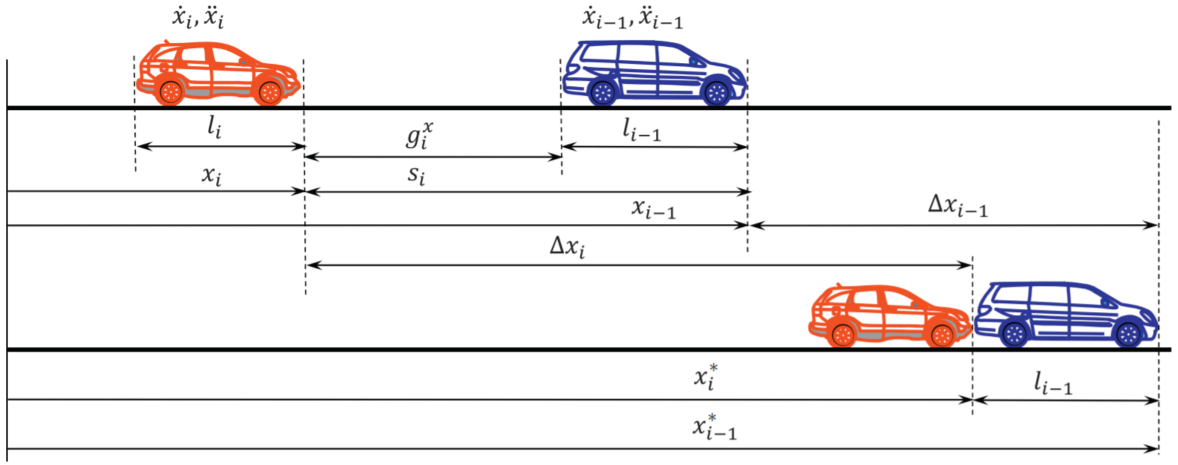
- Previously introduced models are all single-regime models: they have only one equation that applies to the entire driving process and do not consider different driving scenarios or regimes.
- Such models are simple and mathematically attractive.
- However, their descriptive power is frequently of concern.
- A driver may encounter different regimes such as start-up, speedup, free flow, cutoff, following, stop and go, trailing, approaching, and stopping. A one-equation model may or may not apply to all regimes.
- Multiregime models might be helpful in capturing different driving scenarios.

Gipps Model

- The Gipps car-following model is based on the following assumption:

  "The driver of the following vehicle selects his speed to ensure that he can bring his vehicle to a safe stop should the vehicle ahead come to a sudden stop"

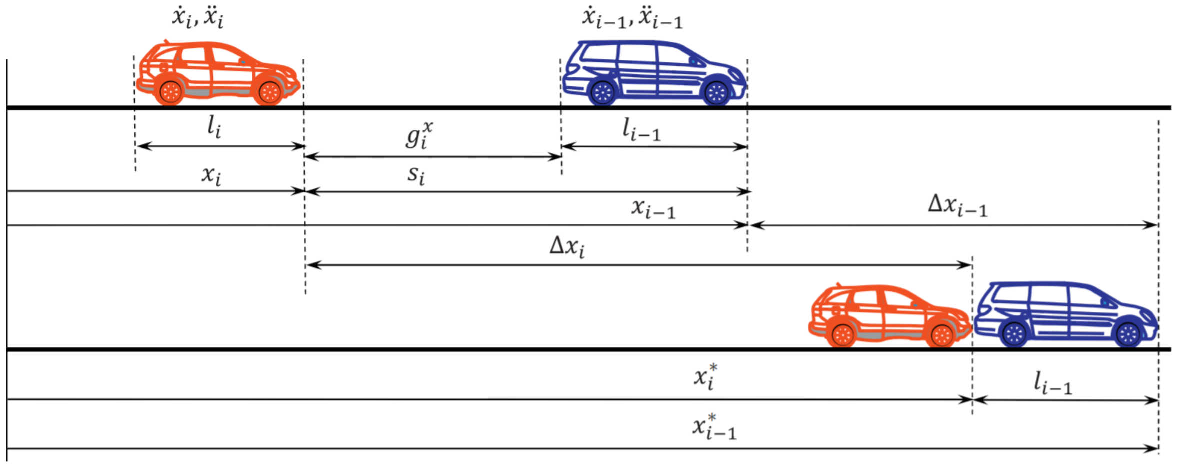
- Put another way, at any moment the following driver should leave enough safe distance in front such that in case the leading vehicle commences an emergency brake, the subject driver has time to respond and decelerate to a stop behind the leading vehicle without a collision.
- This scenario is depicted in Figure:



Gipps car-following scenario

Gipps Model

- At time t, vehicle i is located at $x_i(t)$ and the leading vehicle $i-1$ is at $x_{i-1}(t)$.
- At this moment, vehicle $i-1$ at speed $\dot{x}_{i-1}(t)$ commences an emergency brake at a rate of B_{i-1}.
- Alerted by the braking light in front, driver i at speed $\dot{x}_i(t)$goes through a perception-reaction process of duration τ_i, trying to understand the situation, evaluate potential options, and then decides to brake as well at a tolerable rate of b_i.
- Hence, the vehicle starts to decelerate from $\dot{x}_i(t+\tau_i)$ to a stop, with the most adverse situation being stopped right after vehicle $i-1$.



Gipps car-following scenario

Gipps Model

- Therefore, the distance traveled by vehicle i − 1 during its emergency brake is:

$$\frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}}$$

- Since B_{i-1} is negative, so the vehicle stops at location:

$$x_{i-1}^* = x_{i-1}(t) - \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}}$$

- Meanwhile, vehicle i travels a certain distance during the perception-reaction time:

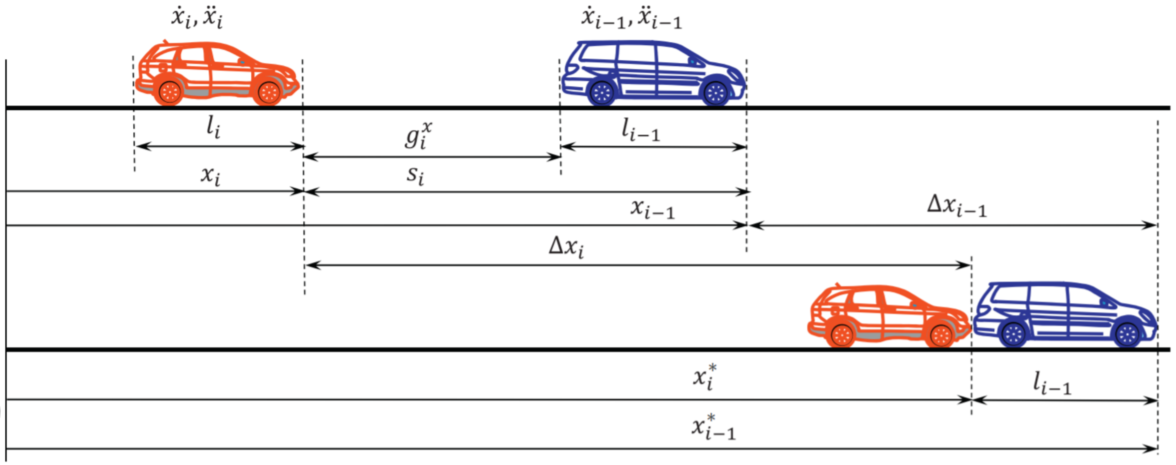
$$\frac{\dot{x}_i(t)+\dot{x}_i(t+\tau_i)}{2}\tau_i$$

- Then travels a braking distance: $\dfrac{\dot{x}_i(t+\tau_i)^2}{2b_i}$
- Hence, the vehicle stops at location:

$$x_i^* = x_i(t) + \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i - \frac{\dot{x}_i^2(t+\tau_i)}{2b_i}$$

- To be conservative, Gipps added an extra buffer time (θ) appended to the perception-reaction time:

$$\boxed{x_i^* = x_i(t) + \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i + \dot{x}_i(t+\tau_i)\theta - \frac{\dot{x}_i^2(t+\tau_i)}{2b_i}}$$



Gipps car-following scenario

Gipps Model

- To ensure safety, the following relationship must hold:

$$x^*_{i-1} - l_{i-1} \geq x^*_i$$

The actual spacing is

$$s_i(t) = x_{i-1}(t) - x_i(t)$$

↓

$$s_i(t) \geq \frac{\dot{x}_i(t) + \dot{x}_i(t+\tau_i)}{2}\tau_i + \dot{x}_i(t+\tau_i)\theta - \frac{\dot{x}_i^2(t+\tau_i)}{2b_i} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1}$$

Therefore, the target speed that the driver tries to achieve next is

$$-\frac{1}{2b_i}\dot{x}_i^2(t+\tau_i) + \left(\frac{\tau_i}{2} + \theta\right)\dot{x}_i(t+\tau_i) + \frac{\dot{x}_i(t)\tau_i}{2} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1} - s_i(t) \leq 0$$

↓

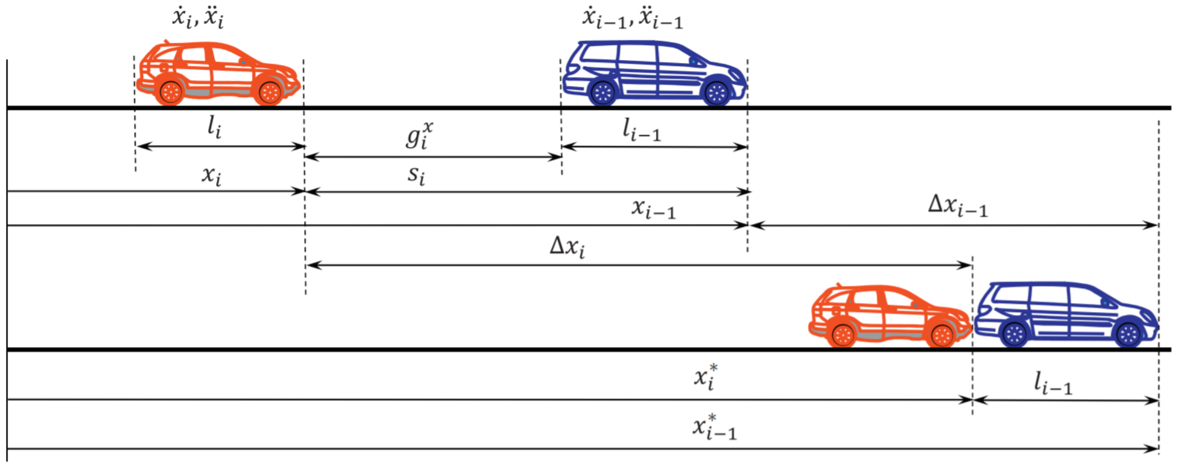
Considering $\theta = \tau_i/2$ as suggested by Gipps, the roots of the above quadratic equation are

$$\dot{x}_i(t+\tau_i) = -b_i\tau_i \pm \sqrt{b_i^2\tau_i^2 - b_i\left[-\dot{x}_i(t)\tau_i - \frac{\dot{x}_{i-1}^2(t)}{B_{i-1}} - 2l_{i-1} + 2s_i(t)\right]}$$

Gipps Model

- Consider the signs of the roots and that speed is a positive value, the above equation translates to the following :

$$\dot{x}_i(t+\tau_i) = \min \begin{cases} \dot{x}_i(t) + 2.5A_i\tau_i(1 - \frac{\dot{x}_i(t)}{v_i})\sqrt{0.025 + \frac{\dot{x}_i(t)}{v_i}} & \text{(free flow)}, \\ -b_i\tau_i + \sqrt{b_i^2\tau_i^2 - b_i[\dot{x}_i(t)\tau_i - \frac{\dot{x}_{i-1}^2(t)}{B_{i-1}} + 2l_{i-1} - 2s_i(t)]} & \text{(car following)}. \end{cases}$$



Gipps car-following scenario

Macroscopic Bridge – Gipps Model

- If one ignores the speed change during the perception-reaction process and the additional buffer time θ, and sets both sides equal, then, rearranging terms yields:

$$s_i(t) = \dot{x}_i(t)\tau_i - \frac{\dot{x}_i^2(t)}{2b_i} + \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} + l_{i-1}$$

- Under equilibrium conditions, the above car-following model leads to the following speed-density relationship:

$$\frac{1}{k} = \gamma v^2 + \tau v + l$$

- Where
  - k is traffic density, $\gamma = -1/2b + 1/2B$,
  - $b < 0$ is the average tolerable braking rate,
  - $B < 0$ is the average emergency braking rate,
  - v is the average traffic speed,
  - τ is the average perception-reaction time,
  - and l is the average nominal vehicle length.

Macroscopic Bridge – Gipps Model

- The corresponding flow-speed relationship is:

$$q = \frac{v}{\gamma v^2 + \tau v + l}$$

- To find the capacity, one takes the first derivative of flow q with respect to v and sets the result to zero:

$$\left.\frac{\mathrm{d}q}{\mathrm{d}v}\right|_{v_\mathrm{m}} = -\left.\frac{\gamma - \frac{l}{v^2}}{(\gamma v + \tau' + \frac{l}{v})^2}\right|_{v_\mathrm{m}} = 0$$

- Solving the equation yields:

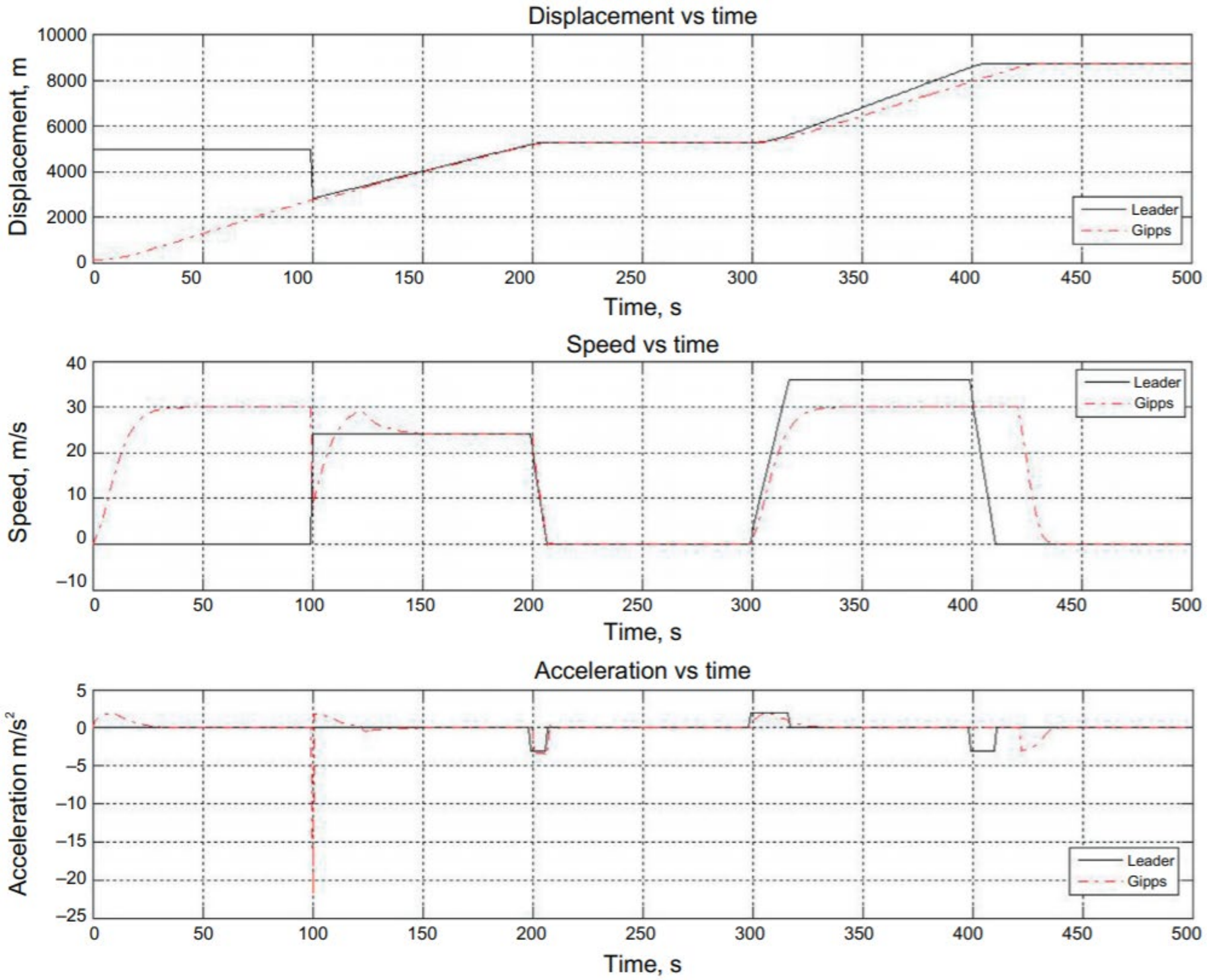
$$v_\mathrm{m} = \sqrt{\frac{l}{\gamma}} \quad \textit{and,} \quad q_\mathrm{m} = \frac{1}{2\sqrt{\gamma l} + \tau}$$

Microscopic Benchmarking – Gipps Model

- ➢ The above benchmarking is based on the set of parameters presented in the table below.
- ➢ ***Start-up:*** The model is able to start the vehicle up from standstill. See when $t > 0\ s$.
- ➢ ***Speedup:*** The model is able to speed the vehicle up realistically to its desired speed. See when $0 < t < 100\ s$.
- ➢ ***Free flow:*** The model is able to reach and settle at the desired speed under free-flow conditions. See when $0 < t < 100\ s$.

l_i	v_i	τ_i	b_i	
6 m	30 m/s	1.0 s	-3.4 m/s^2	
A_i	B_{i-1}	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
1.7 m/s^2	-6.0 m/s^2	120 m	0 m/s	0 m/s^2

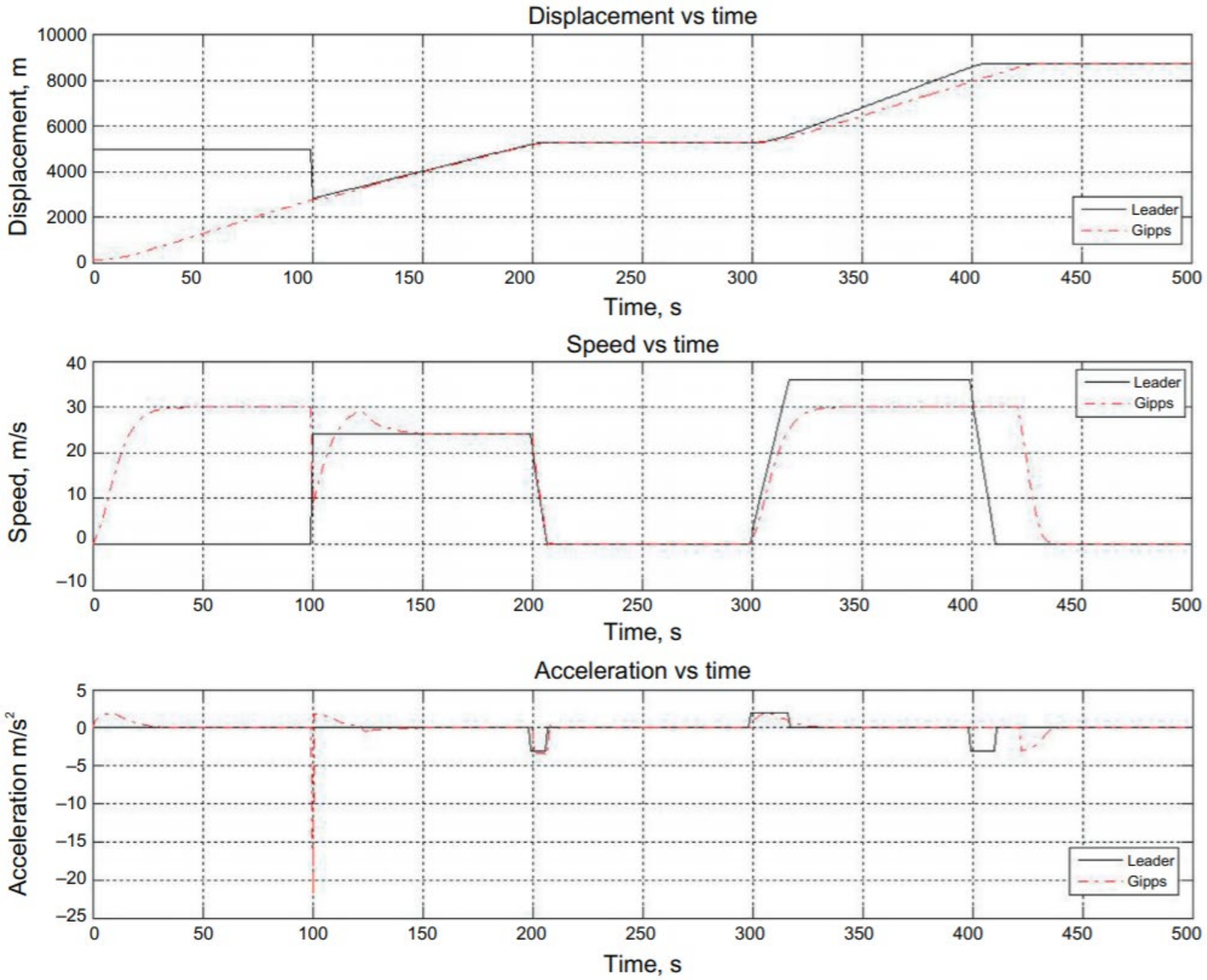
Microscopic benchmarking parameters of the Gipps model



Microscopic benchmarking of Gipps' model

Microscopic Benchmarking – Gipps Model

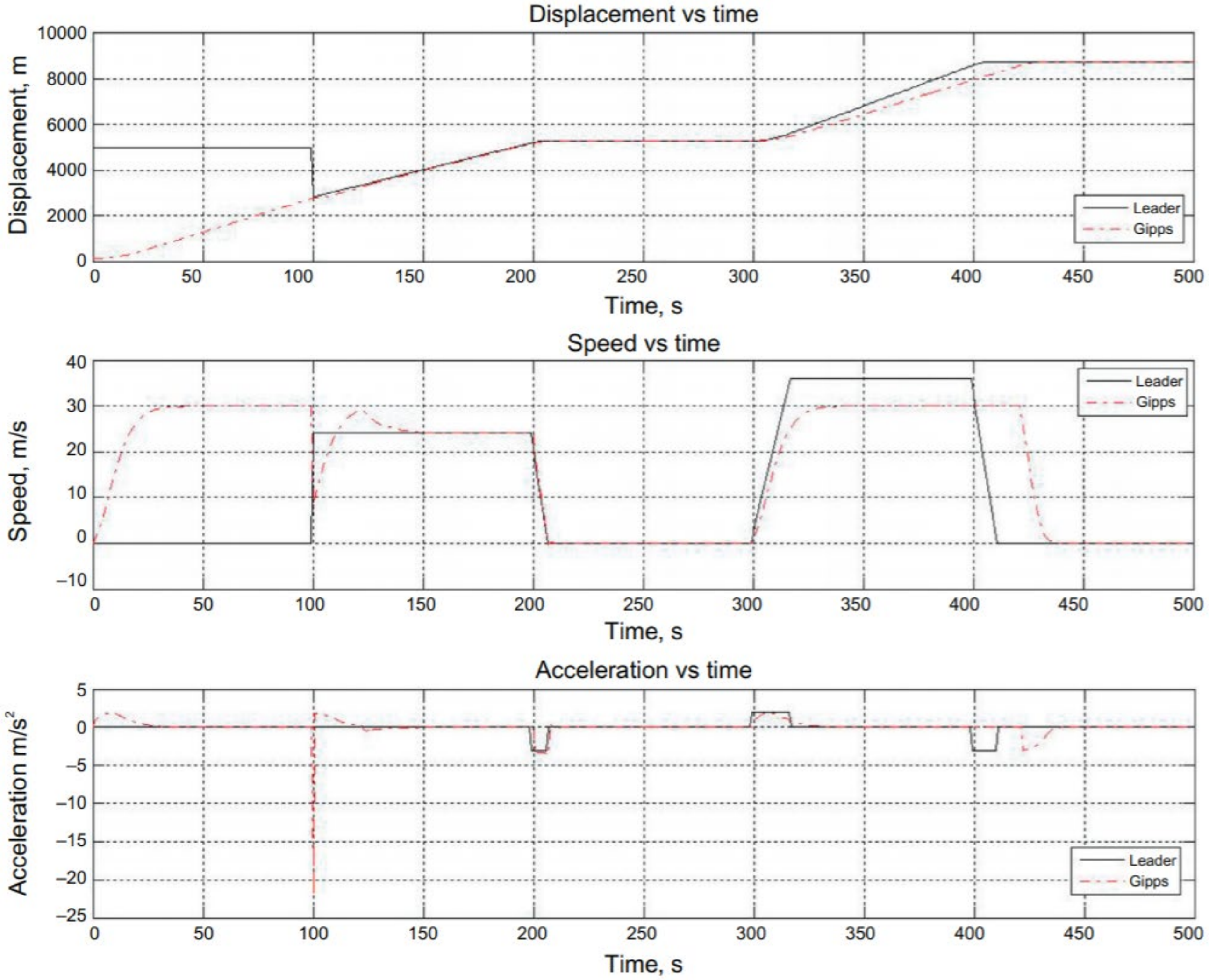
- ***Cutoff:*** The model over decelerates slightly, which causes a small oscillation in speed, but in general the model retains control and responds reasonably when a vehicle cuts in in front.
  See around $t = 100\ s$.
- ***Following:*** The model is able to adopt the leader's speed and follow the leader at a reasonable distance.
  See when $100 < t < 200\ s$.
- ***Stop and go:*** The model is able to stop the vehicle safely behind its leader and start the vehicle moving when the leader departs.
  See when $200 \leq t \leq 300\ s$.



Microscopic benchmarking of Gipps' model

Microscopic Benchmarking – Gipps Model

- ***Trailing:*** The model is able to speed up normally without being tempted to speed up by its speeding leader.
  See when $300 < t < 400\ s$.
- ***Approaching:*** The model is able to decelerate properly when approaching a stationary vehicle at a distance.
  See when $400 \leq t < 420\ s$.
- ***Stopping:*** The model is able to stop the vehicle safely behind the stationary vehicle.
  See when $t \geq 420\ s$.



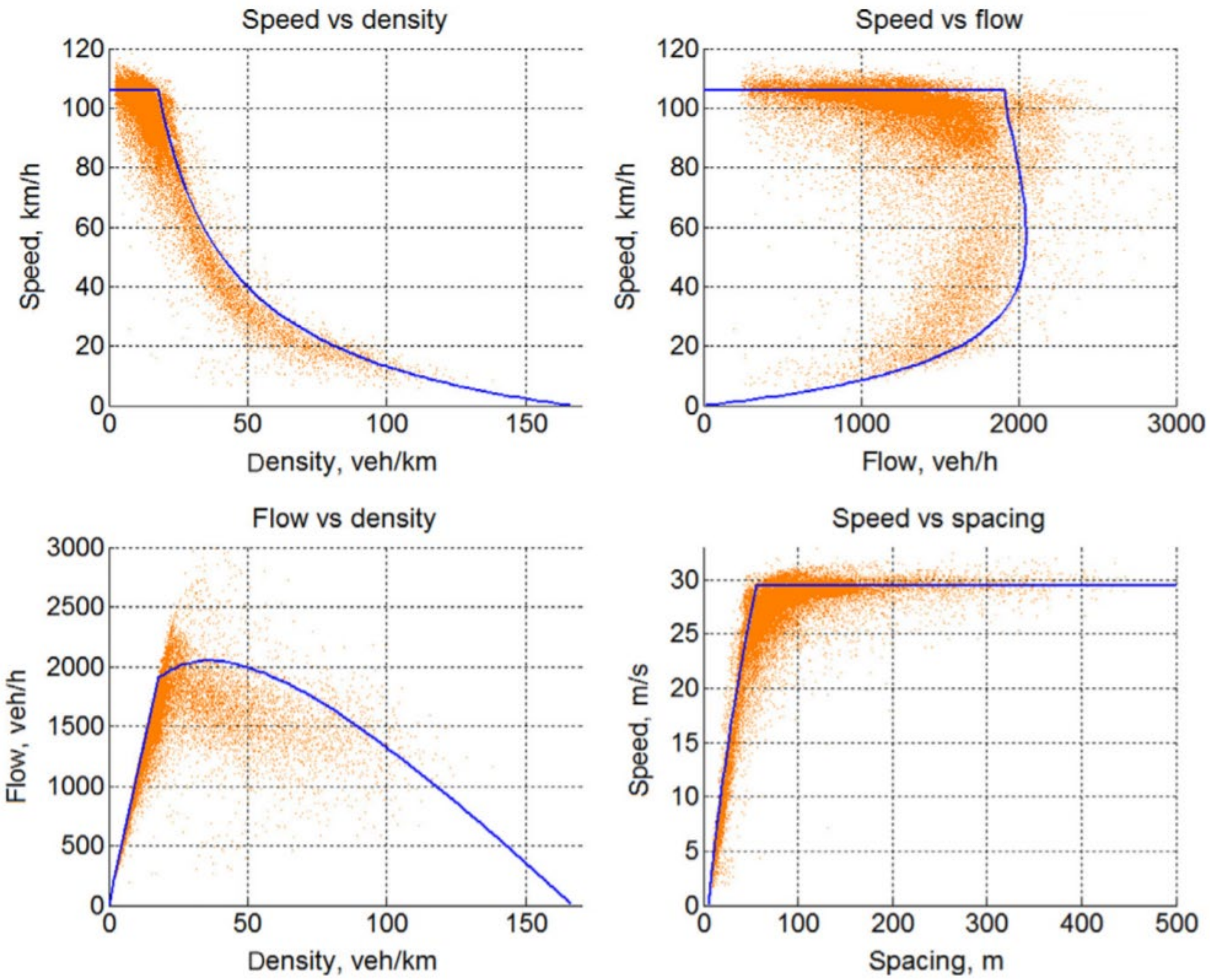
Microscopic benchmarking of Gipps' model

Macroscopic Benchmarking – Gipps Model

- The fundamental diagram implied by the Gipps model is plotted against empirical observations.
- The model parameters are presented in the table below.
- The model fits empirical data reasonably well except for free-flow conditions (i.e., in the low-density range).
- In addition, the critical flow predicted by the Gipps model is much lower than it should be.

b	B	τ	l
$-3.0\,\mathrm{m/s^2}$	$-3.5\,\mathrm{m/s^2}$	$1\,\mathrm{s}$	$6.5\,\mathrm{m}$

Macroscopic benchmarking parameters of the Gipps



Fundamental diagram implied by the Gipps model.

Newell Nonlinear Model

- Newell proposed a car-following models in 1961, which is referred to as the "Newell nonlinear" car-following model.
- It takes the following form:

$$\dot{x}_i(t+\tau_i) = v_i\left(1 - e^{-\frac{\lambda_i}{v_i}(s_i(t)-l_i)}\right)$$

- Where
  - $\dot{x}_i(t)$ is the speed of the vehicle i at time t,
  - τ_i is driver i's perception-reaction time,
  - v_i is driver i's desired speed,
  - λ_i is a parameter associated with driver i (i.e., the slope of driver i's speed-spacing curve evaluated at $\dot{x}_i(t)$),
  - $s_i = x_{i-1} - x_i$ is the spacing between vehicle i and its leader $i-1$,
  - l_i is the minimum value of s_i, which can be viewed as the nominal vehicle length.

- Newell Nonlinear Model is based on empirical studies and it leads to an equilibrium speed-density curve that resembles field observations.

Newell Nonlinear Model

- Under equilibrium conditions, Newell nonlinear model reduces to the following speed-density relationship:

$$v = v_{\mathrm{f}} \left(1 - \mathrm{e}^{-\frac{\lambda}{v_{\mathrm{f}}} \left(\frac{1}{k} - \frac{1}{k_{\mathrm{j}}} \right)} \right)$$

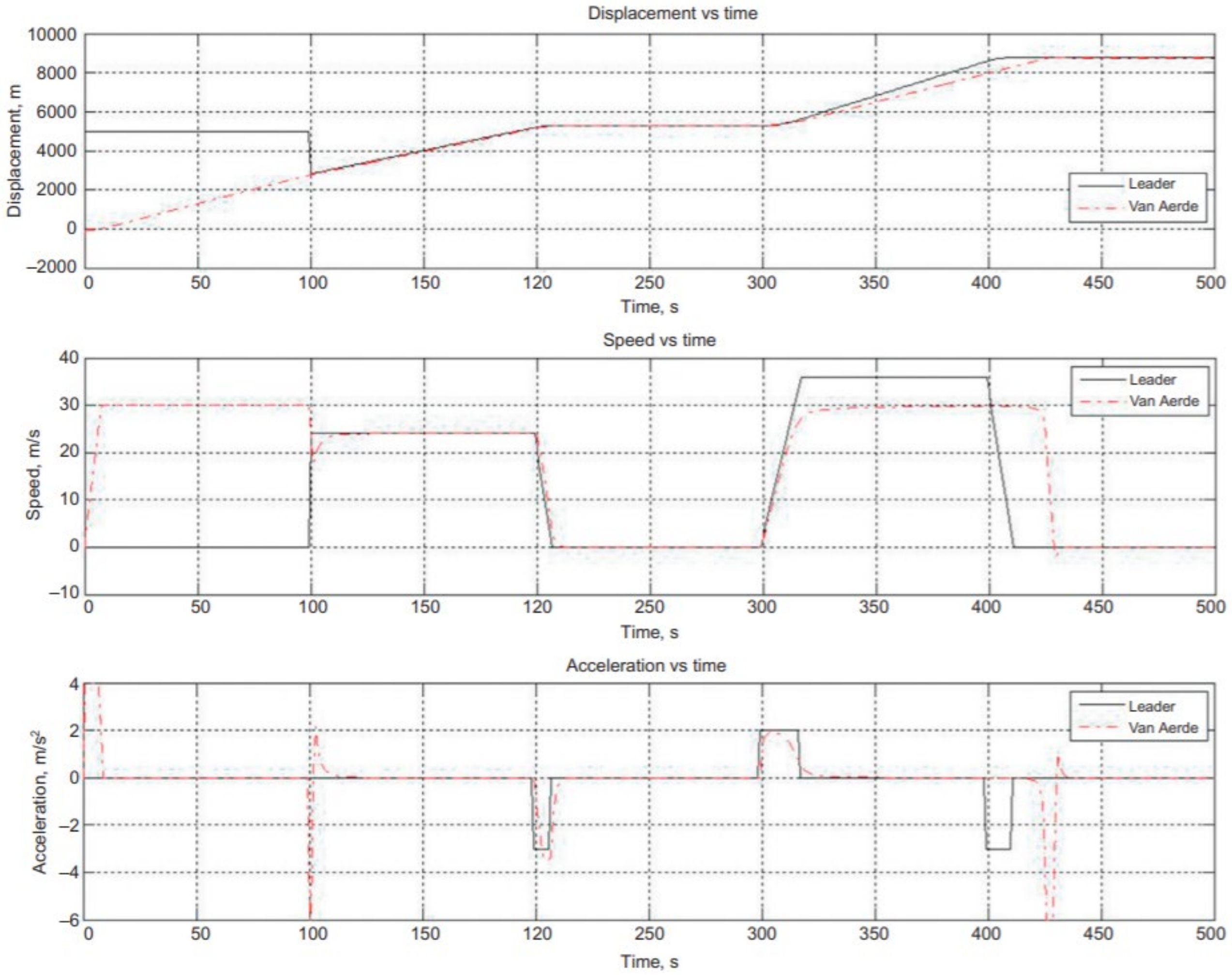
- Where:
  - v is traffic speed, which is aggregated from vehicle speed $\dot{x}_i$,
  - v_f is freeflow speed, which is aggregated fromv_i,
  - λ is a parameter aggregated from λ_i,
  - k is traffic density, which is the reciprocal of average spacing s, which, in turn, is aggregated from spacing s_i,
  - k_j is jam density, which is the reciprocal of average vehicle length l, which, in turn, is aggregated from nominal vehicle length l_i

Microscopic Benchmarking – Newell Nonlinear Model

- The benchmarking result of the Newell nonlinear model is plotted and summarized as follows.
- ***Start-up:*** The model is able to start a vehicle up from standstill.
  See when $t > 0\ s$.
- ***Speedup:*** The model allows the vehicle speed to jump from 0 to 30 m/s in one time-step, resulting in an acceleration of 30 m/s^2. This is unrealistic, so an external logic has to be imposed to limit the maximum acceleration.
  See when $0 < t < 100\ s$.

l_i	v_i	τ_i	λ	–
6 m	30 m/s	1.0 s	7.9	–
A_i	B_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
4.0 m/s^2	6.0 m/s^2	−97 m	0 m/s	0 m/s^2

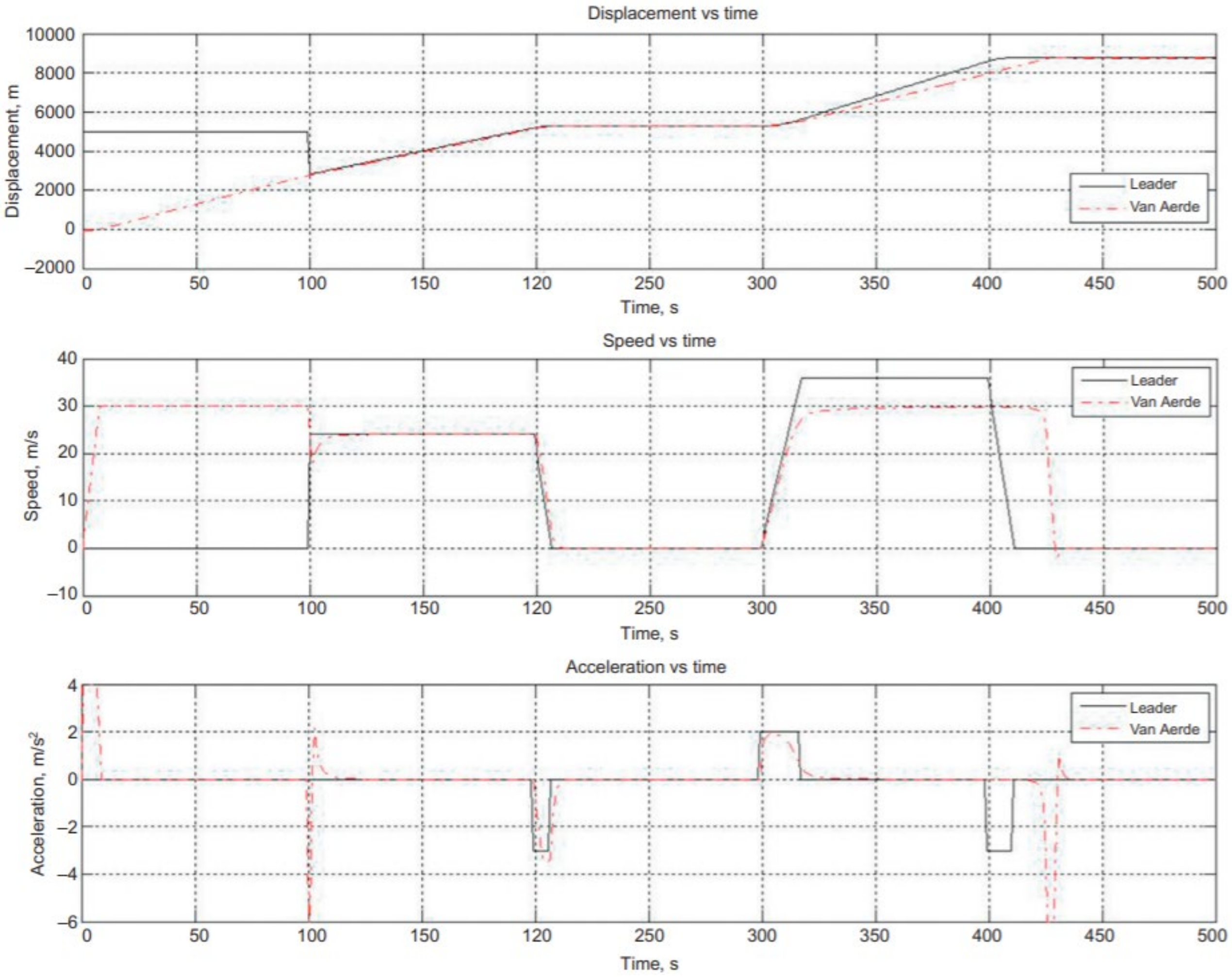
Microscopic benchmarking parameters of the Newell nonlinear model



Microscopic benchmarking of the Newell nonlinear model

Microscopic Benchmarking – Newell Nonlinear Model

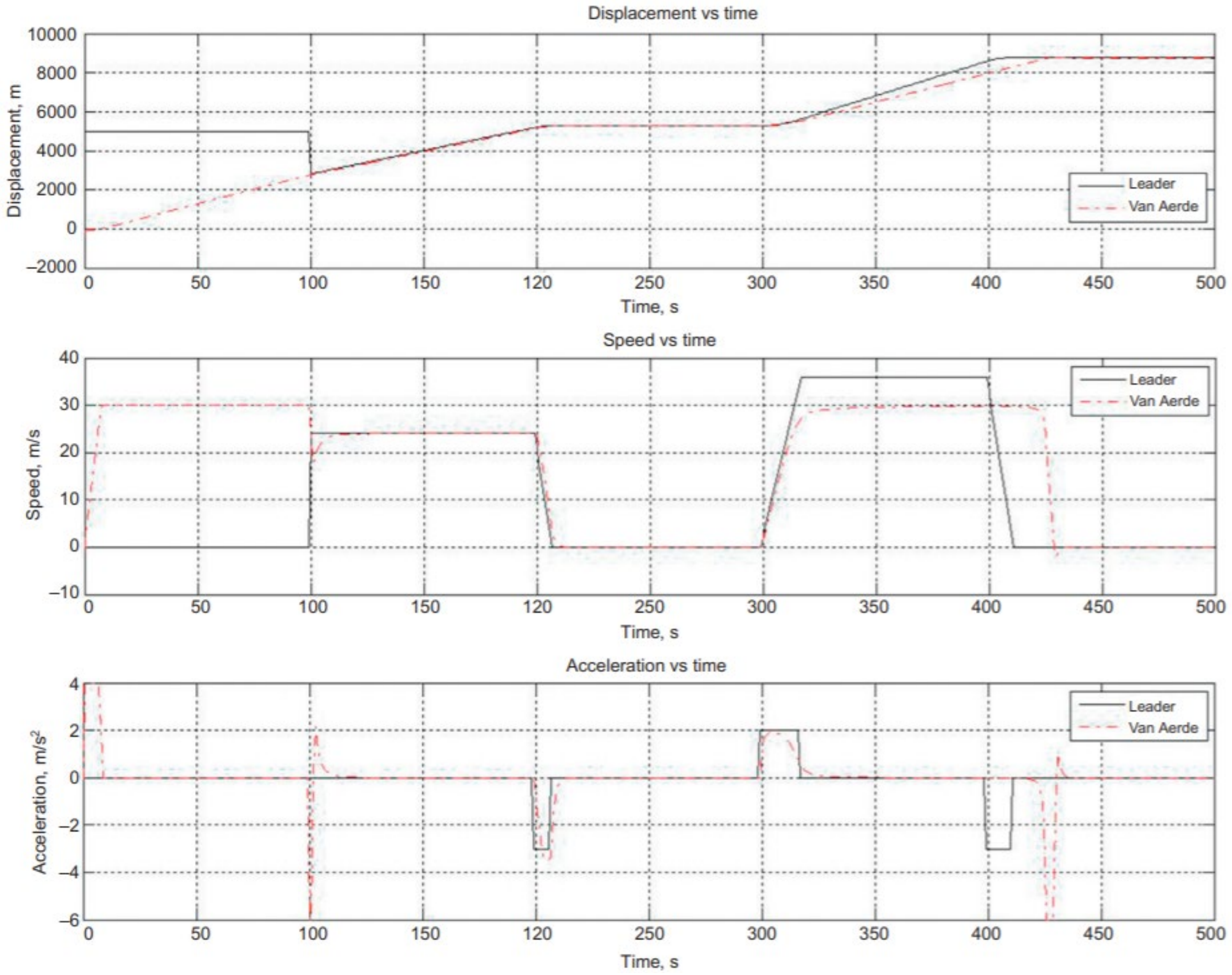
- ***Free flow:*** The model is able to reach and settle at the desired speed under free-flow conditions. See when $0 < t < 100\ s$.
- ***Cutoff:*** By itself, the Newell nonlinear model would predict a deceleration of about $-184.6\ m/s^2$ when the third vehicle cuts in and an acceleration of $182.9\ m/s^2$ in the next time step. This is a very unrealistic jerking, so an external logic has to be imposed to limit the maximum acceleration and deceleration, and the same argument as for speedup applies here.
  See around $t = 100\ s$ after these external conditions have bene incorporated.
- ***Following:*** The model is able to adopt the leader's speed and follow the leader at a reasonable distance.
  See when $100 < t < 200\ s$.



Microscopic benchmarking of the Newell nonlinear model

Microscopic Benchmarking – Newell Nonlinear Model

- ***Stop and go:*** The model is able to stop the vehicle safely behind its leader and start the vehicle moving when the leader departs.
  See when $200 \leq t \leq 300\ s$.
- ***Trailing:*** The model is able to speed up normally without being tempted to speed up by its speeding leader.
  See when $300 < t < 400\ s$.
- ***Approaching:*** With the above external logic on limiting deceleration, the model is able to decelerate properly when approaching a stationary vehicle.
  See when $400 \leq t < 420\ s$.
- ***Stopping:*** The model is able to stop the vehicle safely behind the stationary vehicle.
  See when $t \geq 420\ s$.



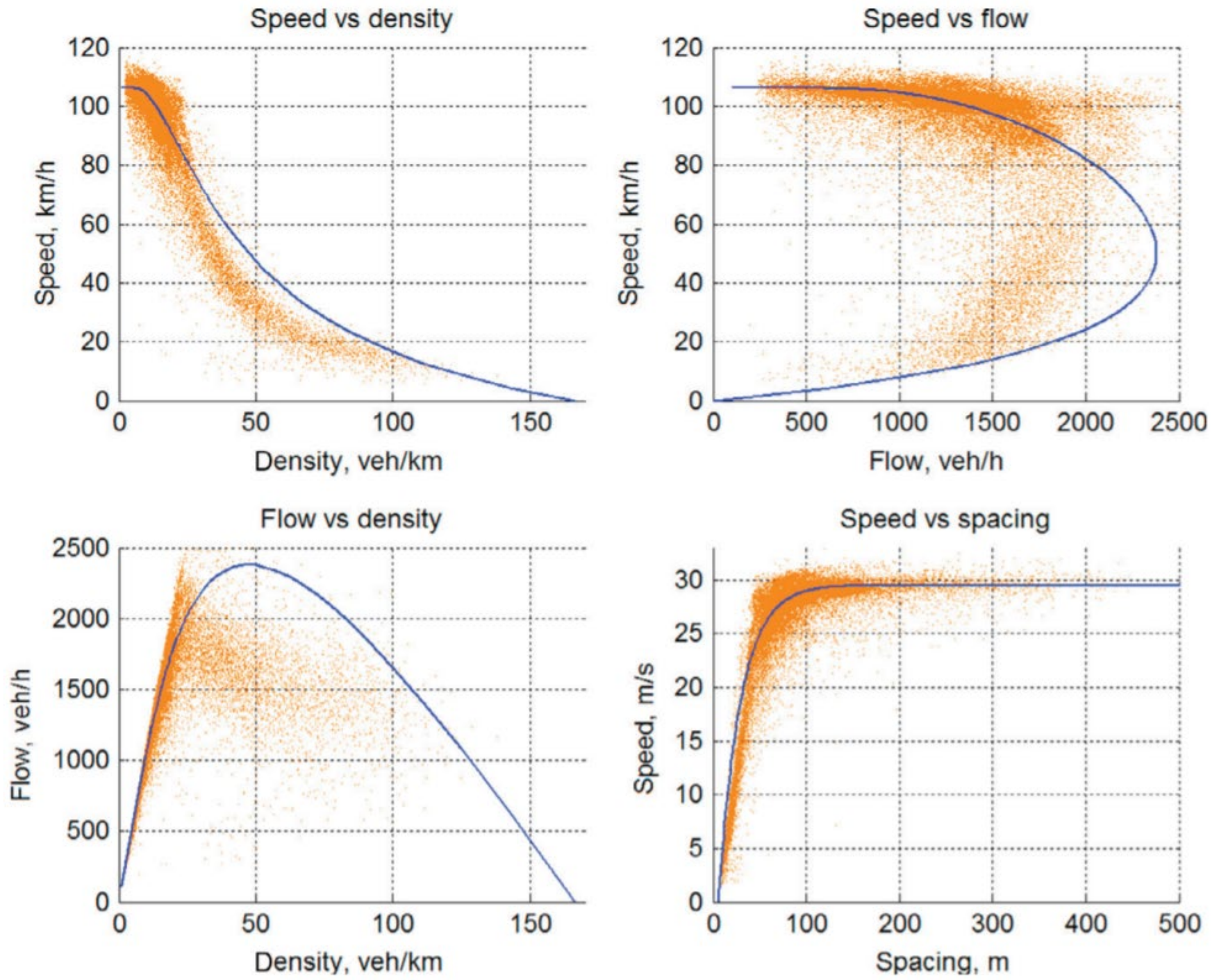
Microscopic benchmarking of the Newell nonlinear model

Macroscopic Benchmarking – Newell Nonlinear Model

- The fundamental diagram implied by the Newell nonlinear model is depicted.
- The model meets the boundary conditions at $(k = 0, v = v_f)$ and $(k = k_j, v = 0)$.
- The flow-density exhibits a concave shape, and the fitting quality is reasonably good given that only three parameters are employed.

v_f	k_j	λ
29.5 m/s	0.2 vehicles/m	0.8

Macroscopic benchmarking parameters of the Newell nonlinear model



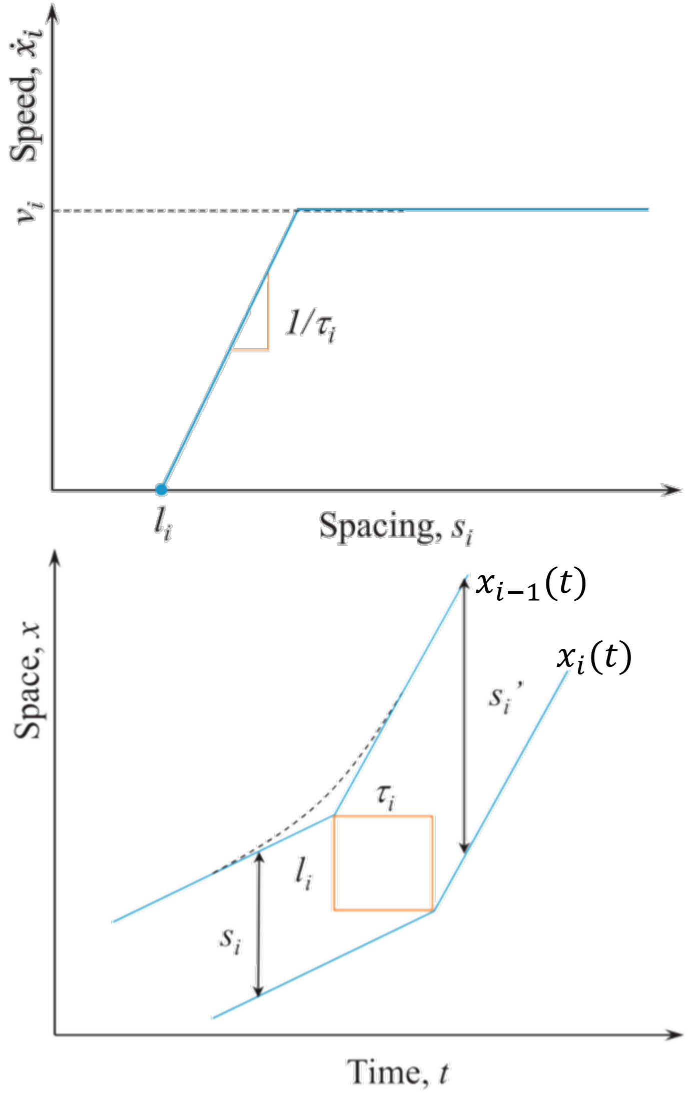
Fundamental diagram implied by the Newell nonlinear model.

Newell Simplified Model

- After about 40 years, Newell published a simplified car-following model
- It simply translates the leading vehicle's trajectory
- If vehicle $i-1$'s trajectory $x_{i-1}(t)$ is given, vehicle i's trajectory can be directly determined by the following equation:

$$x_i(t+\tau_i) = x_{i-1}(t) - l_i$$

- Graphically, this means translating trajectory $x_{i-1}(t)$to the right by a horizontal distance of τ_i and then downward by a vertical distance of l_i ➔ One can squeeze a rectangle with dimensions $\tau_i \times l_i$ between the two trajectories.
  - The physical meaning of li is the minimum value of the spacing, that is, the nominal vehicle length.
  - τi is the reciprocal of the tangent to the speed-spacing relationship drawn at point $(0, li)$.
  - Evidence shows that τi can most likely be interpreted as the perception-reaction time of driver i



Newell simplified car-following model

Newell Simplified Model

It can be seen from the figure:

$$s_i(t) = x_{i-1}(t) - x_i(t)$$

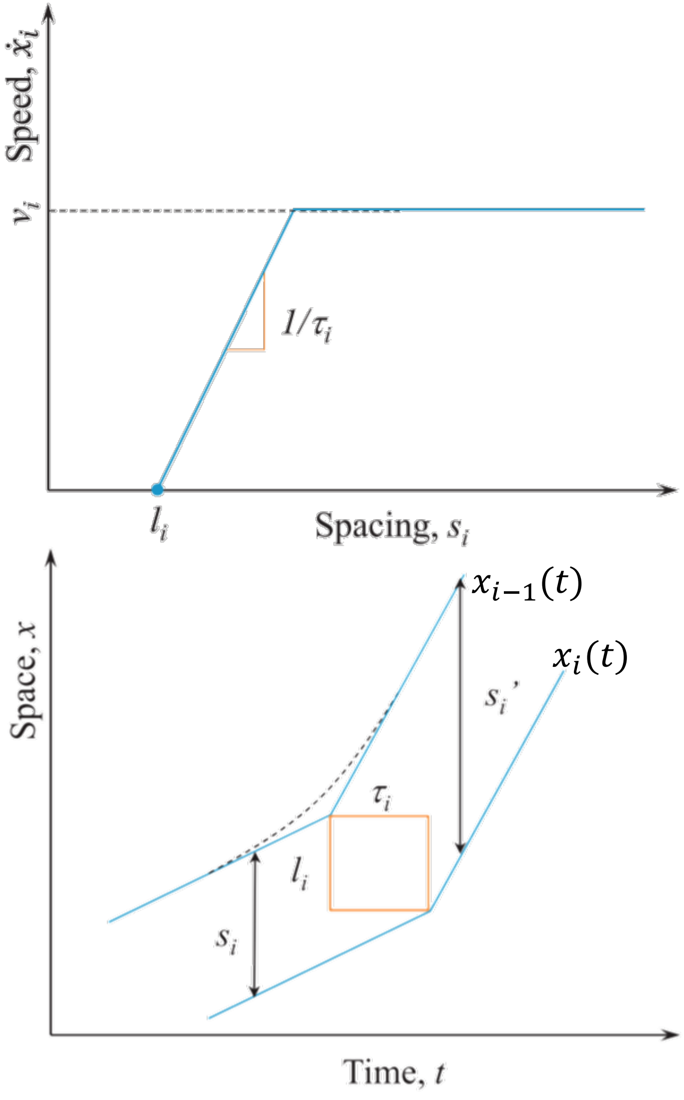
In addition, the locations of vehicle i at time t and t+τi can be related as

$$x_i(t + \tau_i) = x_i(t) + \dot{x}_i(t)\tau_i$$

Combining the above three equations, we get:

$$s_i(t) = \dot{x}_i(t)\tau_i + l_i$$

- This is the same as the Pipes/Forbes model, which in turn, is equivalent to GM1.



Newell simplified car-following model

Intelligent Driver Model

- The intelligent driver model (IDM) is expressed as follows:

$$\ddot{x}_i(t+\tau_i) = A_i\left[1 - \left(\frac{\dot{x}_i}{v_i}\right)^{\delta} - \left(\frac{s_i^*}{s_i}\right)^2\right]$$

- Where
  - $\ddot{x}_i$ is driver i's acceleration,
  - A_i is driver i's maximum acceleration when starting from standstill,
  - δ is the acceleration exponent,
  - $s_i = x_{i-1} - x_i$ is the spacing between vehicle i and its leader $i-1$,
  - The desired spacing s_i^* is a function of speed $\dot{x}_i$ and relative speed $(\dot{x}_i - \dot{x}_{i-1})$:

$$s_i^* = s_0 + s_1\sqrt{\frac{\dot{x}_i}{v_i}} + T_i\dot{x}_i + \frac{\dot{x}_i[\dot{x}_i - \dot{x}_{i-1}]}{2\sqrt{g_i b_i}}$$

    - Where s_0, s_1, and T_i are parameters

Macroscopic Bridge – IDM

- Under equilibrium conditions, IDM model reduces to the following density-speed relationship:

$$k = \frac{1}{(s_0 + vT)\left[1 - \left(\frac{v}{v_\mathrm{f}}\right)^{\delta}\right]^{-1/2}}$$

- If one further assumes that $s_0 = s_1 = 0$ and $\delta = 1$, a special case results:

$$v = \frac{(s-L)^2}{2v_\mathrm{f}T^2}[-1 + \sqrt{1 + \frac{4T^2v_\mathrm{f}^2}{(s-L)^2}}]$$

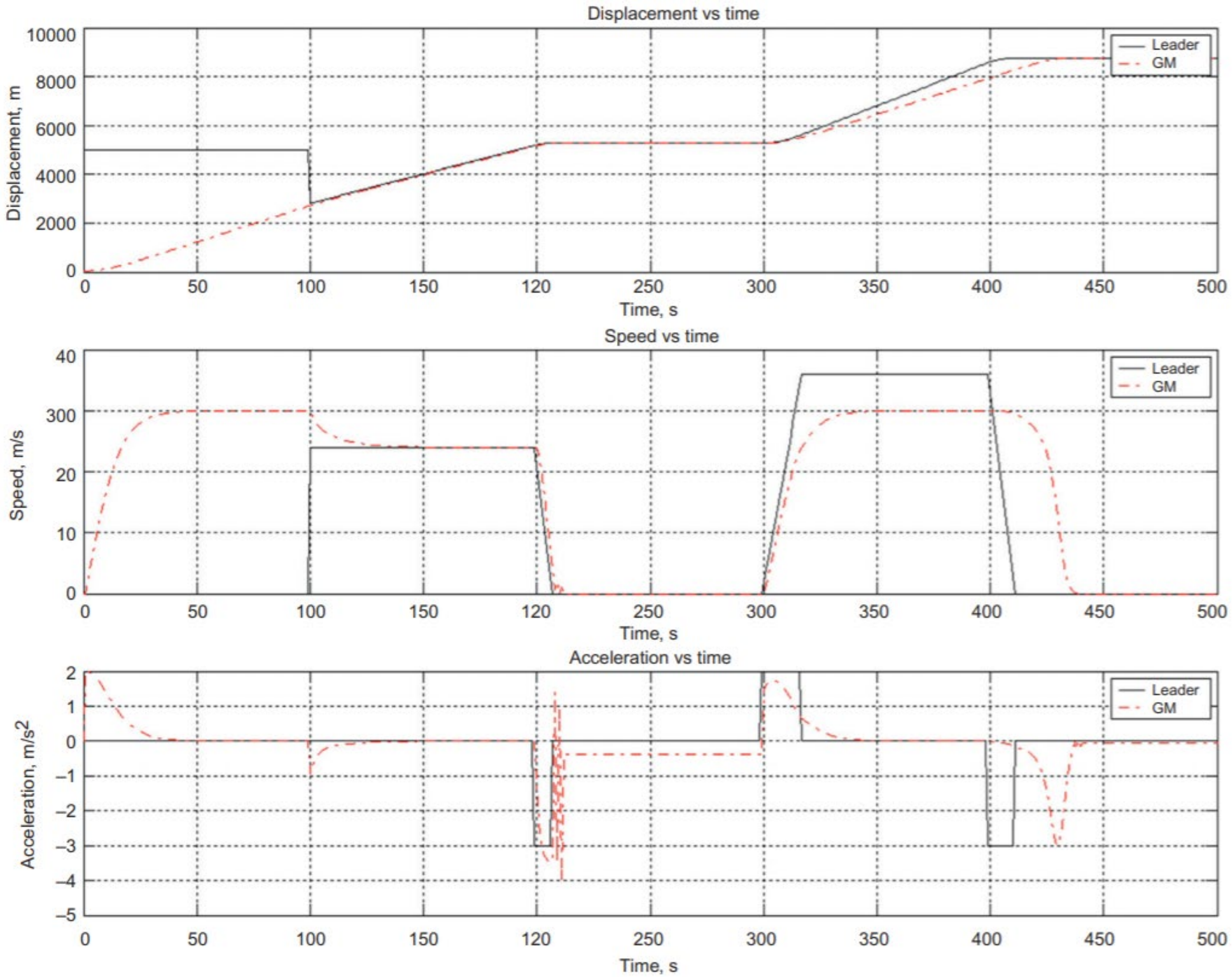
- Where T is the average safe time headway, $s = 1/k$ is the average spacing, and k is traffic density.

Microscopic Benchmarking – IDM

- The benchmarking result of the IDM is plotted, and the set of parameters is presented in table below.
- ***<u>Start-up:</u>*** The model is able to start the vehicle up from standstill. See when $t > 0\ s$.
- ***<u>Speedup:</u>*** The model can speed the vehicle up realistically to its desired speed. See when $0 < t < 100\ s$.
- ***<u>Free flow:</u>*** The model can reach and settle at the desired speed under free-flow conditions. See when $0 < t < 100\ s$.

l_i	v_i	τ_i	δ	s_0
6 m	30 m/s	1.0 s	2	2 m
A_i	b_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
2.0 m/s^2	4.0 m/s^2	39.5 m	0 m/s	0 m/s^2

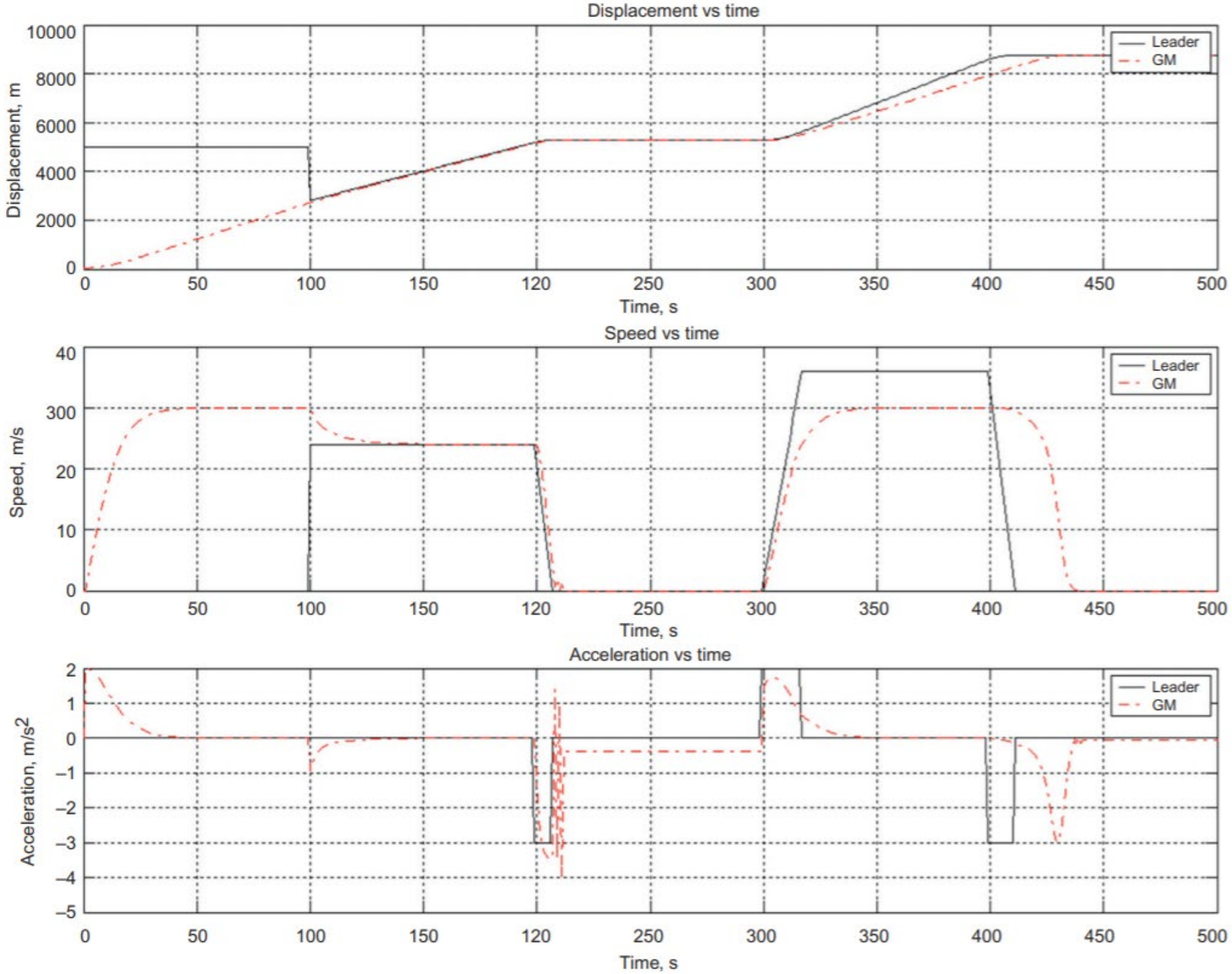
Microscopic benchmarking parameters of the IDM



Microscopic benchmarking of the IDM

Microscopic Benchmarking – IDM

- ***Cutoff:*** The model retains control and responds reasonably when a vehicle cuts in in front. See around $t = 100$ s.
- ***Following:*** The model can adopt the leader's speed and follow the leader at a reasonable distance. See when $100 < t < 200\ s$.
- ***Stop and go:*** The model exhibits some oscillation in acceleration, stopping behind the leading vehicle. The model can start moving when the leader departs. See when $200 \leq t \leq 300\ s$.
- ***Trailing:*** The model can speed up normally without being tempted to speed up by its speeding leader. See when $300 < t < 400\ s$.
- ***Approaching:*** The model is able to decelerate properly when approaching a stationary vehicle. See when $400 \leq t < 420\ s$.
- ***Stopping:*** The model is able to stop the vehicle safely behind the stationary vehicle. See when $t \geq 420\ s$



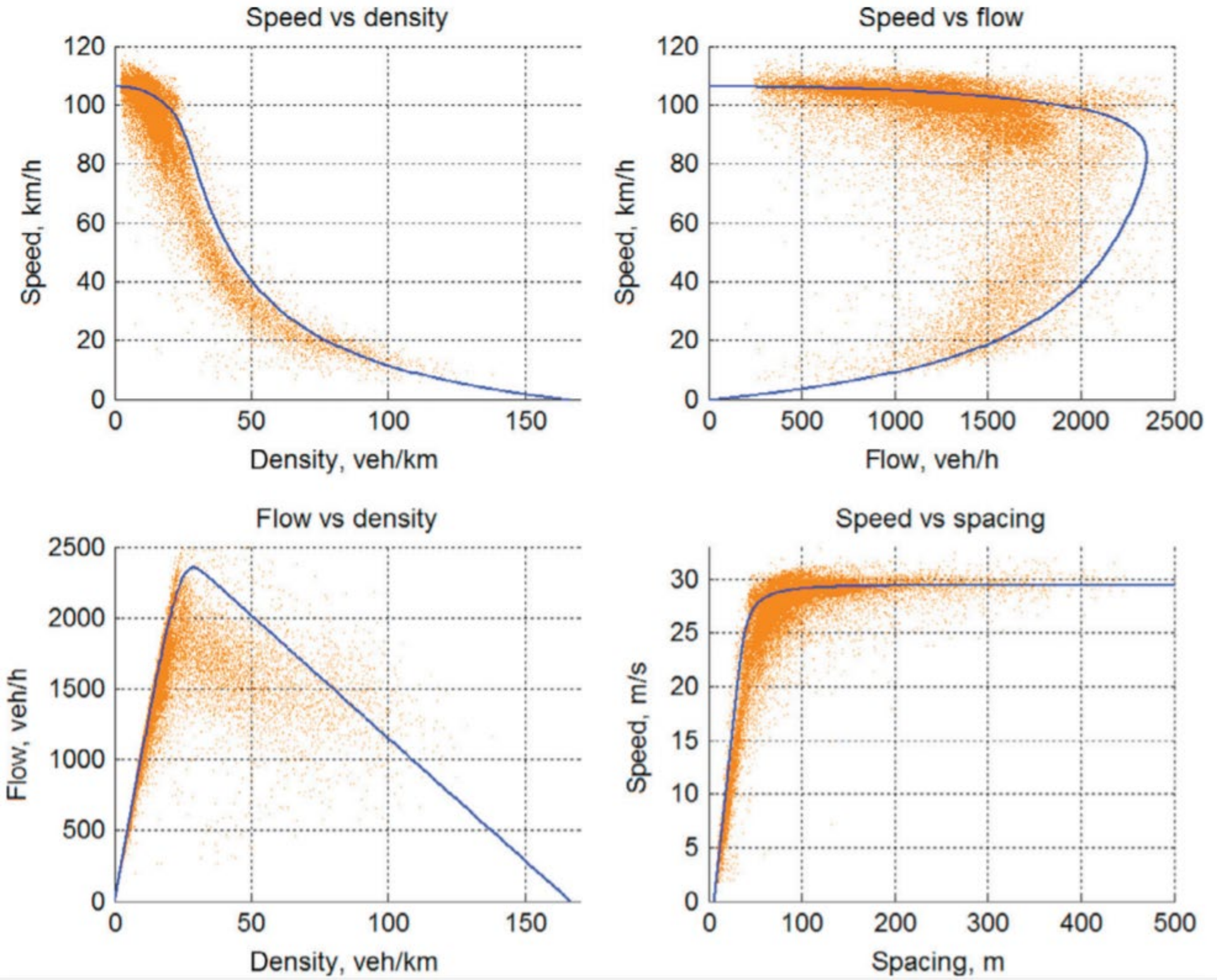
Microscopic benchmarking of the IDM

Macroscopic Benchmarking – IDM

- The fundamental diagram implied by the IDM is depicted.
- The model employs four parameters and exhibits a desirable shape with good fitting quality.
- The above benchmarking is based on the set of parameters in Table below
- The outcome may differ for a different set of parameters.
- The model meets the boundary conditions at $(k = 0, v = v_f)$ and $(k = k_j, v = 0)$.

v_f	T	δ	s_0
29.5 m/s	1.7 s	15	4 m

Macroscopic benchmarking parameters of the IDM



Fundamental diagram implied by the IDM

Van Aerde Model

- The Van Aerde car-following model combines the Pipes model and the Greenshields model into a single equation:

$$s_i = c_1 + c_3 \dot{x}_i + c_2/(v_f - \dot{x}_i)$$

$$\begin{cases} c_1 & = \frac{v_\mathrm{f}}{k_\mathrm{j} v_\mathrm{m}^2}(2v_\mathrm{m} - v_\mathrm{f}), \\ c_2 & = \frac{v_\mathrm{f}}{k_\mathrm{j} v_\mathrm{m}^2}(v_\mathrm{f} - v_\mathrm{m})^2, \\ c_3 & = \frac{1}{q_\mathrm{m}} - \frac{v_\mathrm{f}}{k_\mathrm{j} v_\mathrm{m}^2}, \end{cases}$$

- where v_f is the free-flow speed of the roadway facility, k_j is the jam density, and v_m is the optimal speed at capacity q_m.

Macroscopic Bridge – Van Aerde Model

- Under equilibrium conditions, the Van Aerde model reduces to the following density-speed relationship:

$$k = \frac{1}{c_1 + c_3 v + c_2/(v_f - v)}$$

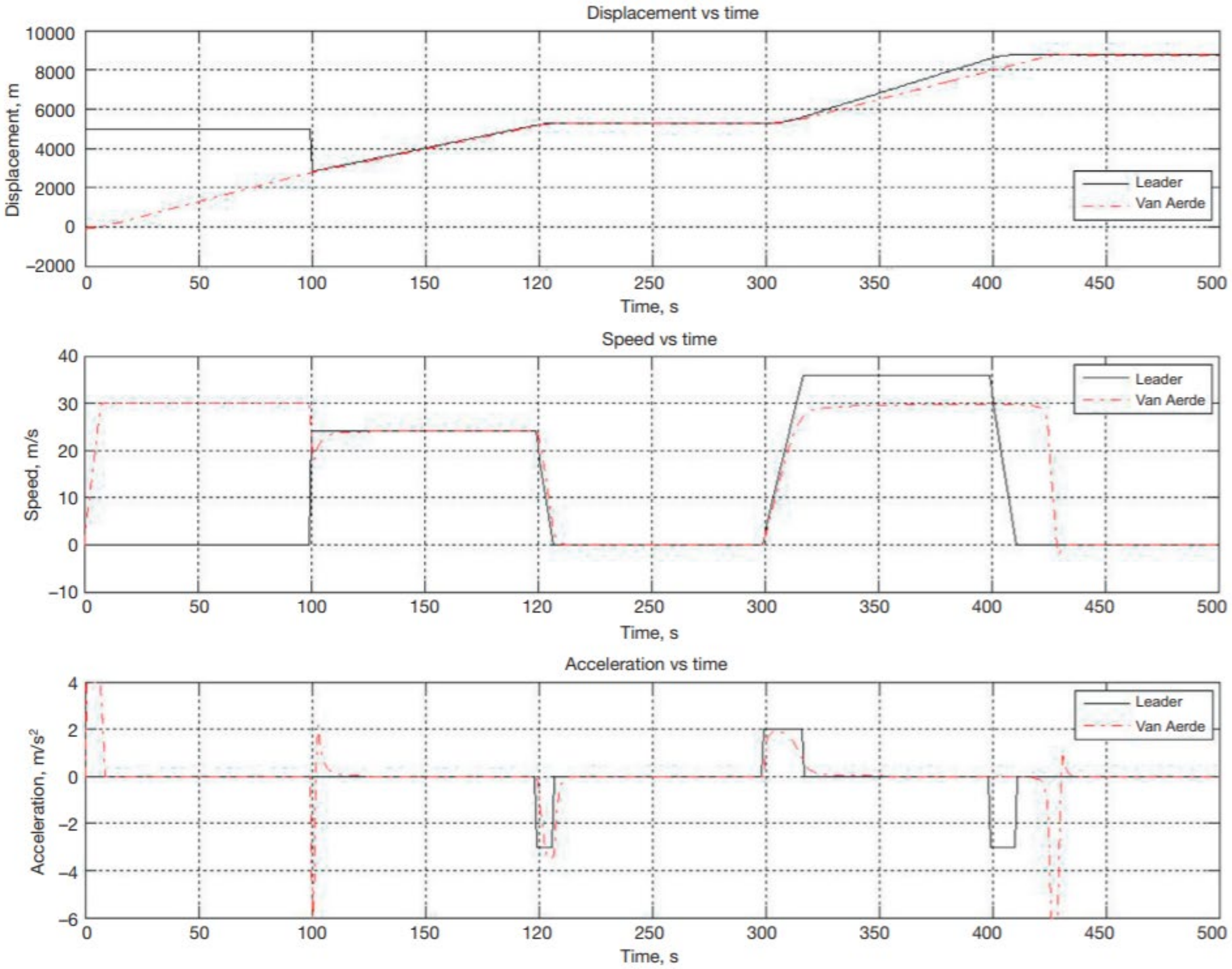
$$\begin{cases} c_1 & = \frac{v_f}{k_j v_m^2}(2v_m - v_f), \\ c_2 & = \frac{v_f}{k_j v_m^2}(v_f - v_m)^2, \\ c_3 & = \frac{1}{q_m} - \frac{v_f}{k_j v_m^2}, \end{cases}$$

Microscopic Benchmarking – Van Aerde Model

- The benchmarking result of the Van Aerde model is plotted, and the set of parameters is presented in table below.
- ***Start-up:*** The model is able to start the vehicle up from standstill. See when $t > 0$ s.
- ***Speedup:*** The same argument as in the corresponding part for the Newell nonlinear car-following model applies here. See when $0 < t < 100$ s.
- ***Free flow:*** The model is able to reach and settle at the desired speed under free-flow conditions. See when $0 < t < 100$ s.

k_j	v_f	τ_i	v_m	q_m
1/6 vehicles/m	30 m/s	1.0 s	25 m/s	1800 vehicles/h
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$		
–99.4 m	0 m/s	0 m/s^2		

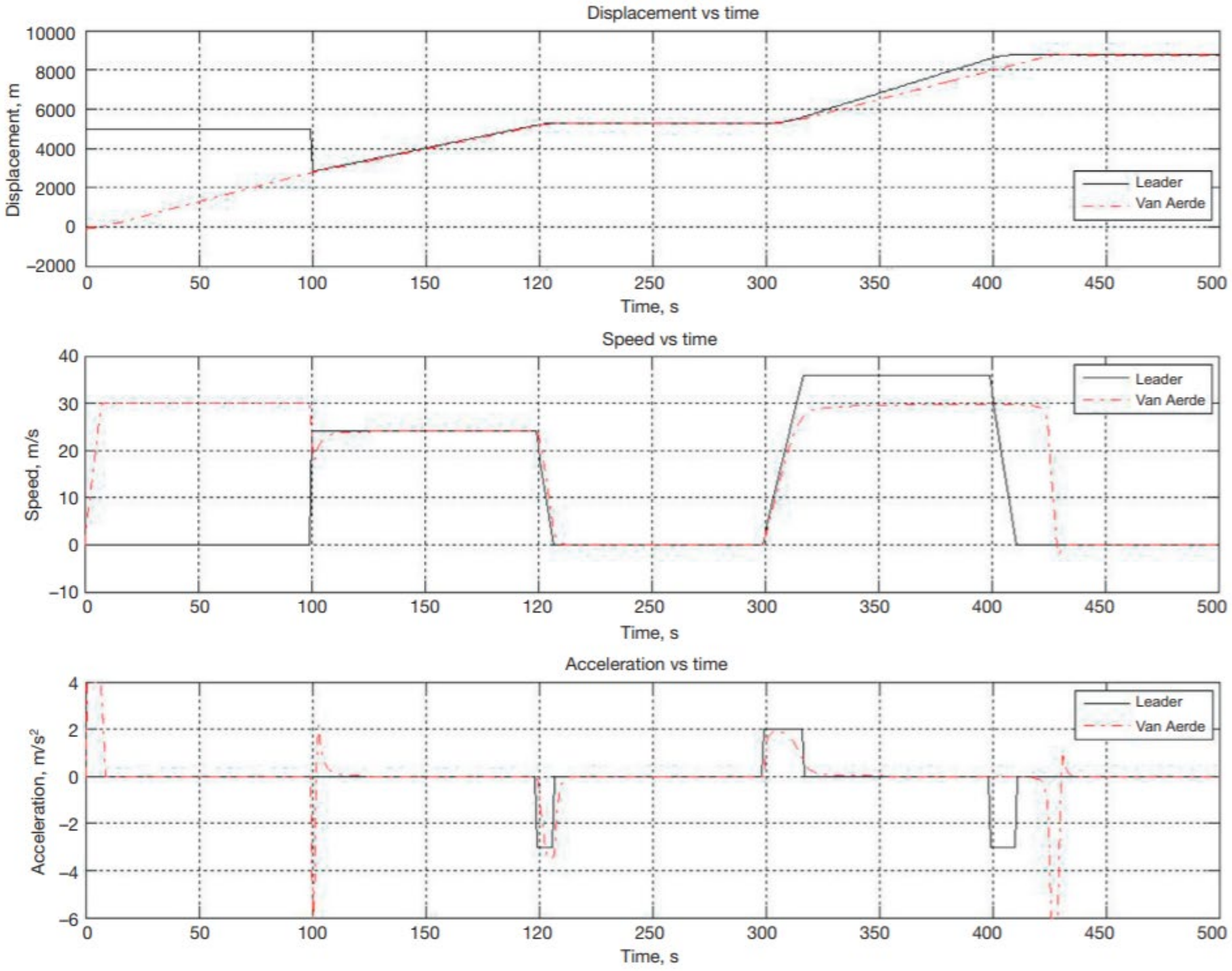
Microscopic benchmarking parameters of the Van Aerde model



Microscopic benchmarking of the Van Aerde model.

Microscopic Benchmarking – Van Aerde Model

- ***Cutoff:*** The same argument as in the corresponding part for the Newell nonlinear car-following model applies here. See around $t = 100$ s.
- ***Following:*** The model is able to adopt the leader's speed and follow the leader at a reasonable distance. See when $100 < t < 200$ s.
- ***Stop and go:*** The model is able to stop the vehicle safely behind its leader and start the vehicle moving when the leader departs. See when $200 \le t \le 300$ s.
- ***Trailing:*** The model is able to speed up normally without being tempted to speed up by its speeding leader. See when $300 < t < 400$ s.
- ***Approaching:*** With limiting deceleration, the model is able to decelerate properly when approaching a stationary vehicle. See when $400 \le t < 420$ s.
- ***Stopping:*** The model is able to stop the vehicle safely behind the stationary vehicle. See when $t \ge 420$ s.



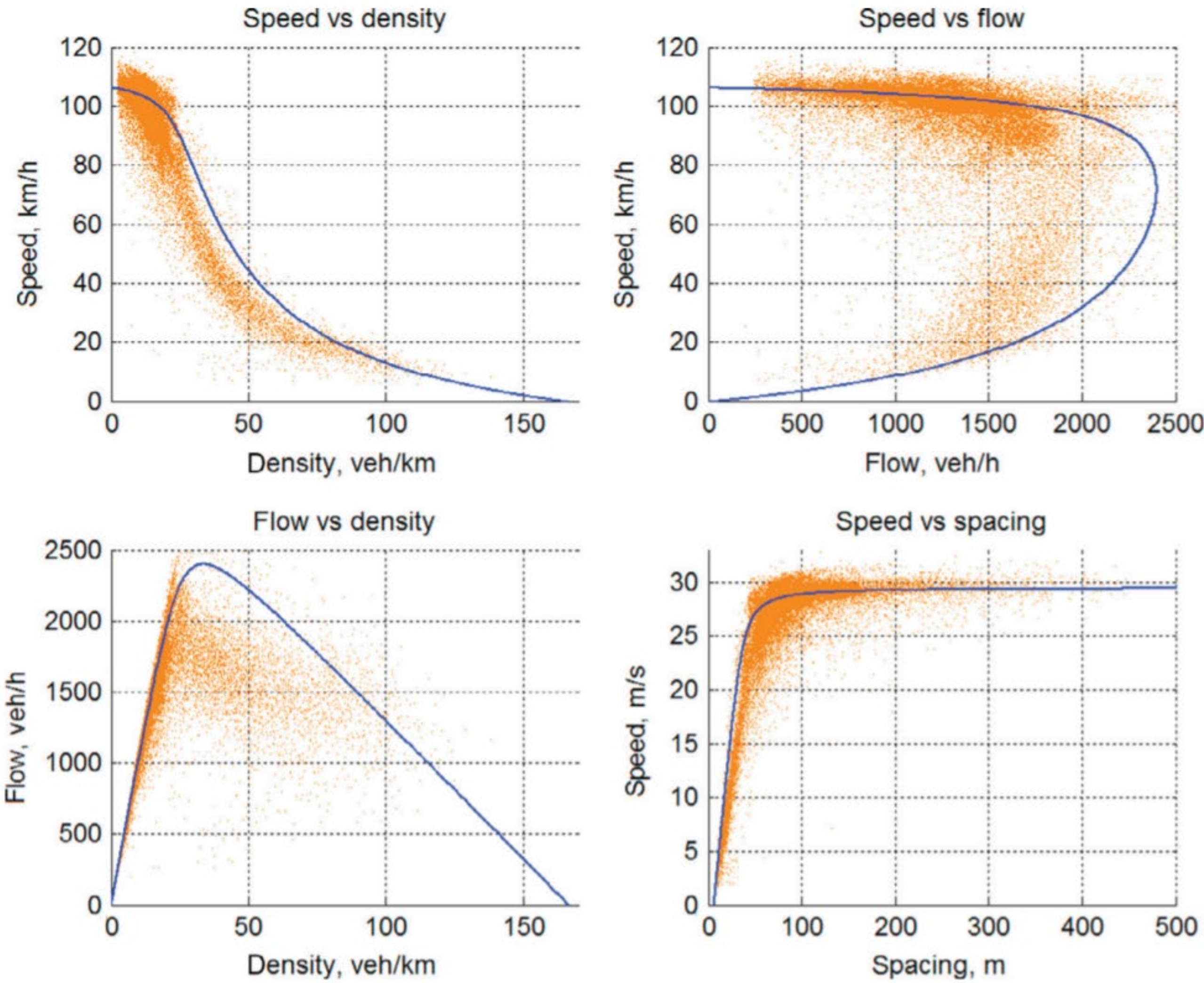
Microscopic benchmarking of the Van Aerde model.

Macroscopic Benchmarking – Van Aerde Model

- The fundamental diagram implied by the Van Aerde model is depicted.
- The model employs four parameters and exhibits a desirable shape with good fitting quality.

v_f	k_j	v_m	q_m
29.5 m/s	0.25 vehicles/m	20 m/s	1950 vehicles/h

Macroscopic benchmarking parameters of the Van Aerde model



Fundamental diagram implied by the Van Aerde model.

Psychophysical Model

- The psychophysical model got its name because it involves both psychological activities (such as perception-reaction threshold and unconscious car following) and physical behavior (e.g., accelerating and decelerating efforts).
- Compared with the models introduced before, this model captures more driving regimes explicitly, such as
  - free flow (no reaction area),
  - approaching (reaction area),
  - following (car following area),
  - and decelerating (deceleration area).
- A typical psychophysical model is the one proposed by Wiedemann in 1974.

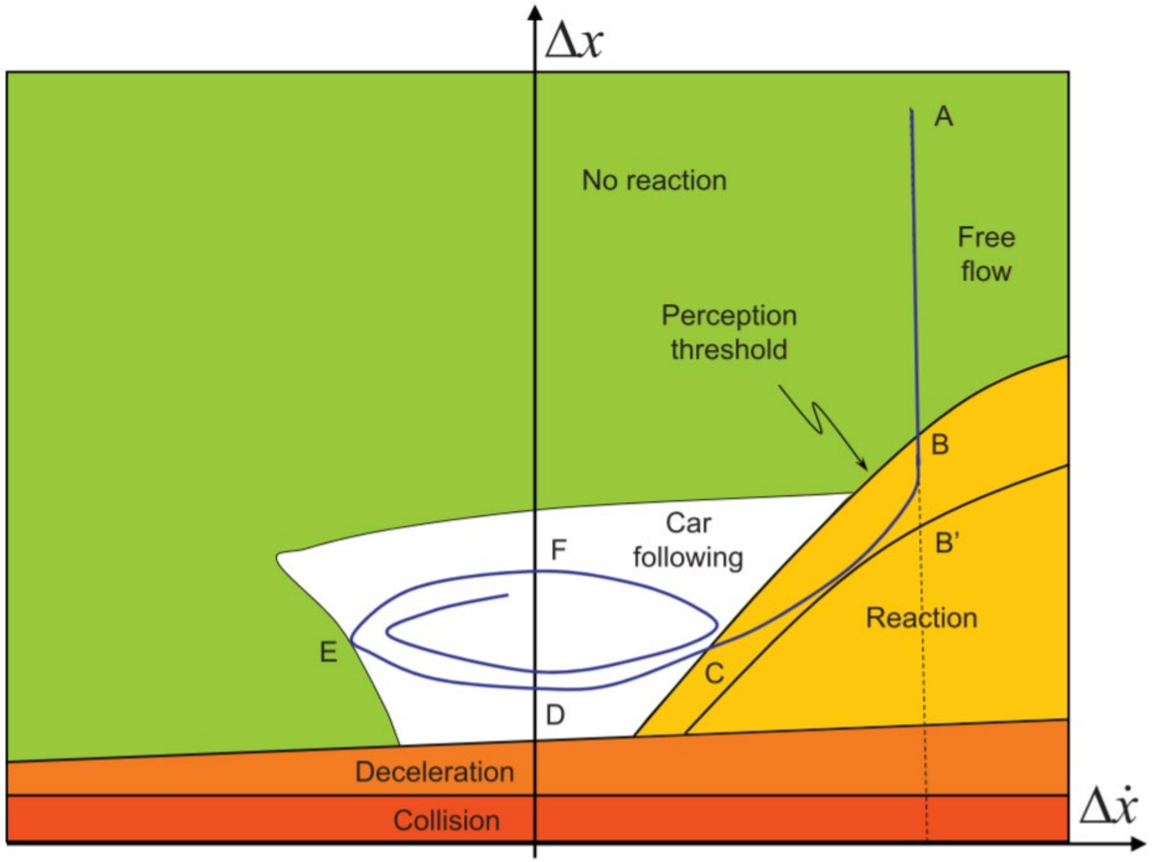


Illustration of a psychophysical model

Psychophysical Model

- The model considers two major factors influencing driver's operational control:
  - relative position $\Delta x = x_{i-1} - x_i$
  - relative speed $\Delta \dot{x} = \dot{x}_i - \dot{x}_{i-1}$.
- Hence, the working principle of the model can be illustrated by a diagram with $\Delta \dot{x}$ as the horizontal axis and Δx as the vertical axis.
- The operating condition of a vehicle i in relation to its leading vehicle $i-1$ can be represented as a point $(\dot{x}, x)$ in the diagram.
- As vehicle i moves, its operating point changes accordingly, leaving a trajectory in the diagram.

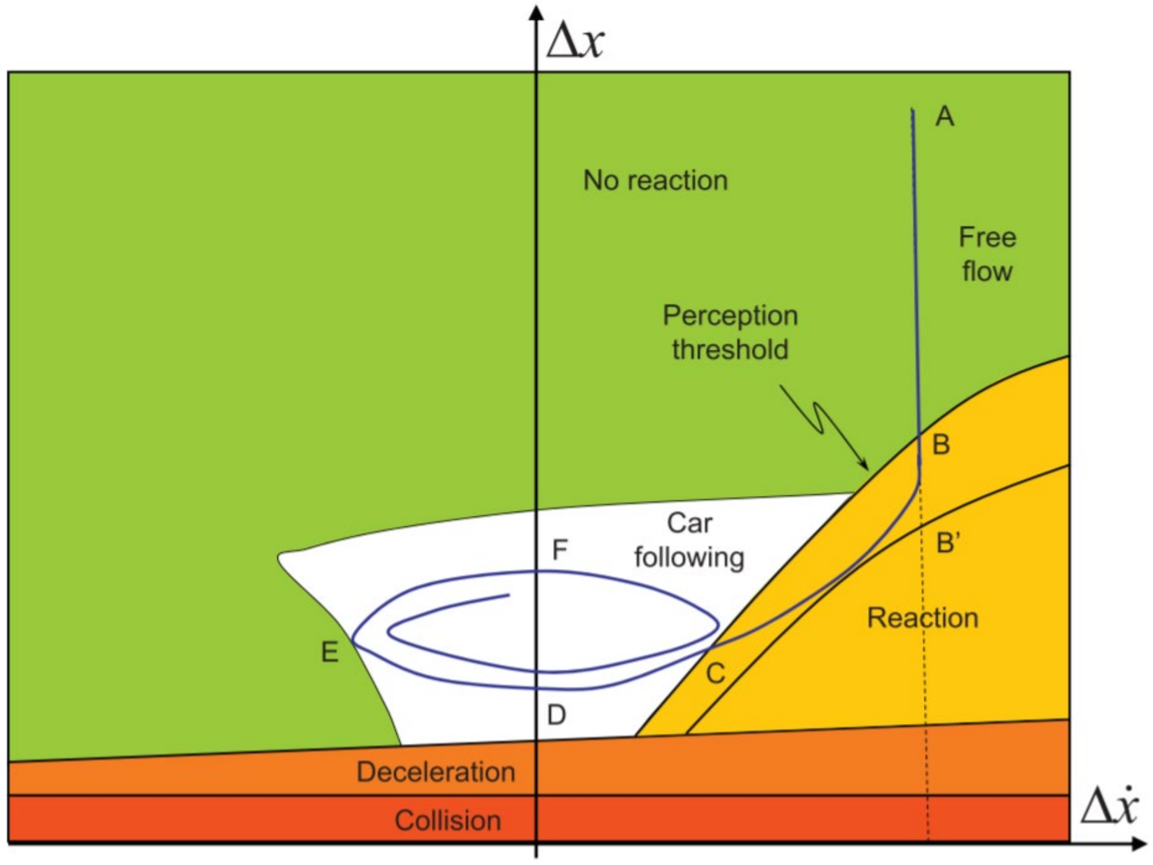


Illustration of a psychophysical model

Psychophysical Model

- If the point is on the negative side of $\Delta\dot{x}$, vehicle i is traveling more slowly than vehicle $i-1$, while the relation is reversed if the point is on the positive side of $\Delta\dot{x}$.
- In addition, the point is always on the positive side of Δx since vehicle $i-1$ is in front.
- The smaller x is, the closer the two vehicles are to each other.
- Hence, the two vehicles collide if Δx is less than one vehicle length l.
- This situation is depicted by the *collision area* in the diagram bounded by the horizontal axis and a horizontal line at $\Delta x = l$.
- On top of this area is another area, denoted the *deceleration area*, where the two vehicles are so close that an imminent collision causes the following vehicle to back up for safety.

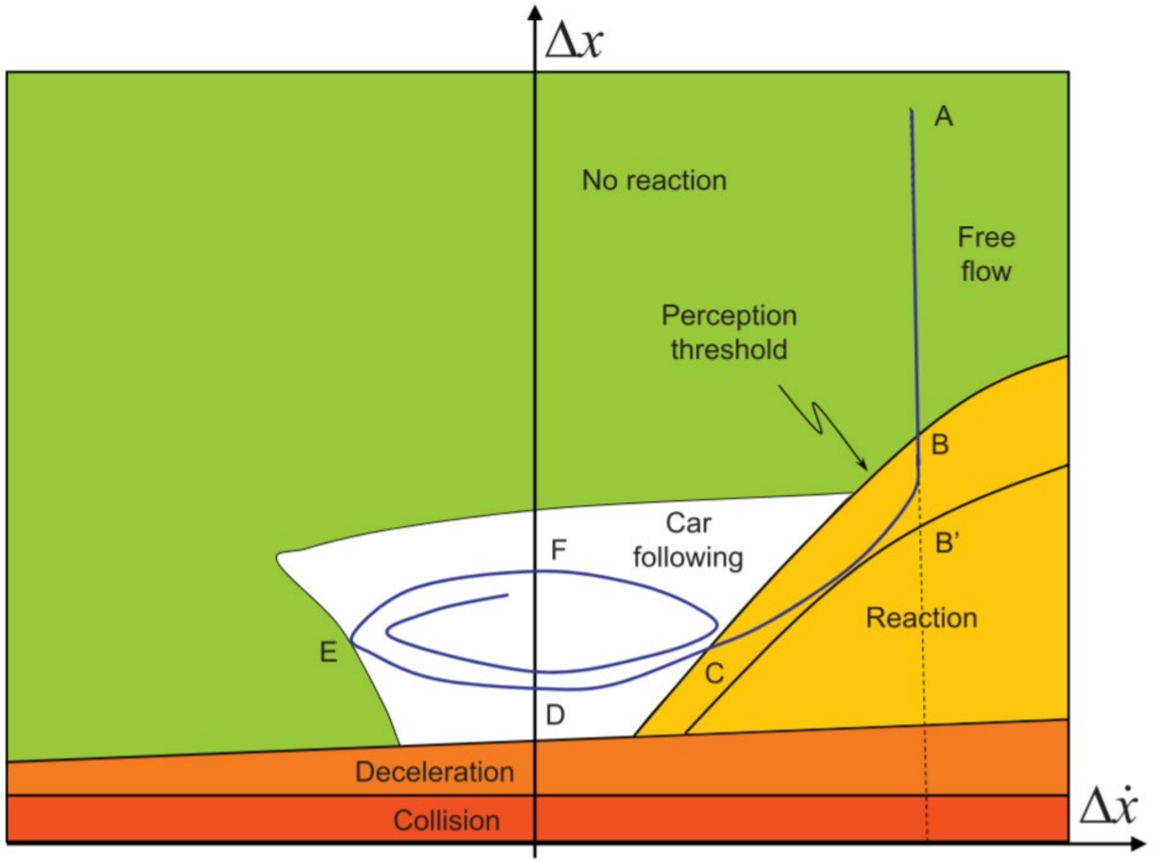


Illustration of a psychophysical model

Psychophysical Model

- Now, suppose vehicle i is traveling on a highway with the leading vehicle $i-1$ far ahead and vehicle i is faster than vehicle $i-1$.
- The operating condition can be represented by point A, which has a large positive Δx and a positive $\Delta \dot{x}$.
- Since vehicle $i-1$ is far ahead, driver i does not have to respond to vehicle $i-1$. This area is denoted as *no reaction* in the diagram.
- As vehicle i keeps moving, the relative speed $\Delta \dot{x}$ remains unchanged, but the relative separation Δx decreases.
- Hence, the operating point moves downward.
- Sooner or later, vehicle i will catch up and begin to respond to vehicle $i-1$ as the gap is closing.

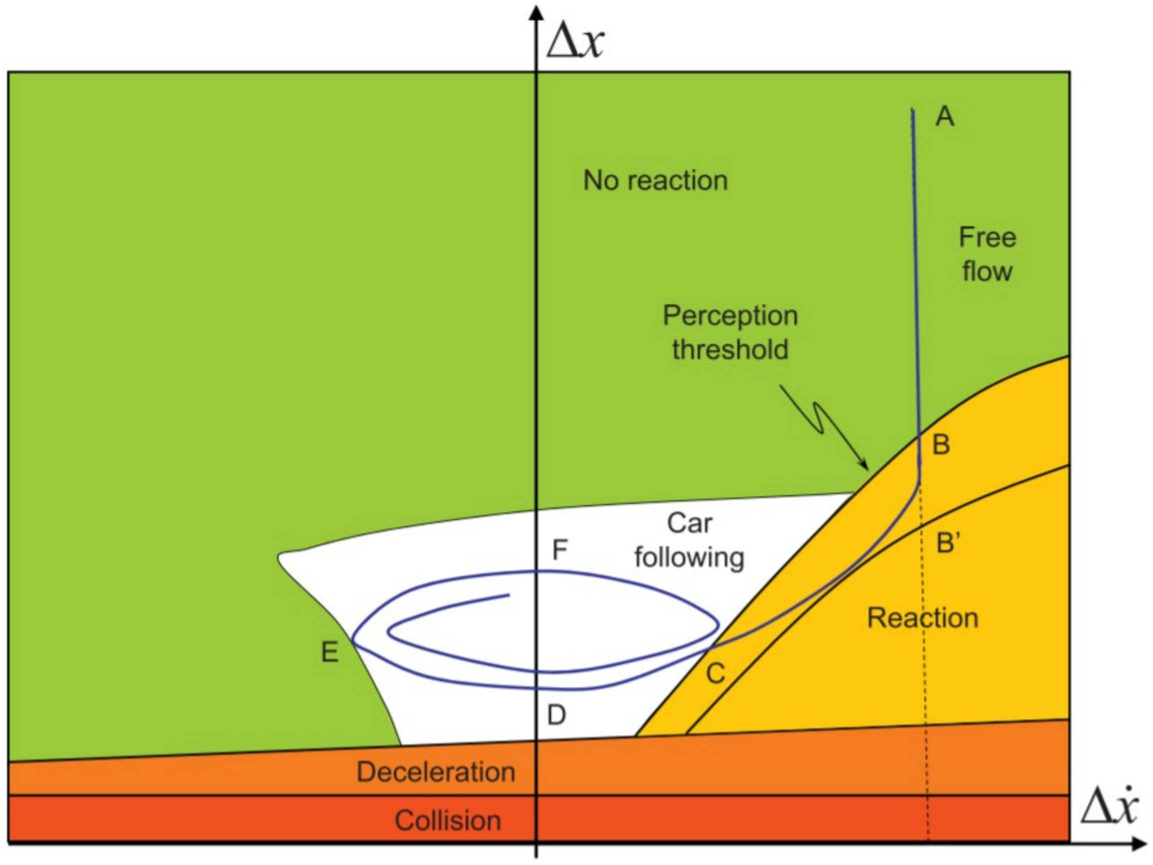


Illustration of a psychophysical model

Psychophysical Model

- However, the cutoff point is rather vague since this is a subjective matter.
- Perhaps a better way to draw the line is to set an upper limit such as point B, before which drivers are less likely to respond, and a lower limit such as point B' , after which drivers definitely need to respond.
- Note that points B and B' vary as $\Delta\dot{x}$ changes.
- The trajectory of point B or point B'under different $\Delta\dot{x}$ separates the *reaction area* from the no reaction area.
- Since driver i is likely to respond to vehicle $i - 1$ by slowing down (if lane change is not an option), the operating point moves downward and left toward to point C and finally to point D when the two vehicles are traveling at the same speed.

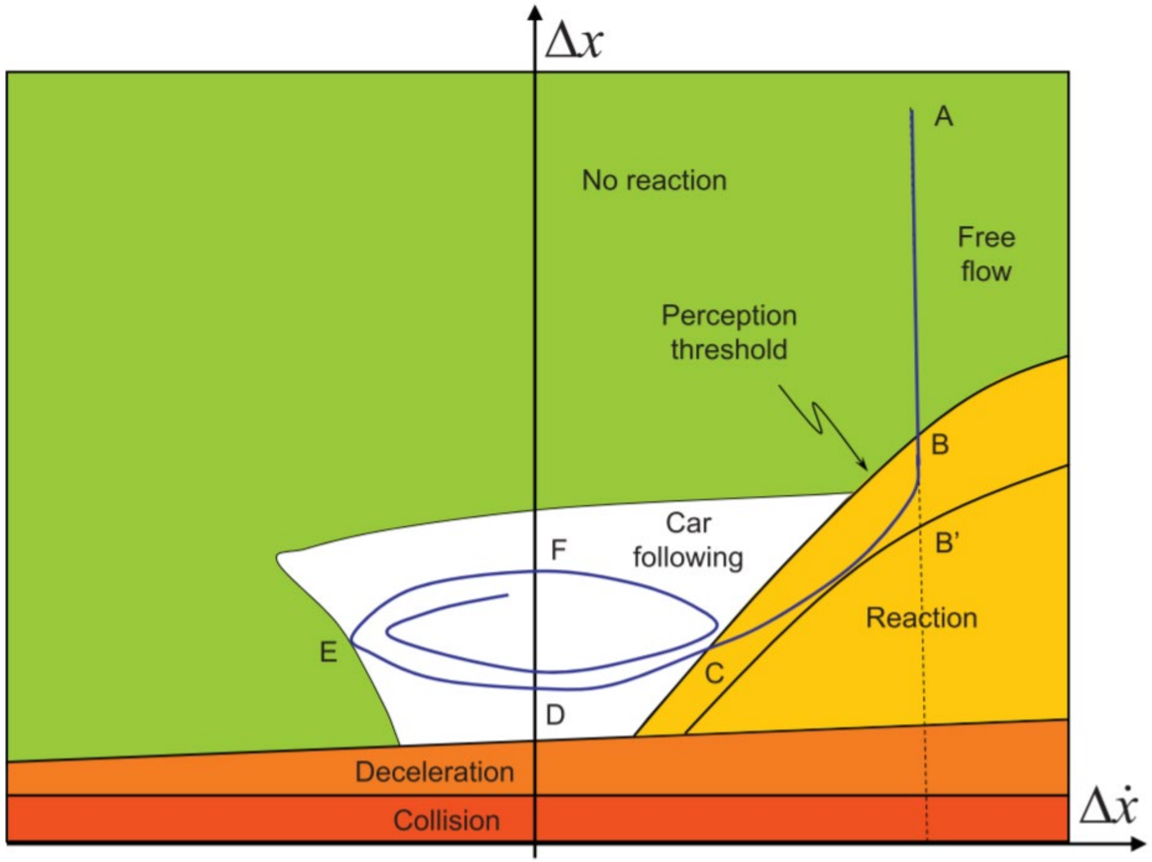


Illustration of a psychophysical model

Psychophysical Model

- Now the two vehicles are in the car-following regime, during which driver i tries to keep the same pace as vehicle $i-1$ separated by a comfortable distance.
- However, drivers are easily bored and distracted, especially during long trips.
- As a result, driver i might slow down unconsciously.
- Consequently, $\Delta\dot{x}$ becomes negative and keeps decreasing while Δx increases.
- As such, the operating point moves from D toward E, at which point the opening gap reminds driver i that he or she is falling behind.
- Hence, the driver begins to catch up.

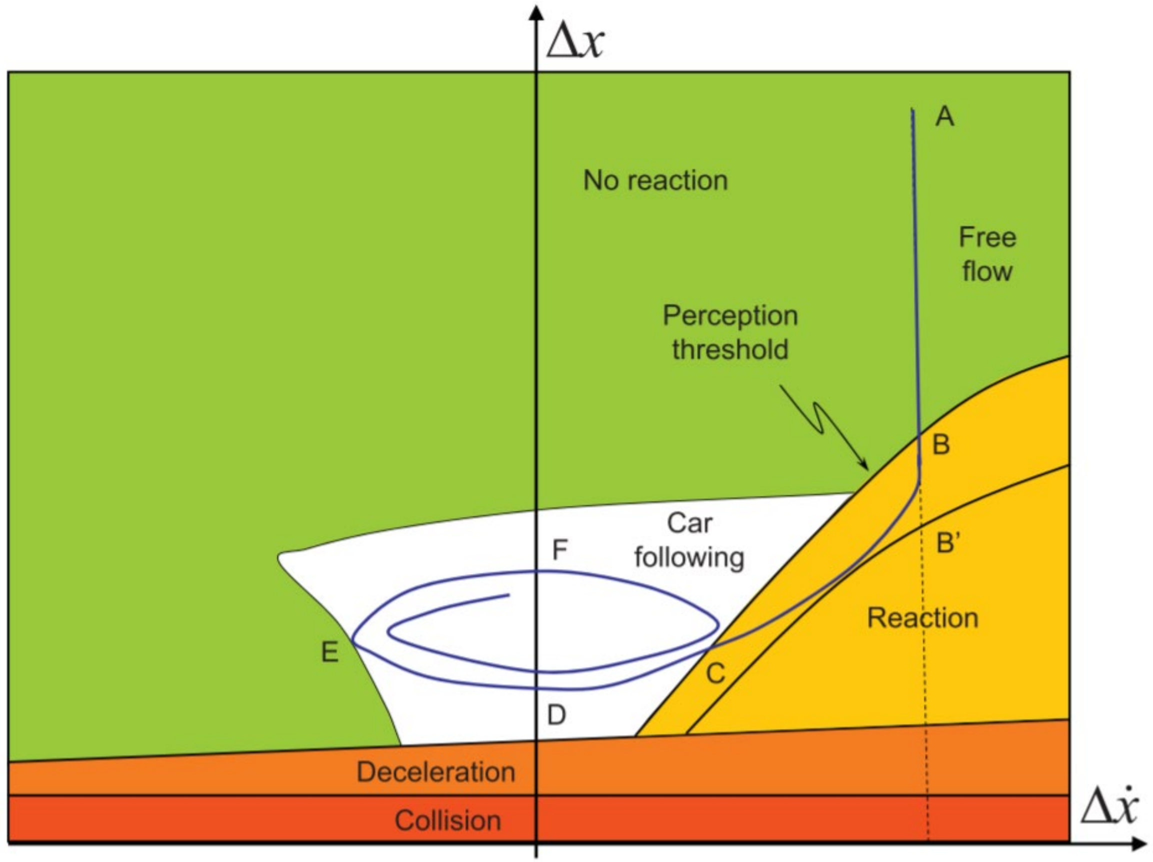


Illustration of a psychophysical model

Psychophysical Model

- This corresponds to a transition from E to F, when the two vehicles are again traveling at the same speed but with a large gap in between.
- Next, driver i may want to keep accelerating in order to shorten the gap to a comfortable level, which is denoted as a transition from F back to C.
- Therefore, as the driver oscillates back and forth around his or her comfortable car-following distance, the operating point drifts around within an area in the diagram denoted as *car following*.

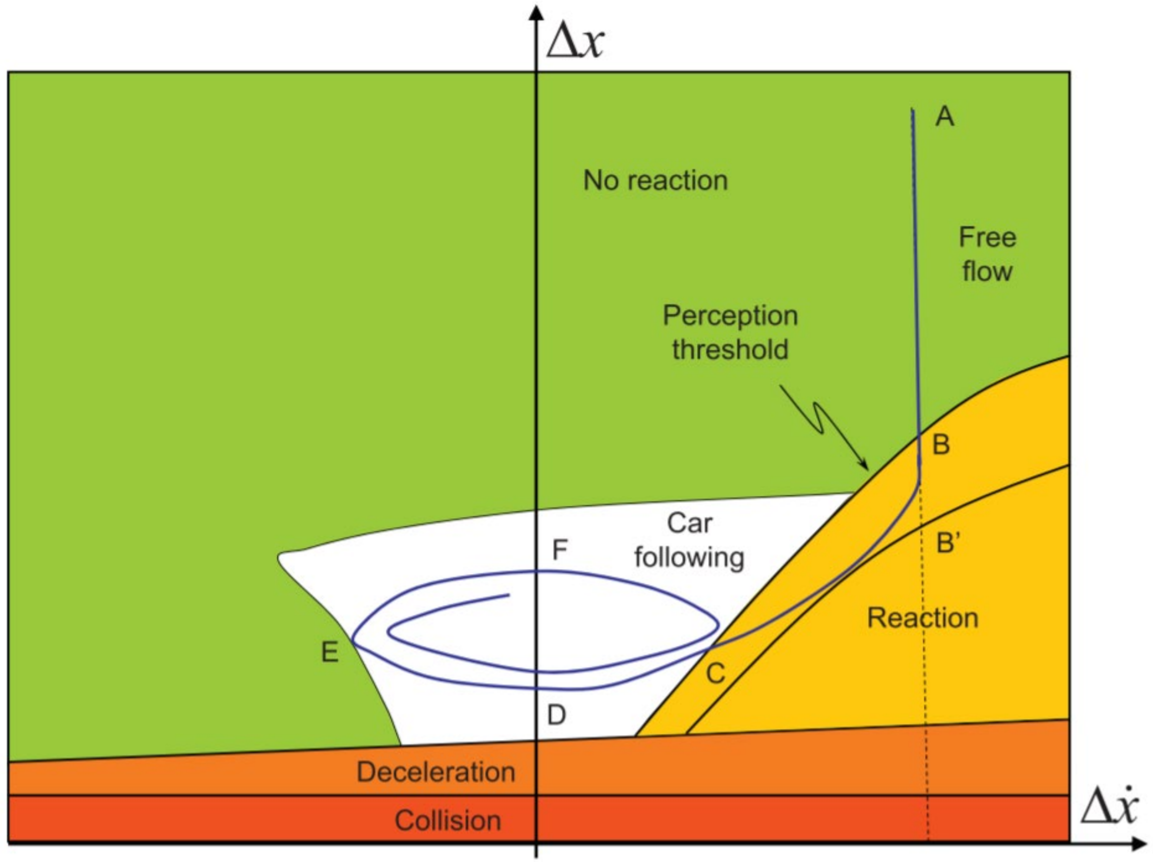


Illustration of a psychophysical model

Neural Network Model

- Perhaps the approach that best mimics driver behavior is artificial neural networks.
- This is because artificial neural networks are capable of associating, recognizing, organizing, memorizing, learning, and adapting.
- A neural network typically consists of many interconnected working units called neurons.
- A neuron receives inputs $x_1, x_2, \dots, x_n$ which are weighted $w_1, w_2, \dots, w_n$, respectively.
- The total input to the neuron is the weighted sum of individual inputs: $z = \sum_{i=1}^{n} w_i x_i$
- The output of the neuron y depends not only on z but also on the threshold of the neuron θ.
- The neuron outputs 1 if $z \geq \theta$ and 0 otherwise.

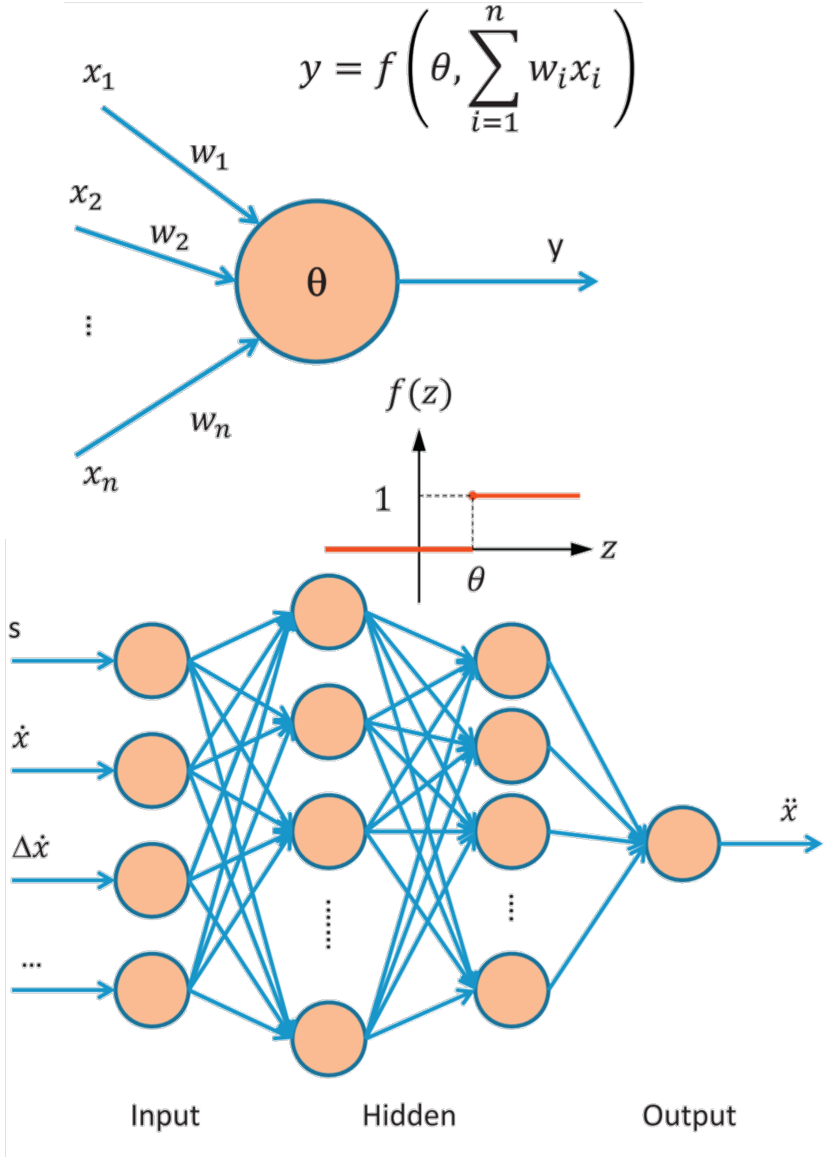


Illustration of a neural network model

Neural Network Model

- Neurons with such a simple functionality can be organized into neural networks of varying complexity and topology.
- Figure illustrates an example of a back-propagation neural network.
- The network consists of one input layer (which in turn consists of a set of neurons), one output layer, and one or more hidden layers.
- Each neuron feeds its output only forward to neurons in the next layer, without backward feeding and cross-layer connection.

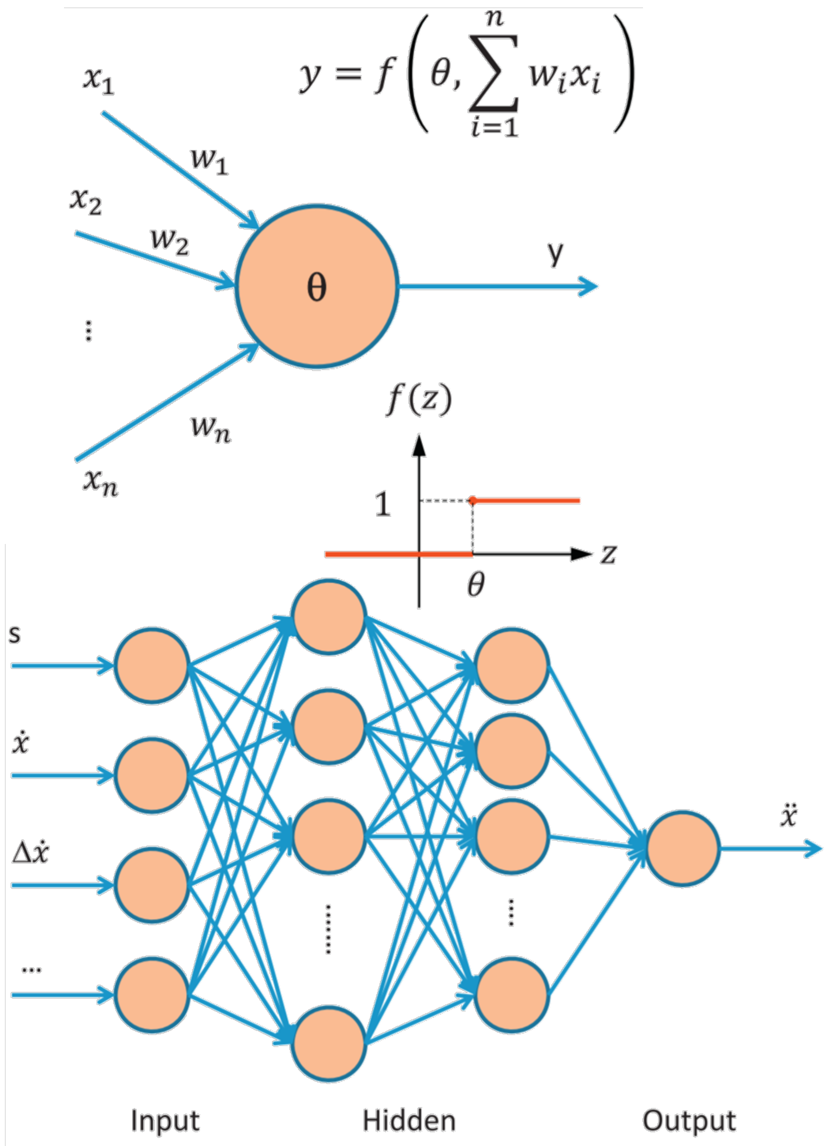


Illustration of a neural network model

Neural Network Model

- To apply neural networks to the modeling of car-following behavior, one first identifies a set of factors to be considered that influence the driver's operational control.
- For example, these influencing factors can be spacing s, speed $\dot{x}$, relative speed $\Delta\dot{x}$, etc.
- It is also possible to include other factors not considered before, such as a tailgating vehicle behind, weather, and lighting conditions, time of day, etc.
- These factors are represented by neurons in the input layer.
- The output layer in this example consists of only one neuron: acceleration/deceleration.
- If one needs to model not only longitudinal but also lateral motion, a second neuron is necessary to represent steering effort.
- Between input and output layers lie one or more hidden layers.
- The more hidden layers the network has, the more flexible it is, but the more complex it becomes.

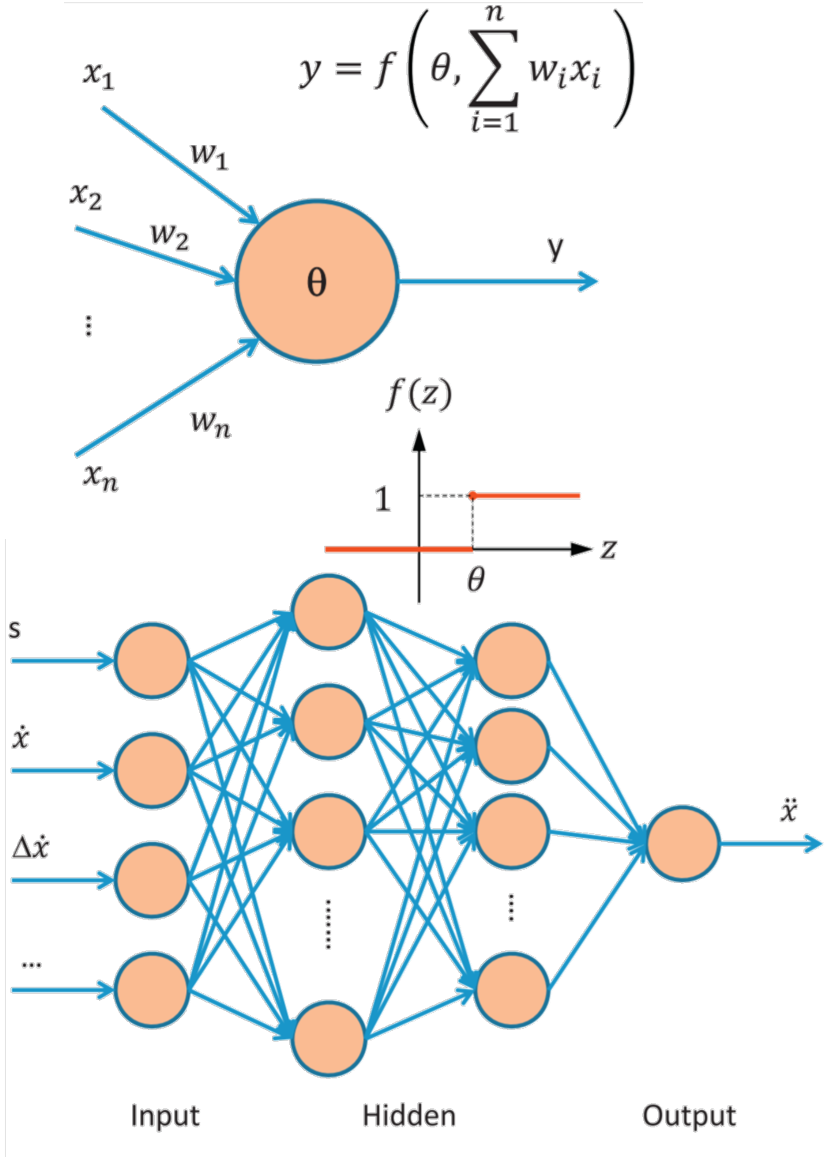


Illustration of a neural network model

Neural Network Model

- After the neural network has been constructed, it needs to be trained before it can be useful.
- The training process starts with data collection.
- For example, from field experiments, one observes that, at time t_1, a vector of input $[s(1), \dot{x}(1), \Delta\dot{x}(1), \dots]$ results in driver operational control $[\ddot{x}(1)]$, and more patterns are observed at $t_2, t_3, \dots, t_m$.

$$\begin{bmatrix} s(1), \dot{x}(1), \Delta\dot{x}(1), \dots \\ s(2), \dot{x}(2), \Delta\dot{x}(2), \dots \\ \dots \\ s(m), \dot{x}(m), \Delta\dot{x}(m), \dots \end{bmatrix} \Rightarrow \begin{bmatrix} \ddot{x}(1) \\ \ddot{x}(2) \\ \dots \\ \ddot{x}(m) \end{bmatrix}$$

- After initializing the neural network (i.e., assigning initial values to connection weights and neuron thresholds), if the computed output is different from the observed output, the error is propagated backward layer by layer to adjust their connection weights and neuron thresholds.
- This is why networks of this kind are called back-propagation networks.

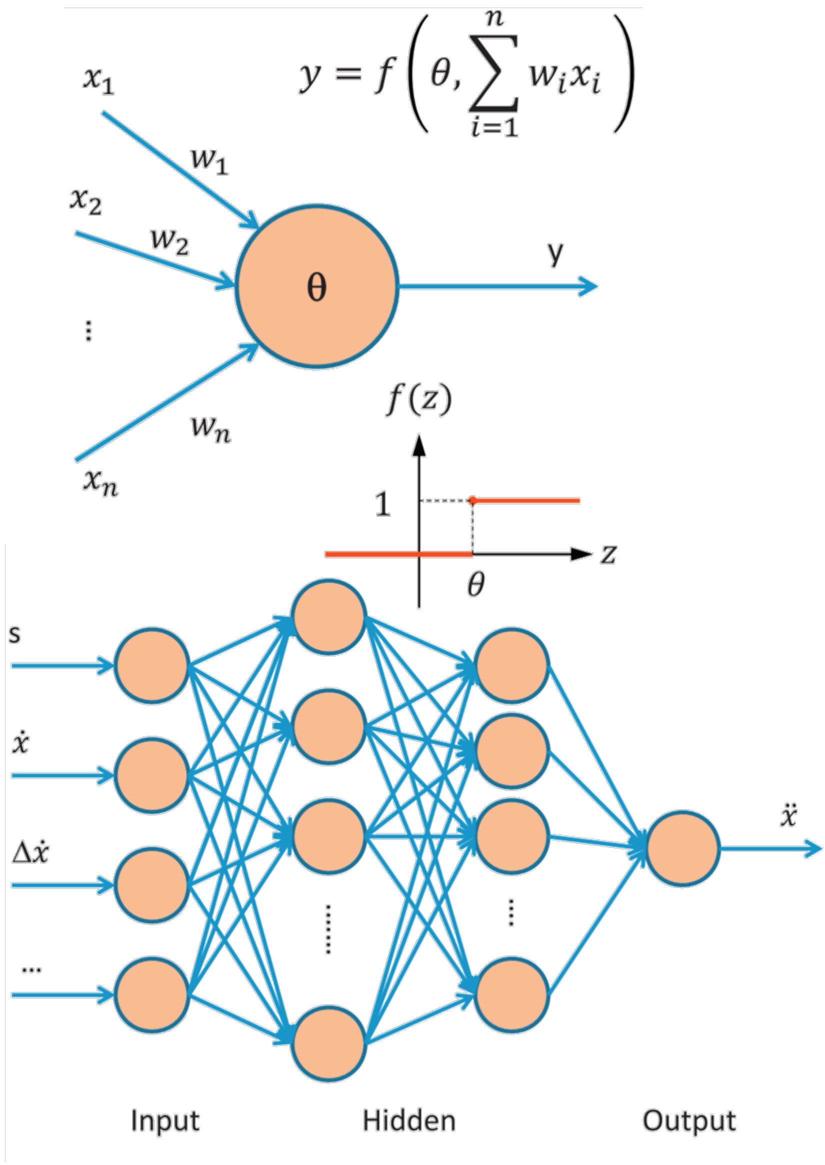


Illustration of a neural network model

Neural Network Model

- After the error has been propagated backward, the same input is imposed again at the input layer and the network computes a new output.
- This time, the output error, if any, should be smaller than in the previous round.
- Again, the error needs to be propagated back, and all the weights and thresholds are adjusted for a new round of learning.
- The process continues until the computed output becomes sufficiently close or equal to the observed output.
- This completes the learning of the first input-output pattern.
- Next, one continues with the training of the second row, the third row, etc.
- The training is completed after all data in the set have been trained and the neural network is able to associate the correct output with the corresponding input.
- The trained neural network is now ready to be applied to vehicle operational control.
- In addition, the neural network may continue learning while working, and hence adapt to a new environment which it has never encountered before.

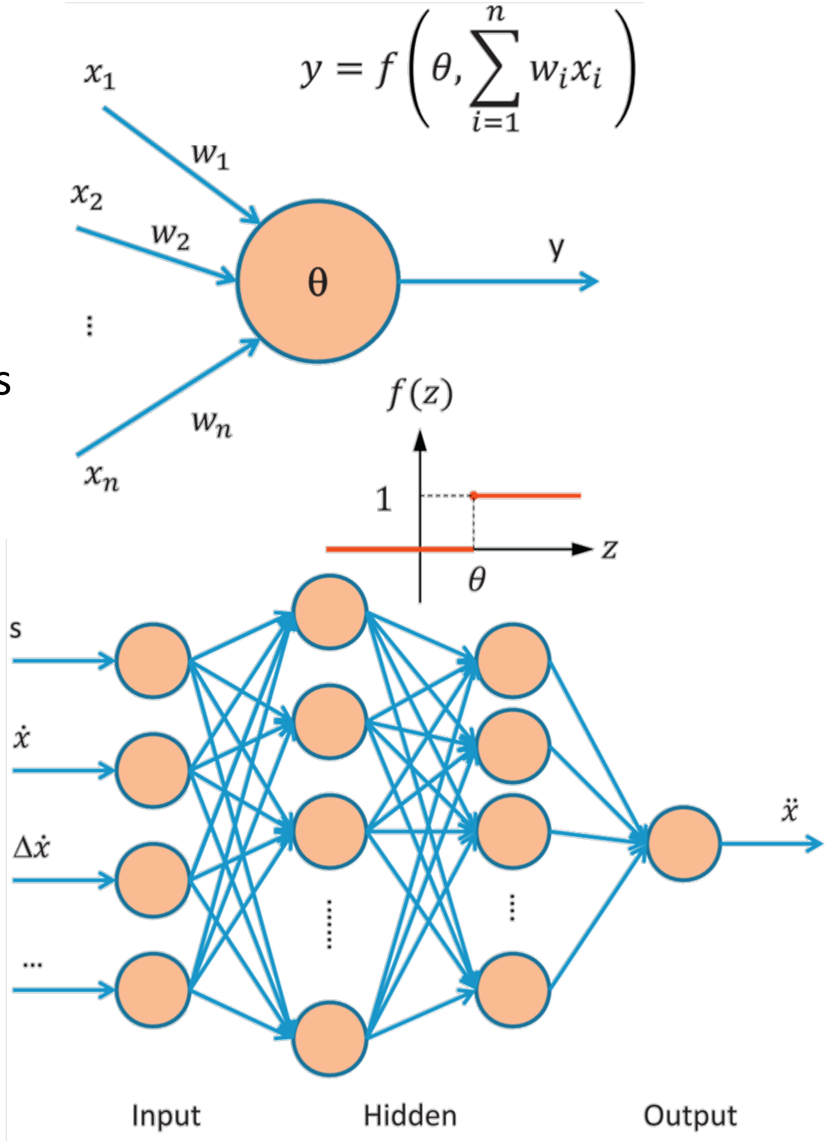


Illustration of a neural network model

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Thank
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