Microscopic Models of Traffic Flow – A

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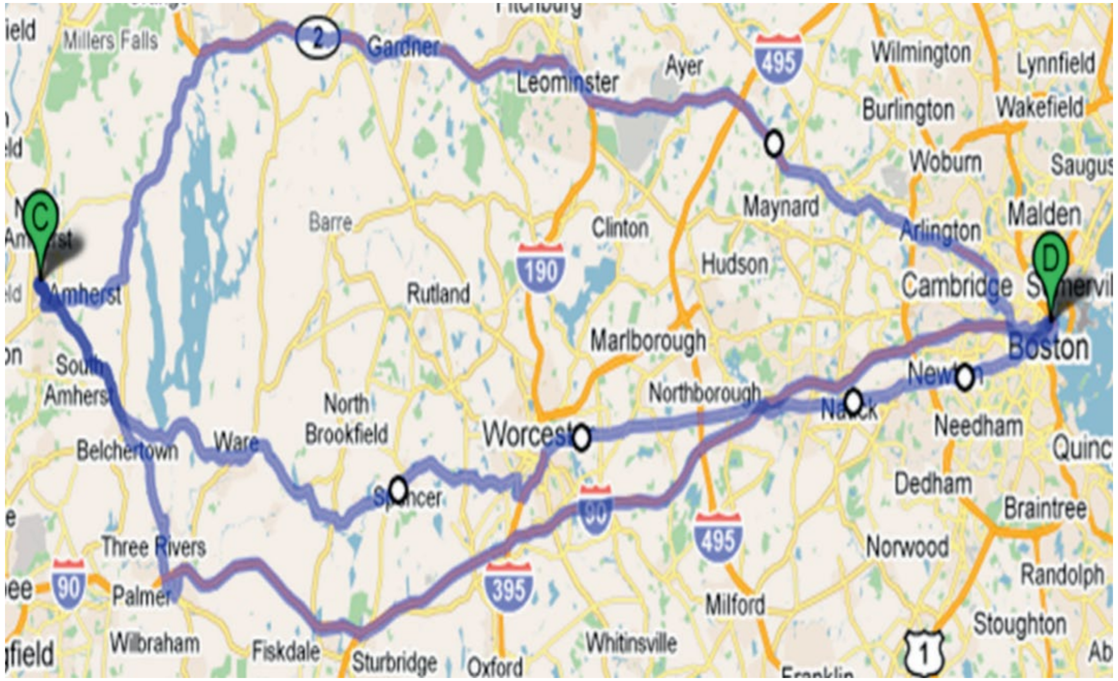
Introduction

- The models presented previously emphasize collective and average behavior of vehicles: flow, speed, and density
- They consider traffic flow as a compressible fluid.
- Central to these models are the relationships among flow, speed, and density as well as how they vary dynamically over time and space.
- Such models are termed macroscopic, and they are capable of capturing the number of vehicles flowing into and out of roadway segments over time, rather than tracking each and every vehicle as it moves along the roadway.
- In contrast, microscopic models emphasize the behavior of individual vehicles, and are capable of capturing the motion of and interaction among these vehicles.
- Unlike macroscopic models, which treat vehicles as a fluid, microscopic models represent a driver-vehicle unit as a particle without mass.
- Such a particle is sometimes referred to as an “active” particle since it is capable of making decisions based on rules stipulated in microscopic models.

Introduction

- Depending on the geographical scope and time frame involved, driving decisions can be categorized at three levels
  - Strategic,
  - Tactical, and
  - Operational.

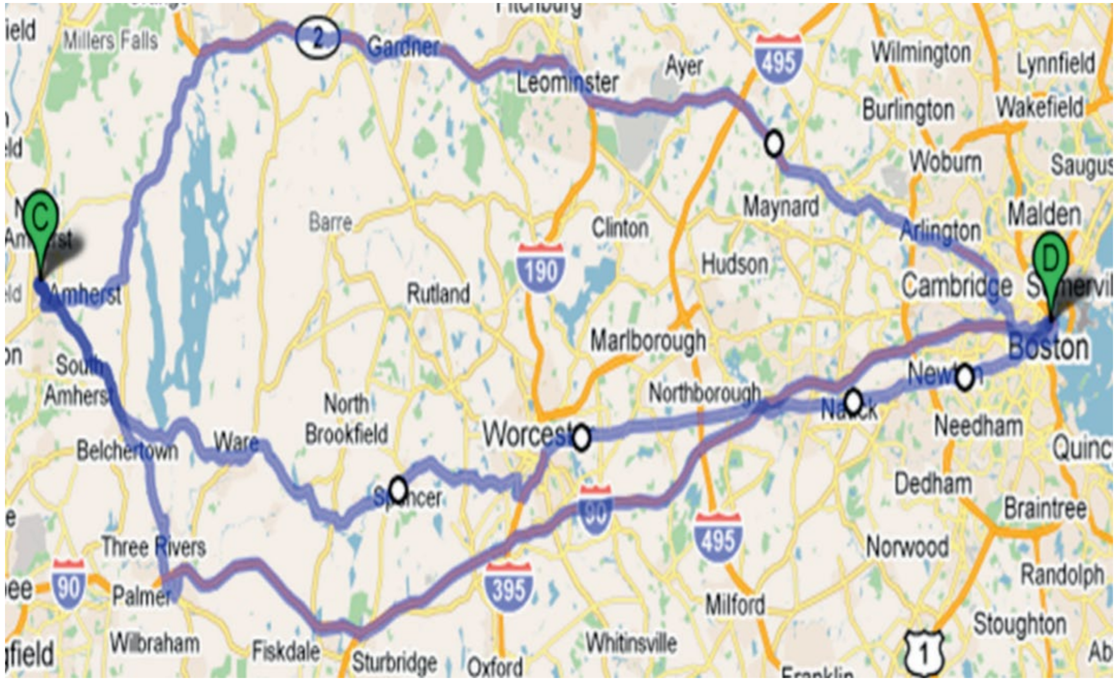
- Driving decisions at the strategic level involve a large geographical scope and a long-time frame.
- For example, the decision-making scenario faced by a driver who is about to travel from the University of Massachusetts Amherst (point C) to Boston (point D) is illustrated.



Strategic level of decision making

Introduction

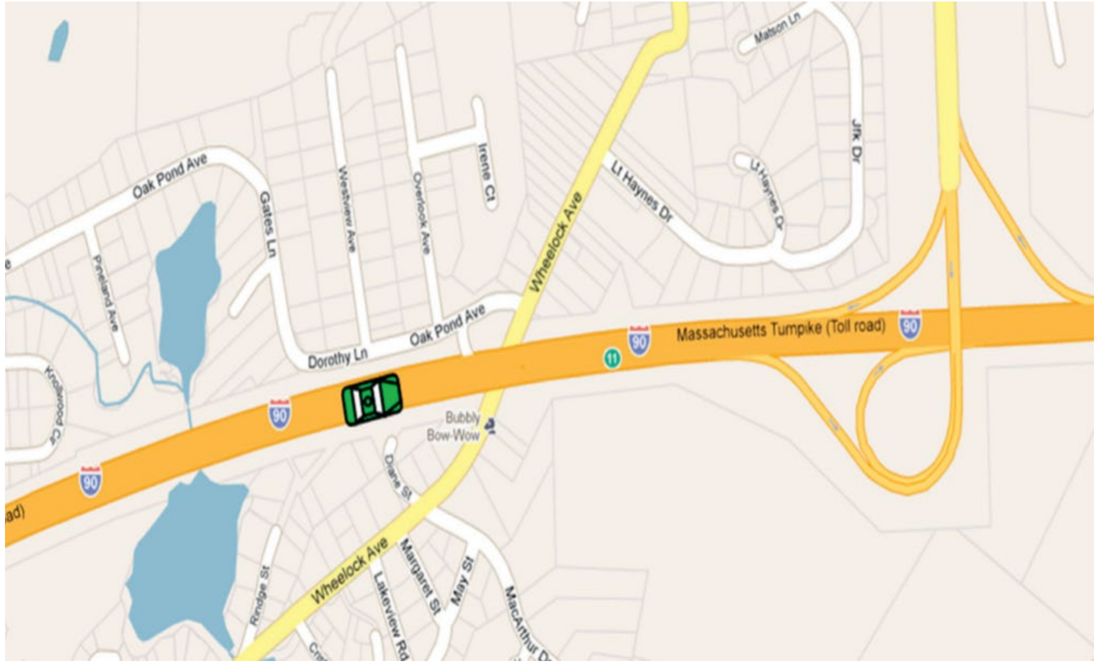
- The driver has at least three options:
  - The bottom route, which is the fastest route if there is no congestion, and the toll is about $5.
  - The top route, which is a scenic, rural highway that is rarely congested.
  - The middle route, which is the shortest route, but it goes through many town centers and traffic signals.
- This scenario constitutes a route-choice decision that involves a geographical scope of about 100 km and a time frame of a few hours.
- A microscopic model that describes how drivers make a **route choice** decision is called a route-choice model.
- Such a model is typically a discrete choice model which chooses one of a set of options based on some utilities and constraints.



Strategic level of decision making

Introduction

- After the driver has chosen a route and is traveling down the road, a tactical decision will have to be made sooner or later that involves a medium geographical scope and a medium time frame.
- For example, the driver needs to decide when and where to change to the side lane in preparation for using the upcoming exit.
- Such a case constitutes a lane-changing decision with a geographical scope of a few kilometers and a time frame of a few minutes.
- Again, a **lane-changing** model is typically a discrete choice model that determines the choice of a target lane from available options based on the driver's objective and constraints.



Tactical level of decision making

Introduction

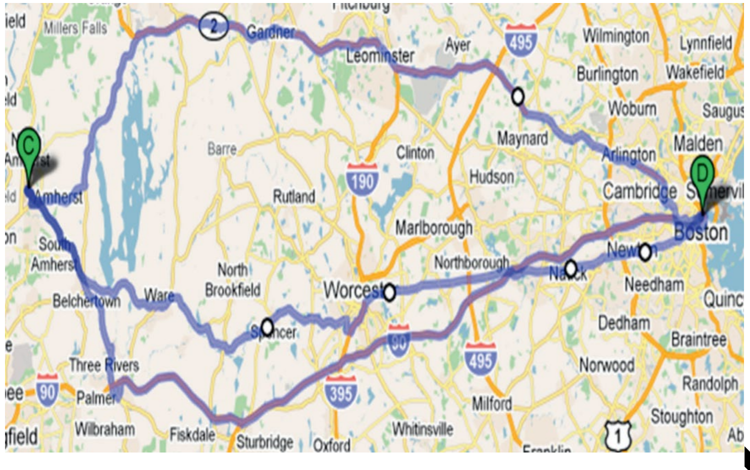
- An operational decision involves the driver's operational control of the vehicle in order to ensure safety and maintain mobility within a small geographical scope and a short time frame.
- For example, the driver in the circled vehicle in the figure is following another vehicle in a context of a geographical scope of tens of meters and a time frame of a few seconds.
- The driver needs to make a **car-following** decision on how to operate his or her vehicle (e.g., determine speed and acceleration in the next second) so as to avoid colliding with the leading vehicle.
- Meanwhile, if the driver feels stressed following the slow leading vehicle, the driver may want to change to another lane to improve his or her mobility.
- As such, the driver makes a **gap-acceptance** decision by looking for gaps in the adjacent lane and switching to that lane when an acceptable gap becomes available.

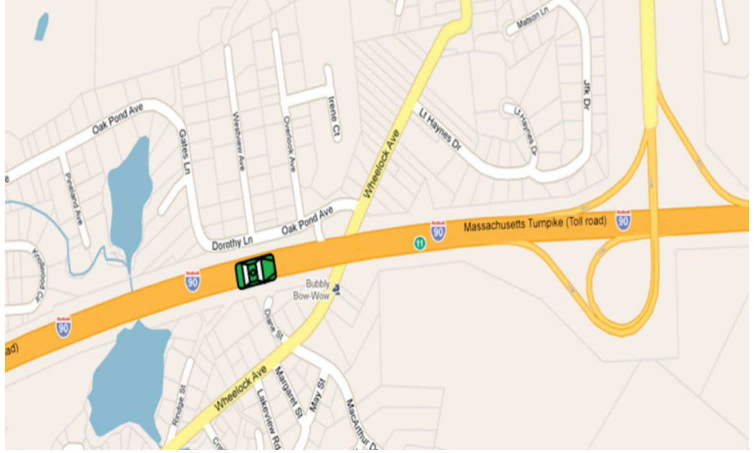
Operational level of decision making

Introduction

- Therefore, on the basis of the geographical scope and time frame involved, microscopic models can fall into one of the following three broad categories:
    1. at the strategic level: route-choice models;
    2. at the tactical level: lane-changing models;
    3. at the operational level: car-following and gap-acceptance models.

- We first introduce car-following models.
- More specifically, drivers' operational control when following another vehicle on a single-lane highway will be considered where no passing is allowed.
- We then introduce route choice models to address the strategic level.

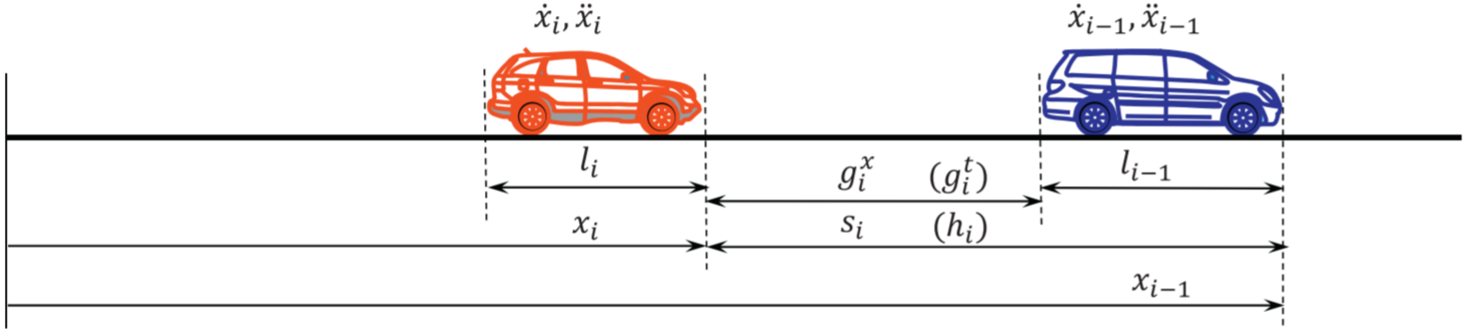




Car-Following Models

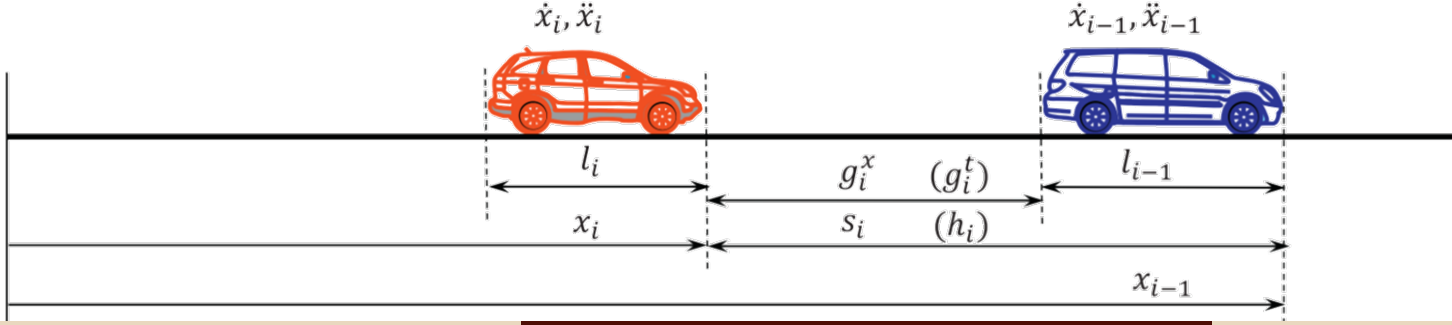
Notation

- Before the formal discussion of car-following models, it is helpful to summarize the notation to be used.
- Two vehicles traveling on a one-lane highway are depicted.
- These vehicles $(1, 2, \ldots, i-1, i, i+1, \ldots, I)$ are numbered cumulatively with lower-numbered vehicles in front—for example, vehicle 1 leads vehicle 2.
- The locations and displacements of vehicles are measured from a common but arbitrary reference point.



Notation

- i vehicle ID, $i = 1, 2, \dots, I$.
- $x_i(t)$ the location of vehicle i at time t.
- $\dot{x}_i(t)$ the speed of vehicle i at time t.
- v_i desirable speed that driver i is willing to travel at whenever possible.
- $\ddot{x}_i(t)$ the acceleration of vehicle i at time t.
- A_i the maximum acceleration that vehicle i is able to apply. $A_i > 0$.
- B_i the maximum deceleration that vehicle i is able to apply. $B_i < 0$.
- l_i the length of vehicle i.
- τ_i the perception-reaction time of driver i.
- $s_i(t)$ the spacing between vehicle i and the leading vehicle $i-1$ at time t.
- $g_i^x(t)$ the distance between vehicle i and the vehicle in front of it at time t.
- $h_i(t)$ the headway between vehicle i and the vehicle in front of it at time t.
- $g_i^t(t)$ the time gap between vehicle i and the vehicle in front of it at time t.

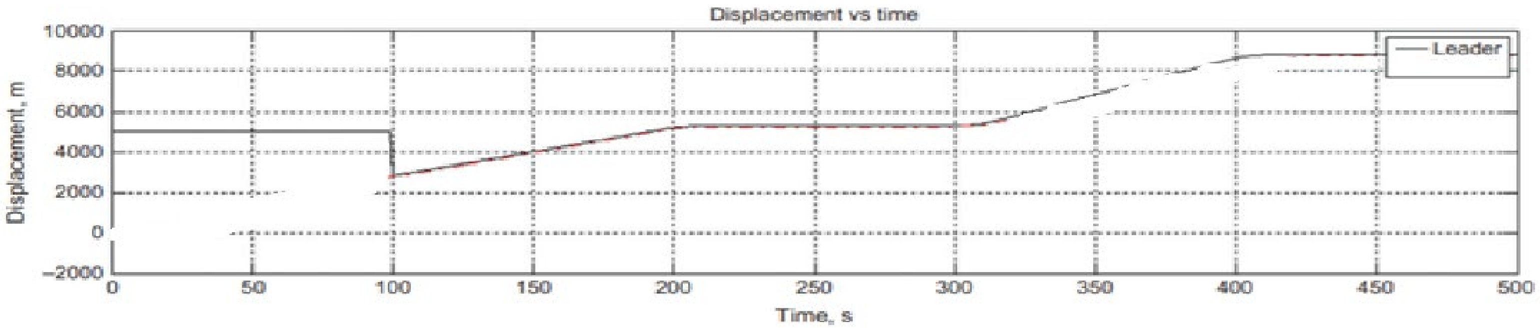


Benchmarking Scenarios

- Various microscopic car-following models will be discussed.
- These models were formulated with a variety of modeling philosophies and appeared in different forms.
- It would be very interesting and informative if these models could be cross-compared on the basis of a common ground.
- Such a process is called benchmarking.
- We are going to compare the models based on two scenarios : one microscopic and the other macroscopic.
- The purpose of microscopic benchmarking is to illustrate the performance of these car-following models in different regimes so that their operational control under various conditions can be examined.

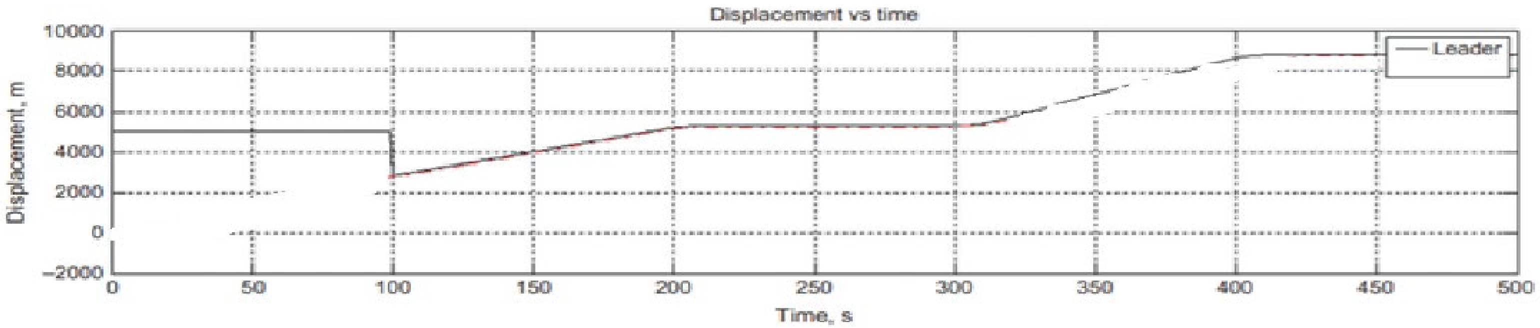
Microscopic Benchmarking

- Microscopic benchmarking employs a concrete example consisting of a set of hypothetical driving regimes.
- The example involves two vehicles: a leading vehicle $i-1$ and a following vehicle i.
- The motion of the leader is predetermined and that of the follower is governed by the car-following model under study.
- Initially ($t\ = 0$), vehicle $i-1$ stands still at $5000\ m$ from the reference point ($x_{i-1}(0)\ =\ 5000\ m$, $\dot{x}_{i-1}(0)\ =\ 0\ m/s$, and $\ddot{x}_{i-1}(0) =\ 0\ m/s^2$).
- Vehicle i, which is also still ($\dot{x}_i(0) =\ 0\ m/s$) and $\ddot{x}_i(0)\ =\ 0\ m/s^2$), stands somewhere near the reference point, with the exact location to be determined case by case in different car-following models.
- When the scenario starts ($t > 0$), vehicle $i-1$ remains still, while vehicle i starts to move.
- Since vehicle $i-1$ is far ahead, vehicle i is entitled to accelerate freely.



Microscopic Benchmarking

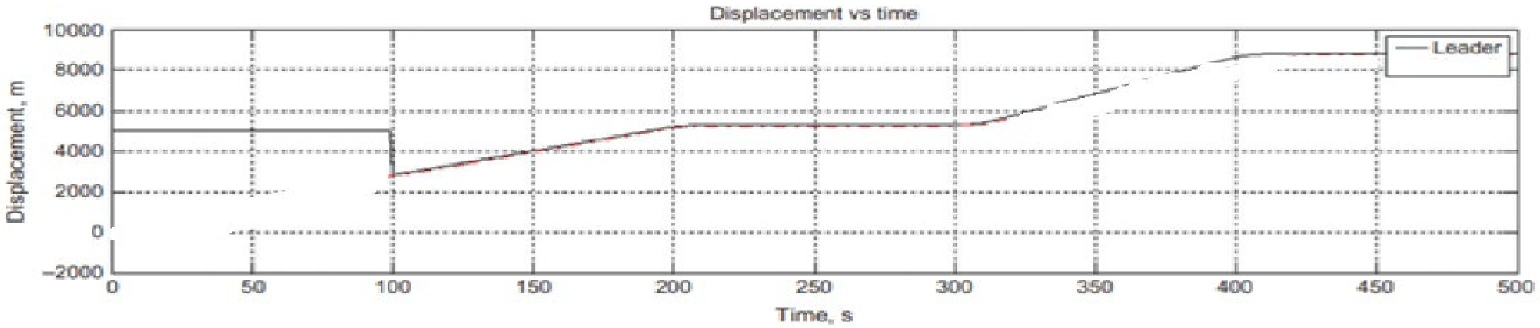
- At time $t = 100\ s$, vehicle i is at somewhere about $x_i(100) \approx 2770\ m$.
- At this moment, a third vehicle previously moving in the adjacent lane at $24\ m/s$ changes to the subject lane at location $2810\ m$ and takes over as the new leading vehicle, assuming ID $i - 1$—that is,$x_{i-1}(100) = 2810\ m$, $\dot{x}_{i-1}(100) = 24\ m/s$, and $\ddot{x}_{i-1}(100) = 0\ m/s^2$.
- This change is designed to mimic the effect that a vehicle cuts in in front of another vehicle with a spacing of about $40\ m$.
- Meanwhile, the previous, stationary leading vehicle is removed from the road.
- The new leading vehicle keeps moving at that speed up to $t = 200\ s$, and then undergoes deceleration at a rate of $\ddot{x}_{i-1} = -3\ m/s^2$until it comes to a complete stop.
- After that, vehicle $i - 1$ remains stopped up to $t = 300\ s$.
- Then, it begins to accelerate at a constant rate of $\ddot{x}_{i-1} = 2\ m/s^2$, and eventually settles at its full speed of $\dot{x}_{i-1} = 36\ m/s$.



Microscopic Benchmarking

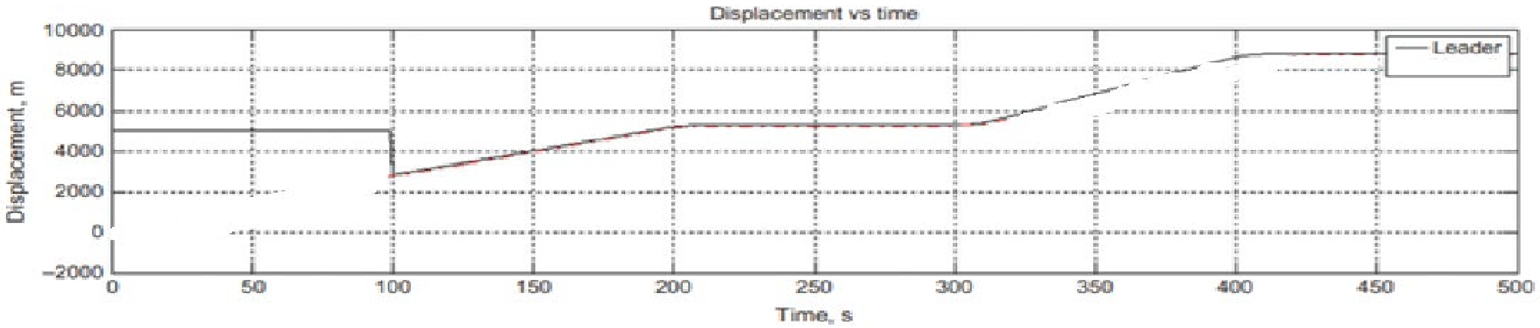
- At time $t = 400\ s$, the vehicle starts to decelerate again at a constant rate of $\ddot{x}_{i-1} = -3\ m/s^2$ until it comes to another full stop, and remains there.
- During all the time, the motion of the follower i is completely stipulated by the studied car-following model.
- The above scenario is formulated as follows:

$$
\begin{cases}
x_{i-1} = 5000\,\mathrm{m}, \dot{x}_{i-1} = 0\,\mathrm{m/s}, \ddot{x}_{i-1} = 0\,\mathrm{m/s^2} & \text{when } 0\,\mathrm{s} \le t < 100\,\mathrm{s}, \\
x_{i-1} = 2810\,\mathrm{m}, \dot{x}_{i-1} = 24\,\mathrm{m/s} & \text{when } t = 100\,\mathrm{s}, \\
\ddot{x}_{i-1} = 0\,\mathrm{m/s^2} & \text{when } 100\,\mathrm{s} \le t < 200\,\mathrm{s}, \\
\ddot{x}_{i-1} = -3\,\mathrm{m/s^2} & \text{when } 200\,\mathrm{s} \le t < 208\,\mathrm{s}, \\
\ddot{x}_{i-1} = 0\,\mathrm{m/s^2} & \text{when } 208\,\mathrm{s} \le t < 300\,\mathrm{s}, \\
\ddot{x}_{i-1} = 2\,\mathrm{m/s^2} & \text{when } 300\,\mathrm{s} \le t < 318\,\mathrm{s}, \\
\ddot{x}_{i-1} = 0\,\mathrm{m/s^2} & \text{when } 318\,\mathrm{s} \le t < 400\,\mathrm{s}, \\
\ddot{x}_{i-1} = -3\,\mathrm{m/s^2} & \text{when } 400\,\mathrm{s} \le t < 412\,\mathrm{s}, \\
\ddot{x}_{i-1} = 0\,\mathrm{m/s^2} & \text{when } t \ge 412\,\mathrm{s}.
\end{cases}
$$



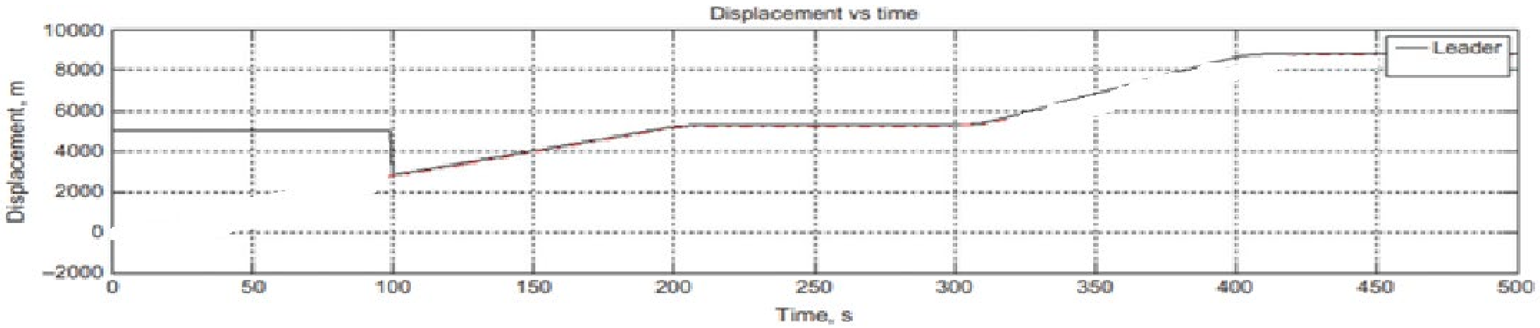
Microscopic Benchmarking

- The driving regimes involved in the above process include the following:
  - Start-up: Vehicle i starts to move from standstill, when the process begins ($t > 0\ s$).
  - Speedup: After start-up, vehicle i continues to accelerate to higher speeds ($0\ s < t < 100\ s$).
  - Free flow: As vehicle i speeds up, it settles at its desired speed if it is unimpeded ($0\ s < t < 100\ s$).
  - Cutoff: A sudden decrease in spacing owing to the new leader $i-1$ cutting in ($t = 100\ s$).
  - Following: Vehicle i has to adopt vehicle $i-1$'s speed so as to avoid a collision ($100\ s < t < 200\ s$).
  - Stop and go: Vehicle i is forced to stop and go owing to vehicle $i-1$'s brief stopping ($200\ s \le t \le 300\ s$).
  - Trailing: Vehicle i is following a speeding leader ($300\ s < t < 400\ s$).
  - Approaching: Vehicle i is getting close to a slower or stationary leader ($400\ s \le t < 420\ s$).
  - Stopping: Vehicle i tries to stop behind a stationary object separated by a minimum spacing ($t \ge 420\ s$).



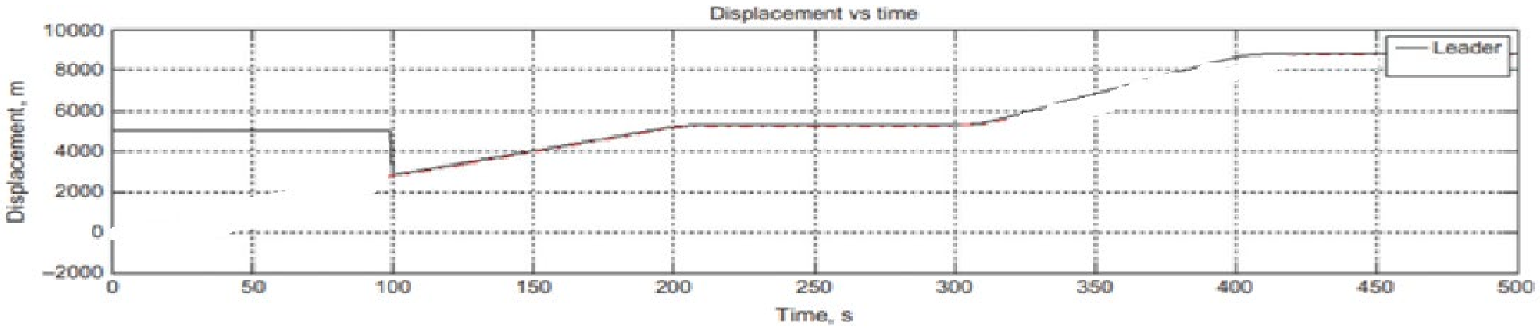
Microscopic Benchmarking

- In other words, we need to test the proposed models to analyze whether a model makes physical sense facing the following situations:
  - Start-up: Whether the model itself is sufficient to start the vehicle up without involving any additional, external logic.
  - Speedup: Whether the model generates speed and acceleration profiles that make physical sense.
  - Free flow: Whether the model settles at its desired speed without overshooting or undershooting.
  - Cutoff: Whether the model loses control or responds with a reasonable control maneuver.
  - Following: Whether the model is able to adopt the leader's speed and follow the leader at a reasonable distance.



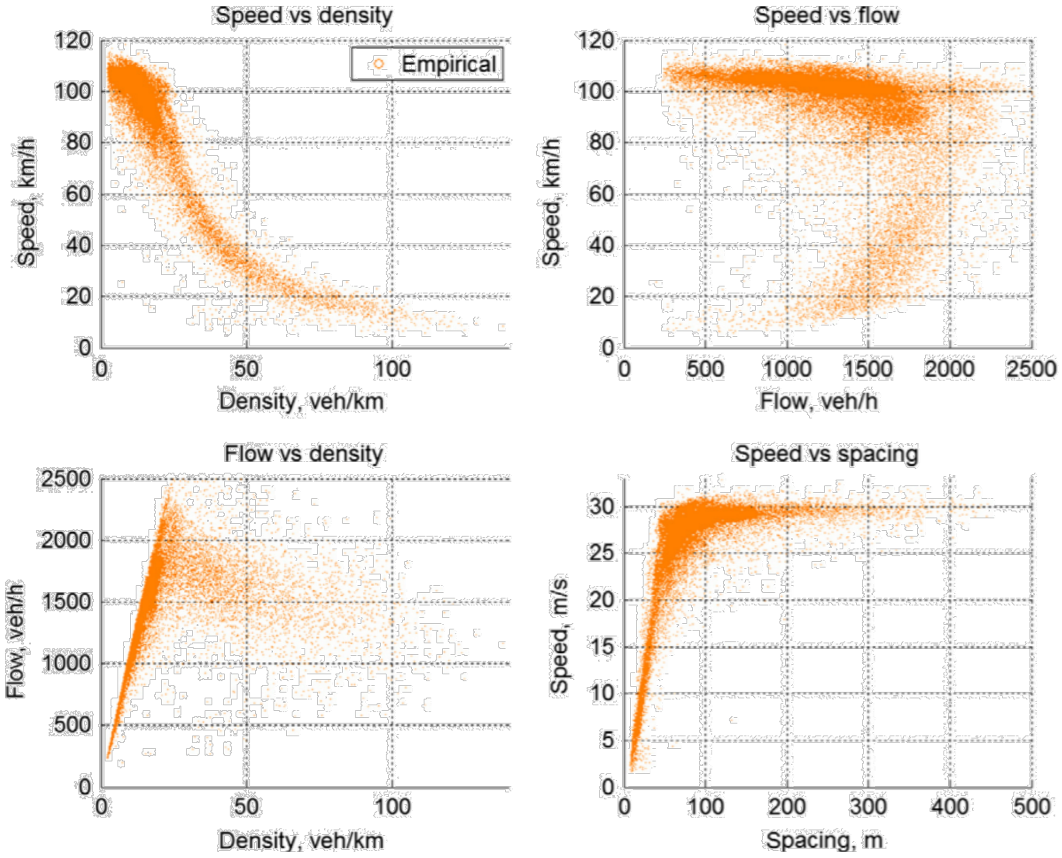
Microscopic Benchmarking

- In other words, we need to test the proposed models to analyze whether a model makes physical sense facing the following situations:
  - Stop and go: Whether the model is able to stop the vehicle safely behind its leader and start moving again when the leader resumes motion.
  - Trailing: Whether the model is able to speed up normally without being tempted to speed up by its speeding leader—that is, a vehicle is attracted to excessively high speeds by its speeding leader.
  - Approaching: Whether the model is able to adjust the vehicle properly when the intervehicle spacing closes up.
  - Stopping: Whether the model is able to stop the vehicle properly behind a stationary object separated by a minimum spacing, without overshooting or undershooting, and causing speed and acceleration to return to zero naturally when stopped.



Macroscopic Benchmarking

- Macroscopic benchmarking employs a set of empirical data obtained from Georgia 400, a toll road with freeway by design located in Atlanta, Georgia, USA.
- The data contain 1-years' worth of field observations at one station across four lanes.
- This figure contains a set of four plots that illustrate speed-density, speed-flow, flow-density, and speed-spacing relationships.
- The "cloud" contains field observations of flow, speed, and density aggregated to $\approx$ 5 min intervals.



Pipes Model

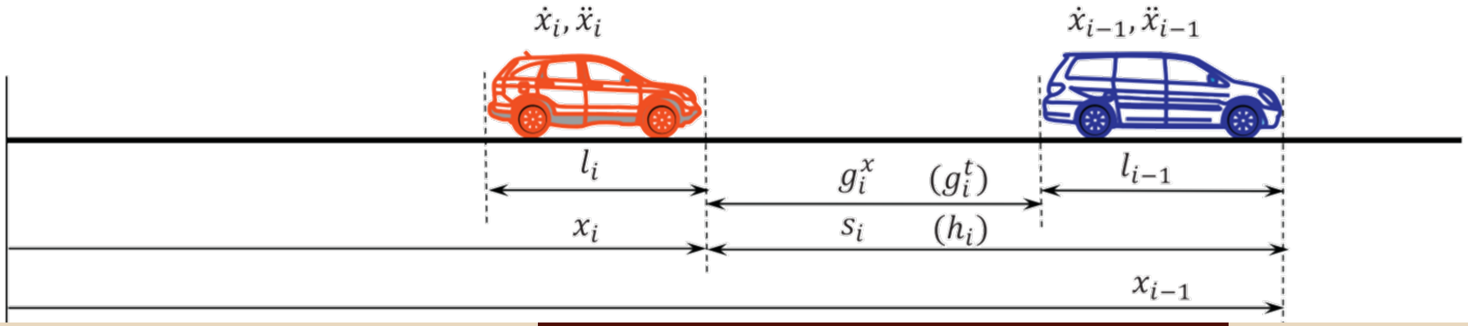
- The Pipes model is based on a safe driving rule coined in the California Vehicle Code:

  A good rule for following another vehicle at a safe distance is to allow yourself at least the length of a car between your vehicle and the vehicle ahead of you for every ten mile per hour of speed at which you are traveling.

- Putting this in mathematical language, we get

$$g_i^x(t)_{\min} = [(x_{i-1}(t) - x_i(t)) - l_{i-1}]_{\min} = (s_i(t) - l_{i-1})_{\min} = \frac{\dot{x}_i(t)}{0.447 \times 10} l_i$$

- where $\dot{x}_i(t)$ is in meters per second, and $g_i^x(t)$, $x_{i-1}(t)$, and $x_i(t)$ are measured in meters.



Pipes Model

- The Pipes model is formulated as

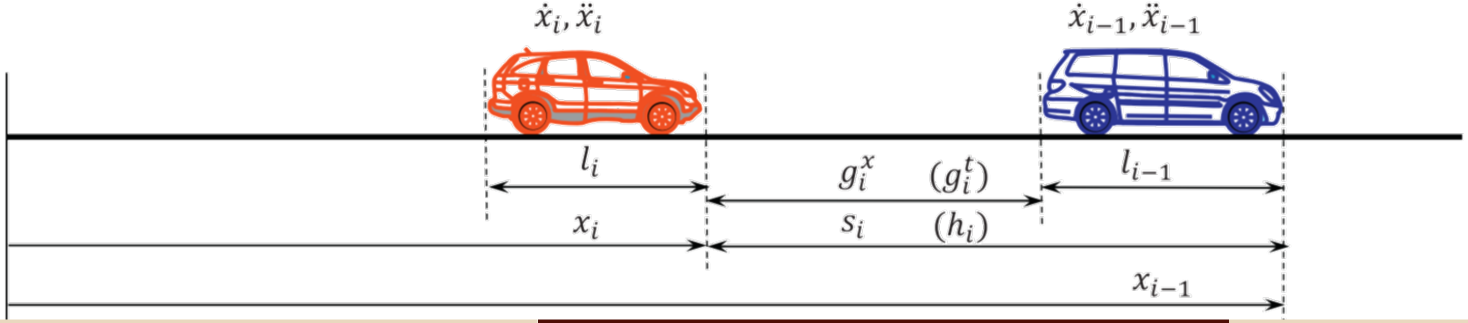
$$s_i(t)_{\min} = \frac{l_i}{4.47}\dot{x}_i(t) + l_{i-1}$$

- If we assume a vehicle length of 6 m, the model reduces to:

$$s_i(t)_{\min} = 1.34\dot{x}_i(t) + 6$$

- or

$$h_i(t)_{\min} = 1.34 + \frac{6}{\dot{x}_i(t)}$$



Cruise Control

- Perhaps the simplest form of automatic driving is cruise control.
- As an in-vehicle system, cruise control automatically controls the speed of a motor vehicle so that the vehicle maintains a constant speed set by its driver.
- Cruise control makes it easier to drive on long road trips, and hence is a popular car feature.
- As more and more vehicles join the traffic and the road becomes crowded, the driver has to switch cruise control on and off so frequently that cruise control becomes less useful.
- To adapt to the dynamics of the vehicle in front, it is desirable that the cruise control system be able to adjust speed accordingly to maintain a safe car-following distance.
- Hence, an adaptive or autonomous cruise control system has been developed.
- With the aid of distance sensors such as radar or laser sensors, autonomous cruise control allows the vehicle to slow down when approaching another vehicle and accelerate to the preset speed when traffic conditions permit.

Cruise Control and Pipes Model

- To make this happen, the system requires an internal logic which relates the vehicle speed to the distance to the vehicle in front.
- Simple car-following models such as the Pipes model can be employed as the basis of such an internal logic.
- More specifically, Pipes's equation can be rearranged as follows:

$$\dot{x}_i(t) \leq \frac{0.447 \times 10}{l_i} g_i^x(t)$$

- For a vehicle length of 6 m, the above control logic becomes $\dot{x}_i(t) \leq 0.745\ g_i^x(t)$.
- Therefore, the autonomous cruise control works as follows.
- At any moment t, the distance sensor measures the gap between the two vehicles $g_i^x(t)$.
- Then, the target speed that the vehicle needs to adapt to is set as $0.745 g_i^x(t)$ or less.

Cruise Control and Pipes Model

- Obviously, the target speed can easily go out of bound as the gap $g_i^x(t)$ becomes sufficiently large.
- Therefore, it is necessary to set an upper bound to the target speed, which is usually referred to as the desirable speed v_i.
- Therefore, the target speed is actually the minimum of (1) the desirable speed v_i and (2) the speed constrained by the vehicle in front $\dot{x}_i(t)$:

$$\dot{x}_i(t) \leq \min\{v_i, \frac{0.447 \times 10}{l_i} g_i^x(t)\}$$

Properties of the Pipes Model

- In mathematical modeling, it is always interesting to understand how a system's microscopic behavior relates to its macroscopic behavior, or alternatively to interpret the microscopic basis of a macroscopic phenomenon.
- In traffic flow theory, microscopic car-following models are typically related to macroscopic speed-density relationships and further the fundamental diagram.
- Typically, the linkage between microscopic and macroscopic models can be addressed in two ways.
- One approach is to run a simulation based on the microscopic model.
- Such a microscopic simulation typically involves random variables such as perception-reaction time, desired speed, and acceleration rate.
- As a result, simulation results vary in different runs.
- Hence, the macroscopic behavior implied by the microscopic model can be obtained by a statistical analysis of these simulation results.

Properties of the Pipes Model

- The other approach is analytical—that is, one tries to aggregate or integrate the microscopic model under some equilibrium or steady-state assumptions.
- If a system is in the steady state, any property of the system is unchanging in time.
- More specifically, a traffic system in the steady state would consist of homogeneous vehicles which exhibit uniform behavior over time and space.
- Therefore, under steady-state conditions:
  - Vehicles lose their identities (e.g., $l_i \rightarrow l$),
  - Vehicles travel at uniform speed (i.e., $\dot{x}_i = \dot{x}_j \rightarrow v$ and $\ddot{x}_i \rightarrow 0$),
  - Drivers' desired speeds converge to the free-flow speed (i.e.,$v_i \rightarrow v_f$),
  - The vehicle spacing $s_i(t)$ reduces to s, which, in turn, is replaced by the reciprocal of traffic density $1/k$.
  - Uniform vehicle length l is equivalent to the reciprocal of jam density kj, that is, $1/kj$.

Properties of the Pipes Model

- Hence, the Pipes model reduces to:

$$\frac{k_j}{k} = \frac{v}{4.47} + 1 \text{ or } v = 4.47\left(\frac{k_j}{k} - 1\right)$$

where k is measured in vehicles per meter and v is measured in meters per second.

- With $q = k \times v$, the above speed-density relationship gives rise to the following flow-density and speed-flow relationships:

$$q = 4.47(k_j - k) \quad \Rightarrow \quad v = \frac{q}{k_j - 0.22q}$$

- These equations constitute the mathematical representation of the fundamental diagram implied by the Pipes model.

Forbes Model

- Rather than ensuring a safe distance between vehicles as the Pipes model does, Forbes stipulates that:

  To ensure safety, the time gap between a vehicle and the vehicle in front of it should be always greater than or equal to reaction time.

- This safety rule can be formulated as:

$$g_i^t(t) = h_i(t) - \frac{l_i}{\dot{x}_i} \geq \tau_i$$

- For a reaction time of 1.5 s and a vehicle length 6 m, the model becomes:

$$h_i(t) \geq 1.5 + \frac{6}{\dot{x}_i} \quad \rightarrow \quad s_i(t) \geq 1.5\dot{x}_i + 6$$

- This is very similar to the Pipes model except for a slight difference in the coefficient of the speed term, which is interpreted as perception-reaction time τi.

Forbes Model

- Therefore, the Pipes model and the Forbes model are essentially equivalent and can be generically expressed as

$$s_i(t) \geq \tau_i \dot{x}_i + l_i$$

where τ_i and vehicle length l_i are model parameters.

- Note that applications and properties of the Pipes model discussed before, also apply to the Forbes model.
- In addition, the fundamental diagram implied by the Pipes and Forbes models can be generically expressed as:

$$v = \frac{1}{\tau k} - \frac{l}{\tau} \quad , \quad q = \frac{1}{\tau} - \frac{l}{\tau}k \quad , \quad v = \frac{ql}{1 - \tau q}$$

where τ is the average perception-reaction time and l is the average vehicle length.

Microscopic Benchmarking - Pipes Model

- Since the Pipes and Forbes models are essentially equivalent, the following discussion addresses only the Pipes model with the understanding that the result applies to the Forbes model as well.

$$\dot{x}_i(t + \Delta t) = \frac{s_i(t) - l_i}{\alpha}$$

where Δt is the simulation time step and α is a constant resulting from unit conversion ($\alpha = 1.34$ if speed is in meters per second and $l_i = 6\ m$). $s_i(t)$ is the spacing between vehicle i and the leading vehicle at time t.

Microscopic Benchmarking - Pipes Model

Vehicle Acceleration

- The model has a problem with vehicle acceleration.
- Suppose that initially the leading vehicle is located at $x_{i-1}(0) = 5000\ m$ and the subject vehicle is at $x_i(0) = -102\ m$ and both vehicles are standing still.
- When the simulation begins, vehicle i starts to move according to the Pipes model.
- A spacing of $s_i\ (0) = 5102\ m$ results in a speed of about $3800\ m/s$ at the next time step (assuming $t = 1\ s$), which requires an acceleration of $3800\ m/s^2$.
- It follows that an infinite speed and acceleration would result if there is no leading vehicle in front.
- Therefore, the following external logic has to be imposed on the Pipes model in order to limit its maximum acceleration:

$$\ddot{x}_i(t) = \frac{\dot{x}_i(t + \Delta t) - \dot{x}_i(t)}{\Delta t} \leq A_i$$

- where A_i is the maximum acceleration of vehicle i, for example, $A_i = 4m/s^2$.

Microscopic Benchmarking - Pipes Model

Vehicle Speed

- Even though an external logic is added to bound the maximum acceleration of vehicle, the Pipes model still has a problem with the maximum speed.
- It is true that the acceleration no longer exceeds A_i, but the vehicle can still reach unrealistically high speeds.
- Therefore, another external logic has to be imposed to limit the speed

$$\dot{x}_i \leq v_i$$

where v_i is driver i's desired speed.

Microscopic Benchmarking - Pipes Model

Vehicle Deceleration

- The third problem is unrealistic deceleration.
- For example, at time $t = 424$, vehicle i is located at about $x_i = 8734\,m$ moving at speed $\dot{x}_i(t) = 30\,m/s$, while vehicle $i-1$ stops at $x_{i-1} = 8762\,m$.
- According to the Pipes model, vehicle $i's$ speed at the next step would be $\dot{x}_i(t) \approx 16.42\,m/s$. As such, the deceleration rate is $\ddot{x}_i(t) = -13.58\,m/s^2$.
- Hence, a third external logic has to be imposed to limit maximum deceleration B_i (e.g., $-6\,m/s^2$):

$$\ddot{x}_i(t) = \frac{\dot{x}_i(t+\Delta t) - \dot{x}_i(t)}{\Delta t} \geq B_i$$

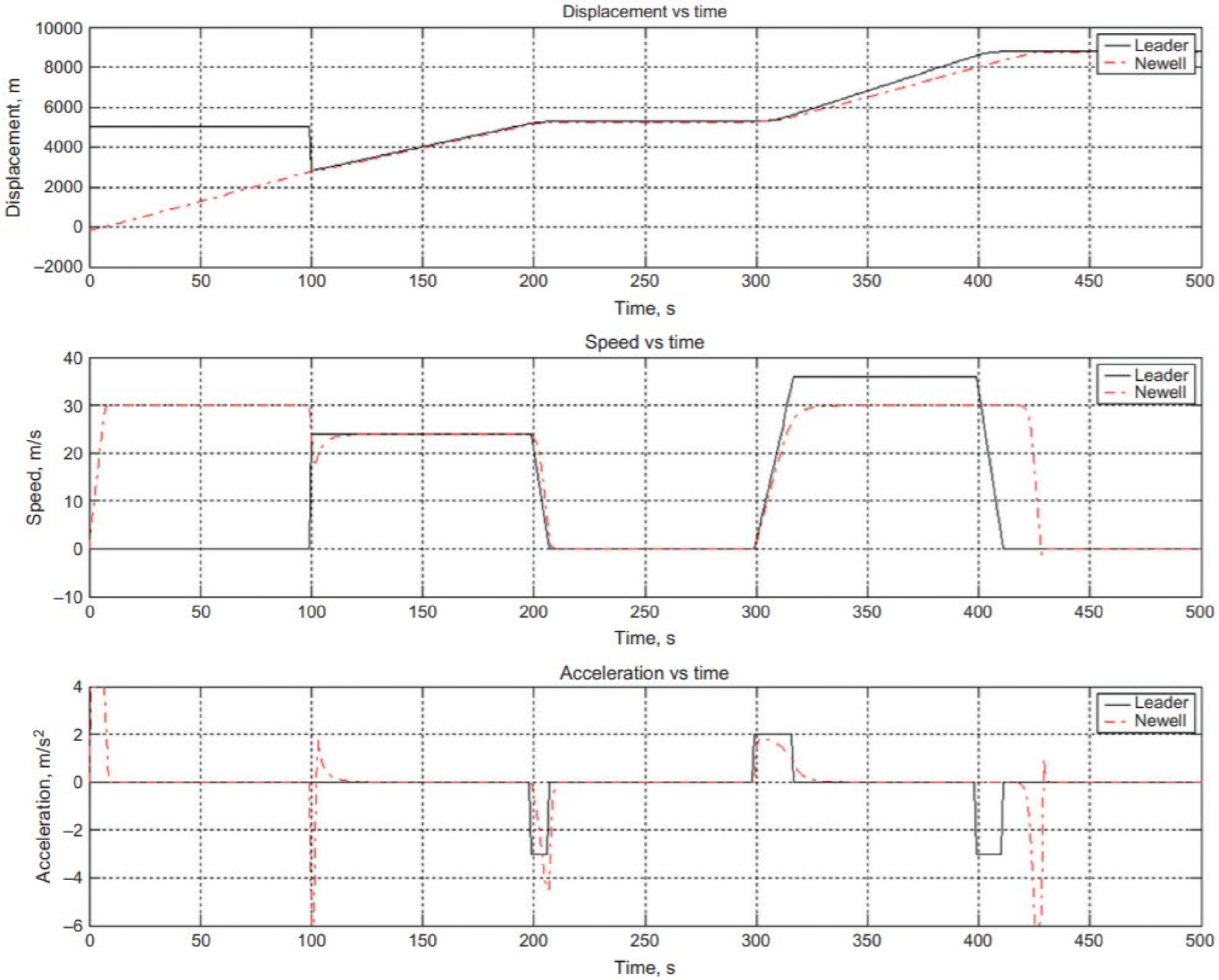
- This addition introduces a new problem. For example, vehicle i's speed at the next step becomes $\dot{x}_i(t) = 30 - 6 = 24\,m/s^2$and its location is $x_i = 8758\,m$. This would leave a spacing of $s_i = 4\,m$, which is less than a vehicle length $l_i - 1 = 6\,m$, that is, vehicle i has collided with vehicle $i-1$.
- Unfortunately, there is no easy remedy to the problem except for accepting the unrealistic deceleration behavior.

Microscopic Benchmarking - Pipes Model

- The performance of the constrained Pipes model is summarized as follows.
- ***Start-up:*** The model is able to start the vehicle up from standstill. See when $t > 0\ s$.
- ***Speedup:*** The model is able to speed up the vehicle. However, its acceleration profile (i.e., acceleration as a function of speed) is unrealistic because the vehicle is able to retain maximum acceleration at high speeds. Normally, maximum acceleration is available only when a vehicle starts up. As the vehicle speeds up, acceleration decreases. See when $0\ s < t < 100\ s$.

l_i	v_i	τ_i	α	–
6 m	30 m/s	1.0 s	1.34	–
A_i	B_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
4.0 m/s^2	6.0 m/s^2	-120 m	0 m/s	0 m/s^2

Microscopic benchmarking parameters



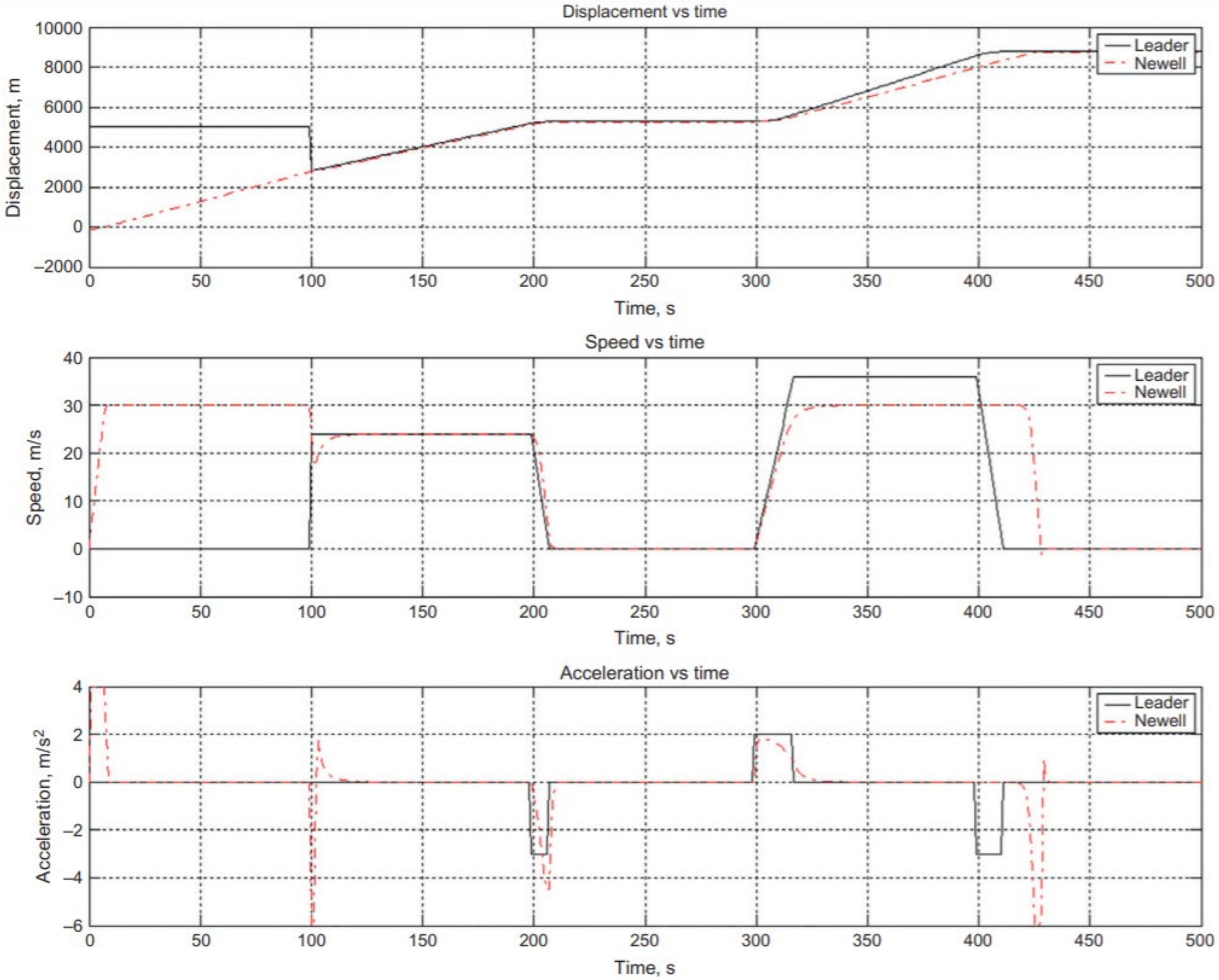
Microscopic benchmarking of the Pipes model.

Microscopic Benchmarking - Pipes Model

- ***Free flow:*** An external logic has to be imposed to limit the maximum speed under the free-flow condition. See when $0s < t < 100\ s$.
- ***Cutoff:*** The model retains control and responds reasonably when a vehicle cuts in in front. See around $t = 100\ s$.
- ***Following:*** The model is able to adopt the leader's speed and follow the leader by a reasonable distance. See when $100\ s < t < 200\ s$.
- ***Stop and go:*** The model is able to stop the vehicle safely behind its leader and start it moving when the leader departs. See when $200\ s \le t \le 300\ s$.

l_i	v_i	τ_i	α	–
6 m	30 m/s	1.0 s	1.34	–
A_i	B_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
4.0 m/s^2	6.0 m/s^2	-120 m	0 m/s	0 m/s^2

Microscopic benchmarking parameters



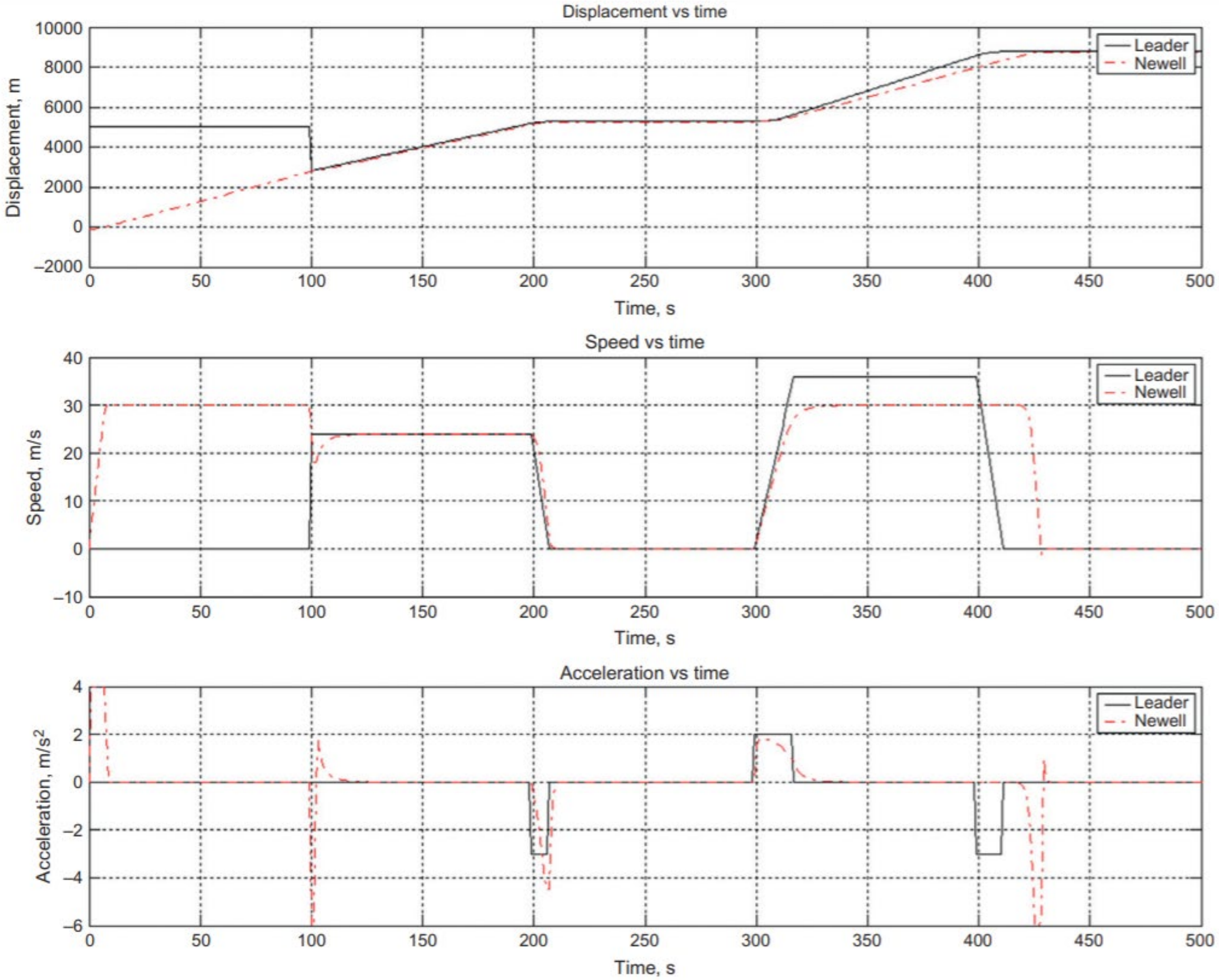
Microscopic benchmarking of the Pipes model.

Microscopic Benchmarking - Pipes Model

- ***Trailing:*** The model is able to speed up normally without being tempted to speed up by its speeding leader. See when $300\ s < t < 400\ s$.
- ***Approaching:*** The model is unable to decelerate properly when approaching a stationary vehicle at a distance. The vehicle might collide with its leader when maximum deceleration is imposed. See when $400\ s < t < 420\ s$.
- ***Stopping:*** This portion is invalid since approaching fails. See when $t \geq 420\ s$.

l_i	v_i	τ_i	α	–
6 m	30 m/s	1.0 s	1.34	–
A_i	B_i	$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
4.0 m/s^2	6.0 m/s^2	-120 m	0 m/s	0 m/s^2

Microscopic benchmarking parameters



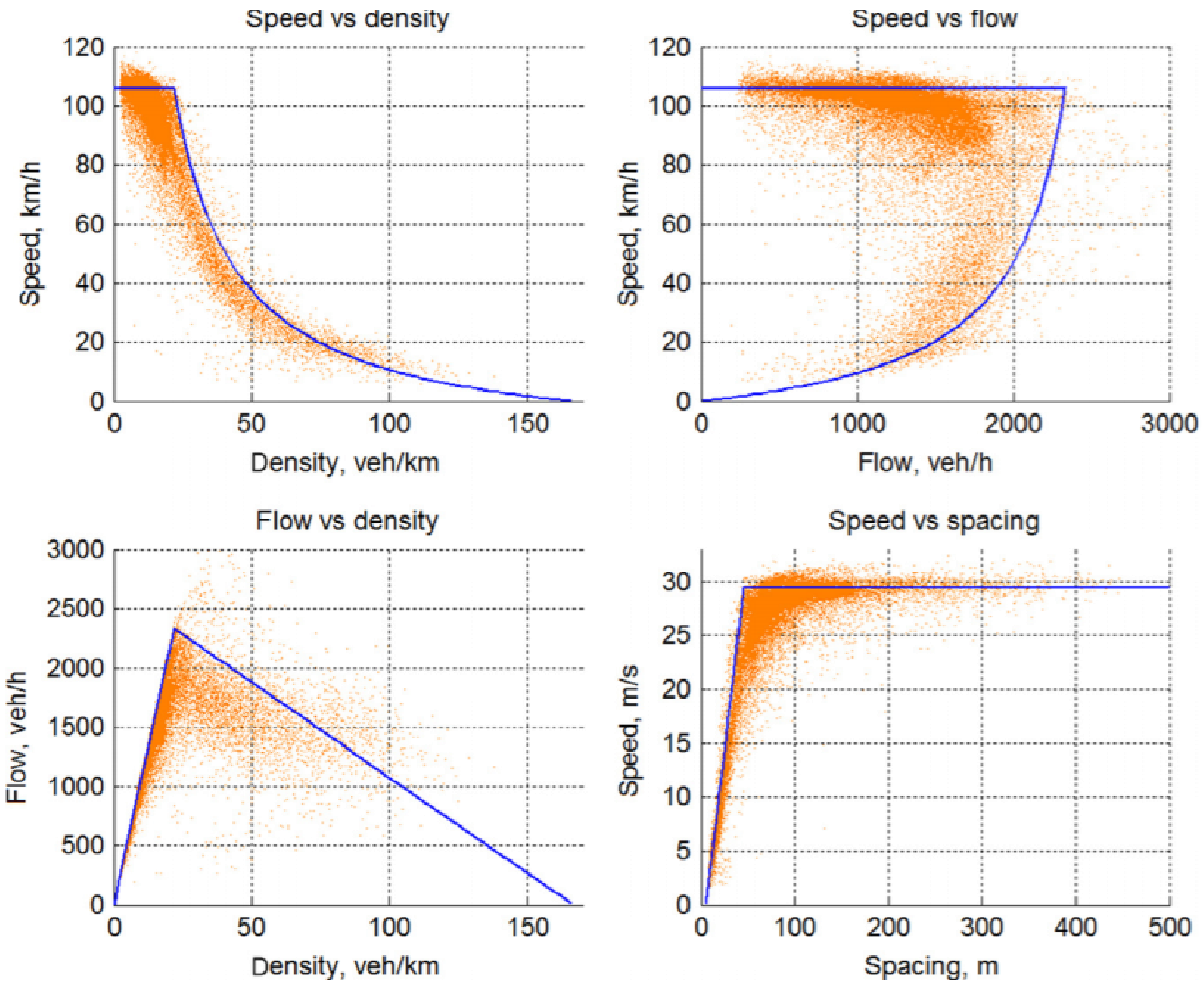
Microscopic benchmarking of the Pipes model.

Macroscopic Benchmarking - Pipes Model

- The cloud contains 5 min empirical observations.
- Curves are the equilibrium relationships implied by the Pipes model.
- These curves roughly fit the empirical data in the middle to upper range of density (e.g., $k > 20$ vehicles per kilometer), but do not apply to the low-density range (e.g., $k < 20$ vehicles per kilometer).
- Pipes model predicts that traffic speed would increase to infinity as density approaches zero.

α (τ)	l
1.34 s	6 m

Macroscopic benchmarking parameters



Fundamental diagram implied by the Pipes model

General Motors Models

- A seminal work in the early history of car following models, GM models spawned and inspired numerous research efforts that have shaped today's traffic flow theory, and thus their importance cannot be overestimated.
- GM models assume that a driver's control maneuver is a result of not only external stimuli such as the dynamics of the subject vehicle and its leading vehicle, but also the driver's sensitivity.
- Hence such a relationship can be expressed as: $Response = f\ (sensitivity,\ stimuli)$
- When formulating the above relationship, GM researchers chose the subject vehicle's acceleration (deceleration is negative acceleration) produced after a reaction time, $\ddot{x}_i(t+\tau)$, as the response
- The consideration of stimuli and sensitivity evolved over time and resulted in a family of models.

GM1

- Originally, General Motors researchers observed that drivers responded to the relative speed between the subject vehicle i and its leading vehicle $i\ -1$, $\dot{x}_{i-1}(t) - \dot{x}_i(t)$.
- If sensitivity is treated as a coefficient that is multiplicative to the stimulus, the subject driver's operational control can be formulated as:

$$\ddot{x}_i(t + \tau_i) = \alpha[\dot{x}_{i-1}(t) - \dot{x}_i(t)]$$

- The above model is the first-generation model which can be used to interpret some car-following phenomena effectively.
- For example, when the subject vehicle approaches its leading vehicle (e.g., $\dot{x}_i(t) = 120\ km/h$ and $\dot{x}_{i-1}(t) = 100\ km/h$), the relative speed is negative, and hence the driver will decelerate since $\ddot{x}_i(t + \tau_i) < 0$ (assuming that α is a positive constant).
- In contrast, if the subject vehicle is falling behind its leading vehicle (e.g., $\dot{x}_i(t) = 100\ km/h$ and $\dot{x}_{i-1}(t) = 120\ km/h$), the subject vehicle will accelerate because the relative speed now becomes positive.

GM1

- However, the model has difficulty distinguishing scenarios with large and small car-following distances.
- For example, the model predicts the same deceleration response to the following two scenarios:
  - Scenario 1: $\dot{x}_i(t) = 120\ km/h$, $\dot{x}_{i-1}(t) = 100\ km/h$, and $s_i\ (t) = 50\ m$
  - Scenario 2: $\dot{x}_i(t) = 120\ km/h$, $\dot{x}_{i-1}(t) = 100\ km/h$, and $s_i\ (t) = 5000\ m$
- Since both scenarios have a speed difference of $-20\ km/h$, intuitively, the subject driver in scenario 1 would brake much harder than the driver in scenario 2 because the former is facing an imminent collision.

GM2

- The effect of spacing motivated General Motors researchers to choose different sensitivity coefficients, and hence the second-generation model resulted:

$$\ddot{x}_i(t+\tau_i) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} [\dot{x}_{i-1}(t) - \dot{x}_i(t)]$$

- Field experiments revealed that the sensitivity coefficient α ranges between 0.17 and 0.74.
- In $GM2$, a high sensitivity value α_1 is chosen when the two vehicles are close, while a low sensitivity value α_2 is employed when the two vehicles are far apart.

GM3

- The effect of spacing was partially address in GM2 because one has to frequently calibrate the sensitivity coefficient depending on car-following distances.
- The inconvenience seemed to suggest that spacing should be explicitly included in the model, which led to the formulation of the third-generation model:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]}$$

- Although the issue of spacing has been suppressed, another problem pops up. The model is unable to predict any difference between the following scenarios:
  - Scenario 1: One vehicle is following another at a spacing of 100 m with speeds $\dot{x}_i(t) = 30\ km/h$, $\dot{x}_{i-1}(t) = 10\ km/h$.
  - Scenario 2: One vehicle is following another at a spacing of 100 m with speeds $\dot{x}_i(t) = 130\ km/h$, $\dot{x}_{i-1}(t) = 110\ km/h$.

The driver from the second scenario is under pressure to maintain safety during car following.

In contrast, the driver from the first scenario has peace of mind because he can always slam on the brake to stop the vehicle.

Hence, our daily driving experiences suggest that the response in scenario 2 be greater than that in scenario 1. However, GM3 predicts no difference because in both scenarios the speed difference is 20 km/h and the spacing is 100 m.

GM4

- GM3's inability to differentiate high-speed and low-speed car-following scenarios motivated General Motors researchers to further explore unexplained factors that can be extracted from the sensitivity coefficient.
- A dimension analysis reveals that the sensitivity coefficient has the same unit as speed in GM3.
- Since there is a need to explicitly consider speed as a stimulus, it seems ideal to extract speed from the sensitivity coefficient, leaving the remainder as a new, dimensionless coefficient.
- This gives rise to the fourth-generation model:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]}$$

GM5

- To generalize the results of the above GM models and, as becomes clear later, to facilitate finding “the bridge” between microscopic and macroscopic models, a generic form of GM models is proposed as the fifth model:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_i(t+\tau_i)]^m}{[x_{i-1}(t) - x_i(t)]^l}[\dot{x}_{i-1}(t) - \dot{x}_i(t)]$$

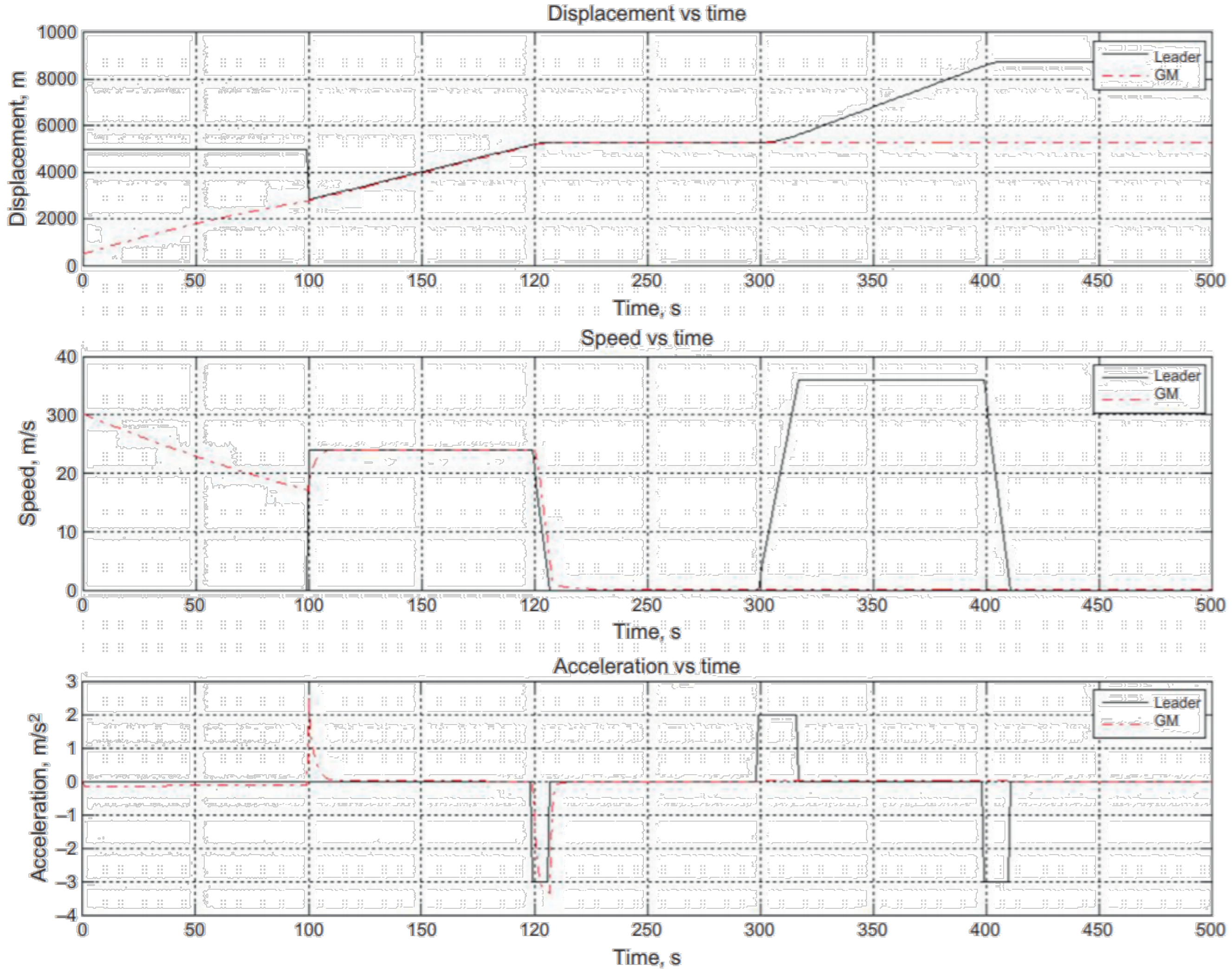
- Where x_i, $\dot{x}_i$, and $\ddot{x}_i$ are the displacement, speed, and acceleration of the subject vehicle i.
- Similar notation applies to its leader $i-1$.
- τ is the perception-reaction time that applies to all drivers, α is a dimensionless sensitivity coefficient, and m and l are speed and spacing exponents, respectively.

Microscopic Benchmarking – GM4

- The performance of the GM4 model is summarized as follows.
- ***Start-up:*** the model is unable to start a vehicle from standstill. Therefore, an external logic has to be imposed to assign an initial speed $\dot{x}_i(0)$ to the subject vehicle i.
- Note that the initial speed $\dot{x}_i(0)$ has to be set at the desired speed v_i. Otherwise, vehicle i will not be able to reach that speed by itself.
- See when $t > 0\ s$.

τ_i	α	–
1.0 s	0.8	–
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
467 m	30 m/s	0 m/s^2

Microscopic benchmarking parameters



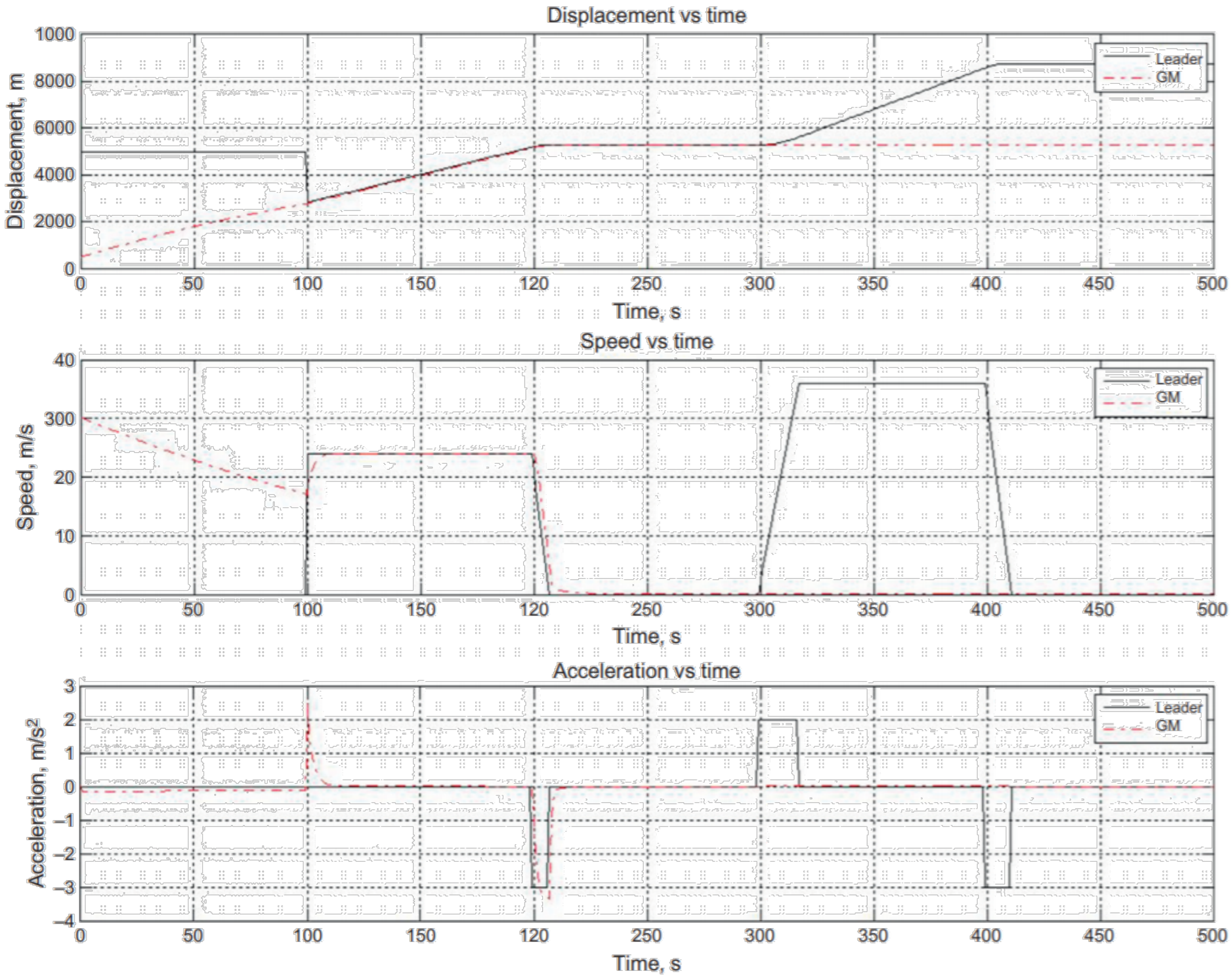
Microscopic benchmarking of GM4

Microscopic Benchmarking – GM4

- ***Speedup:*** rather than speeding up vehicle i as drivers normally do in the real world, the model predicts a deceleration by driver i even though its leading vehicle $i-1$ is thousands of meters ahead. See when $0\ s < t < 100\ s$.
- ***Free flow:*** the model predicts that vehicle i is unable to attain the free flow condition by itself unless it is set to do so by an external logic.
- As long as it follows a slower leader, the model constantly decelerates the vehicle until it adopts the leader's speed. See when $0\ s < t < 100\ s$.

τ_i	α	–
1.0 s	0.8	–
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
467 m	30 m/s	0 m/s^2

Microscopic benchmarking parameters



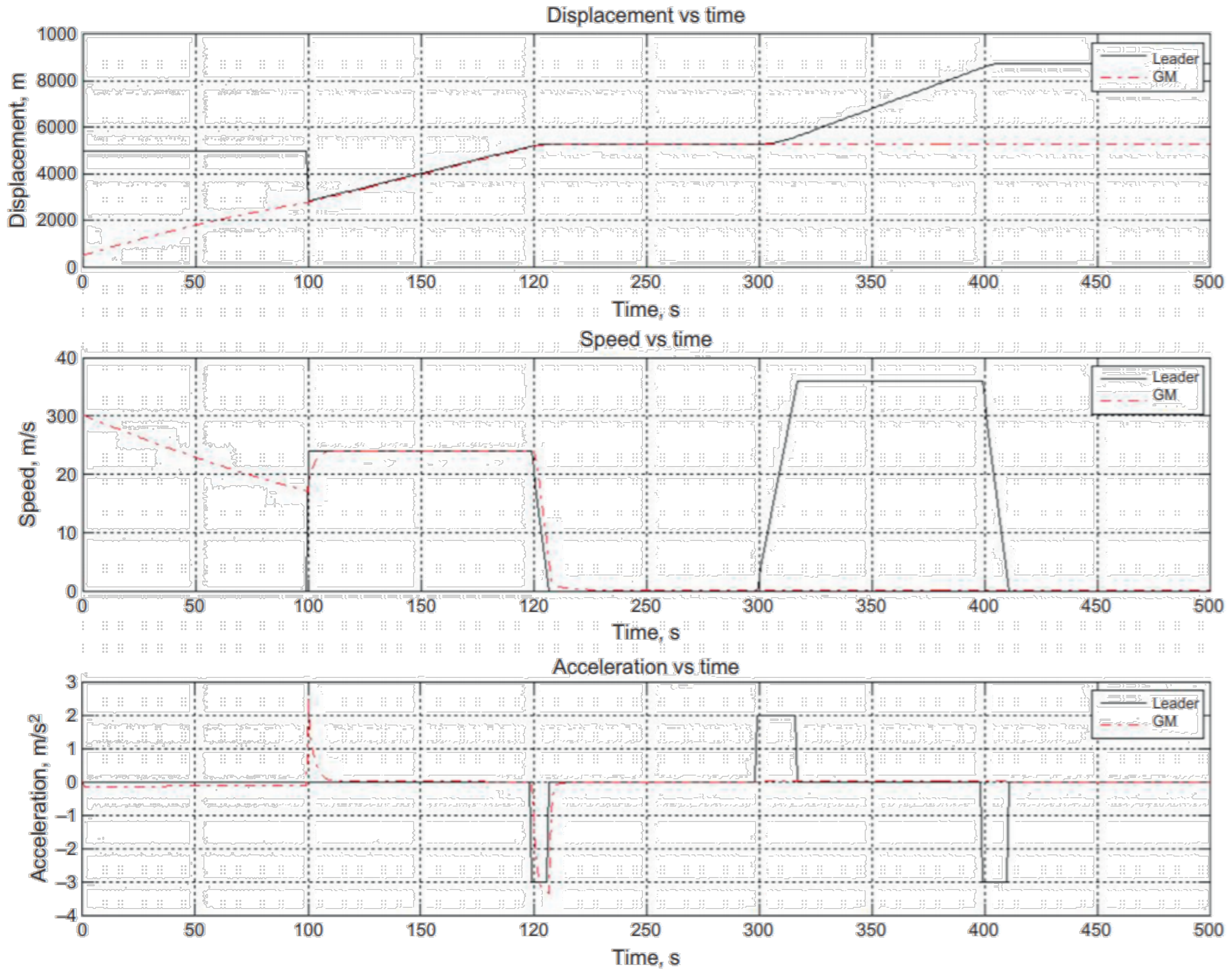
Microscopic benchmarking of GM4

Microscopic Benchmarking – GM4

- **_Cutoff:_** when the third vehicle suddenly takes over as the new leader $40\ m$ ahead at $24\ m/s$, the model predicts a sudden acceleration, while in the real-world drivers may or may not decelerate the vehicle. See around $t = 100\ s$.
- **_Following:_** the model is able to adopt the leader's speed and follow the leader at a reasonable distance. See when $100\ s < t < 200\ s$.

τ_i	α	–
1.0 s	0.8	–
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
467 m	30 m/s	0 m/s^2

Microscopic benchmarking parameters



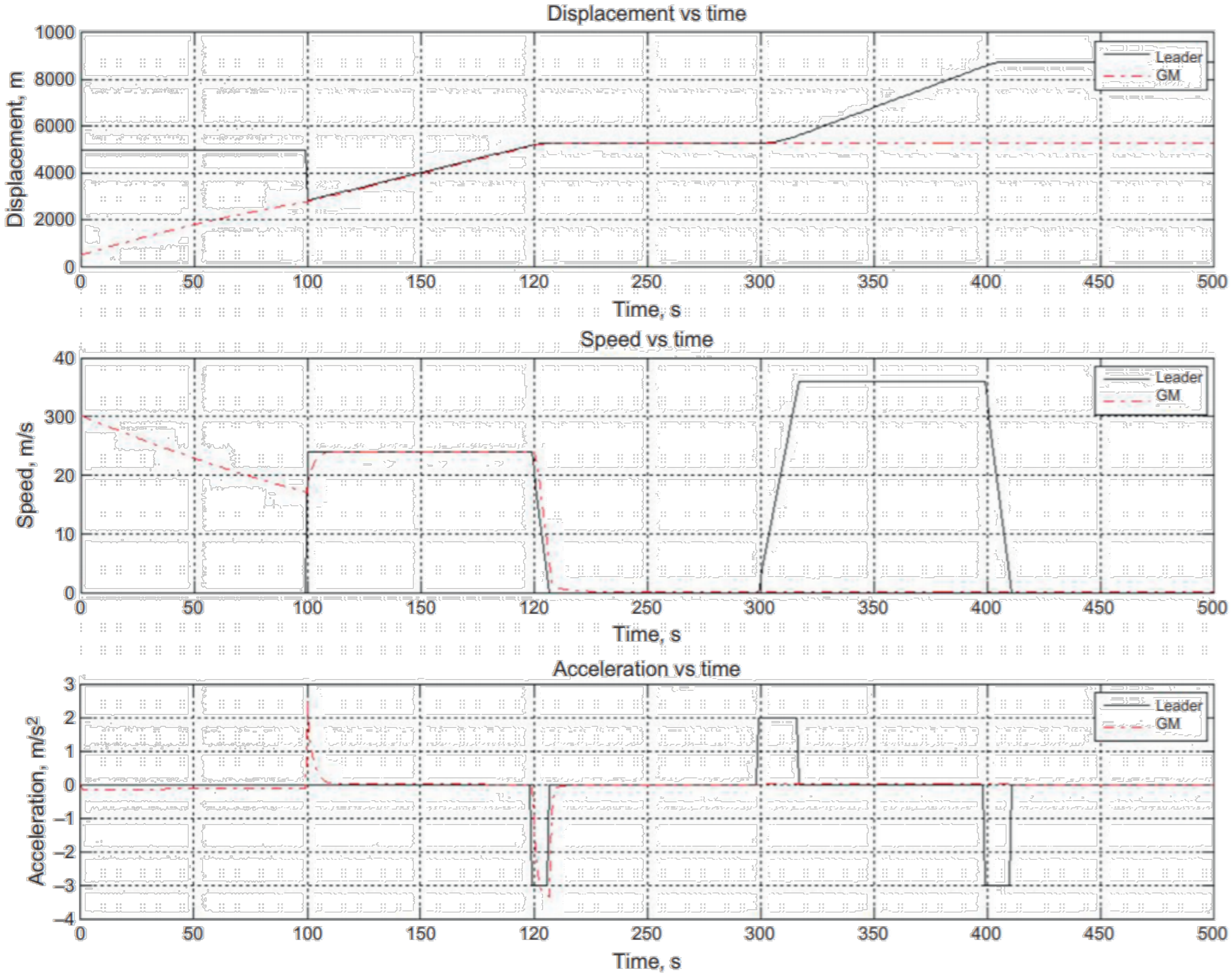
Microscopic benchmarking of GM4

Microscopic Benchmarking – GM4

- ***Stop and go:*** the model predicts that vehicle i will gradually but surely collide with its leader while maintaining a speed, regardless of how low the speed is.
- When the leader resumes motion, vehicle i will be stuck there because of its infinitesimally low speed. See when $200\ s\ \geq\ t\ \leq\ 300\ s$.
- ***Trailing:*** vehicle i is stuck and fails to catch up with its speeding leader, unless another external logic brings it out of being stuck. See when $300\ s < t < 400\ s$.
- ***Approaching and Stopping:*** the simulation fails to be reasonable beyond t = 300 s.

τ_i	α	–
1.0 s	0.8	–
$x_i(0)$	$\dot{x}_i(0)$	$\ddot{x}_i(0)$
467 m	30 m/s	0 m/s^2

Microscopic benchmarking parameters



Microscopic benchmarking of GM4

Microscopic Macroscopic Bridge – GM

Greenberg Model

- The Greenberg's stream model can be derived from the GM5 model.
- The GM model is given as:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_i(t+\tau_i)]^m}{[x_{i-1}(t) - x_i(t)]^l}[\dot{x}_{i-1}(t) - \dot{x}_i(t)]$$

- Setting $m = 0$ and $l = 1$, GM5 reduces to GM3:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t). - x_i(t)]}$$

- At steady state condition, the equation becomes:

$$\ddot{x}_i = \alpha \frac{(\dot{x}_{i-1} - \dot{x}_i)}{x_{i-1} - x_i} \xrightarrow[\text{respect to t}]{\text{integrating both sides with}} \int \ddot{x}_i dt = \int \alpha \frac{(\dot{x}_{i-1} - \dot{x}_i)}{x_{i-1} - x_i} dt \xrightarrow{\text{Change of variable}} \left\{ \begin{array}{l} u = x_{i-1} - x_i \\ \dfrac{du}{dt} = \dot{x}_{i-1} - \dot{x}_i \\ du = (\dot{x}_{i-1} - \dot{x}_i)dt \end{array} \right.$$

Microscopic Macroscopic Bridge – GM

- Therefore:

$$\int \ddot{x}_i dt = \int \alpha \frac{du}{u} \quad \Rightarrow \quad \begin{aligned} \dot{x}_i &= \alpha \log u + c \\ &= \alpha \log(x_{i-1} - x_i) + c \\ &= \alpha \log(s_i) + c \end{aligned}$$

- where, s_i is the spacing.
- Under steady state conditions, there is no identity for vehicles. Hence, the above expression will be as follows:

$$\dot{x} = \alpha \log(s) + c = \alpha \log(\frac{1}{k}) + c$$

- To find the constant *c*, apply the boundary condition, namely, at jam density, the speed is zero. That is:

$$0 = \alpha \log(\frac{1}{k_j}) + c \Rightarrow \mathrm{c} = -\alpha \log(\frac{1}{k_j}) \Rightarrow \dot{x}_i = \alpha \log(\frac{1}{k}) - \alpha \log(\frac{1}{k_j}) \Rightarrow \dot{x}_i = \alpha \log(\frac{1}{k}) + \alpha \log(k_j)$$

$$\Rightarrow \dot{x}_i = \alpha \log(\frac{k_j}{k})$$

Microscopic Macroscopic Bridge – GM

- To find α, we differentiate flow with respect to density and equate to zero :

$$q = k\dot{x} = \alpha k \log(\frac{k_j}{k})$$

$$\frac{dq}{dk} = \alpha k \left[\frac{d}{dk}\log(\frac{k_j}{k})\right] + \alpha(\log\frac{k_j}{k}) \quad \Rightarrow \quad \frac{dq}{dk} = \alpha k \left[\frac{d}{dk}\log k_j - \frac{d}{dk}\log k\right] + \alpha(\log\frac{k_j}{k})$$

$$= \alpha k \left[-\frac{1}{k}\right] + \alpha(\log\frac{k_j}{k})$$

$$\Rightarrow \log\frac{k_j}{k} = 1 \Rightarrow \frac{k_j}{k} = e \quad \Rightarrow \quad k_m = \frac{k_j}{e}$$

$$\Rightarrow \dot{x}_m = \alpha \log\left(\frac{k_j}{k}\right) = \alpha \log e = \alpha$$

- We therefore obtain the Greenberg model: $\boxed{\dot{x} = \dot{x}_m \log\left(\frac{k_j}{k}\right)}$

Microscopic Macroscopic Bridge – GM

Greenshields Model

- Setting $l = 2$ and $m = 0$ in GM5 yields:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^2}$$

- It can be proved that this microscopic model can be transformed into the Greenshields model:

$$v = v_{\mathrm{f}} - \frac{v_{\mathrm{f}}}{k_{\mathrm{j}}} k$$

Microscopic Macroscopic Bridge – GM

Underwood Model

- Setting $l = 2$ and $m = 1$ in GM5 yields:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^2}$$

- It can be proved that this microscopic model can be transformed into the Underwood model:

$$v = v_{\mathrm{f}} \mathrm{e}^{-k/k_{\mathrm{m}}}$$

Microscopic Macroscopic Bridge – GM

Drake et al. Model

- Setting $l = 3$ and $m = 1$ in GM5 yields:

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^3}$$

- It can be proved that this microscopic model can be transformed into the Drake et al. model:

$$v = v_{\mathrm{f}} \mathrm{e}^{-\frac{1}{2}\left(\frac{k}{k_{\mathrm{m}}}\right)^2}$$

Microscopic Macroscopic Bridge – GM

Pipes-Munjal Model / Drew Model

- Setting $l = n + 1$ and $m = 0$ in GM5 yields (n is the exponent determined during calibration):

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]^{n+1}}$$

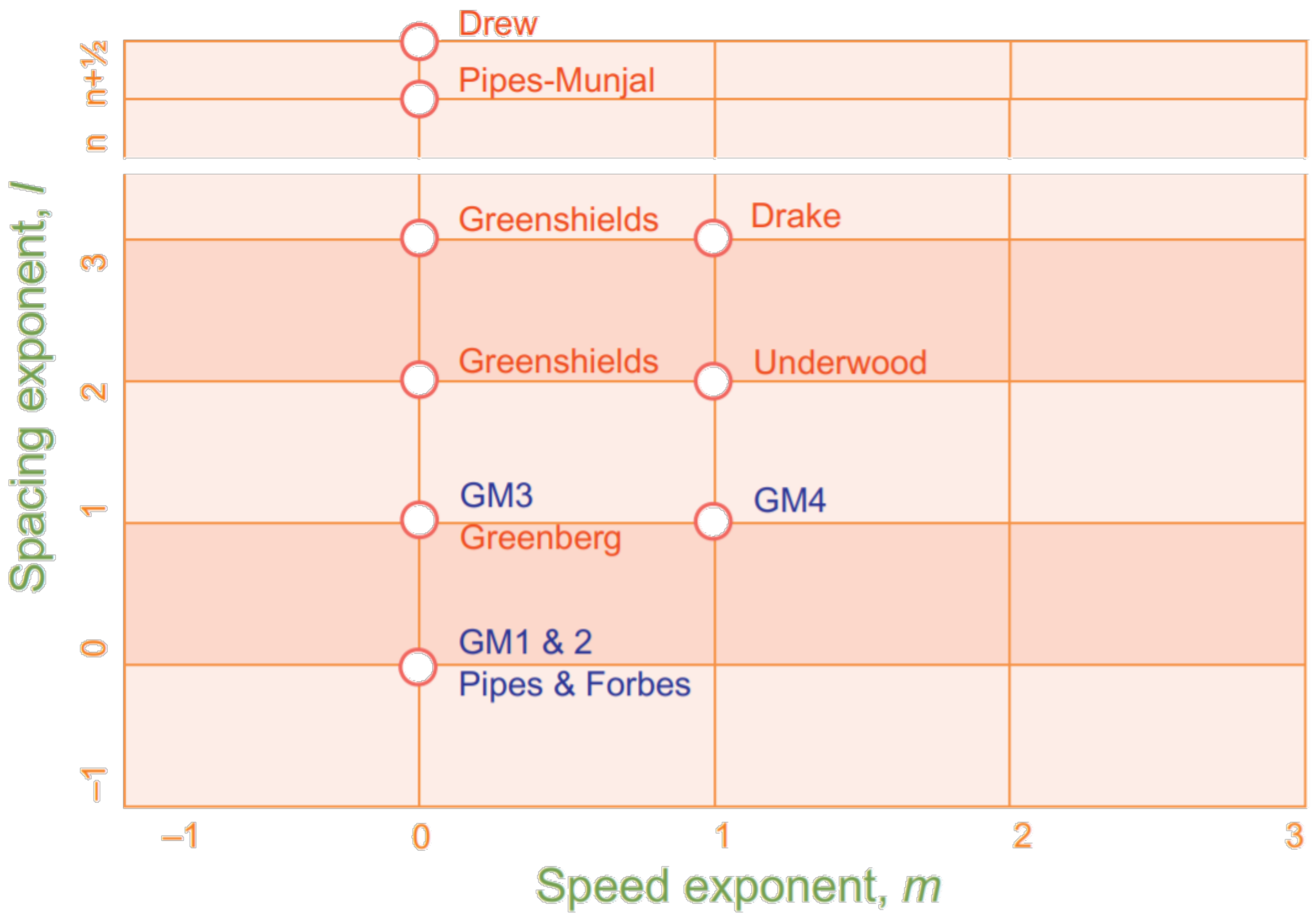
- It can be proved that this microscopic model can be transformed into the Pipes-Munjal model:

$$v = v_{\mathrm{f}}\left[1 - \left(\frac{k}{k_{\mathrm{j}}}\right)^{n}\right]$$

- Since the Drew model and the Pipes-Munjal model are exactly the same except for their exponent, one only needs to replace n with $n + 1/2$ in the above derivation to obtain the Drew model.
- Hence, the Drew model corresponds to GM5 with $l = n + 1.5$ and $m = 0$.

Microscopic Macroscopic Bridge – GM

- Macroscopic equilibrium models are labeled in red and microscopic car-following models are labeled in blue.
- Circles on the grid denote models and their corresponding m and l combination in relation to GM5.



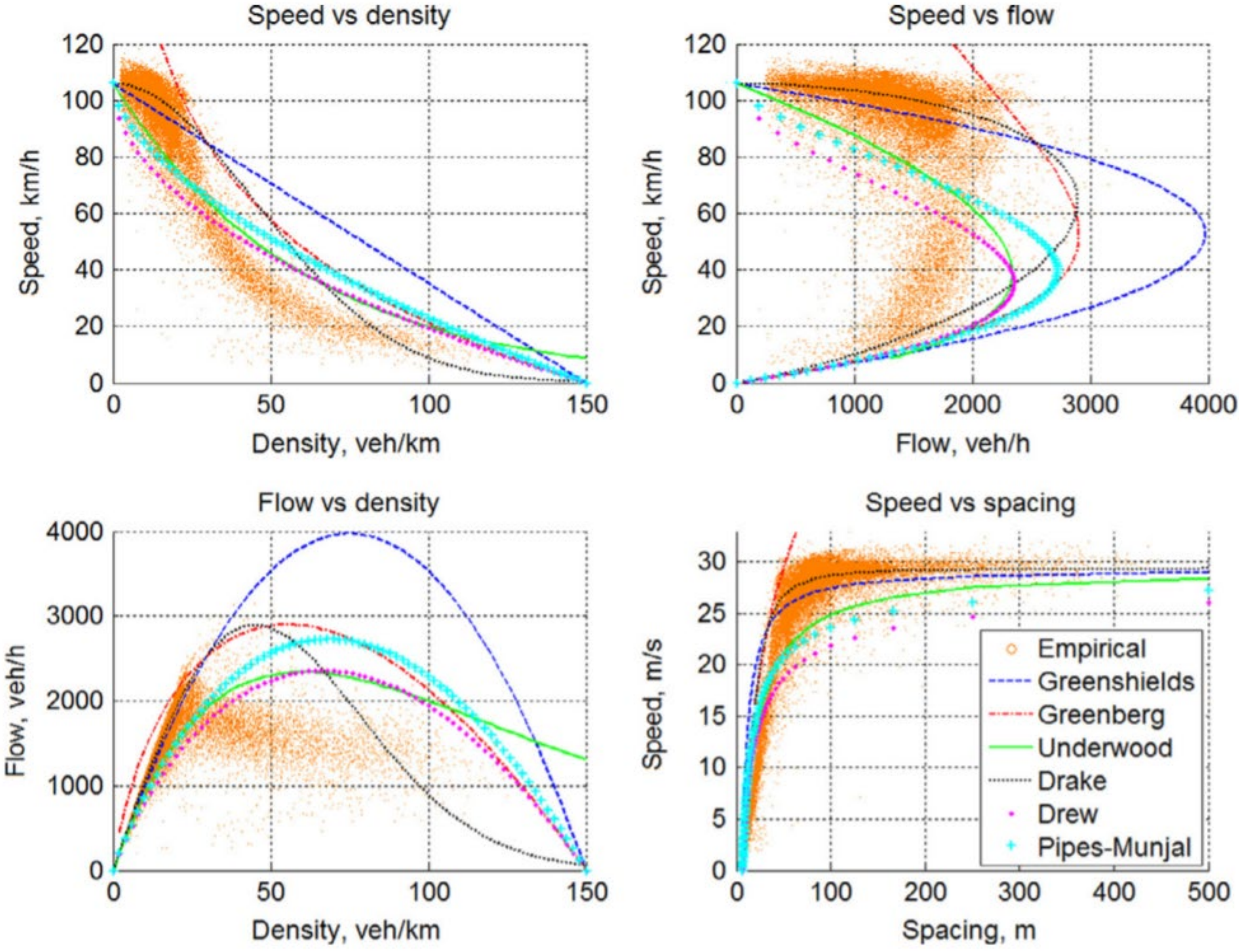
Microscopic-macroscopic bridge

Macroscopic Benchmarking - GM

- Fundamental diagrams implied by GM models and their associated equilibrium models are presented against empirical observations.
- It can be seen that these fundamental diagrams achieve varying success in fitting empirical data, but none of them fit the data reasonably well in the entire range of density.
- This benchmarking is based on the following set of parameters:

Greenshields	v_f 30 m/s	k_j 1/6 vehicles/m	– –
Greenberg	v_m 10.7 m/s	k_j 1/6 vehicles/m	k_c 0.01 vehicles/m
Underwood	v_f 30 m/s	k_m 0.05 vehicles/m	– –
Drake et al. (Northwestern)	v_f 30 m/s	k_m 0.04 vehicles/m	n 2
Drew	v_f 30 m/s	k_j 1/6 vehicles/m	n 0.1
Pipes and Munjal	v_f 30 m/s	k_j 1/6 vehicles/m	n 0.5

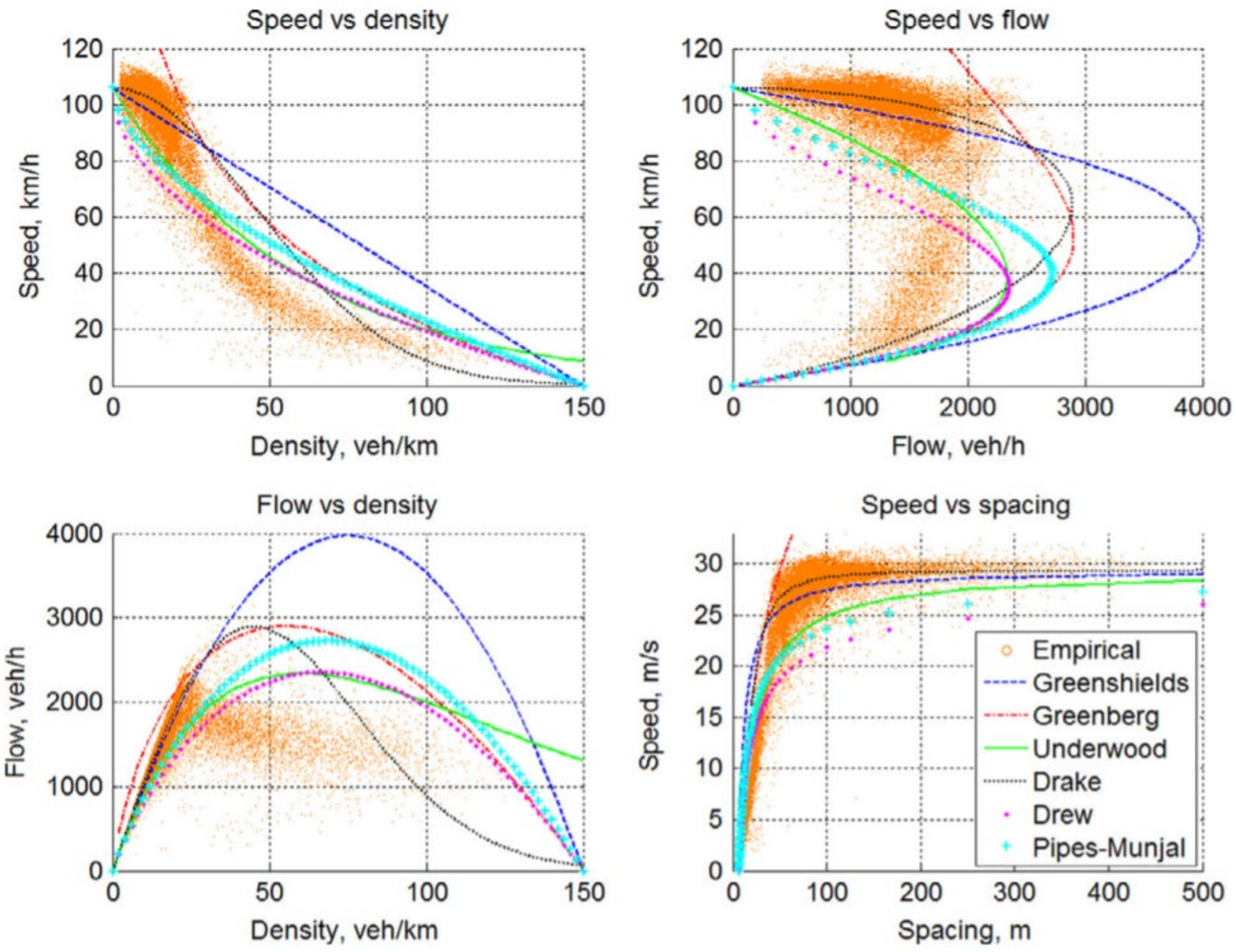
Macroscopic benchmarking parameters - GM models



Fundamental diagrams implied by GM models

Macroscopic Benchmarking - GM

- The Greenshields model overpredicts speed (and hence flow) in the majority of the density range except for the free-flow (i.e., low-density) condition;
- The Greenberg model has a problem fitting the data under the free-flow condition;
- The Underwood model, perhaps the best among the models, underestimates speed at low densities and overestimates speed at high densities, and capacity occurs at much lower speed than it ought to;
- The model of Drake et al. has a flow-density curve that is convex in the high-density range;
- The Drew and Pipes-Munjal models, share the same problem as the Greenshields model but to a lesser extent.



Fundamental diagrams implied by GM models

Limitations of GM Models

- On the one hand, GM4 is mathematically attractive since it has only one equation that covers all situations.
- On the other hand, such a one-regime property stipulates universal car following, which is not realistic.
- For example, the model predicts that a vehicle in Atlanta must be following another vehicle in Boston even though they are over 1000 km apart.

- The GM4 model is very similar to Newton's law of universal gravitation and Coulomb's law:

$$F = G\frac{m_1 m_2}{r^2}$$

where F is the force between two masses, G is the gravitational constant, m_1 is the first mass, m_2 is the second mass, and r is the distance between the masses.

$$F = k_e \frac{q_1 q_2}{r^2}$$

where F is the electrostatic force between two point charges, q_1 is the first point charge, q_2 is the second point charge, r is the distance between the two point charges, and k_e is a proportionality constant.

Limitations of GM Models

- Therefore, GM4 can be interpreted as equivalent to Coulomb's law as follows.

  Vehicle i will be repelled by its leader $i-1$ when vehicle i is approaching vehicle $i-1$ at a higher speed, while vehicle i will be attracted to vehicle $i-1$ should vehicle i fall behind at a slower speed.

- Though the first half of the reasoning seems to make some sense, the second half does not.
- For example, what if the subject vehicle does not have a leader and is the first vehicle to enter the highway. Then the subject vehicle could not start because there would be no vehicle to pull it forward.
- Even if the subject vehicle is following a leader and the gap between them is increasing, it does not feel as if the subject vehicle is attracted to the leader. Actually, the subject vehicle speeds up because one would like to achieve the desired speed.

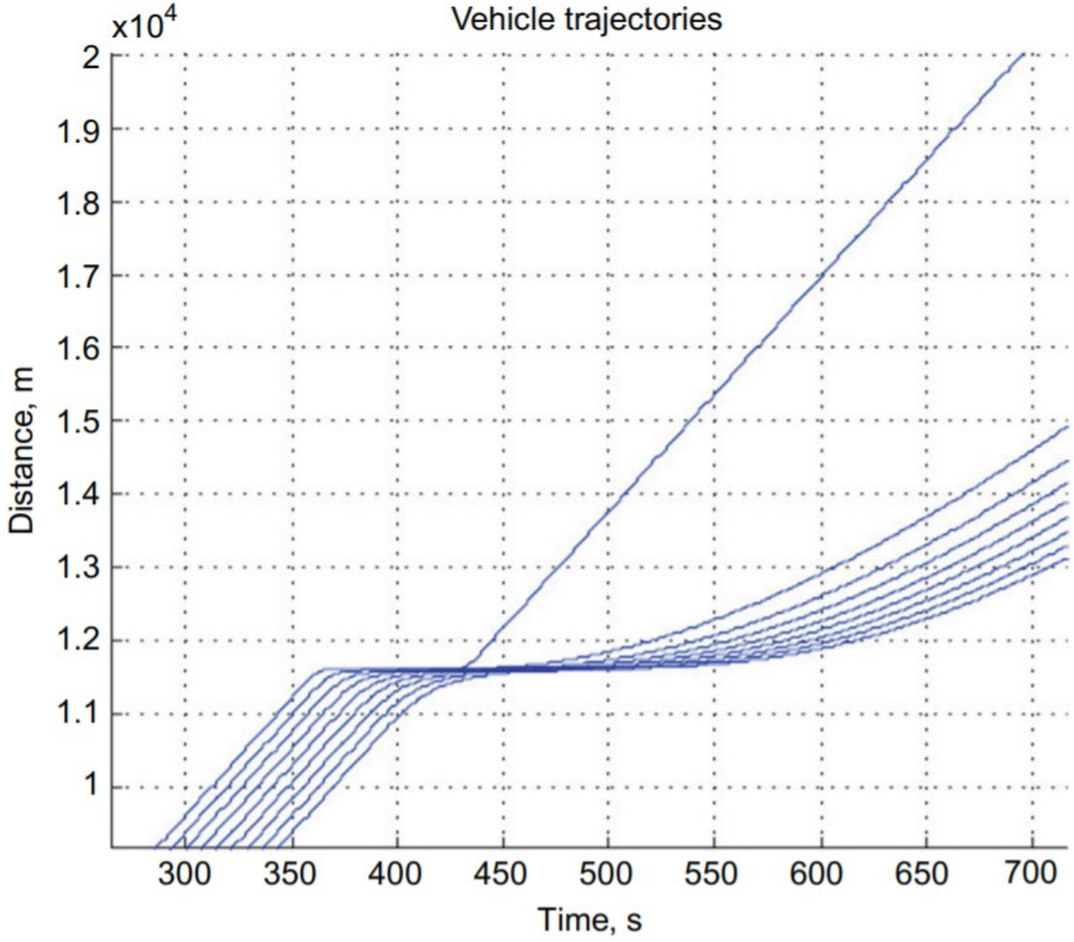
Limitations of GM Models

Slow start

- According to GM4, a vehicle at a stopped position is unable to start.
- This is because the vehicle's speed at the current step ($\dot{x}_i(t) = 0$) determines its acceleration in the next step ($\ddot{x}_i(t + \tau_i) = 0$).

$$\ddot{x}_i(t+\tau_i) = \alpha \frac{\dot{x}_i(t+\tau_i)[\dot{x}_{i-1}(t) - \dot{x}_i(t)]}{[x_{i-1}(t) - x_i(t)]}$$

- Therefore, the vehicle has to maintain a nonzero speed at any time in order to avoid being trapped.

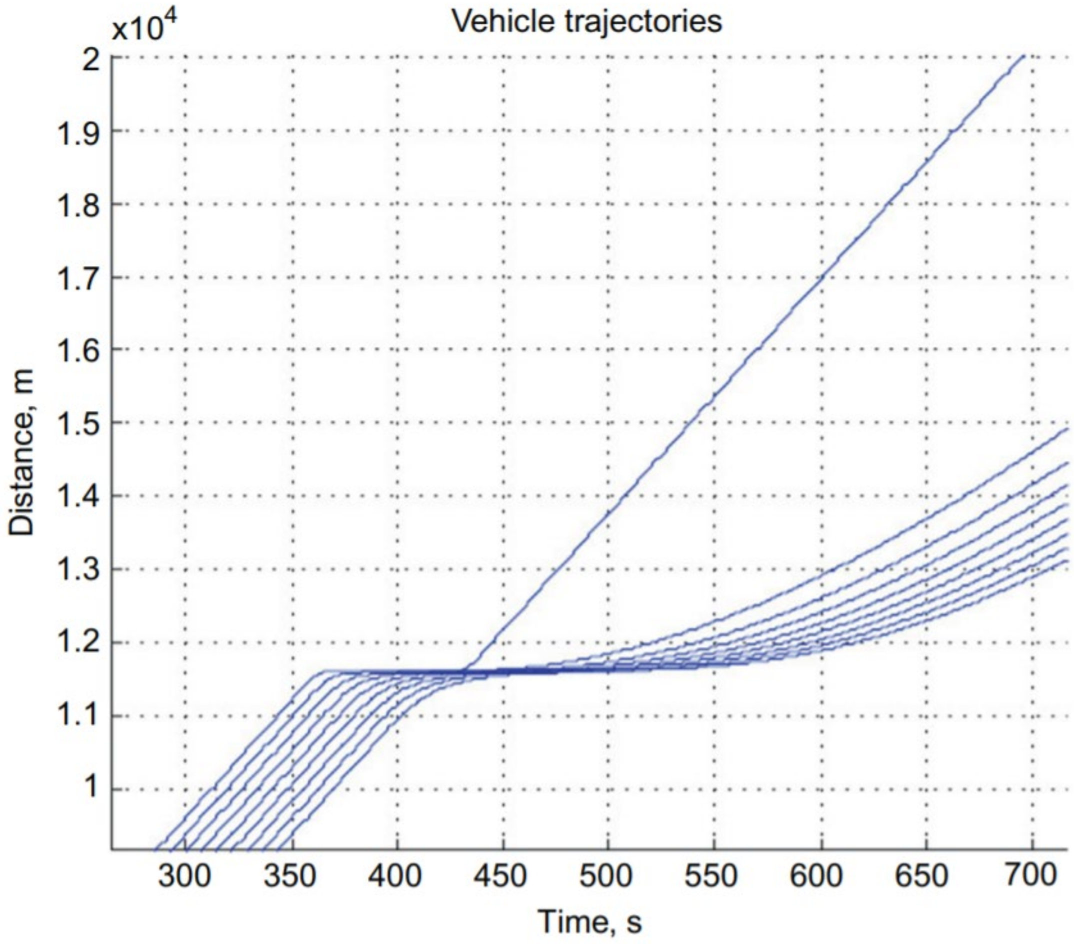


GM4 slow start

Limitations of GM Models

Slow start

- As such, the model fails to apply when a vehicle is stopped by a red light at an intersection or is completely blocked by another vehicle on a highway.
- The subject vehicle has to slow down to an infinitesimal speed rather than to a complete stop in order to avoid being trapped.
- When the light turns green or the leading vehicle resumes motion, the subject vehicle will take a long time to get up to speed.
- This is because the vehicle's infinitesimal speed results in a weak attraction, which is the only mechanism to accelerate the vehicle.
- Such a scenario is illustrated, where a noticeable gap results between the first vehicle and the second vehicle.



GM4 slow start

Limitations of GM Models

Proximity

- According to GM4, two vehicles can get arbitrarily close to each other as long as they are traveling at the same speed.
- It is certainly not reasonable to imply that two vehicles would follow each other with 1-inch gap between them at 120 km/h!
- The reason why GM4 allows such a car-following distance is because, regardless of how close the two vehicles are, the model predicts no response as long as the two vehicles are moving at the same speed.

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Thank you
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