Traffic Flow Characteristics

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Introduction

- Traffic can be directly observed by different methods such as cameras on top of a tall building or mounted on an airplane.
- According to their reporting mechanisms the collected data, traffic sensors can be classified into three categories: mobile sensors, point sensors, and space sensors.
- A mobile sensor resides in a vehicle, moves along with the vehicle, and logs the location of this particular vehicle over time.
- A point sensor sits at a fixed location on a roadway, observes the passage of vehicles, and reports traffic data only at this particular location over time.
- A space sensor observes traffic on a stretch of road, and records the positions of vehicles at an instant of time over this particular stretch of road.
- We will discuss what traffic data reported by these sensors look like and, further, how traffic flow characteristics are determined from these data.

Mobile Sensor Data

- If a vehicle is equipped with a global positioning system (GPS) device, the device can report the vehicle's position as time progresses.

- Suppose GPS signals are received every second, the GPS data may look similar to the data shown in the table.

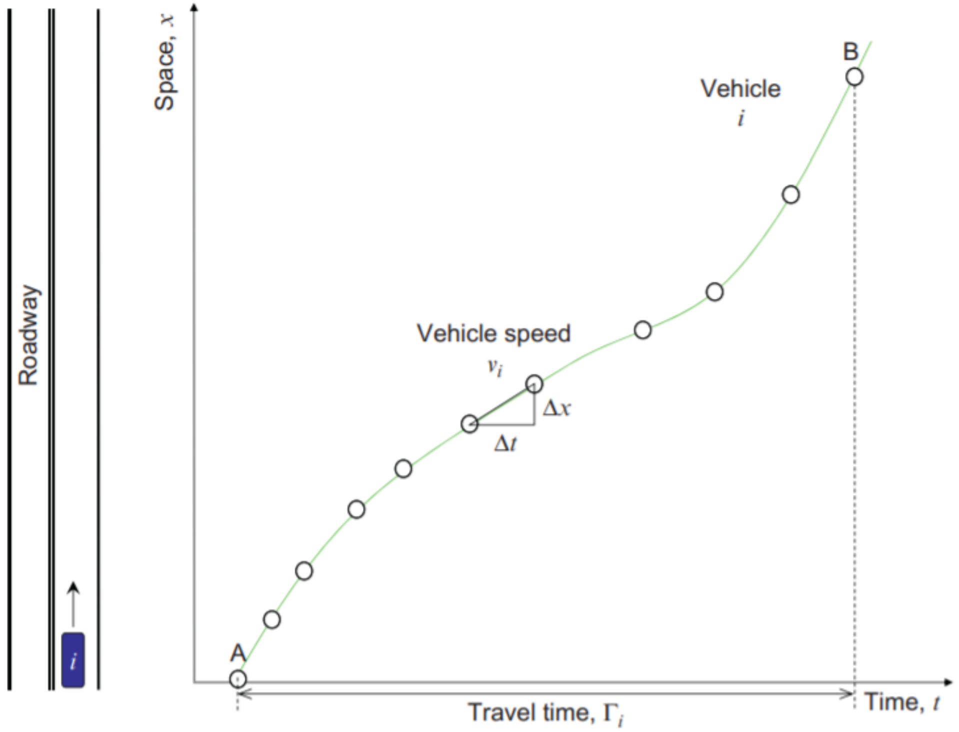
- Vehicle's position is defined by longitudinal *x* and lateral *y* timestamped coordinates.

- In the figure, every circle represents a GPS reading.

- If one connects these circles, the trajectory of this vehicle is obtained—that is, the location of the vehicle as a function of time.

Time	x (feet)	y (feet)	x (m)	y (m)
09:00:00	0	0	0.0	0.0
09:00:01	3	0	0.9	0.0
09:00:02	5	0	1.5	0.0
09:00:03	7	0	2.1	0.0
09:00:04	10	1	3.0	0.3
09:00:05	15	4	4.6	1.2
09:00:06	18	9	5.5	2.7
09:00:07	21	12	6.4	3.7
09:00:08	23	12	7.0	3.7
09:00:09	27	12	8.2	3.7
09:00:10	30	12	30	12

GPS data



Vehicle's trajectory

Mobile Sensor Data

- To calculate the speed of the vehicle:

$$\dot{x}_i = \frac{\Delta x}{\Delta t}.$$

- If the vehicle's trajectory is known and smooth, we can determine $\dot{x}_i$ by taking the first derivative of the trajectory:

$$\dot{x}_i = \frac{\mathrm{d}x}{\mathrm{d}t}.$$

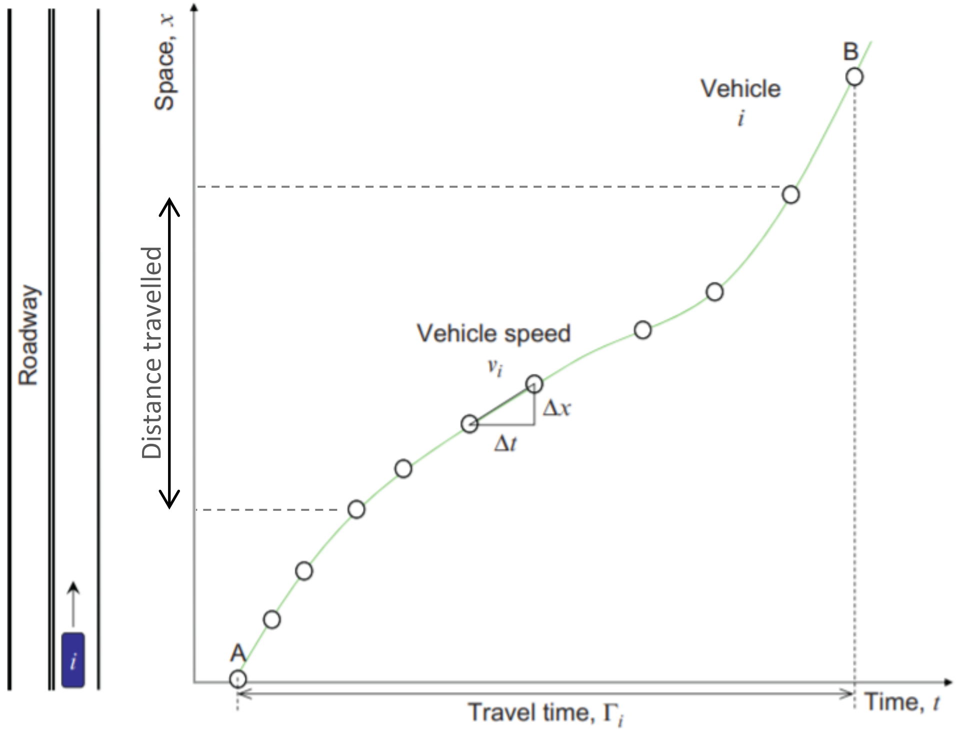
- The vehicle's travel time, between two points A and B can be directly read from the trajectory:

$$\Gamma_i = t_i^{\mathrm{B}} - t_i^{\mathrm{A}}$$

- Distance travelled?

Time	x (feet)	y (feet)	x (m)	y (m)
09:00:00	0	0	0.0	0.0
09:00:01	3	0	0.9	0.0
09:00:02	5	0	1.5	0.0
09:00:03	7	0	2.1	0.0
09:00:04	10	1	3.0	0.3
09:00:05	15	4	4.6	1.2
09:00:06	18	9	5.5	2.7
09:00:07	21	12	6.4	3.7
09:00:08	23	12	7.0	3.7
09:00:09	27	12	8.2	3.7
09:00:10	30	12	30	12

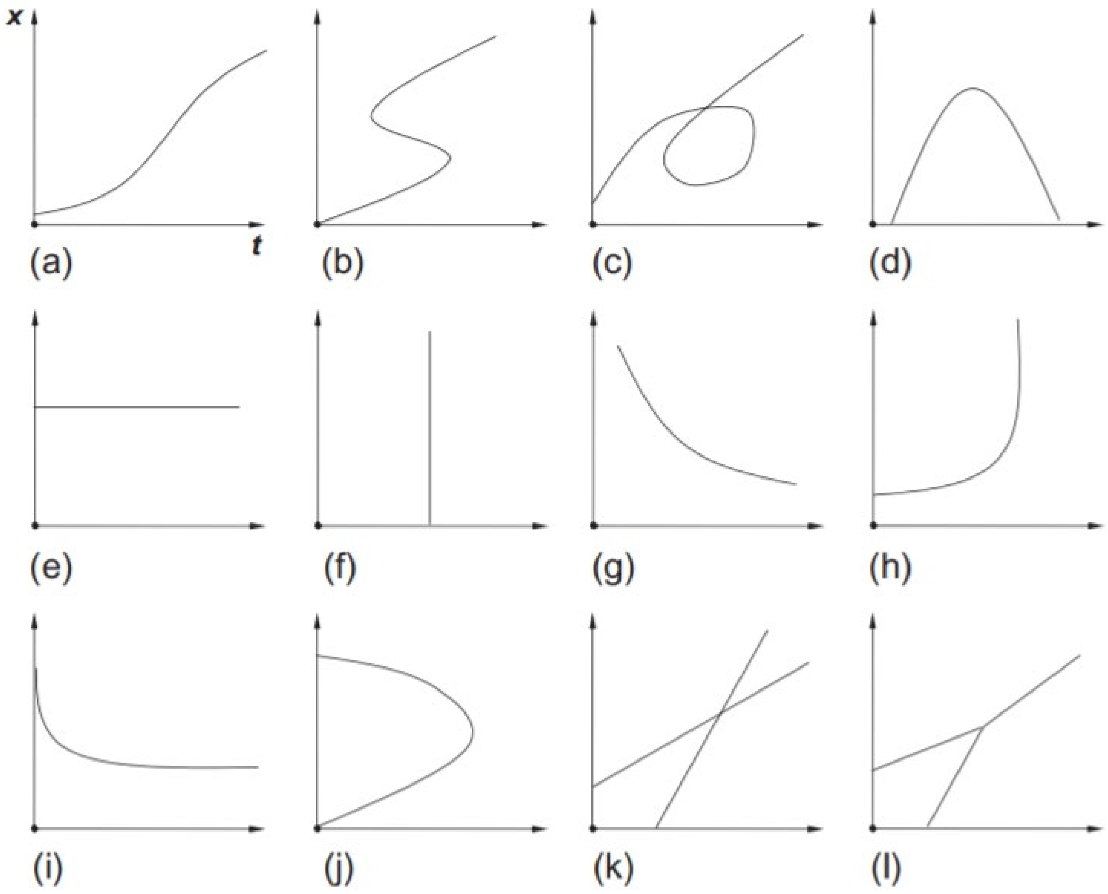
GPS data



Vehicle's trajectory

Mobile Sensor Data

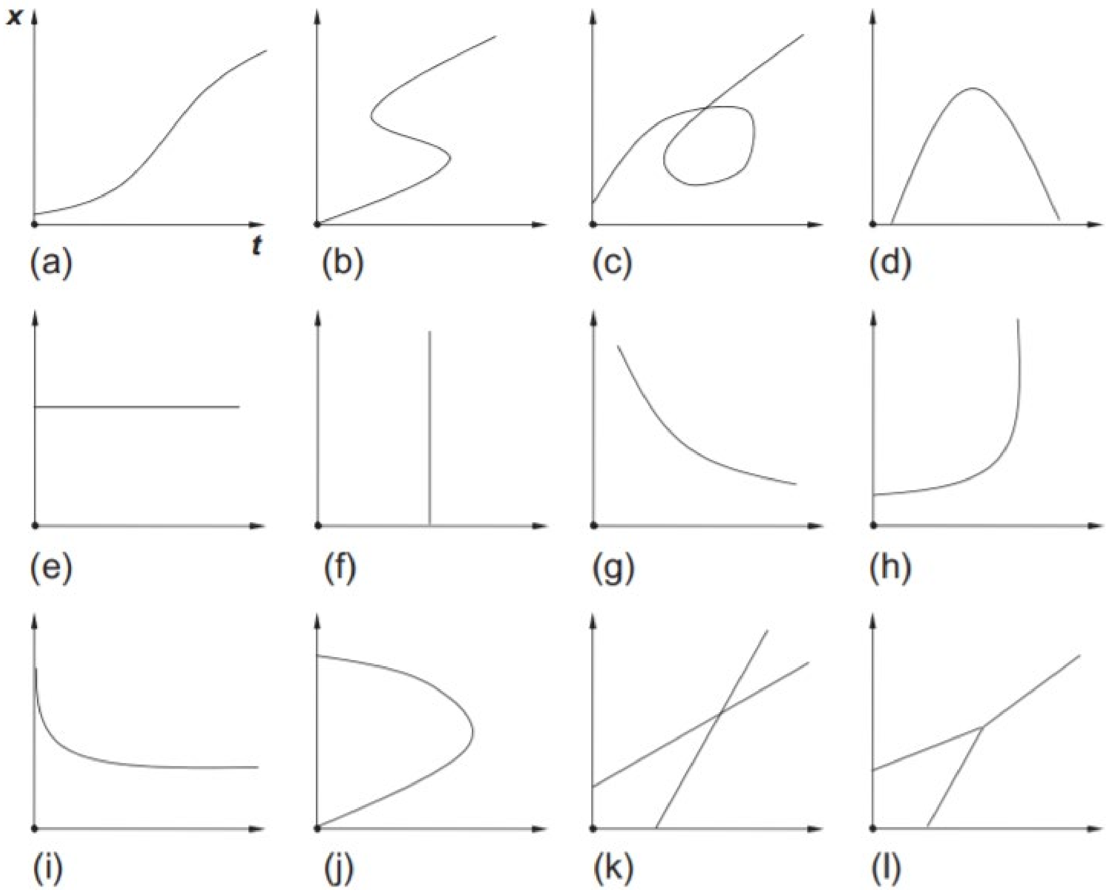
- Plot (a) is valid, and it shows a vehicle moving in the positive x direction over time.
- Plot (b) is not a valid trajectory. If one draws a vertical line, it may intersect the trajectory several times. This means that at an instant of time the vehicle can appear at multiple locations simultaneously, which is impossible.
- For the same reason, plots (c) and (j) are not valid either.
- Plot (d) is valid, and the trajectory suggests that the vehicle first moves forward (i.e., in the positive x direction) and then, at some point in time, reverses.
- Plot (e) is valid, and simply suggests that the vehicle does not move (maybe parked).
- Plot (f) is impossible because it suggests an infinite speed (i.e., the tangent of the trajectory).



Hypothetical vehicle trajectories

Mobile Sensor Data

- Plot (g) is a valid since the vehicle just moves backward.
- Plot (h) is very unusual because the vehicle first moves at reasonable speeds and then almost flies at the end.
- Plot (i) is valid, and the vehicle gradually comes to a stop.
- Plot (k) can be interpreted in two ways:
  - one is a two-lane scenario where a fast vehicle overtakes a slow vehicle;
  - The other is a one-lane scenario where the fast vehicle collides with the slow vehicle.
- Plot (l) is very unlikely and suggests that a fast vehicle catches up with a slow vehicle and then they move as a single unit thereafter.
- It should be noted that the vehicle is considered as a point (not having any length) in these plots.



Hypothetical vehicle trajectories

Point Sensor Data

- If a point sensor (such as a loop detector) is installed on the road at location x, this sensor will be able to observe vehicles passing above or under it.

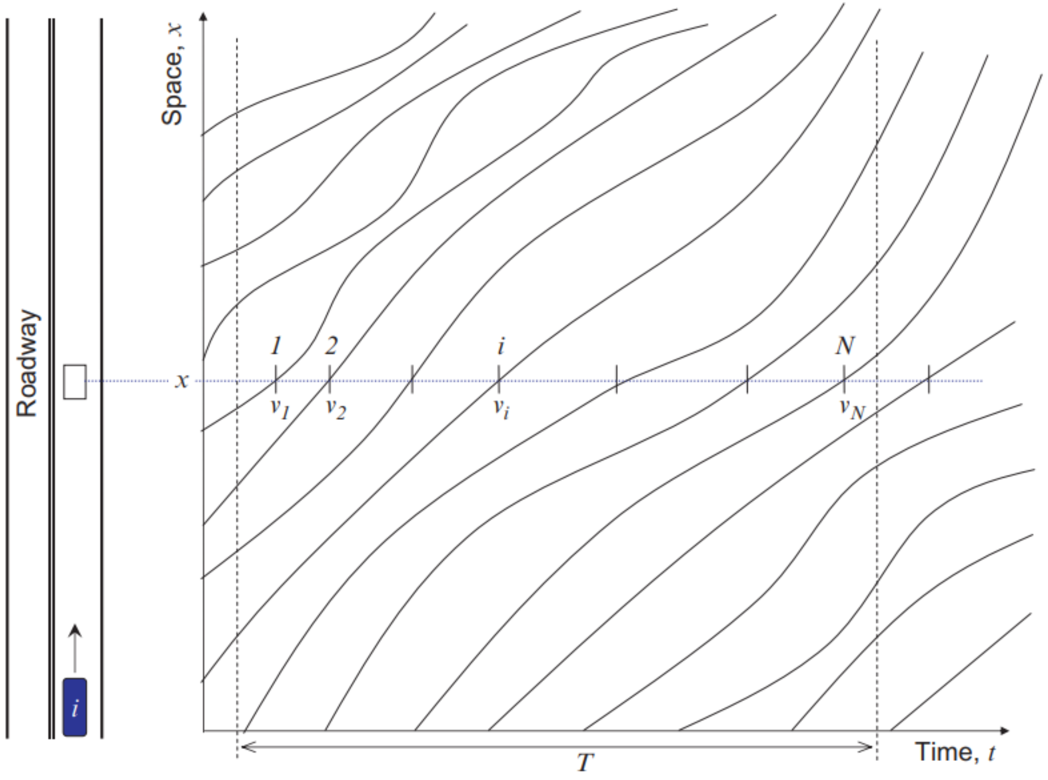
- In a time-space diagram, each vehicle will be counted at this location.

- During an observation period T, a total of N vehicles are counted by the sensor.

- N is referred to as the traffic count

- Traffic count can be converted to the hourly equivalent rate of flow (q hereafter) as follows:

$$q = \frac{N}{T}$$



Point sensor data

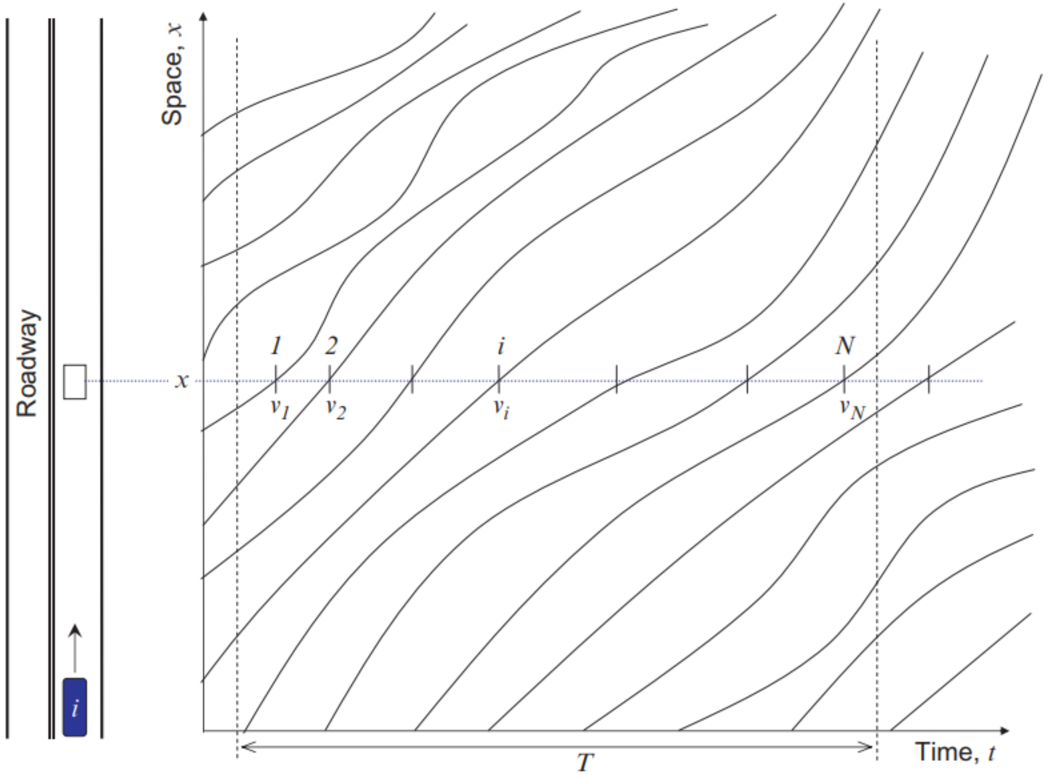
Point Sensor Data

- Headway h_i is defined as the temporal separation between two consecutive vehicles, and can be determined as:

$$h_i = t_i - t_{i-1}$$

- If one ignores the error due to incomplete headways of the first and last vehicles, the observation duration T can be expressed as:

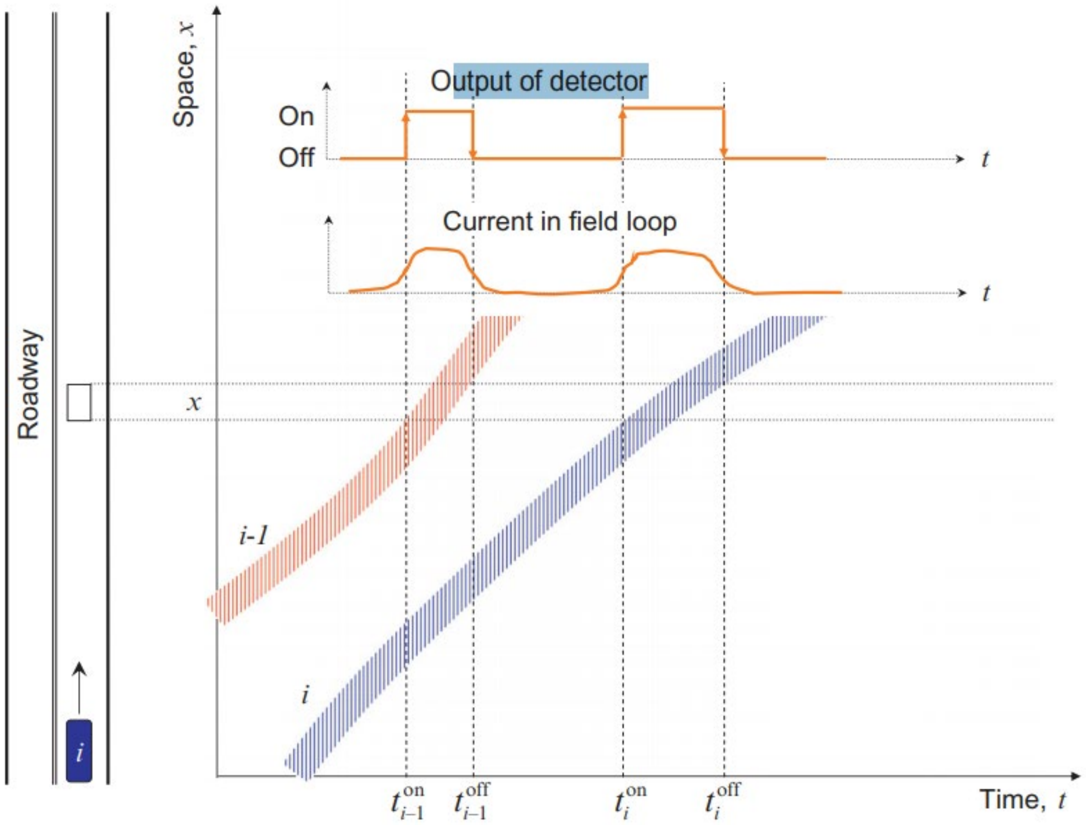
$$T = \sum_{i=1}^{N} h_i$$



Point sensor data

Point Sensor Data

- Both vehicles and point sensors have physical dimensions.
- If the sizes of the vehicles and sensors are taken into consideration, more information can be obtained from the time-space diagram.
- When a vehicle's front bumper enters the detection zone of a loop detector, a detection signal will be generated in the detector according to electromagnetism.
- When the vehicle's rear bumper exits the detection zone, the signal will drop to its normal state.
- If a threshold is set properly, the loop detector outputs two states:
  - "on" when a vehicle is above the loop, and
  - "off " when the loop detects no vehicle over it.



Loop detector data

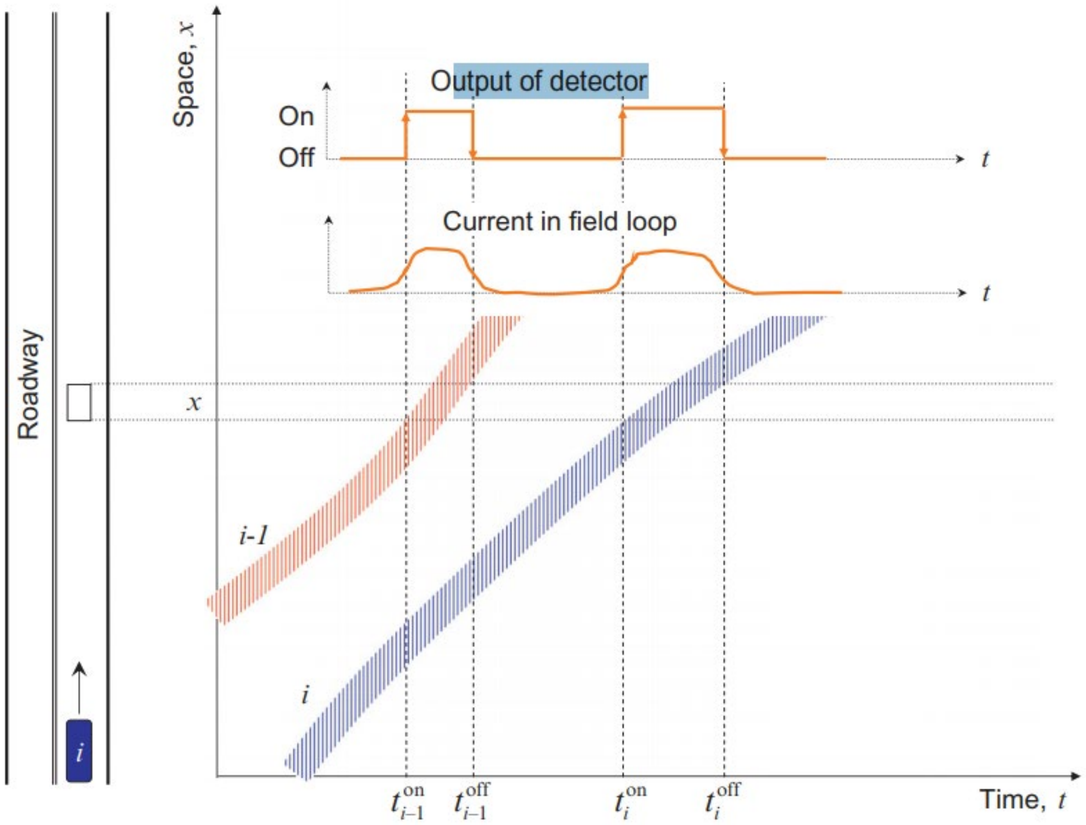
Point Sensor Data

- Since the on state consists of an upward transition and a downward transition of the detector output, one need only count either the upward transition or the downward transition consistently over all vehicles in order to obtain the traffic count N.
- If one chooses reference points on all vehicles consistently (e.g., front bumpers), the headway between vehicles $i - 1$ and i can be calculated as:

$$h_i = t_i^{\text{on}} - t_{i-1}^{\text{on}}$$

- The duration from the moment when a vehicle's front bumper enters the detection zone to the moment when the vehicle's rear bumper exits the detection zone is called the "on time", ξ_i.

$$\xi_i = t_i^{\text{off}} - t_i^{\text{on}}$$



Loop detector data

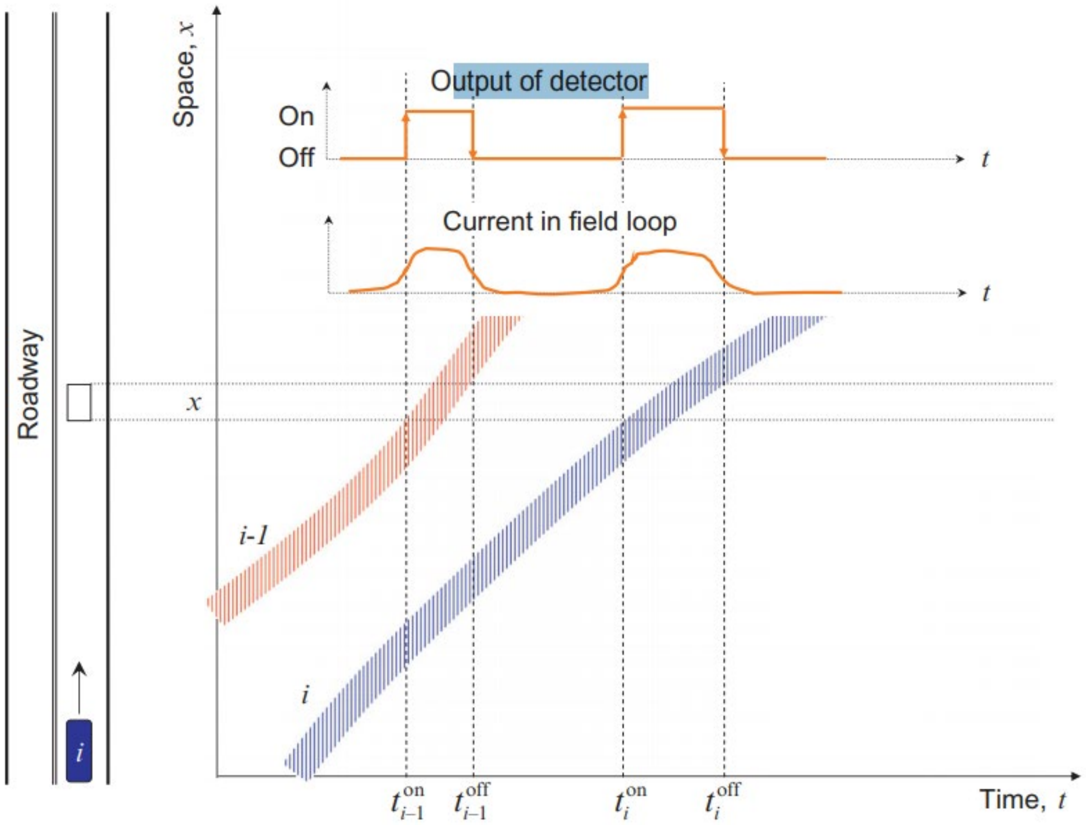
Point Sensor Data

- During the on time, vehicle i travels a distance of $d + l_i$ where d is the width of the loop (typically 6 feet or 1.8 m for small loops) and l_i is the length of the vehicle. Hence, the vehicle's instantaneous speed can be determined as:

$$\dot{x}_i = \frac{d + l_i}{\xi_i} = \frac{d + l_i}{t_i^{\text{off}} - t_i^{\text{on}}}$$

- Occupancy is defined as the percentage of time when a loop is busy—that is, when the loop detects vehicles above it. Hence, if the observation period is T, during which N vehicles are detected, the total "on" time is $\sum_{i=1}^{N} \xi_i$ and the occupancy is determined as:

$$o = \frac{\sum_{i=1}^{N} \xi_i}{T}$$

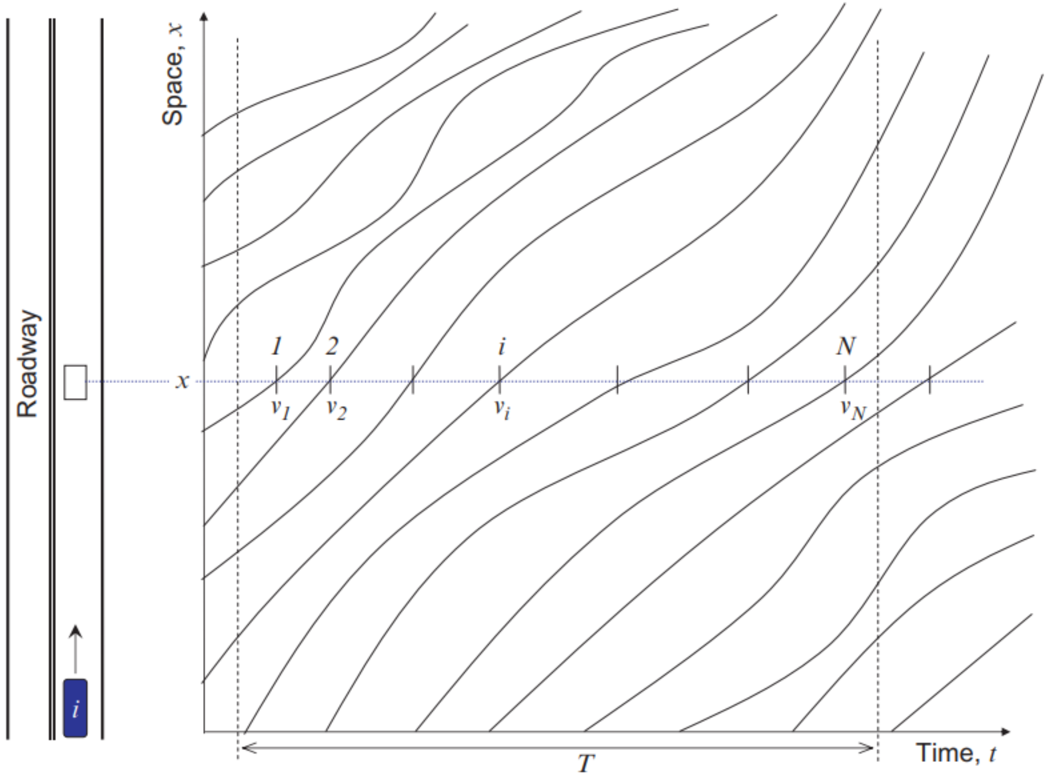


Loop detector data

Point Sensor Data

- If one averages vehicle speeds observed at a point of roadway (by a point sensor), one obtains a mean speed in the time domain, and hence such a mean speed is termed “time-mean speed”.

$$v_t = \frac{1}{N}\sum_{i=1}^{N}\dot{x}_i$$



Point sensor data

Space Sensor Data

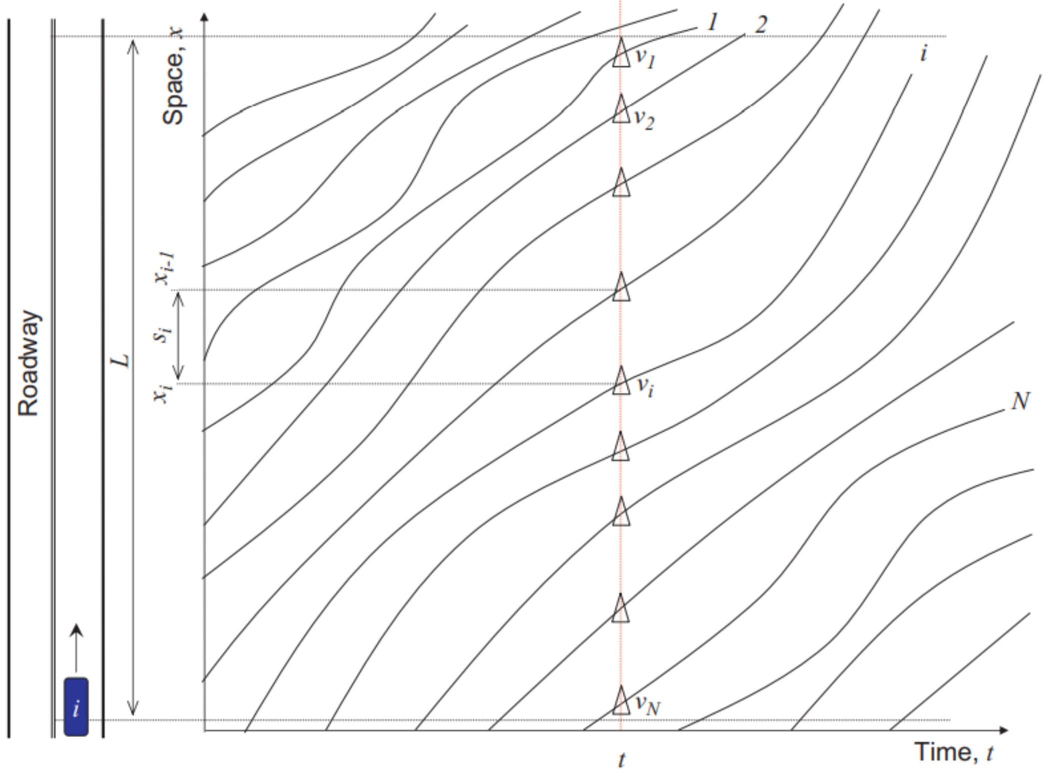
- If one takes aerial photos of a roadway from a helicopter, one can locate vehicles in each of these snapshots.
- Figure illustrates a snapshot taken at time t where vehicles are labeled as triangles.
- The spatial separation, s_i between two consecutive vehicles is called spacing:

$$s_i = x_{i-1} - x_i.$$

- Density k is defined as the number of vehicles observed on a unit length of the road,

$$k = \frac{N}{L}$$

  where L is the length of the stretch of road under observation and N is number of vehicles observed on this stretch of road.



Space sensor data (A snapshot of roadway)

Space Sensor Data

- If one ignores the error due to incomplete spacings of the first and last vehicles, the length of roadway L can be expressed as:

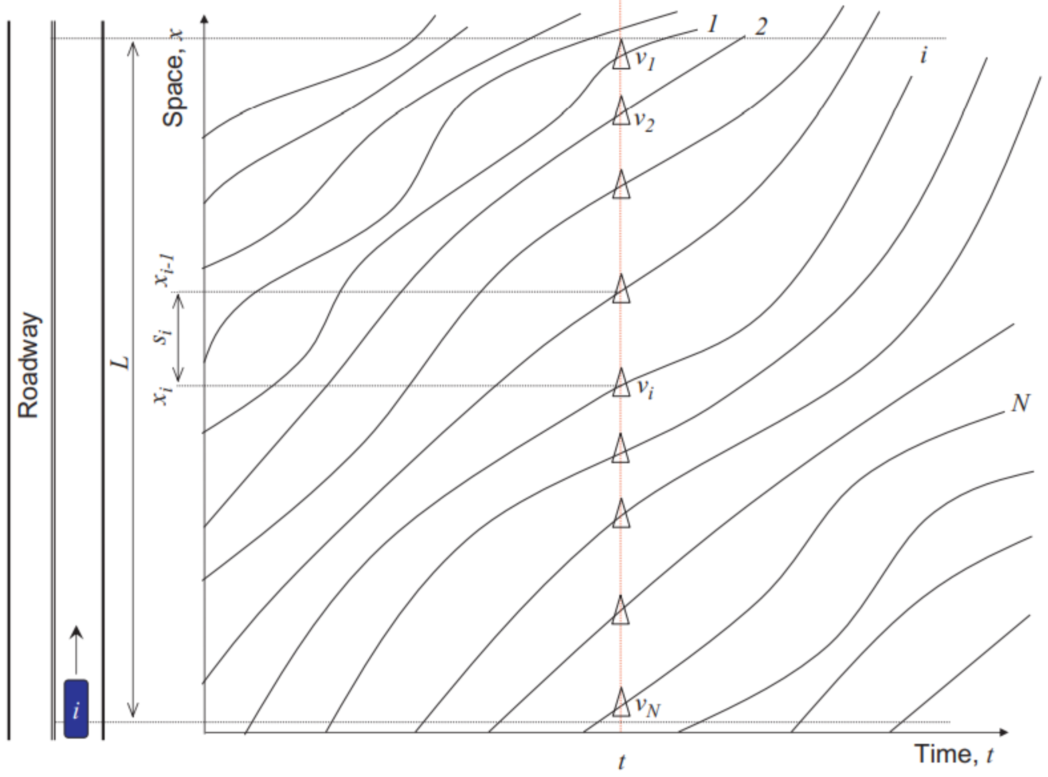
$$L = \sum_{i=1}^{N} s_i.$$

- With two snapshots (at t1 and t2, respectively), one is able to compare vehicle locations and find the distance traversed by each vehicle. Since the time between the two snapshots known, the speed of each vehicle can be determined accordingly:

$$v_i(t) = \frac{\Delta x_i}{\Delta t}.$$

- If one averages vehicle speeds obtained from aerial photos, a mean speed in the space domain results, and hence such a mean speed is termed the "space-mean speed".

$$v_s = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i$$

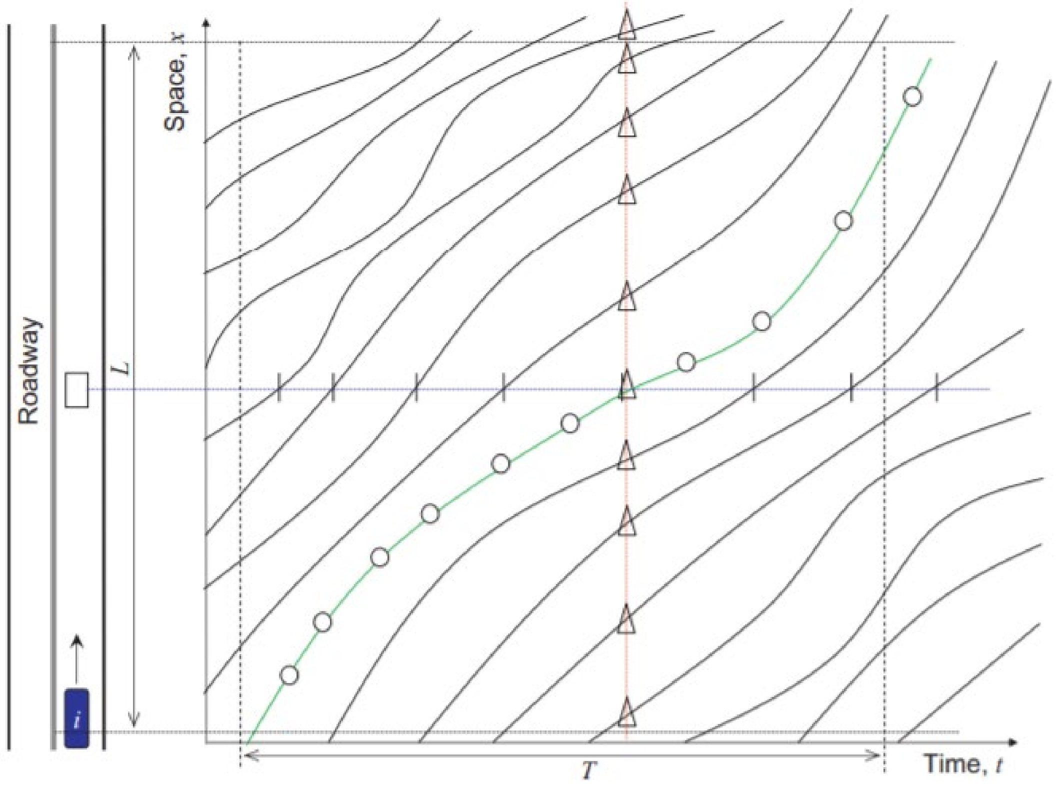


Space sensor data (A snapshot of roadway)

Space-Time Diagram

- The figure illustrates a space-time diagram with vehicle trajectories where the data is reported by the three types of sensors.
- The characteristics are considered at two levels of detail:
  - *Microscopic characteristics* are vehicle specific and hence all bear subscript i,
  - *Macroscopic characteristics* are aggregated measures and the aggregation can be done over vehicles, time, or space.

Category	Sensors	Microscopic characteristics	Macroscopic characteristics
	Mobile	–	–
Flux	Point	h_i	N, q
	Space	–	–
	Mobile	$\dot{x}_i$	–
Speed	Point	$\dot{x}_i$	v_t
	Space	$\dot{x}_i$	v_s
	Mobile	–	–
Concentration	Point	ξ_i	o
	Space	s_i	N, k



Time-space diagram and data from the three types of sensors

Wardrop Equation

$$v_t = v_s + \frac{\sigma^2}{v_s}$$

- Wardrop demonstrated (from field data) that the following relationship between time-mean speed and space-mean speed always holds:
- Where σ^2 is the variance of vehicle speeds.
- Time-mean speed v_t is always greater than or equal to space-mean speed v_s.
- They are equal only if the traffic is uniform—that is, all vehicles are traveling at the same speed ($\sigma = 0$).
- The difference between these speeds tends to decrease as the absolute values of speeds increase.

Time-Mean Speed and Space-Mean Speed

- Time mean speed ($\bar{v}_t$) is the arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time.
- Where:
  - n = number of vehicles passing a point on the highway
  - v_i = the speed of the i^{th} vehicle (m/s)

$$\bar{v}_t = \frac{1}{n}\sum_{i=1}^{n} v_i$$

- Space mean speed ($\bar{v}_s$) is the harmonic mean of the speeds of vehicles passing a point on a highway during an interval of time.
- Where:
  - n = Number of vehicles
  - v_i = The speed of the i^{th} vehicle (m/s)
  - t_i = Time it takes the i^{th} vehicle to travel across a section of highway (s)
  - L = Length of section of highway (m)

$$\bar{v}_s = \frac{n}{\sum_{i=1}^{n}(\frac{1}{v_i})} = \frac{nL}{\sum_{i=1}^{n} t_i}$$

Time-Mean Speed and Space-Mean Speed

Example 1

If the spot speeds are 50, 40, 60, 54 and 45, then find the time mean speed, the space mean speed, and verify the Wardrop relation.

$$v_t = \frac{45 + 50 + 60 + 55 + 40}{5} = 50\ km/h$$

$$v_s = \frac{5}{\frac{1}{45} + \frac{1}{50} + \frac{1}{60} + \frac{1}{55} + \frac{1}{40}} = 48.98\ km/h$$

Note that $v_s < v_t$

$$\sigma_s^2 = \frac{1}{5}((45 - 48.98)^2 + (50 - 48.98)^2 + (60 - 48.98)^2 + (55 - 48.98)^2 + (40 - 48.98)^2) = 51.02$$

$$v_t = 48.98 + \frac{51.02}{48.98} \approx 50\ km/hr$$

Time-Mean Speed and Space-Mean Speed

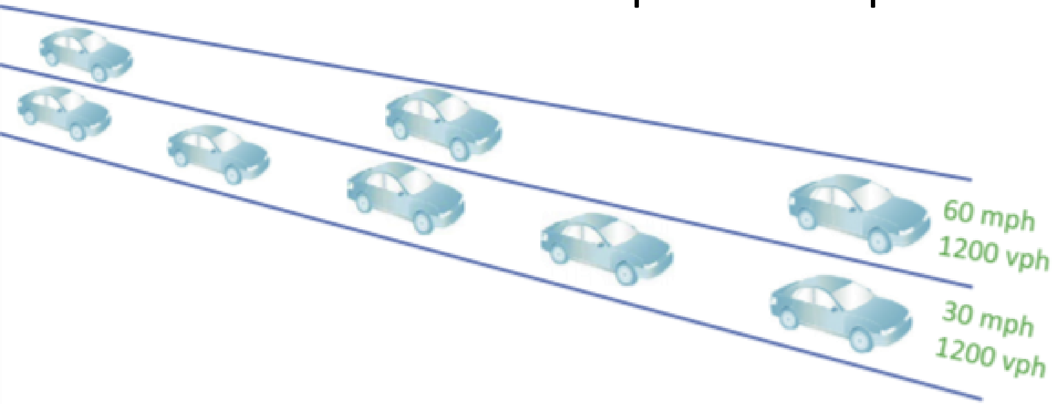
Example 2

Consider two lanes of traffic which are perfectly controlled so that there are only two streams of traffic:

- fast vehicles all travel at 60 miles per hour in the inner lane
- slow vehicles all move at 30 miles per hour in the outer lane.

Traffic flow in each lane is 1200 vehicles per hour, and lane change is prohibited.

What is the time-mean speed and space-mean speed of traffic in both lanes?



Time-Mean Speed and Space-Mean Speed

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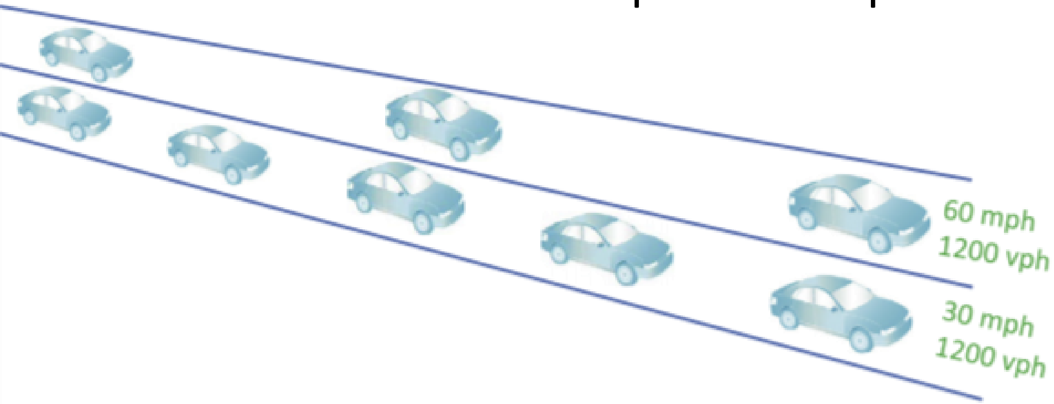

In 1 mile of the road, one observes a total of 60 vehicles, of which 20 vehicles are in the inner lane (1200 vehicles per hour/60 miles per hour) and 40 vehicles in the outer lane (1200 vehicles per hour/30 miles per hour). Therefore, space-mean speed is determined as:

$$v_s = \frac{20 \times 60\,\mathrm{mi/h} + 40 \times 30\,\mathrm{mi/h}}{60} = 40\,\mathrm{mi/h}.$$

Time-Mean Speed and Space-Mean Speed

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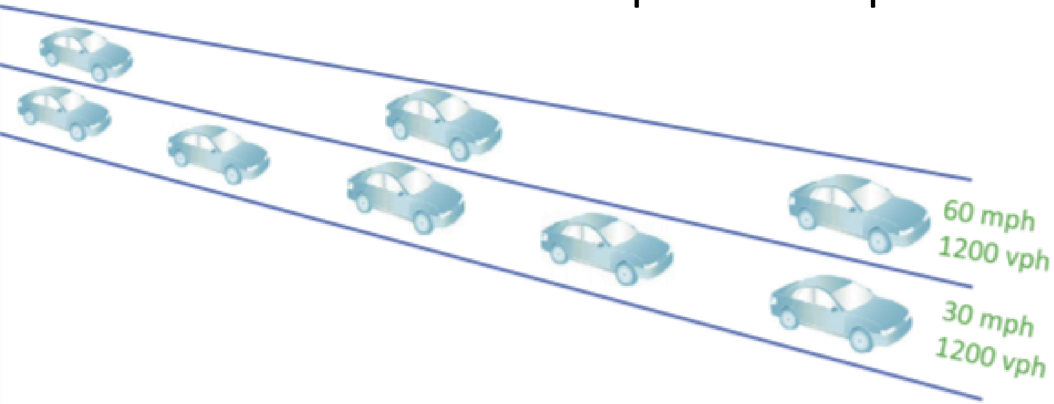


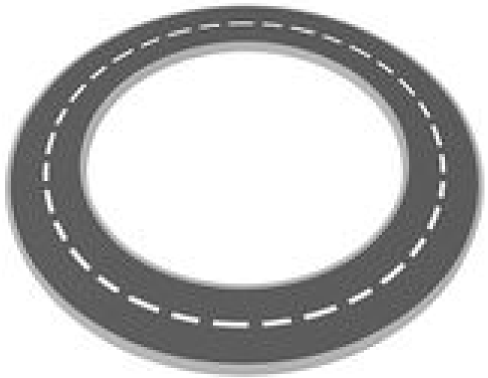
For time-mean speed, one must imagine a hypothetical observer standing at the roadside watching vehicles passing in front of him. As a result, the observer records 2400 vehicles in 1 h, of which 1200 vehicles are in the inner lane and 1200 vehicles are in the outer lane. Hence, by definition, time-mean speed is

$$v_t = \frac{1200 \times 60\,\mathrm{mi/h} + 1200 \times 30\,\mathrm{mi/h}}{2400} = 45\,\mathrm{mi/h}$$

Time-Mean Speed and Space-Mean Speed

Example 3

- Consider a circular road track with a $2km$ length.
- 3 vehicles are spotted with speeds respectively at 100, 120 and 140 km/h.
- Determine the density, hourly flow, average spatial and temporal velocities and check the Wardrop relation.



- Density $k = \frac{3}{2}\frac{veh}{km}$

- At one point on the track, for one hour, the vehicle traveling at 100 km/h will pass 50 times, the second traveling at 120 km / h 60 times and the third 70 times. Therefore:
  - $q = 50 + 60 + 70 = 180\ veh/h$
  - $v_s = \frac{100+120+140}{3} = 120\ km/h$, also $v_s = \frac{180}{\frac{50}{100}+\frac{60}{120}+\frac{70}{140}} = 120\ km/h$
  - $v_t = \frac{50\times100 + 60\times120 + 70\times140}{180} = 122\,\frac{km}{h}$

- Note that $v_s < v_t$

Levels of description – Microscopic vs Macroscopic

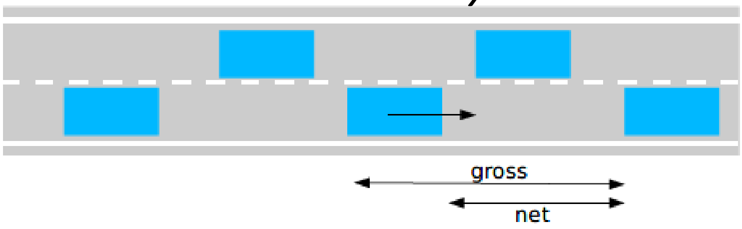
Microscopic

- In a microscopic traffic description, the vehicle-driver combinations (often referred to as "vehicle") are described individually.

- Full information of a vehicle is given in its trajectory, i.e. the specification of the position of the vehicle at all times.

- Basic variables in the microscopic representation are speed, headway (time headway), and space headway (gap).

Headway



Macroscopic

- In a macroscopic traffic description, one describes for each road section the aggregated variables.
- Density k, flow q and average speed u are the basic variables in the macroscopic representation of traffic.

Microscopic – Macroscopic Links

Flow and Headway:

Flow q is the reciprocal of average headway h:

$$q = \frac{N}{T} \qquad T = \sum_{i=1}^{n} h_i \qquad \longrightarrow \qquad q = \frac{N}{\sum_{i=1}^{n} h_i} = \frac{1}{\frac{1}{N}\sum_{i=1}^{n} h_i} = \frac{1}{h}$$

For example, a flow of 1200 vehicles per hour suggests an average headway of:

$$\frac{1}{1200 \text{ vehicles per hour}} = \frac{3600 \text{ s/h}}{1200 \text{ vehicles per hour}} = 3 \text{ s.}$$

Density and Spacing:

Therefore, density k is the reciprocal of average spacing s:

$$k = \frac{N}{L} \qquad L = \sum_{i=1}^{n} s_i \qquad \longrightarrow \qquad k = \frac{N}{\sum_{i=1}^{n} s_i} = \frac{1}{\frac{1}{N}\sum_{i=1}^{n} s_i} = \frac{1}{s}$$

For example, a density of 25 vehicles per kilometer suggests an average spacing of:

$$\frac{1}{25 \text{ vehicles per kilometer}} = \frac{1000 \text{ m/km}}{25 \text{ vehicles per kilometer}} = 40\text{m}$$

Microscopic – Macroscopic Links

Occupancy and Density

- The use of one loop has brought about the introduction of the characteristic occupancy rate.
- A vehicle passing over a loop temporarily 'occupies' it, approximately from the moment the front of the car is at the beginning of the loop until its rear is at the end of the loop.
- The approximately equal sign is based on the assumption of uniform vehicle length, $l_i = l$.
- Occupancy o is proportional to density k, and the proportion coefficient c_k is the sum of loop width d and uniform vehicle length l.
- If a mix of passenger cars and trucks is present, then the meaning of c_k is less obvious.
- If one uses two loops it is better to calculate density k from flow q and the harmonic mean of the local speeds.

$$
\begin{aligned}
o &= \frac{1}{T}\sum_{i=1}^{N}\tau_i = \frac{1}{T}\sum_{i=1}^{N}\frac{d+l_i}{\dot{x}_i} \approx \frac{d+l}{T}\sum_{i=1}^{N}\frac{1}{\dot{x}_i} \\
&= (d+l)\frac{1}{T}\sum_{i=1}^{N}\frac{1}{\dot{x}_i} = (d+l)\left(\frac{N}{T}\right)\left(\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\dot{x}_i}\right) \\
&= (d+l)q\frac{1}{v_s} = (d+l)k = c_k k
\end{aligned}
$$

Homogeneous vs Stationary Flow Conditions

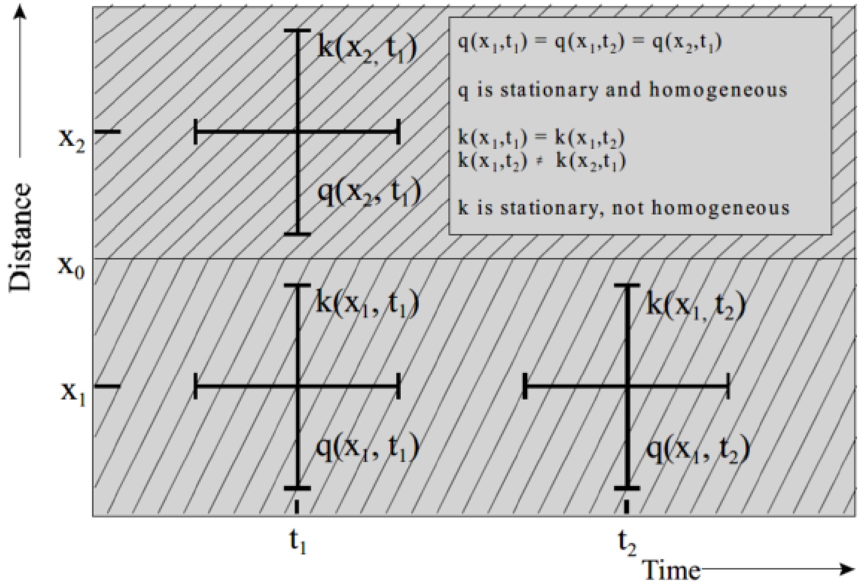
- A traffic flow is composed of vehicles. Movements of different vehicles are a function of position and time (each vehicle has its own trajectory).

- The characteristics of a traffic flow, such as intensity, density, and mean speed, are an aggregation of characteristics of the individual vehicles and can consequently also be dependent on position and time.

- Consider a variable $z(x,t)$. We define this variable z to be:
  - Homogeneous, if $z(x,t) = z(t)$; i.e. the variable z does not depend on position.
  - Stationary, if $z(x,t) = z(x)$; i.e. the variable z is independent of time.

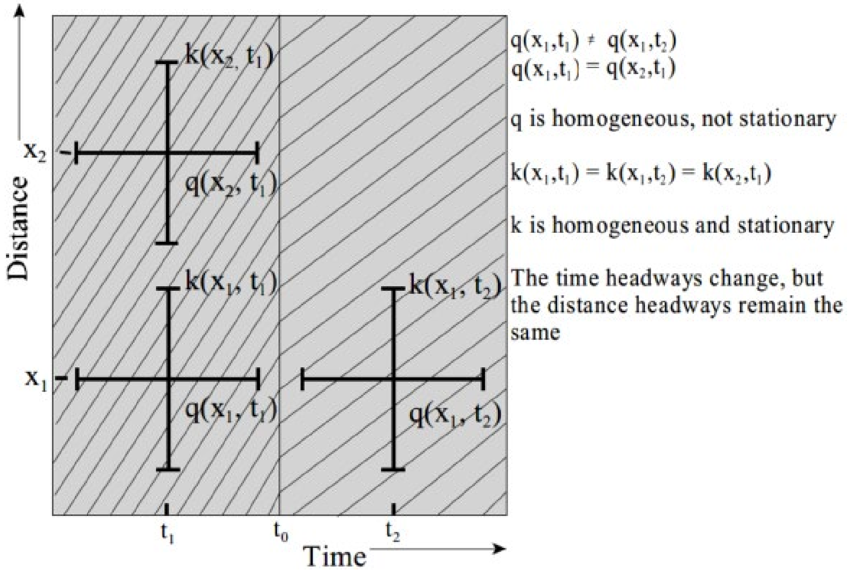
Homogeneous vs Stationary Flow Conditions

- At the spot $x = x_0$ the road profile changes drastically, and as a result all vehicles reduce their speed when passing x_0.
- In this case the distance headways change but the time headways remain the same. This means that flow q is stationary and homogeneous and density k is stationary but not homogeneous.



Effects of drastic change of road profile at position x_0

- At the moment t_0, the weather changes drastically.
- All vehicles reduce their speed at that moment.
- Then the time headways change but the distance headways remain the same.
- This means that flow q is homogeneous but not stationary and density k is stationary and homogeneous.



Effects of substational weather change at moment t_0

Microscopic – Macroscopic Links

Flow, Speed, and Density

- Considering a traffic flow in a stationary and homogeneous ‘state’, the following relation, referred to as the “fundamental relation”, is valid:

$$q = k \times v_s$$

- Flow q is the product of density k and space-mean speed v_s.
- It is important to understand that the three variables vary simultaneously.
- For example, when the average speed increases, the space headway (gap) increases and therefore the density decreases.

Time-Mean Speed and Space-Mean Speed

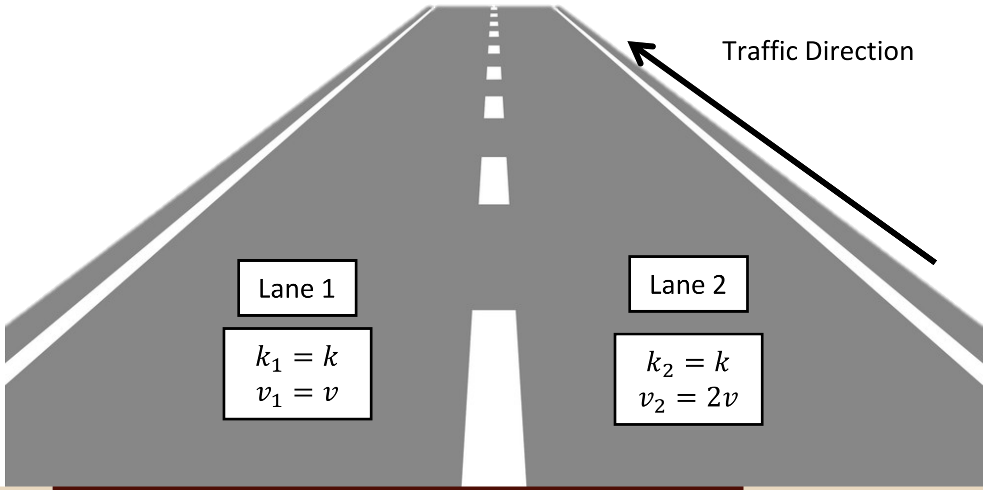
Example 4

- Consider the case shown in figure below. What are the time-mean speed and space-mean speed of traffic?

$$Q = q_1 + q_2 = 3kv \qquad K = 2k \qquad \rightarrow \qquad U = 3v/2$$

$$v_s = \frac{1}{K\Delta x}\sum_{i=1}^{K\Delta x} v_i = \frac{k_1 \Delta x v_1 + k_2 \Delta x v_2}{\Delta x (k_1 + k_2)} = \frac{kv + 2kv}{k + 2k} = \frac{3}{2} v = U$$

$$v_t = \frac{1}{Q\Delta t}\sum_{i=1}^{Q\Delta t} v_i = \frac{q_1 \Delta t v_1 + q_2 \Delta t v_2}{\Delta t (q_1 + q_2)} = \frac{kv \times v + 2kv \times 2v}{3kv} = \frac{5}{3} v > U$$



Generalized Definition

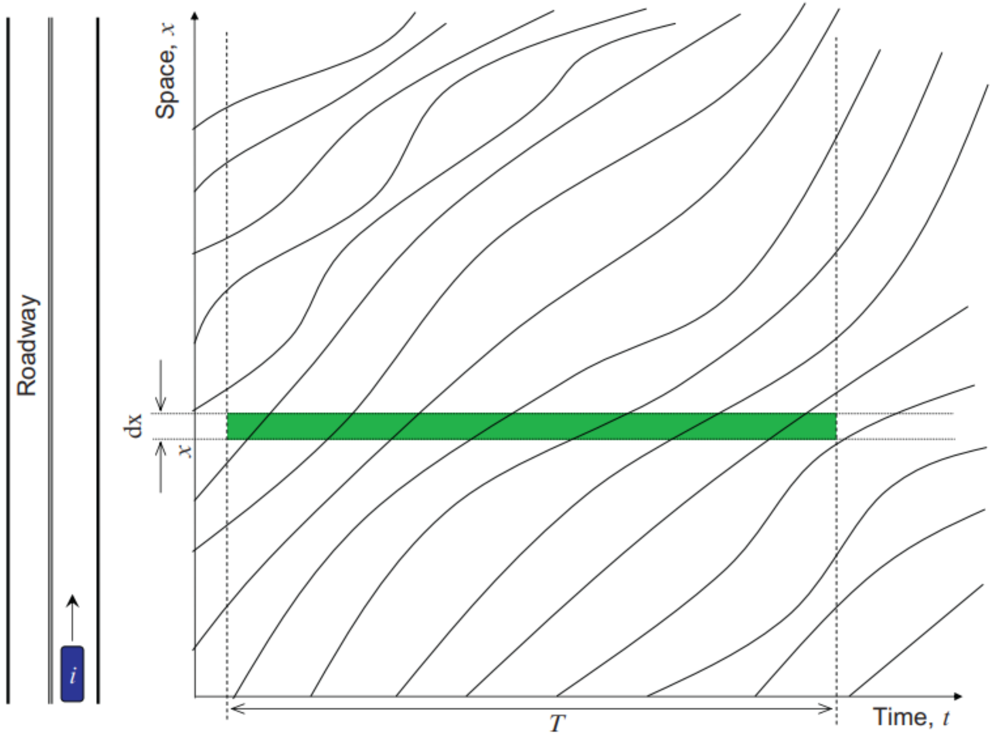
$$q = \frac{N}{T} = \frac{N \times \mathrm{d}x}{T \times \mathrm{d}x}$$

- dx denotes an infinitesimal distance.
- The physical meaning of the numerator is the sum of the distances traversed by all vehicles in area A during time period T

$$\underbrace{\mathrm{d}(A)}_{\textit{Total travelled distance}} = N \times \mathrm{d}x = \sum_{i=1}^{N} \Delta x_i$$

- The denominator simply means the area of the time-space rectangle A bounded by T and dx.
- Hence, the definition of q can alternatively be expressed as the total distance traversed by all vehicles within A divided by the area of A.

$$q = \frac{\mathrm{d}(A)}{|A|}$$



Time-space diagram

Generalized Definition

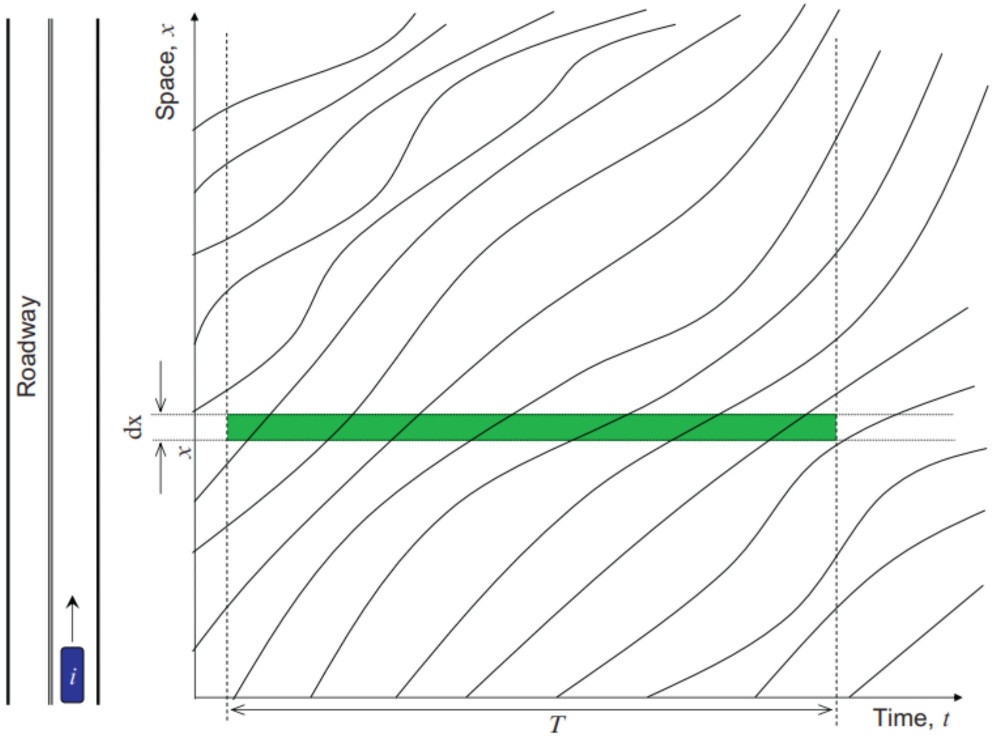
- According to the HCM, the mean speed of vehicles is the total distance traveled by all vehicles divided by the total travel time of these vehicles.
- The total distance traveled by all vehicles within rectangle A is $\mathrm{d}(A) = N \times \mathrm{d}x$
- The total time spent by all vehicles within A is

$$t(A) = \sum_{i=1}^{N} \frac{\mathrm{d}x}{\dot{x}_i}$$

- Therefore:

$$v = \frac{\mathrm{d}(A)}{t(A)} = \frac{N \times \mathrm{d}x}{\sum_{i=1}^{N} \frac{\mathrm{d}x}{\dot{x}_i}} = \frac{N \times \mathrm{d}x}{\mathrm{d}x \times \sum_{i=1}^{N} \frac{1}{\dot{x}_i}} = \frac{1}{\frac{1}{N} \sum_{i=1}^{N} \frac{1}{\dot{x}_i}}$$

- This is the harmonic mean, which corresponds to the space mean speed presented before.



Time-space diagram

Microscopic – Macroscopic Links

$$k = \frac{N}{L} = \frac{N \times \mathrm{d}t}{L \times \mathrm{d}t}$$

- dt denotes an infinitesimal duration.
- L and dt define a time-space rectangle A.
- The numerator is the sum of the times spent by all vehicles within A, $t(A)$
- The denominator is the area of the rectangle, $|A|$:

$$t(A) = N \times \mathrm{d}t = \sum_{i=1}^{N} \mathrm{d}t_i \quad \text{, and} \quad |A| = L \times \mathrm{d}t$$

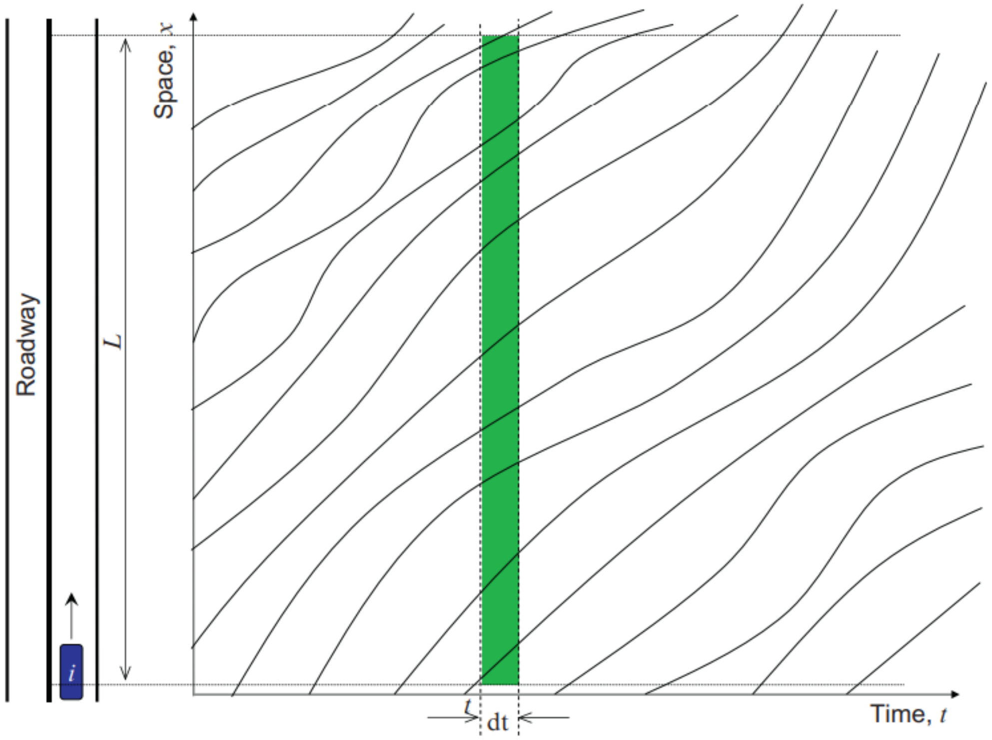
- Therefore:

$$k = \frac{t(A)}{|A|}$$

- The total distance traveled by all vehicles within A: $\sum_{i=1}^{N} \mathrm{d}t \times v_i$
- Hence, the mean speed

$$v = \frac{\sum_{i=1}^{N} \mathrm{d}t \times v_i}{N \times \mathrm{d}t} = \frac{1}{N} \sum_{i=1}^{N} \dot{x}_i.$$

- This is the arithmetic mean, which corresponds to the time mean speed, presented before.



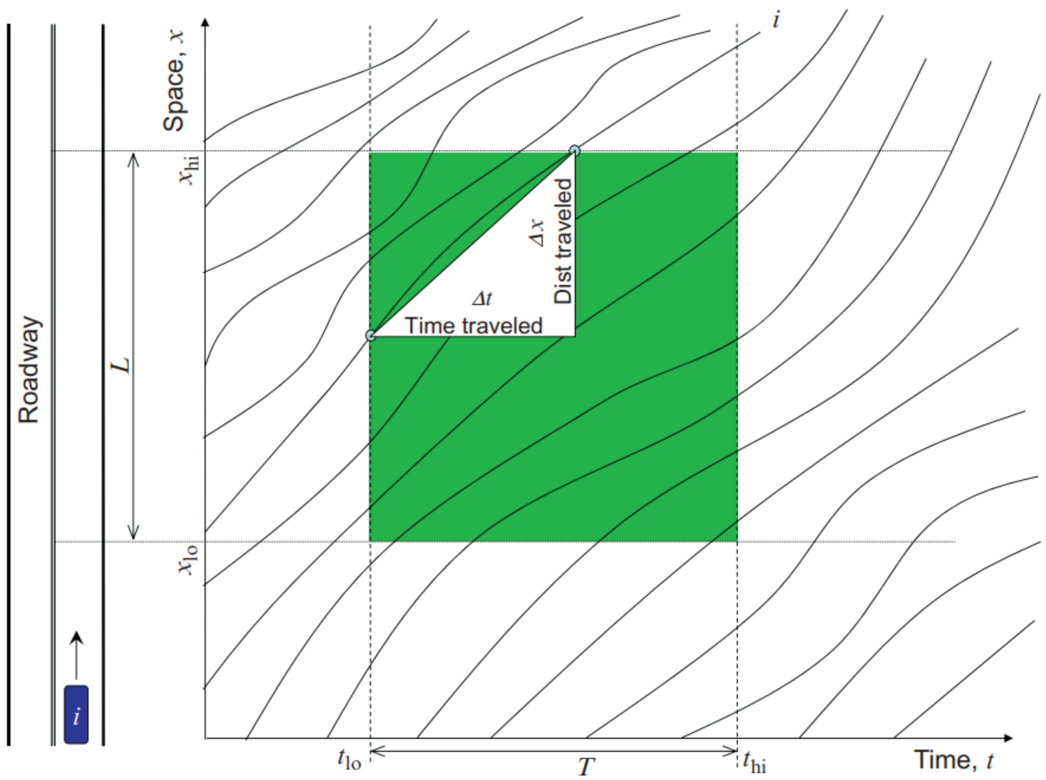
Time-space diagram

Microscopic – Macroscopic Links

- The above discussion suggests that a time-space rectangle may serve as the common ground to unify the definition of flow q, mean speed v, and density k.
- A general time-space rectangle A covering length L (bounded by upstream location x_{lo} and downstream locationx_{hi}) and duration T (bounded by instants $t_{lo}$and t_{hi}).
- Based on A, the three traffic flow characteristics can be defined as follows:

$$q(A) = \frac{d(A)}{|A|} \qquad k(A) = \frac{t(A)}{|A|} \qquad v(A) = \frac{d(A)}{t(A)}$$

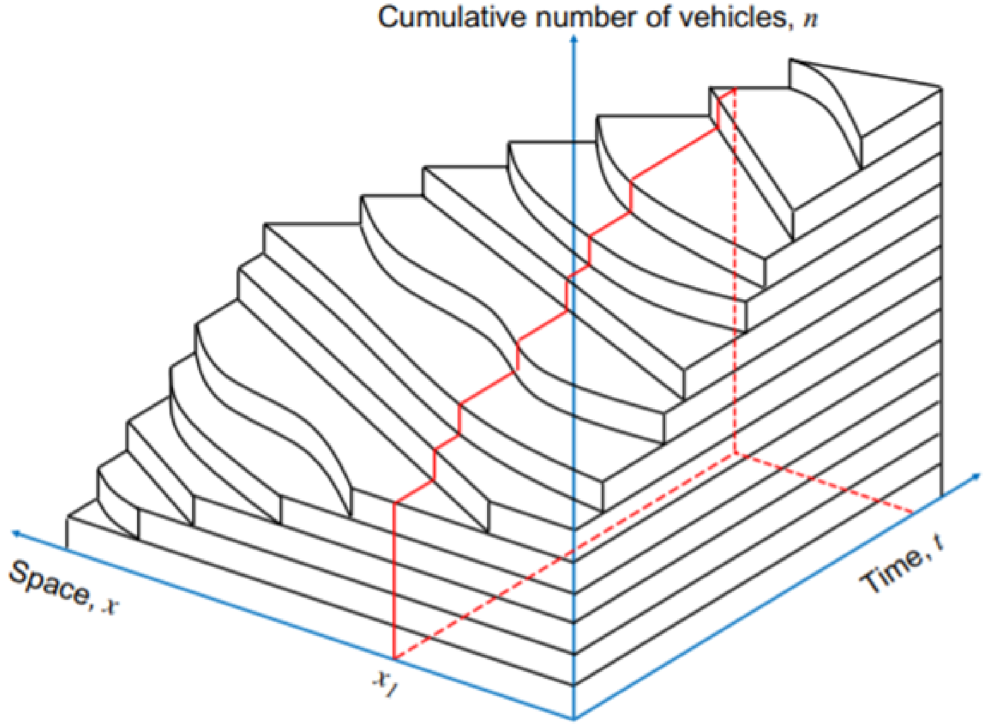
- The generalized definition of flow, mean speed, and density based on a time-space region was originally proposed by Edie



Time-space diagram

Three-Dimensional Representation

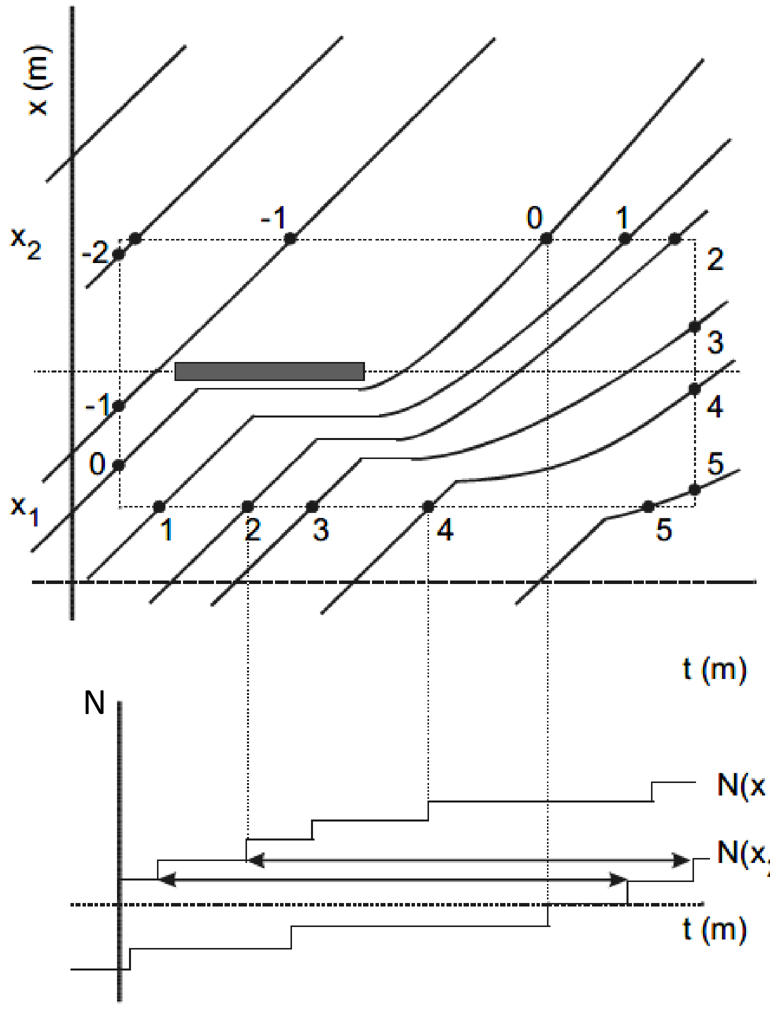
- A time-space diagram is a two-dimensional representation, and it can be more informative if we adopt a three-dimensional perspective.
- Suppose that vehicles are numbered cumulatively (i.e., $ID = 1, 2, 3, \ldots$) in the order they appear on the road.
- In a three-dimensional representation, each vehicle is elevated along the third dimension to the height corresponding to the vehicle's ID (i.e., vehicle 1 raised to height 1, vehicle 2 raised to 2, and so on).
- Let us call the third dimension the cumulative number of vehicles (N) and denote the surface that passes these elevated vehicle trajectories $N(x, t)$.



Three-dimensional representation including the cumulative number of vehicles

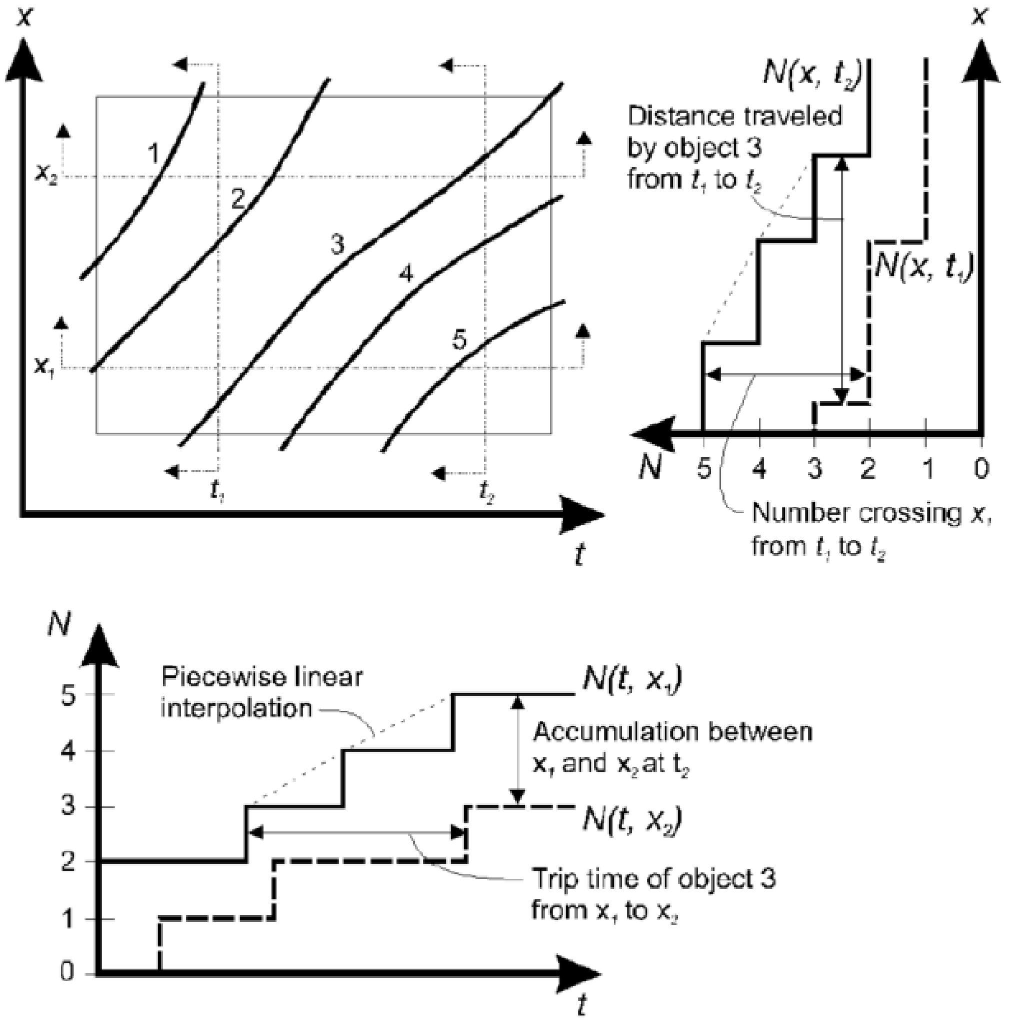
Three-Dimensional Representation

- A cumulative plot of vehicles is a function $N(x, t)$ that represents the number of vehicles that has passed a cross section x from an arbitrary starting moment.
- The example shows vehicles being stopped at a controlled intersection.
- Figure shows a couple of vehicle trajectories which are numbered in increasing order.
- The cumulative vehicle plots $N(x_1, t)$ and $N(x_2, t)$ are determined for two cross-sections x_1 and x_2 as a function of time.
- The arrows in the lower figure indicate the travel times of vehicles 1 and 2, including their delay due to the controlled intersection.



Two-dimensional representation including the cumulative number of vehicles

Three-Dimensional Representation



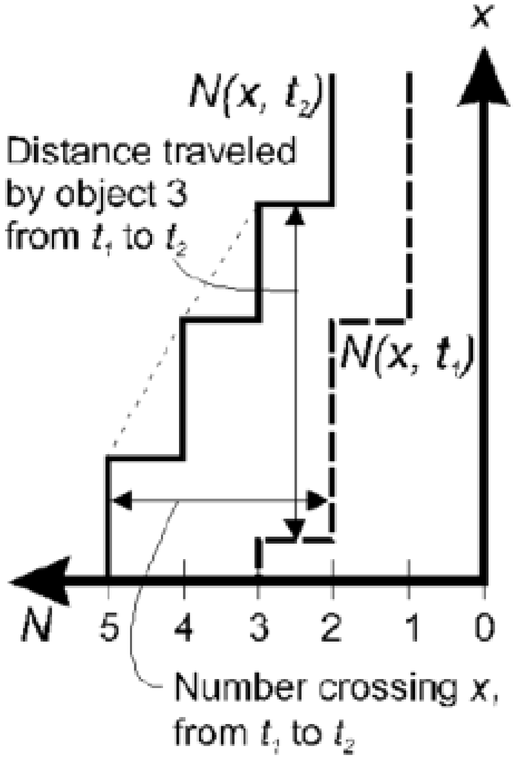
Cumulative plots, flow, density and speed

- The flow measured at a certain cross-section x during period t_1 to t_2 equals:

$$q(x, t_1 \text{ to } t_2) = \frac{N(x,t_2) - N(x,t_1)}{t_2 - t_1}$$

- Since the vehicle are indivisible objects, $N(x,t)$ is a step function.
- However, in most practical problems it is not needed to have solutions with an accuracy of one vehicle.
- This allows us to approximate the step function $N(x,t)$ by a smooth function $\tilde{N}(x,t)$ that is continuous and can be differentiated.
- Taking the limit for $(t_2 - t_1) \rightarrow 0$ results in: $$q(x,t) = \frac{\partial \tilde{N}(x,t)}{\partial t}$$

- As the position x is a continuous variable, we have now introduced a concept of a local and instantaneous flow.



Cumulative flow function $N(x,t)$

Cumulative plots, flow, density and speed

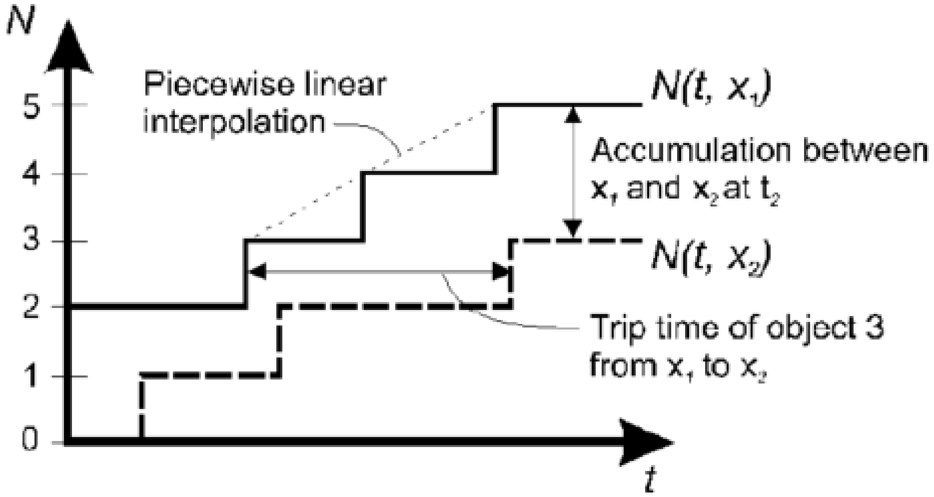
- Now consider two cumulative plots at position x_1 and x_2. Then at time instant t, the average density is:

$$k(x_1 \text{ to } x_2, t) = \frac{N(x_1, t) - N(x_2, t)}{x_2 - x_1}$$

- Taking the limit for $(x_2 - x_1) \rightarrow 0$ results in:

$$k(x, t) = -\frac{\partial \tilde{N}(x, t)}{\partial x}$$

- As the position x is a continuous variable, we have now introduced a concept of a local and instantaneous density.



Cumulative flow function
$N(x, t)$

Cumulative plots, flow, density and speed

- Finally, we can define the mean speed at the spot x and instant t as:

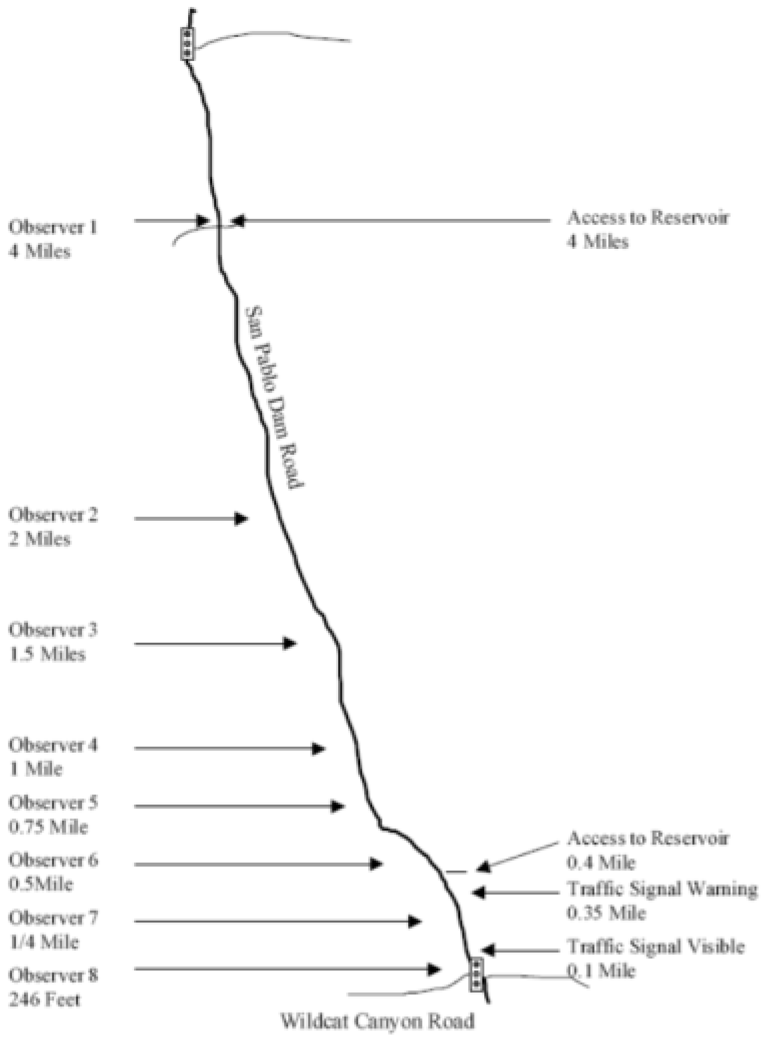
$$u(x,t) = q(x,t)/k(x,t)$$

- In this way, all three main macroscopic characteristics of a traffic flow can be handled as continuous functions of the position x and time t.

- This property will be very useful when considering macroscopic traffic flow models.

Cumulative plots – Example

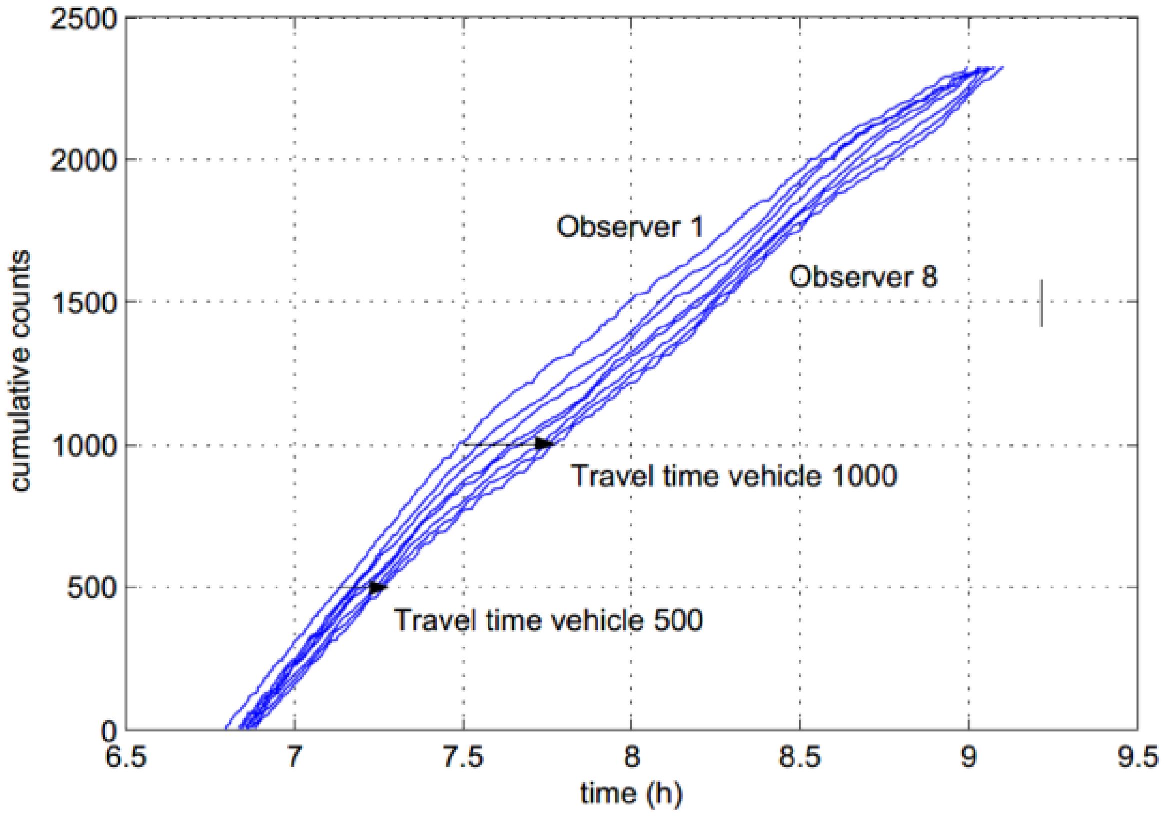
- The data collection site is a two-lane rural highway in California, with a controlled intersection at its end (Wildcat Canyon Road).
- The site has limited overtaking opportunities. There are no entry and exit points.
- At 8 observation points, passage times of vehicles have been collected and stored.
- Note that at the observer 8 site, a traffic responsive traffic signal is present.



Data collection site

Cumulative plots – Example

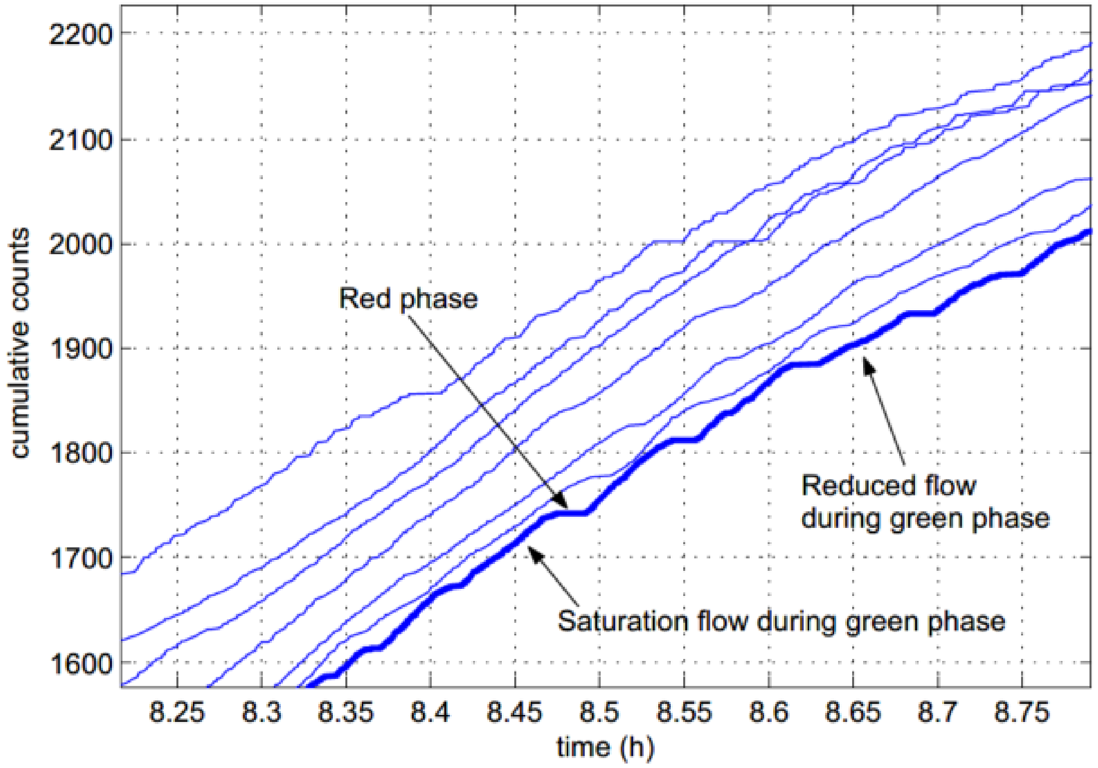
- Using the collected data, cumulative curves have been determined.
- In the same figure, travel times are also indicated.
- It is illustrated how vehicle 1000 experiences a higher travel time than say vehicle 500.
- Notice how the gradient of the cumulative curve of observer 8 decreases after some time (approximately at 7:20).
- The reason for this is the growth in the conflicting traffic streams at Wildcat Canyon Road.



Cumulative counts

Cumulative plots – Example

- The cumulative curves in more detail
- We can observe clearly the flows during the green phase and the zero flow during the red phase.
- Reduction in the average flow is caused by a reduction in the length of the green phase as a result of the conflicting streams. (More time dedicated to the conflicting streams)
- Congestion from downstream spills back over the intersection.



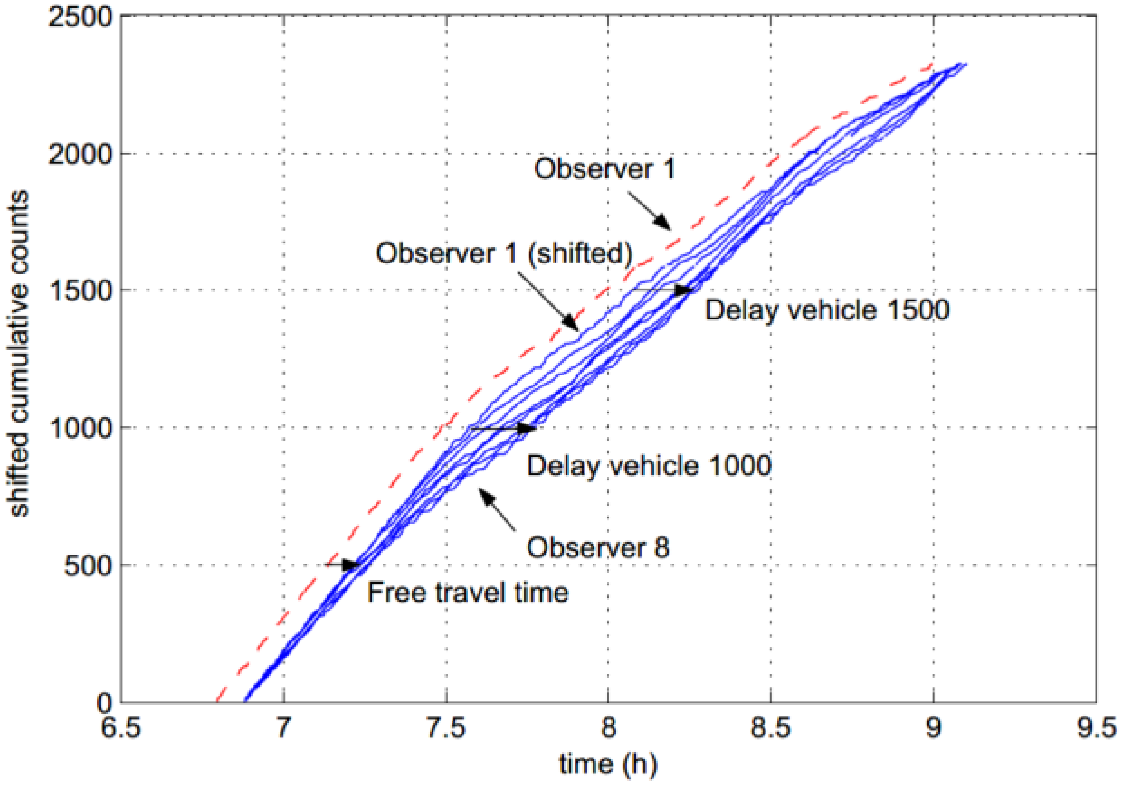
Cumulative counts

Cumulative plots – Example

- Shifted cumulative curves are plotted.

- What for?

- The curves are shifted along the time-axis by the free travel time. The resulting curves can be used to determine the delays per vehicle, as well as the total and average delays.



Shifted cumulative curves

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Thank you
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