Machines et entraînements électriques
ELE8401

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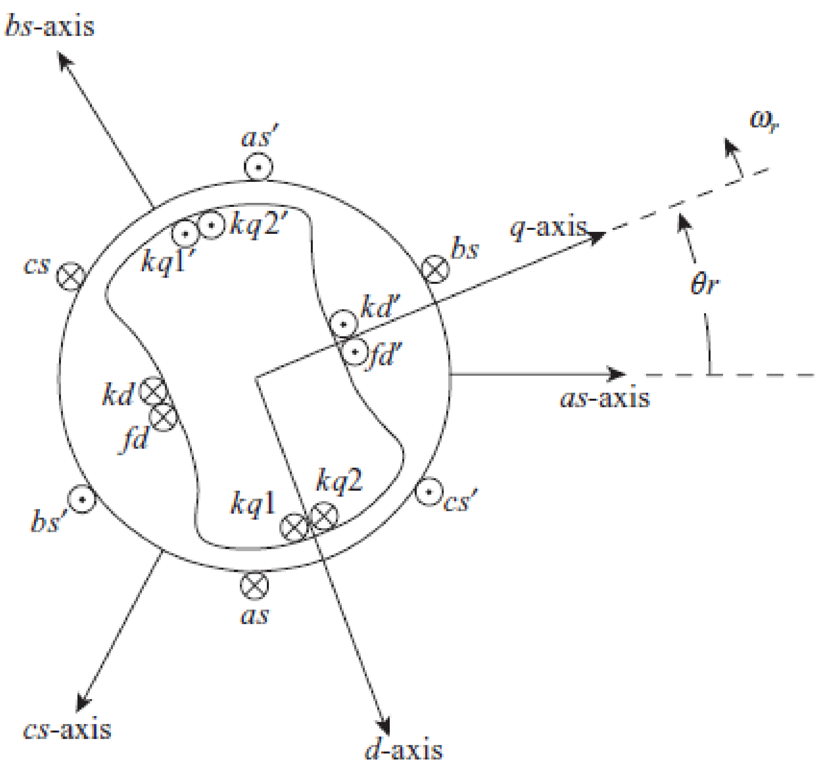
Transient Modeling of Synchronous Machines

Voltage Equations in Machine Variables

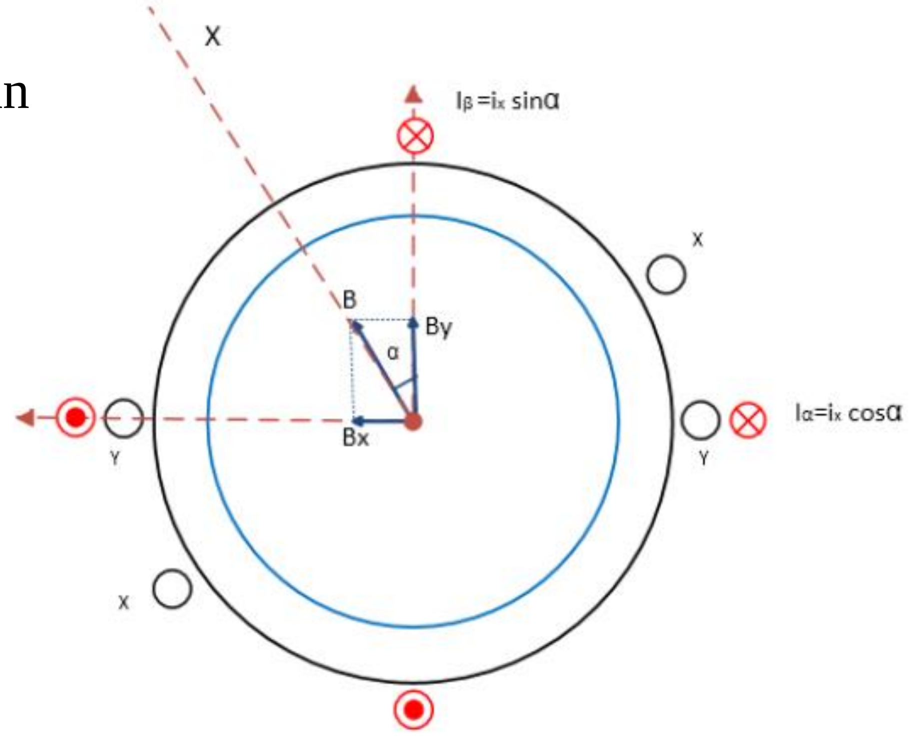
All stator and rotor windings are sinusoidally distributed.
field winding (*fd* winding) with turns N_{fd} and resistance r_{fd}
damping winding (*kd* winding) with turns N_{kd} and resistance r_{kd}
damping winding (*kq* winding) with turns N_{kq} and resistance r_{kq}
Note: Damping windings model the eddy currents



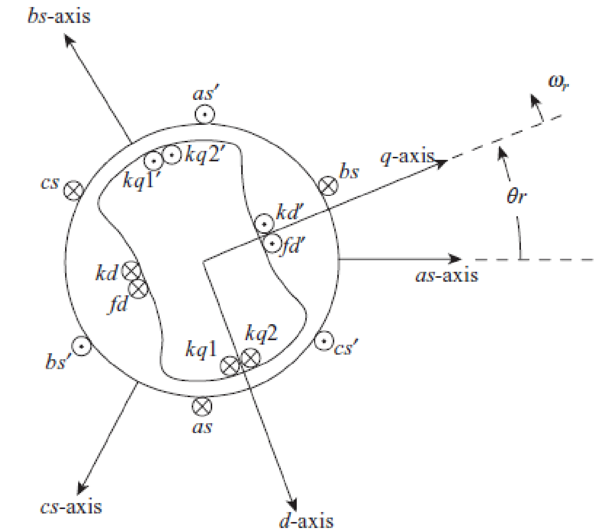
Two-pole, three-phase, wye-connected salient-pole synchronous machine

Transient Modeling of Synchronous Machines

i_x is an eddy current whose winding can be modeled by two windings along d-axis and q-axis.



Transient Modeling of Synchronous Machines



Two-pole, three-phase, wye-connected salient-pole synchronous machine

Voltage Equations in Machine Variables

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

$$\mathbf{v}_{qdr} = \mathbf{r}_r \mathbf{i}_{qdr} + p\boldsymbol{\lambda}_{qdr}$$

$$(\mathbf{f}_{abcs})^T = [f_{as} \quad f_{bs} \quad f_{cs}]$$

$$(\mathbf{f}_{qdr})^T = [f_{kq1} \quad f_{kq2} \quad f_{fd} \quad f_{kd}]$$

$$\mathbf{r}_s = \mathrm{diag}[r_s \quad r_s \quad r_s]$$

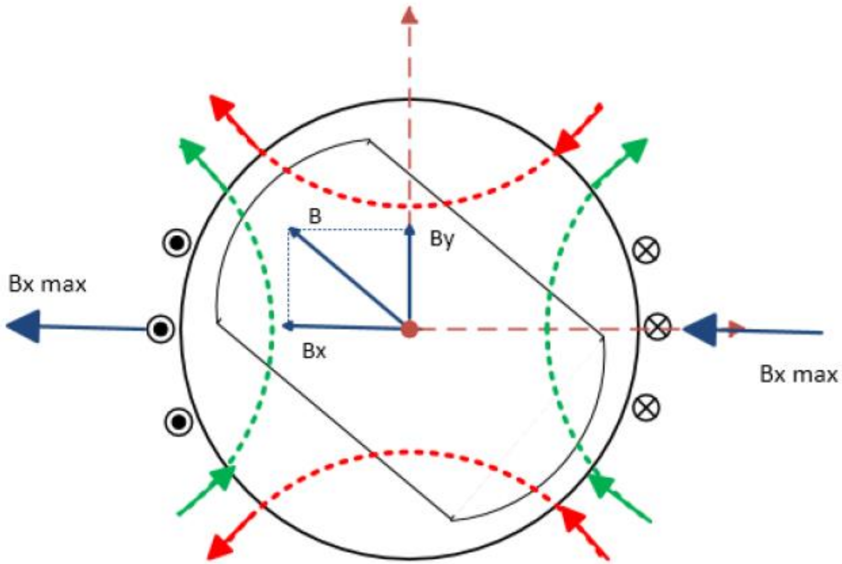
$$\mathbf{r}_r = \mathrm{diag}[r_{kq1} \quad r_{kq2} \quad r_{fd} \quad r_{kd}]$$

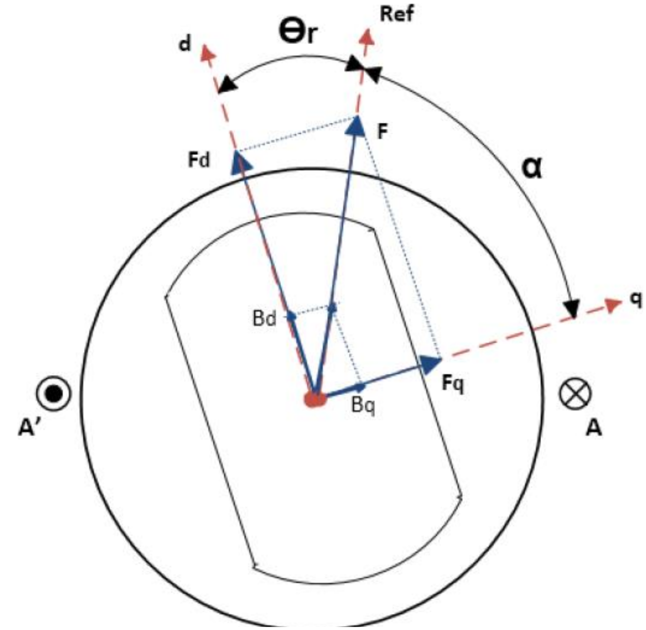
Flux linkage equations for a linear magnetic system

$$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ (\mathbf{L}_{sr})^T & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}_{qdr} \end{bmatrix}$$

Transient Modeling of Synchronous Machines

Stator self inductance





$$F = N_s i_A \;\rightarrow\; \begin{cases} F_d = F\cos\theta_r \;\Rightarrow\; B_d = K_{Bd}F_d \Rightarrow B_{dy} = B_d\cos\theta_r \Rightarrow \lambda_d = K_{\lambda_s} N_s B_{dy} \\ F_q = F\sin\theta_r \;\Rightarrow B_q = K_{Bq}F_q \Rightarrow B_{qy} = B_q\sin\theta_r \Rightarrow \lambda_q = K_{\lambda_s} N_s B_{qy} \end{cases}$$

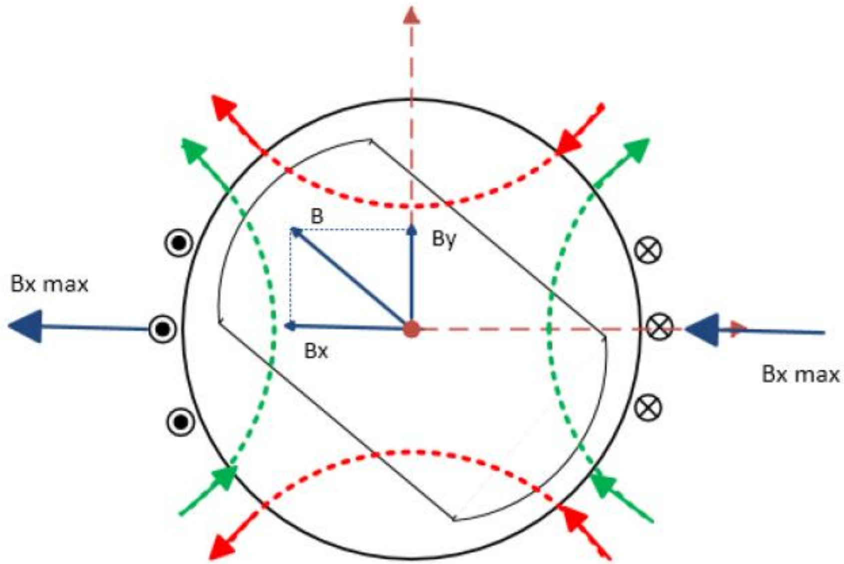
$$\Rightarrow \begin{cases} \lambda_d = K_{\lambda_s} N_s (K_{Bd} N_s i_A \cos\theta_r)\cos\theta_r \\ \lambda_q = K_{\lambda_s} N_s (K_{Bq} N_s i_A \sin\theta_r)\sin\theta_r \end{cases}$$

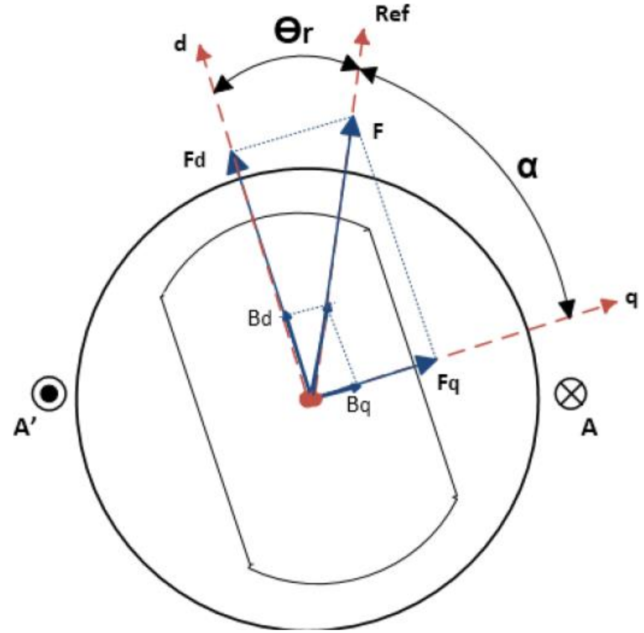
Note:
1- only the y-component accounts for the coupling

2- We consider the Ref axis along y-axis

Transient Modeling of Synchronous Machines

Stator self inductance





$$\Rightarrow \lambda_{A\acute{A}} = \lambda_q + \lambda_d = K_{\lambda_s} {N_s}^2 i_A \left(K_{Bd} \cos^2\theta_r + K_{Bq} \sin^2\theta_r\right) = K_{\lambda_s} {N_s}^2 i_A \left(K_{Bd} \frac{cos\, 2\theta_r + 1}{2} + K_{Bq} \frac{1 - cos\, 2\theta_r}{2}\right)$$

$$= K_{\lambda_s} {N_s}^2 i_A \left(\frac{K_{Bd}+K_{Bq}}{2} + \frac{K_{Bd}-K_{Bq}}{2} cos\, 2\theta_r\right) \Rightarrow L_{m_{A\acute{A}}} = \frac{\lambda_{A\acute{A}}}{i_A} = K_{\lambda_s} {N_s}^2 \left(\frac{K_{Bd}+K_{Bq}}{2} + \frac{K_{Bd}-K_{Bq}}{2} cos\, 2\theta_r\right)$$

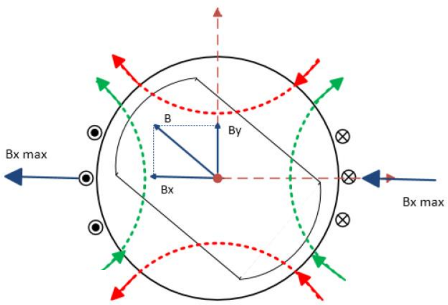
$K_{Bd} > K_{Bq}$

$$L_{A\acute{A}}(\theta_r) = L_{m_{A\acute{A}}}(\theta_r) + l_s \Rightarrow L_{A\acute{A}}(\theta_r) = M_s + L_{ms}\, cos\, 2\theta_r$$

$$L_{B\acute{B}}(\theta_r) = M_s + L_{ms}\, cos\, 2\left(\theta_r - \frac{2\pi}{3}\right) \quad , \quad L_{C\acute{C}}(\theta_r) = M_s + L_{ms}\, cos\, 2\left(\theta_r + \frac{2\pi}{3}\right)$$

Transient Modeling of Synchronous Machines

Stator mutual inductance



$$L_{AB}(\theta_r) = L_{BA}(\theta_r) = -L_A - L_B\, cos\, 2\left(\theta_r + \frac{\pi}{6}\right)$$

$$L_{BC}(\theta_r) = L_{CB}(\theta_r) = -L_A - L_B\, cos\, 2\left(\theta_r + \frac{\pi}{6} - \frac{2\pi}{3}\right) = -L_A - L_B\, cos\, 2\left(\theta_r - \frac{\pi}{2}\right)$$

$$L_{CA}(\theta_r) = L_{AC}(\theta_r) = -L_A - L_B\, cos\, 2\left(\theta_r + \frac{\pi}{6} + \frac{2\pi}{3}\right) = -L_A - L_B\, cos\, 2\left(\theta_r - \frac{\pi}{2}\right)$$

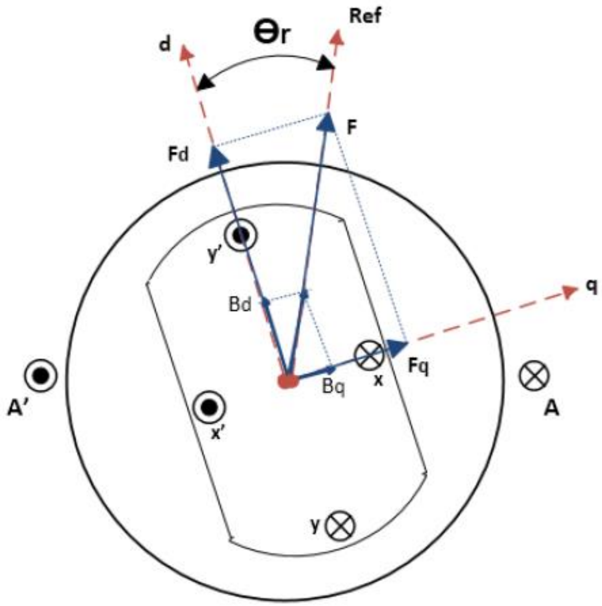
If we write self inductance as

$$L_{mA\acute{A}}(\theta_r) = \underbrace{\underbrace{K_{\lambda_s} N_s^2 \frac{K_{Bd}+K_{Bq}}{2}}_{} + \underbrace{K_{\lambda_s} N_s^2 \frac{K_{Bd}-K_{Bq}}{2} cos\, 2\theta_r}_{L_m} + \underbrace{l_s}_{}}_{L_s}$$

Then mutual inductance can be written in a similar way

$$L_{AB}(\theta_r) = -\underbrace{K_{\lambda_s} N_s^2 \frac{K_{Bd}+K_{Bq}}{4}}_{\frac{L_s - l_s}{2} \triangleq M_s} - \underbrace{K_{\lambda_s} N_s^2 \frac{K_{Bd}-K_{Bq}}{2} cos\, 2\left(\theta_r + \frac{\pi}{6}\right)}_{L_m}$$

Transient Modeling of Synchronous Machines

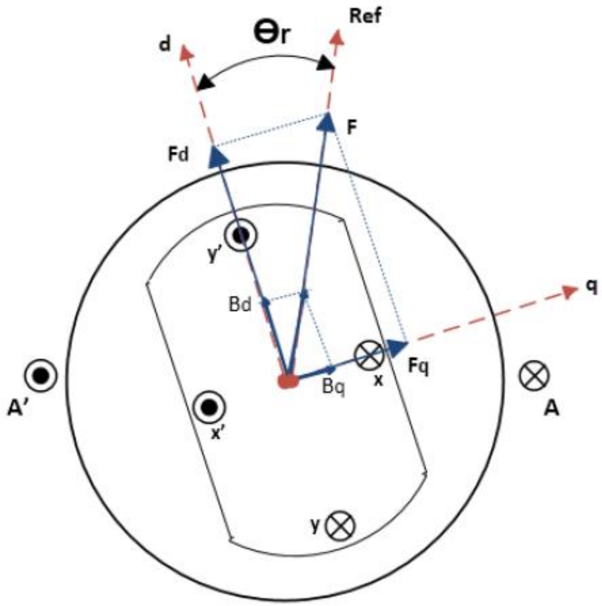


stator-rotor windings mutual inductances

$$F = Ni \;\begin{cases} F_d = N_s i\cos\theta_r \;\Rightarrow\; B_d = K_{Bd} N_s i\cos\theta_r \Rightarrow \lambda_{xA} = K_{\lambda_x} N_x B_d = K_{\lambda_x} B_d N_s N_x i\cos\theta_r \\ F_q = N_s i\sin\theta_r \;\Rightarrow B_q = K_{Bq} N_s i\sin\theta_r \;\Rightarrow \lambda_{yA} = K_{\lambda_y} N_s B_q = K_{\lambda_y} B_q N_s N_y i\sin\theta_r \end{cases}$$

$$\Rightarrow \begin{cases} L_{xA}(\theta_r) = \dfrac{\lambda_{xA}}{i_A} = M_{xA}\cos\theta_r \qquad , x = F, D_1, D_2, \ldots, D_{n-1} \\ L_{yA}(\theta_r) = \dfrac{\lambda_{yA}}{i_A} = M_{yA}\sin\theta_r \qquad , y = D_1, D_2, \ldots, D_n \end{cases}$$

Transient Modeling of Synchronous Machines



stator-rotor windings mutual inductances

$$L_{xA}(\theta_r) = M_x \cos\theta_r$$

$$L_{xB}(\theta_r) = L_{xA}\left(\theta_r - \tfrac{2\pi}{3}\right) = M_x cos(\theta_r - \tfrac{2\pi}{3})$$

$$L_{xC}(\theta_r) = L_{xA}\left(\theta_r + \tfrac{2\pi}{3}\right) = M_x cos(\theta_r + \tfrac{2\pi}{3})$$

$$L_{yA}(\theta_r) = M_x \sin\theta_r$$

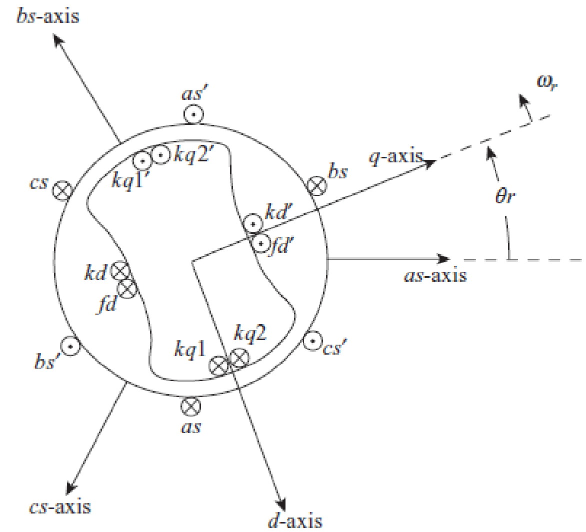
$$L_{yB}(\theta_r) = L_{yA}\left(\theta_r - \tfrac{2\pi}{3}\right) = M_x \sin(\theta_r - \tfrac{2\pi}{3})$$

$$L_{yC}(\theta_r) = L_{yA}\left(\theta_r + \tfrac{2\pi}{3}\right) = M_x \sin(\theta_r + \tfrac{2\pi}{3})$$

Transient Modeling of Synchronous Machines

Voltage Equations in Machine Variables

$$\mathbf{L}_s = \begin{bmatrix} L_{ls} + L_A - L_B\cos 2\theta_r & -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_A - L_B\cos 2\left(\theta_r - \frac{2\pi}{3}\right) & -\frac{1}{2}L_A - L_B\cos 2(\theta_r + \pi) \\ -\frac{1}{2}L_A - L_B\cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B\cos 2(\theta_r + \pi) & L_{ls} + L_A - L_B\cos 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$



Two-pole, three-phase, wye-connected salient-pole synchronous machine

$$\mathbf{L}_{sr} = \begin{bmatrix} L_{skq1}\cos\theta_r & L_{skq2}\cos\theta_r & L_{sfd}\sin\theta_r & L_{skd}\sin\theta_r \\ L_{skq1}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{skq2}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{sfd}\sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{skd}\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{skq1}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{skq2}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{sfd}\sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{skd}\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

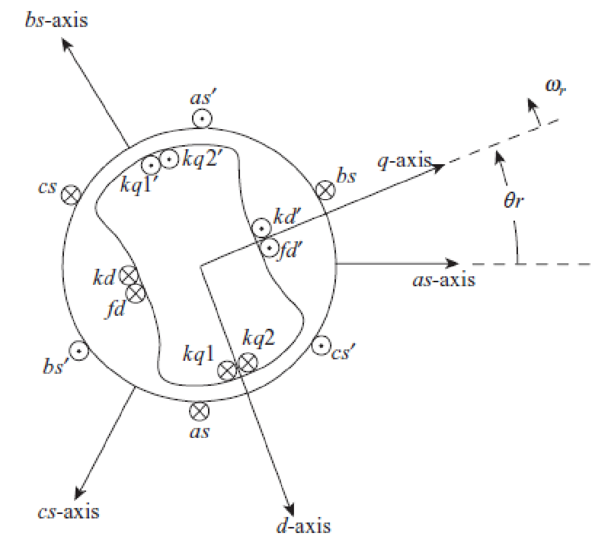
$L_A > L_B$ and L_B is zero for a round rotor machine

$$\mathbf{L}_r = \begin{bmatrix} L_{lkq1} + L_{mkq1} & L_{kq1kq2} & 0 & 0 \\ L_{kq1kq2} & L_{lkq2} + L_{mkq2} & 0 & 0 \\ 0 & 0 & L_{lfd} + L_{mfd} & L_{fdkd} \\ 0 & 0 & L_{fdkd} & L_{lkd} + L_{mkd} \end{bmatrix}$$

The magnetizing inductances are:

$$\boxed{\begin{aligned} L_{mq} &= \frac{3}{2}(L_A - L_B) \\ L_{md} &= \frac{3}{2}(L_A + L_B) \end{aligned}}$$

Transient Modeling of Synchronous Machines



Voltage Equations in Machine Variables

It can be shown that:

$$L_{skq1} = \left(\frac{N_{kq1}}{N_s}\right)\left(\frac{2}{3}\right)L_{mq}$$

$$L_{skq2} = \left(\frac{N_{kq2}}{N_s}\right)\left(\frac{2}{3}\right)L_{mq}$$

$$L_{sfd} = \left(\frac{N_{fd}}{N_s}\right)\left(\frac{2}{3}\right)L_{md}$$

$$L_{skd} = \left(\frac{N_{kd}}{N_s}\right)\left(\frac{2}{3}\right)L_{md}$$

$$L_{mkq1} = \left(\frac{N_{kq1}}{N_s}\right)^2\left(\frac{2}{3}\right)L_{mq}$$

$$L_{mkq2} = \left(\frac{N_{kq2}}{N_s}\right)^2\left(\frac{2}{3}\right)L_{mq}$$

$$L_{mfd} = \left(\frac{N_{fd}}{N_s}\right)^2\left(\frac{2}{3}\right)L_{md}$$

$$L_{mkd} = \left(\frac{N_{kd}}{N_s}\right)^2\left(\frac{2}{3}\right)L_{md}$$

$$L_{kq1kq2} = \left(\frac{N_{kq2}}{N_{kq1}}\right)L_{mkq1} = \left(\frac{N_{kq1}}{N_{kq2}}\right)L_{mkq2}$$

$$L_{fdkd} = \left(\frac{N_{kd}}{N_{fd}}\right)L_{mfd} = \left(\frac{N_{fd}}{N_{kd}}\right)L_{mkd}$$

To refer the rotor variables to the stator windings:

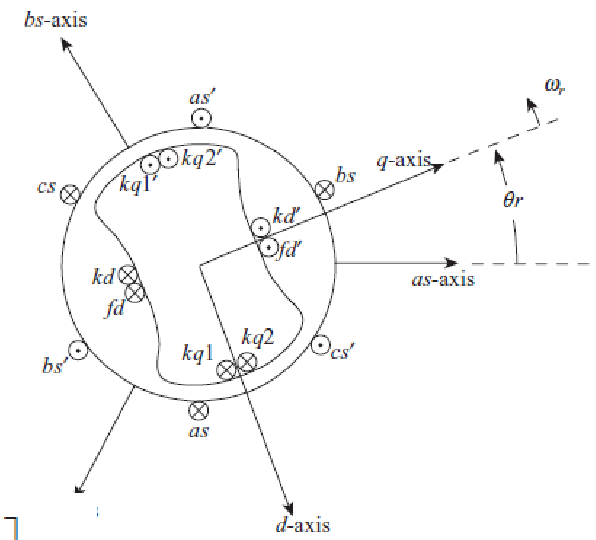
$$i'_j = \left(\frac{2}{3}\right)\left(\frac{N_j}{N_s}\right)i_j$$

$$v'_j = \left(\frac{N_s}{N_j}\right)v_j$$

$$\lambda'_j = \left(\frac{N_s}{N_j}\right)\lambda_j$$

where j may be $kq1$, $kq2$, fd, or kd.

Transient Modeling of Synchronous Machines



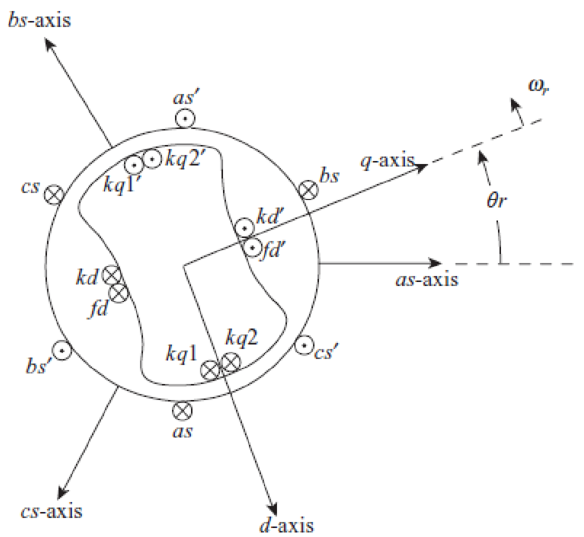
Voltage Equations in Machine Variables

$$\begin{bmatrix} \boldsymbol{\lambda}_{abcs} \\ \boldsymbol{\lambda}'_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}'_{sr} \\ \frac{2}{3}(\mathbf{L}'_{sr})^T & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}'_{qdr} \end{bmatrix}$$

$$\mathbf{L}'_{sr} = \begin{bmatrix} L_{mq}\cos\theta_r & L_{mq}\cos\theta_r & L_{md}\sin\theta_r & L_{md}\sin\theta_r \\ L_{mq}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{mq}\cos\left(\theta_r - \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{mq}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{mq}\cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{md}\sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$\mathbf{L}'_r = \begin{bmatrix} L'_{lkq1} + L_{mq} & L_{mq} & 0 & 0 \\ L_{mq} & L'_{lkq2} + L_{mq} & 0 & 0 \\ 0 & 0 & L'_{lfd} + L_{md} & L_{md} \\ 0 & 0 & L_{md} & L'_{lkd} + L_{md} \end{bmatrix}$$

Transient Modeling of Synchronous Machines



Voltage Equations in Machine Variables

The voltage equations expressed in terms of machine variables referred to the stator windings are:

$$\begin{bmatrix} \mathbf{v}_{abcs} \\ \mathbf{v}'_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_s + p\mathbf{L}_s & p\mathbf{L}'_{sr} \\ \frac{2}{3}p(\mathbf{L}'_{sr})^T & \mathbf{r}'_r + p\mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abcs} \\ \mathbf{i}'_{qdr} \end{bmatrix}$$

$$r'_j = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_j}\right)^2 r_j$$

$$L'_{lj} = \left(\frac{3}{2}\right)\left(\frac{N_s}{N_j}\right)^2 L_{lj}$$

where, again, j may be $kq1$, $kq2$, fd, or kd.

Transient Modeling of Synchronous Machines

Torque Equations in Machine Variables

The energy stored in the coupling field of a synchronous machine may be expressed as:

$$W_f = \frac{1}{2}(\mathbf{i}_{abcs})^T \mathbf{L}_s \mathbf{i}_{abcs} + (\mathbf{i}_{abcs})^T \mathbf{L}'_{sr} \mathbf{i}'_{qdr} + \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)(\mathbf{i}'_{qdr})^T \mathbf{L}'_r \mathbf{i}'_{qdr}$$

$$f_e(\mathbf{i}, x) = \frac{\partial W_c(\mathbf{i}, x)}{\partial x}$$

$$f_e(\boldsymbol{\lambda}, x) = -\frac{\partial W_f(\boldsymbol{\lambda}, x)}{\partial x}$$

$$W_f = W_c,$$

$$\theta_r = \frac{P}{2}\theta_{rm}$$

$$T_e = \left(\frac{P}{2}\right)\left\{\frac{1}{2}(\mathbf{i}_{abcs})^T \frac{\partial}{\partial\theta_r}[\mathbf{L}_s]\mathbf{i}_{abcs} + (\mathbf{i}_{abcs})^T \frac{\partial}{\partial\theta_r}[\mathbf{L}'_{sr}]\mathbf{i}'_{qdr}\right\}$$

$$\begin{aligned} T_e = \left(\frac{P}{2}\right)\Bigg\{&\frac{(L_{md}-L_{mq})}{3}\Bigg[\left(i_{as}^2 - \frac{1}{2}i_{bs}^2 - \frac{1}{2}i_{cs}^2 - i_{as}i_{bs} - i_{as}i_{cs} + 2i_{bs}i_{cs}\right)\sin 2\theta_r \\ &+ \frac{\sqrt{3}}{2}(i_{bs}^2 - i_{cs}^2 - 2i_{as}i_{bs} + 2i_{as}i_{cs})\cos 2\theta_r\Bigg] \\ &+ L_{mq}(i'_{kq1} + i'_{kq2})\left[\left(i_{as} - \frac{1}{2}i_{bs} - \frac{1}{2}i_{cs}\right)\sin\theta_r - \frac{\sqrt{3}}{2}(i_{bs} - i_{cs})\cos\theta_r\right] \\ &- L_{md}(i'_{fd} + i'_{kd})\left[\left(i_{as} - \frac{1}{2}i_{bs} - \frac{1}{2}i_{cs}\right)\cos\theta_r + \frac{\sqrt{3}}{2}(i_{bs} - i_{cs})\sin\theta_r\right]\Bigg\} \end{aligned}$$

Transient Modeling of Synchronous Machines

Stator Voltage Equations in Arbitrary Reference Frame Variables

$$\mathbf{v}_{qd0s} = \mathbf{r}_s \mathbf{i}_{qd0s} + \omega \boldsymbol{\lambda}_{dqs} + p\boldsymbol{\lambda}_{qd0s} \qquad (\boldsymbol{\lambda}_{dqs})^T = [\lambda_{ds} \quad -\lambda_{qs} \quad 0]$$

The rotor windings of a synchronous machine are asymmetrical; therefore, a change of variables offers no advantage in the analysis of the rotor circuits. Since the rotor variables are not transformed, the rotor voltage equations are expressed only in the rotor reference frame.

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs}$$

$$\mathbf{v}_{qdr} = \mathbf{r}_r \mathbf{i}_{qdr} + p\boldsymbol{\lambda}_{qdr}$$

$$\mathbf{v}'^{r}_{qdr} = \mathbf{r}'_r \mathbf{i}'^{r}_{qdr} + p\boldsymbol{\lambda}'^{r}_{qdr}$$

$$\begin{bmatrix} \boldsymbol{\lambda}_{qd0s} \\ \boldsymbol{\lambda}'^{r}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_s \mathbf{L}_s (\mathbf{K}_s)^{-1} & \mathbf{K}_s \mathbf{L}'_{sr} \\ \frac{2}{3}(\mathbf{L}'_{sr})^T (\mathbf{K}_s)^{-1} & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{qd0s} \\ \mathbf{i}'^{r}_{qdr} \end{bmatrix}$$

Transient Modeling of Synchronous Machines

Stator Voltage Equations in Arbitrary Reference Frame Variables

$$\begin{bmatrix} \boldsymbol{\lambda}_{qd0s} \\ \boldsymbol{\lambda}'^{r}_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_s \mathbf{L}_s (\mathbf{K}_s)^{-1} & \mathbf{K}_s \mathbf{L}'_{sr} \\ \frac{2}{3}(\mathbf{L}'_{sr})^T (\mathbf{K}_s)^{-1} & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{qd0s} \\ \mathbf{i}'^{r}_{qdr} \end{bmatrix}$$

It can be shown that all terms of the inductance matrix of this inductance matrix are sinusoidal in nature except $\mathbf{L}_r'$

For example:

$$\mathbf{K}_s \mathbf{L}'_{sr} = \begin{bmatrix} L_{mq}\cos(\theta-\theta_r) & L_{mq}\cos(\theta-\theta_r) & -L_{md}\sin(\theta-\theta_r) & -L_{md}\sin(\theta-\theta_r) \\ L_{mq}\sin(\theta-\theta_r) & L_{mq}\sin(\theta-\theta_r) & L_{md}\cos(\theta-\theta_r) & L_{md}\cos(\theta-\theta_r) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The inductances become constant only if $\omega = \omega_r$.

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables

$$\mathbf{v}^r_{qd0s} = \mathbf{r}_s \mathbf{i}^r_{qd0s} + \omega_r \boldsymbol{\lambda}^r_{dqs} + p\boldsymbol{\lambda}^r_{qd0s}$$

$$\mathbf{v}'^r_{qdr} = \mathbf{r}'_r \mathbf{i}'^r_{qdr} + p\boldsymbol{\lambda}'^r_{qdr}$$

$$(\boldsymbol{\lambda}^r_{dqs})^T = [\lambda^r_{ds} \quad -\lambda^r_{qs} \quad 0]$$

$$\boxed{\begin{bmatrix} \boldsymbol{\lambda}^r_{qd0s} \\ \boldsymbol{\lambda}'^r_{qdr} \end{bmatrix} = \begin{bmatrix} \mathbf{K}^r_s \mathbf{L}_s (\mathbf{K}^r_s)^{-1} & \mathbf{K}^r_s \mathbf{L}'_{sr} \\ \frac{2}{3}(\mathbf{L}'_{sr})^T (\mathbf{K}^r_s)^{-1} & \mathbf{L}'_r \end{bmatrix} \begin{bmatrix} \mathbf{i}^r_{qd0s} \\ \mathbf{i}'^r_{qdr} \end{bmatrix}}$$

$$\left\{ \begin{aligned} \mathbf{K}^r_s \mathbf{L}_s (\mathbf{K}^r_s)^{-1} &= \begin{bmatrix} L_{ls} + L_{mq} & 0 & 0 \\ 0 & L_{ls} + L_{md} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} \\ \mathbf{K}^r_s \mathbf{L}'_{sr} &= \begin{bmatrix} L_{mq} & L_{mq} & 0 & 0 \\ 0 & 0 & L_{md} & L_{md} \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \frac{2}{3}(\mathbf{L}'_{sr})^T (\mathbf{K}^r_s)^{-1} &= \begin{bmatrix} L_{mq} & 0 & 0 \\ L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 \\ 0 & L_{md} & 0 \end{bmatrix} \end{aligned} \right\}$$

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables

$$v_{qs}^r = r_s i_{qs}^r + \omega_r \lambda_{ds}^r + p\lambda_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \omega_r \lambda_{qs}^r + p\lambda_{ds}^r$$

$$v_{0s} = r_s i_{0s} + p\lambda_{0s}$$

$$v_{kq1}'^r = r_{kq1}' i_{kq1}'^r + p\lambda_{kq1}'^r$$

$$v_{kq2}'^r = r_{kq2}' i_{kq2}'^r + p\lambda_{kq2}'^r$$

$$v_{fd}'^r = r_{fd}' i_{fd}'^r + p\lambda_{fd}'^r$$

$$v_{kd}'^r = r_{kd}' i_{kd}'^r + p\lambda_{kd}'^r$$

$$\mathbf{L}_r' = \begin{bmatrix} L_{lkq1}' + L_{mq} & L_{mq} & 0 & 0 \\ L_{mq} & L_{lkq2}' + L_{mq} & 0 & 0 \\ 0 & 0 & L_{lfd}' + L_{md} & L_{md} \\ 0 & 0 & L_{md} & L_{lkd}' + L_{md} \end{bmatrix}$$

$$\mathbf{K}_s^r \mathbf{L}_s (\mathbf{K}_s^r)^{-1} = \begin{bmatrix} L_{ls} + L_{mq} & 0 & 0 \\ 0 & L_{ls} + L_{md} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

$$\mathbf{K}_s^r \mathbf{L}_{sr}' = \begin{bmatrix} L_{mq} & L_{mq} & 0 & 0 \\ 0 & 0 & L_{md} & L_{md} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{2}{3}(\mathbf{L}_{sr}')^T (\mathbf{K}_s^r)^{-1} = \begin{bmatrix} L_{mq} & 0 & 0 \\ L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 \\ 0 & L_{md} & 0 \end{bmatrix}$$

$$\lambda_{qs}^r = L_{ls} i_{qs}^r + L_{mq}(i_{qs}^r + i_{kq1}'^r + i_{kq2}'^r)$$

$$\lambda_{ds}^r = L_{ls} i_{ds}^r + L_{md}(i_{ds}^r + i_{fd}'^r + i_{kd}'^r)$$

$$\lambda_{0s} = L_{ls} i_{0s}$$

$$\lambda_{kq1}'^r = L_{lkq1}' i_{kq1}'^r + L_{mq}(i_{qs}^r + i_{kq1}'^r + i_{kq}$$

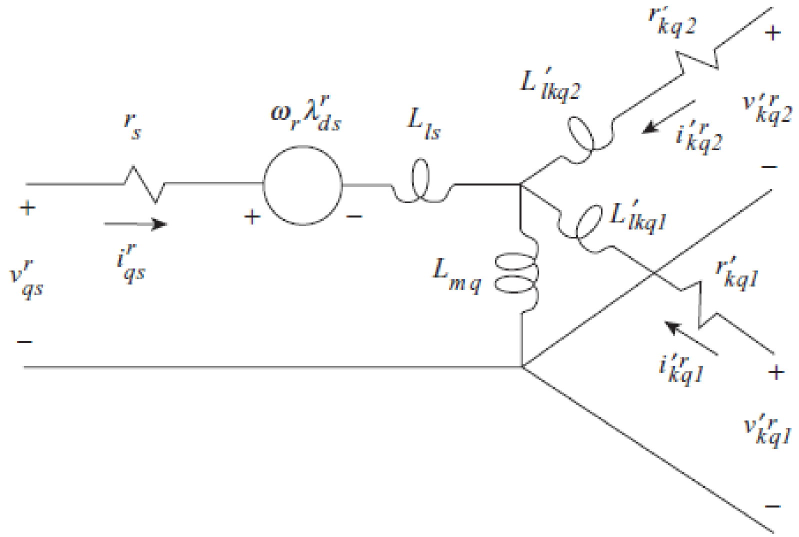
$$\lambda_{kq2}'^r = L_{lkq2}' i_{kq2}'^r + L_{mq}(i_{qs}^r + i_{kq1}'^r + i_{k}$$

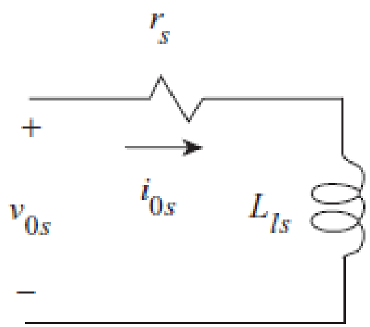
$$\lambda_{fd}'^r = L_{lfd}' i_{fd}'^r + L_{md}(i_{ds}^r + i_{fd}'^r + i_{kd}'^r)$$

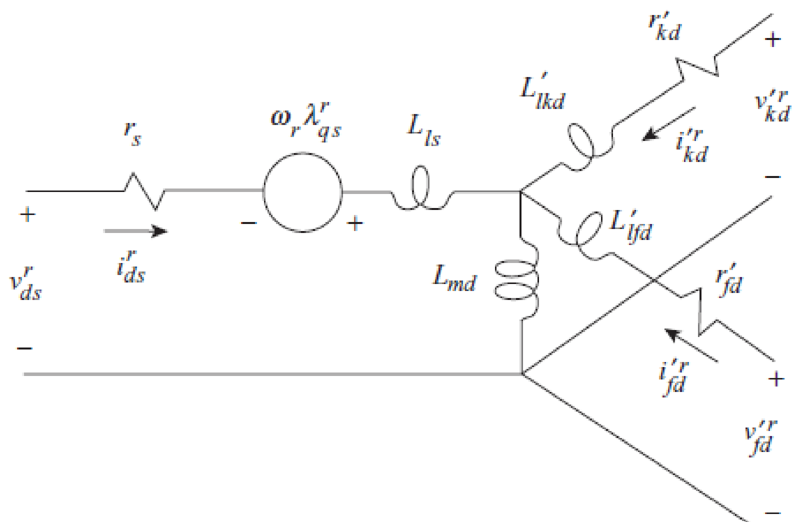
$$\lambda_{kd}'^r = L_{lkd}' i_{kd}'^r + L_{md}(i_{ds}^r + i_{fd}'^r + i_{kd}'^r)$$

Transient Modeling of Synchronous Machines

Equivalent circuits of a three-phase synchronous machine with the reference frame fixed in rotor: Park's equations.







Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables Using Reactances

$$v_{qs}^r = r_s i_{qs}^r + \frac{\omega_r}{\omega_b}\psi_{ds}^r + \frac{p}{\omega_b}\psi_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \frac{\omega_r}{\omega_b}\psi_{qs}^r + \frac{p}{\omega_b}\psi_{ds}^r$$

$$v_{0s} = r_s i_{0s} + \frac{p}{\omega_b}\psi_{0s}$$

$$v_{kq1}'^r = r_{kq1}' i_{kq1}'^r + \frac{p}{\omega_b}\psi_{kq1}'^r$$

$$v_{kq2}'^r = r_{kq2}' i_{kq2}'^r + \frac{p}{\omega_b}\psi_{kq2}'^r$$

$$v_{fd}'^r = r_{fd}' i_{fd}'^r + \frac{p}{\omega_b}\psi_{fd}'^r$$

$$v_{kd}'^r = r_{kd}' i_{kd}'^r + \frac{p}{\omega_b}\psi_{kd}'^r$$

where ω_b is the base electrical angular velocity used to calculate the inductive reactances.

The flux linkages per second are:

$$\psi_{qs}^r = X_{ls} i_{qs}^r + X_{mq}(i_{qs}^r + i_{kq1}'^r + i_{kq2}'^r)$$

$$\psi_{ds}^r = X_{ls} i_{ds}^r + X_{md}(i_{ds}^r + i_{fd}'^r + i_{kd}'^r)$$

$$\psi_{0s} = X_{ls} i_{0s}$$

$$\psi_{kq1}'^r = X_{lkq1}' i_{kq1}'^r + X_{mq}(i_{qs}^r + i_{kq1}'^r + i_{kq2}'^r)$$

$$\psi_{kq2}'^r = X_{lkq2}' i_{kq2}'^r + X_{mq}(i_{qs}^r + i_{kq1}'^r + i_{kq2}'^r)$$

$$\psi_{fd}'^r = X_{lfd}' i_{fd}'^r + X_{md}(i_{ds}^r + i_{fd}'^r + i_{kd}'^r)$$

$$\psi_{kd}'^r = X_{lkd}' i_{kd}'^r + X_{md}(i_{ds}^r + i_{fd}'^r + i_{kd}'^r)$$

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables Using Reactances

Park's equations are generally written without the superscript *r*, the subscript *s*, and the primes, which denote referred quantities.

Also, it is convenient to define:

$$v_{fd}'^{r} = r_{fd}' i_{fd}'^{r} + \frac{p}{\omega_b}\psi_{fd}'^{r}$$

Field voltage

$$e_{xfd}'^{r} = v_{fd}'^{r}\frac{X_{md}}{r_{fd}'} \quad \Longrightarrow \quad e_{xfd}'^{r} = \frac{X_{md}}{r_{fd}'}\left(r_{fd}' i_{fd}'^{r} + \frac{p}{\omega_b}\psi_{fd}'^{r}\right)$$

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables

$$\begin{bmatrix} v_{qs}^{r} \\ v_{ds}^{r} \\ v_{0s} \\ v_{kq1}'^{r} \\ v_{kq2}'^{r} \\ e_{xfd}'^{r} \\ v_{kd}'^{r} \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b}X_q & \frac{\omega_r}{\omega_b}X_d & 0 & \frac{p}{\omega_b}X_{mq} & \frac{p}{\omega_b}X_{mq} & \frac{\omega_r}{\omega_b}X_{md} & \frac{\omega_r}{\omega_b}X_{md} \\ -\frac{\omega_r}{\omega_b}X_q & r_s + \frac{p}{\omega_b}X_d & 0 & -\frac{\omega_r}{\omega_b}X_{mq} & -\frac{\omega_r}{\omega_b}X_{mq} & \frac{p}{\omega_b}X_{md} & \frac{p}{\omega_b}X_{md} \\ 0 & 0 & r_s + \frac{p}{\omega_b}X_{ls} & 0 & 0 & 0 & 0 \\ \frac{p}{\omega_b}X_{mq} & 0 & 0 & r_{kq1}' + \frac{p}{\omega_b}X_{kq1}' & \frac{p}{\omega_b}X_{mq} & 0 & 0 \\ \frac{p}{\omega_b}X_{mq} & 0 & 0 & \frac{p}{\omega_b}X_{mq} & r_{kq2}' + \frac{p}{\omega_b}X_{kq2}' & 0 & 0 \\ 0 & \frac{X_{md}}{r_{fd}'}\left(\frac{p}{\omega_b}X_{md}\right) & 0 & 0 & 0 & \frac{X_{md}}{r_{fd}'}\left(r_{fd}' + \frac{p}{\omega_b}X_{fd}'\right) & \frac{X_{md}}{r_{fd}'}\left(\frac{p}{\omega_b}X_{md}\right) \\ 0 & \frac{p}{\omega_b}X_{md} & 0 & 0 & 0 & \frac{p}{\omega_b}X_{md} & r_{kd}' + \frac{p}{\omega_b}X_{kd}' \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \\ i_{0s} \\ i_{kq1}'^{r} \\ i_{kq2}'^{r} \\ i_{fd}'^{r} \\ i_{kd}'^{r} \end{bmatrix}$$

$$X_q = X_{ls} + X_{mq}$$

$$X_d = X_{ls} + X_{md}$$

$$X_{kq1}' = X_{lkq1}' + X_{mq}$$

$$X_{kq2}' = X_{lkq2}' + X_{mq}$$

$$X_{fd}' = X_{lfd}' + X_{md}$$

$$X_{kd}' = X_{lkd}' + X_{md}$$

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables

$$\begin{bmatrix} \psi_{qs}^{r} \\ \psi_{ds}^{r} \\ \psi_{0s} \\ \psi_{kq1}^{\prime r} \\ \psi_{kq2}^{\prime r} \\ \psi_{fd}^{\prime r} \\ \psi_{kd}^{\prime r} \end{bmatrix} = \begin{bmatrix} X_q & 0 & 0 & X_{mq} & X_{mq} & 0 & 0 \\ 0 & X_d & 0 & 0 & 0 & X_{md} & X_{md} \\ 0 & 0 & X_{ls} & 0 & 0 & 0 & 0 \\ X_{mq} & 0 & 0 & X_{kq1}^{\prime} & X_{mq} & 0 & 0 \\ X_{mq} & 0 & 0 & X_{mq} & X_{kq2}^{\prime} & 0 & 0 \\ 0 & X_{md} & 0 & 0 & 0 & X_{fd}^{\prime} & X_{md} \\ 0 & X_{md} & 0 & 0 & 0 & X_{md} & X_{kd}^{\prime} \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{ds}^{r} \\ i_{0s} \\ i_{kq1}^{\prime r} \\ i_{kq2}^{\prime r} \\ i_{fd}^{\prime r} \\ i_{kd}^{\prime r} \end{bmatrix}$$

If the flux linkages or flux linkages per second are selected as independent variables, it is convenient to first express:

$$\begin{bmatrix} \psi_{qs}^{r} \\ \psi_{kq1}^{\prime r} \\ \psi_{kq2}^{\prime r} \end{bmatrix} = \begin{bmatrix} X_q & X_{mq} & X_{mq} \\ X_{mq} & X_{kq1}^{\prime} & X_{mq} \\ X_{mq} & X_{mq} & X_{kq2}^{\prime} \end{bmatrix} \begin{bmatrix} i_{qs}^{r} \\ i_{kq1}^{\prime r} \\ i_{kq2}^{\prime r} \end{bmatrix}$$

$$\begin{bmatrix} \psi_{ds}^{r} \\ \psi_{fd}^{\prime r} \\ \psi_{kd}^{\prime r} \end{bmatrix} = \begin{bmatrix} X_d & X_{md} & X_{md} \\ X_{md} & X_{fd}^{\prime} & X_{md} \\ X_{md} & X_{md} & X_{kd}^{\prime} \end{bmatrix} \begin{bmatrix} i_{ds}^{r} \\ i_{fd}^{\prime r} \\ i_{kd}^{\prime r} \end{bmatrix}$$

$$\psi_{0s} = X_{ls} i_{0s}$$

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables

$$\begin{bmatrix} \psi_{qs}^r \\ \psi_{kq1}^{\prime r} \\ \psi_{kq2}^{\prime r} \end{bmatrix} = \begin{bmatrix} X_q & X_{mq} & X_{mq} \\ X_{mq} & X'_{kq1} & X_{mq} \\ X_{mq} & X_{mq} & X'_{kq2} \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{kq1}^{\prime r} \\ i_{kq2}^{\prime r} \end{bmatrix} \qquad \begin{bmatrix} \psi_{ds}^r \\ \psi_{fd}^{\prime r} \\ \psi_{kd}^{\prime r} \end{bmatrix} = \begin{bmatrix} X_d & X_{md} & X_{md} \\ X_{md} & X'_{fd} & X_{md} \\ X_{md} & X_{md} & X'_{kd} \end{bmatrix} \begin{bmatrix} i_{ds}^r \\ i_{fd}^{\prime r} \\ i_{kd}^{\prime r} \end{bmatrix}$$

$$\psi_{0s} = X_{ls} i_{0s}$$

$$\begin{bmatrix} i_{qs}^r \\ i_{kq1}^{\prime r} \\ i_{kq2}^{\prime r} \end{bmatrix} = \frac{1}{D_q} \begin{bmatrix} X'_{kq1}X'_{kq2} - X_{mq}^2 & -X_{mq}X'_{kq2} + X_{mq}^2 & -X_{mq}X'_{kq1} + X_{mq}^2 \\ -X_{mq}X'_{kq2} + X_{mq}^2 & X_q X'_{kq2} - X_{mq}^2 & -X_q X_{mq} + X_{mq}^2 \\ -X_{mq}X'_{kq1} + X_{mq}^2 & -X_q X_{mq} + X_{mq}^2 & X_q X'_{kq1} - X_{mq}^2 \end{bmatrix} \begin{bmatrix} \psi_{qs}^r \\ \psi_{kq1}^{\prime r} \\ \psi_{kq2}^{\prime r} \end{bmatrix}$$

$$\begin{bmatrix} i_{ds}^r \\ i_{fd}^{\prime r} \\ i_{kd}^{\prime r} \end{bmatrix} = \frac{1}{D_d} \begin{bmatrix} X'_{fd}X'_{kd} - X_{md}^2 & -X_{md}X'_{kd} + X_{md}^2 & -X_{md}X'_{fd} + X_{md}^2 \\ -X_{md}X'_{kd} + X_{md}^2 & X_d X'_{kd} - X_{md}^2 & -X_d X_{md} + X_{md}^2 \\ -X_{md}X'_{fd} + X_{md}^2 & -X_d X_{md} + X_{md}^2 & X_d X'_{fd} - X_{md}^2 \end{bmatrix} \begin{bmatrix} \psi_{ds}^r \\ \psi_{fd}^{\prime r} \\ \psi_{kd}^{\prime r} \end{bmatrix}$$

$$i_{0s} = \frac{1}{X_{ls}} \psi_{0s}$$

where

$$D_q = -X_{mq}^2 (X_q - 2X_{mq} + X'_{kq1} + X'_{kq2}) + X_q X'_{kq1} X'_{kq2}$$

$$D_d = -X_{md}^2 (X_d - 2X_{md} + X'_{fd} + X'_{kd}) + X_d X'_{fd} X'_{kd}$$

Transient Modeling of Synchronous Machines

Voltage Equations in Rotor Reference Frame Variables

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \\ v_{0s} \\ v_{kq1}'^r \\ v_{kq2}'^r \\ e_{xfd}'^r \\ v_{kd}'^r \end{bmatrix} = \begin{bmatrix} r_s a_{11} + \frac{p}{\omega_b} & \frac{\omega_r}{\omega_b} & 0 & -r_s a_{12} & -r_s a_{13} & 0 & 0 \\ -\frac{\omega_r}{\omega_b} & r_s b_{11} + \frac{p}{\omega_b} & 0 & 0 & 0 & -r_s b_{12} & -r_s b_{13} \\ 0 & 0 & \frac{r_s}{X_{ls}} + \frac{p}{\omega_b} & 0 & 0 & 0 & 0 \\ r_{kq1}' a_{21} & 0 & 0 & r_{kq1}' a_{22} + \frac{p}{\omega_b} & r_{kq1}' a_{23} & 0 & 0 \\ r_{kq2}' a_{31} & 0 & 0 & r_{kq2}' a_{32} & r_{kq2}' a_{33} + \frac{p}{\omega_b} & 0 & 0 \\ 0 & X_{md} b_{21} & 0 & 0 & 0 & X_{md} b_{22} + \frac{X_{md}}{r_{fd}'} \frac{p}{\omega_b} & X_{md} b_{23} \\ 0 & r_{kd}' b_{31} & 0 & 0 & 0 & r_{kd}' b_{32} & r_{kd}' b_{33} + \frac{p}{\omega_b} \end{bmatrix} \begin{bmatrix} \psi_{qs}^r \\ \psi_{ds}^r \\ \psi_{0s} \\ \psi_{kq1}'^r \\ \psi_{kq2}'^r \\ \psi_{fd}'^r \\ \psi_{kd}'^r \end{bmatrix}$$

a_{ij} belongs to $\frac{1}{D_q}\begin{bmatrix} X_{kq1}' X_{kq2}' - X_{mq}^2 & -X_{mq} X_{kq2}' + X_{mq}^2 & -X_{mq} X_{kq1}' + X_{mq}^2 \\ -X_{mq} X_{kq2}' + X_{mq}^2 & X_q X_{kq2}' - X_{mq}^2 & -X_q X_{mq} + X_{mq}^2 \\ -X_{mq} X_{kq1}' + X_{mq}^2 & -X_q X_{mq} + X_{mq}^2 & X_q X_{kq1}' - X_{mq}^2 \end{bmatrix}$

b_{ij} belongs to $\frac{1}{D_d}\begin{bmatrix} X_{fd}' X_{kd}' - X_{md}^2 & -X_{md} X_{kd}' + X_{md}^2 & -X_{md} X_{fd}' + X_{md}^2 \\ -X_{md} X_{kd}' + X_{md}^2 & X_d X_{kd}' - X_{md}^2 & -X_d X_{md} + X_{md}^2 \\ -X_{md} X_{fd}' + X_{md}^2 & -X_d X_{md} + X_{md}^2 & X_d X_{fd}' - X_{md}^2 \end{bmatrix}$

Transient Modeling of Synchronous Machines

Torque Equations in Rotor Reference Frame Variables

The expression for the positive electromagnetic torque for motor action in terms of rotor reference-frame variables

$$T_e = \left(\frac{P}{2}\right)\left\{\frac{1}{2}(\mathbf{i}_{abcs})^T \frac{\partial}{\partial\theta_r}[\mathbf{L}_s]\mathbf{i}_{abcs} + (\mathbf{i}_{abcs})^T \frac{\partial}{\partial\theta_r}[\mathbf{L}'_{sr}]\mathbf{i}'_{qdr}\right\}$$

$$T_e = \left(\frac{P}{2}\right)[(\mathbf{K}_s^r)^{-1}\mathbf{i}_{qd0s}^r]^T \left\{\frac{1}{2}\frac{\partial}{\partial\theta_r}[\mathbf{L}_s](\mathbf{K}_s^r)^{-1}\mathbf{i}_{qd0s} + \frac{\partial}{\partial\theta_r}[\mathbf{L}'_{sr}]\mathbf{i}'^r_{qdr}\right\}$$

After a considerable work!!!

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)[L_{md}(i_{ds}^r + i'^r_{fd} + i'^r_{kd})i_{qs}^r - L_{mq}(i_{qs}^r + i'^r_{kq1} + i'^r_{kq2})i_{ds}^r]$$

Alternative forms of Torque

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)(\lambda_{ds}^r i_{qs}^r - \lambda_{qs}^r i_{ds}^r)$$

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)(\psi_{ds}^r i_{qs}^r - \psi_{qs}^r i_{ds}^r)$$

Try to obtain this!

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)[(a_{11} - b_{11})\psi_{qs}^r\psi_{ds}^r + \psi_{ds}^r(a_{12}\psi'^r_{kq1} + a_{13}\psi'^r_{kq2}) - \psi_{qs}^r(b_{12}\psi'^r_{fd} + b_{13}\psi'^r_{kd})]$$

Transient Modeling of Synchronous Machines

$$T_e = J\left(\frac{2}{P}\right)p\omega_r + T_L$$

In particular, the rotor angle is the displacement of the rotor generally referenced to the maximum positive value of the fundamental component of the terminal (system) voltage of phase *a* . We discussed that in steady state analysis….

$$\delta = \theta_r - \theta_{ev}$$

The electrical angular velocity of the rotor is ω_r; ω_e is the electrical angular velocity of the terminal voltages.

The rotor angle is often used in relating torque and rotor speed. In particular, if ω_e is constant, then

$$T_e = J\left(\frac{2}{P}\right)p^2\delta + T_L$$

where δ is expressed in electrical radians

Transient Modeling of Synchronous Machines

Per Unit System

The equations for a synchronous machine may be written in per unit where base voltage is generally selected as the rms value of the rated phase voltage for the *abc* variables and the peak value for the *qd*0 variables. However, we will often use the same base value when comparing *abc* and *qd*0 variables. When considering the machine separately, the power base is selected as its volt-ampere rating. When considering power systems, a system power base (system base) is selected that is generally different from the power base of the machine (machine base).

Park's equations written in terms of flux linkages per second and reactances are readily per unitized by dividing each term by the peak of the base voltage (or the peak value of the base current times base impedance).

Transient Modeling of Synchronous Machines

Per Unit System

$$T_B = \frac{P_B}{(2/P)\omega_b}$$

$$= \frac{\left(\frac{3}{2}\right) V_{B(qd0)} I_{B(qd0)}}{(2/P)\omega_b}$$

where ω_b corresponds to rated or base frequency, P_B is the base power, $V_{B(qd0)}$ is the peak value of the base phase voltage, and $I_{B(qd0)}$ is the peak value of the base phase current.

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\frac{1}{\omega_b}\right)(\psi^r_{ds} i^r_{qs} - \psi^r_{qs} i^r_{ds}) \xrightarrow{\text{P.U.}} T_e = \psi^r_{ds} i^r_{qs} - \psi^r_{qs} i^r_{ds}$$

$$T_e = J\left(\frac{2}{P}\right) p\omega_r + T_L \xrightarrow{\text{P.U.}} T_e = 2Hp\frac{\omega_r}{\omega_b} + T_L \xrightarrow{\text{If } \omega_e \text{ is constant}} T_e = \frac{2H}{\omega_b} p^2 \delta + T_L$$

SYNCHRONOUS MACHINES CIGRE Report

Synchronous Machines

Outline

1. Physical Description
2. Mathematical Model
3. Park's "dqo" transportation
4. Steady-state Analysis
   - phasor representation in d-q coordinates
   - link with network equations
5. Definition of "rotor angle"
6. Representation of Synchronous Machines in Stability Studies
   - neglect of stator transients
   - magnetic saturation
7. Simplified Models
8. Synchronous Machine Parameters
9. Reactive Capability Limits

Physical Description of a Synchronous Machine

- Consists of two sets of windings:
  - 3 phase armature winding on the stator distributed with centres 120° apart in space
  - field winding on the rotor supplied by DC
- Two basic rotor structures used:
  - salient or projecting pole structure for hydraulic units (low speed)
  - round rotor structure for thermal units (high speed)
- Salient poles have concentrated field windings; usually also carry damper windings on the pole face.

  Round rotors have solid steel rotors with distributed windings
- Nearly sinusoidal space distribution of flux wave shape obtained by:
  - distributing stator windings and field windings in many slots (round rotor);
  - shaping pole faces (salient pole)

Rotors of Steam Turbine Generators

- Traditionally, North American manufacturers normally did not provide special "damper windings"
  - solid steel rotors offer paths for eddy currents, which have effects equivalent to that of amortisseur currents
- European manufacturers tended to provide for additional damping effects and negative-sequence current capability
  - wedges in the slots of field windings interconnected to form a damper case, or
  - separate copper rods provided underneath the wedges

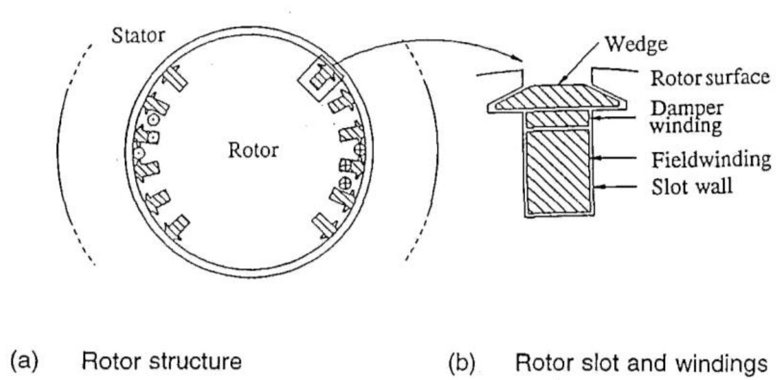
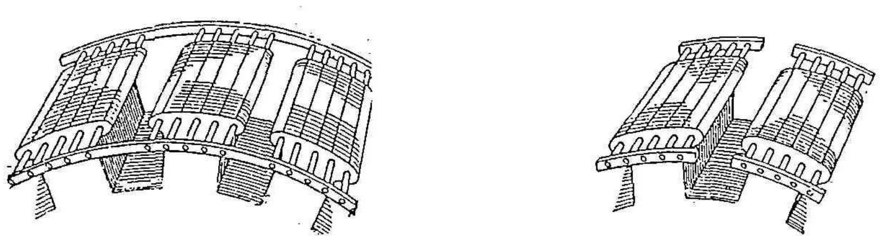


Figure 3.3: Solid round rotor construction

Rotors of Hydraulic Units

- Normally have damper windings or amortisseurs
  - non-magnetic material (usually copper) rods embedded in pole face
  - connected to end rings to form short-circuited windings
- Damper windings may be either continuous or non-continuous
- Space harmonics of the armature mmf contribute to surface eddy current
  - therefore, pole faces are usually laminated



(a) Continuous damper (b) Non-continuous damper

Figure 3.2: Salient pole rotor construction

Balanced Steady State Operation

- Net mmf wave due to the three phase stator windings:
  - travels at synchronous speed
  - appears stationary with respect to the rotor; and
  - has a sinusoidal space distribution

- mmf wave due to one phase:

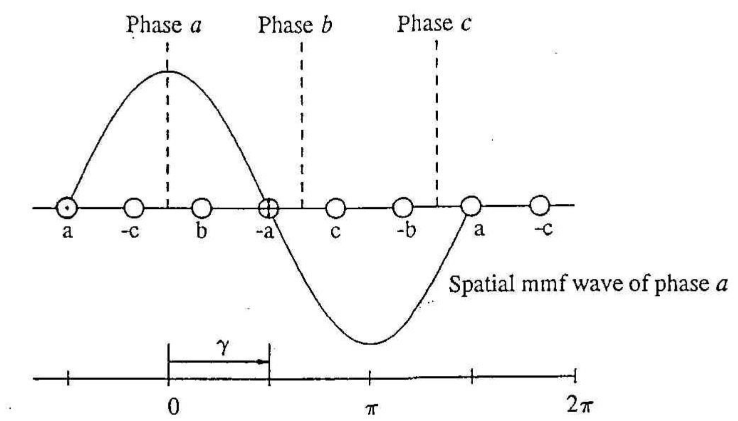


Figure 3.7: Spatial mmf wave of phase a

Balanced Steady State Operation

- The mmf wave due to the three phases are:

$$MMF_a = Ki_a \cos\gamma \qquad i_a = I_m \cos(\omega_s t)$$

$$MMF_b = Ki_b \cos\left(\gamma - \frac{2\pi}{3}\right) \qquad i_b = I_m \cos\left(\omega_s t - \frac{2\pi}{3}\right)$$

$$MMF_c = Ki_c \cos\left(\gamma + \frac{2\pi}{3}\right) \qquad i_a = I_m \cos\left(\omega_s t + \frac{2\pi}{3}\right)$$

$$\begin{aligned} MMF_{total} &= MMF_a + MMF_b + MMF_c \\ &= \frac{3}{2} KI_m \cos(\gamma - \omega_s t) \end{aligned}$$

Balanced Steady State Operation

- Magnitude of stator mmf wave and its relative angular position with respect to rotor mmf wave depend on machine output
  - for generator action, rotor field leads stator field due to forward torque of prime mover;
  - for motor action rotor field lags stator field due to retarding torque of shaft load

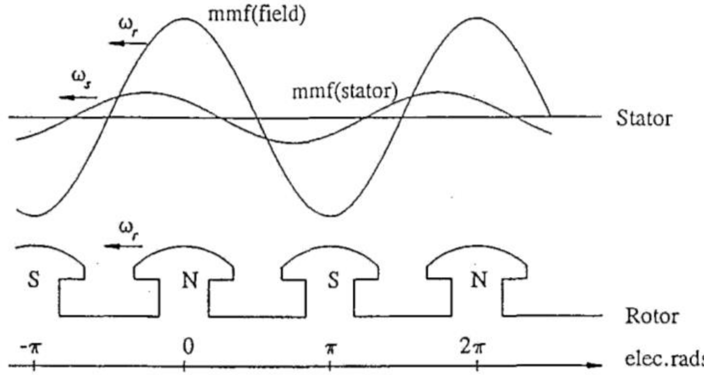


Figure 3.8: Stator and rotor mmf wave shapes

Transient Operation

- Stator and rotor fields may:
  - vary in magnitude with respect to time
  - have different speed
- Currents flow not only in the field and stator windings, but also in:
  - damper windings (if present); and
  - solid rotor surface and slot walls of round rotor machines

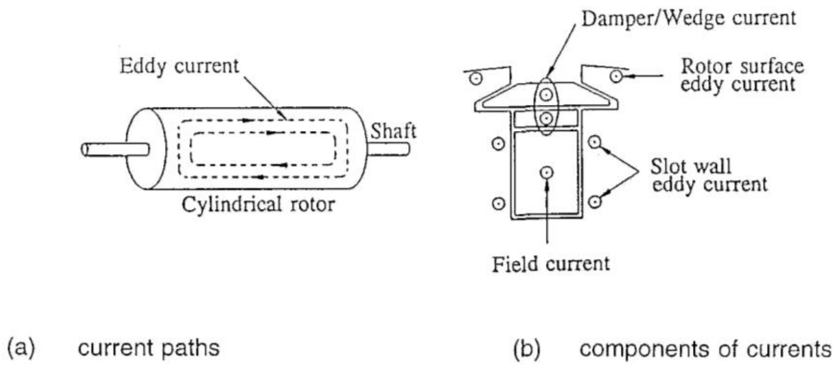


Figure 3.4: Current paths in a round rotor

Direct and Quadrature Axes

- The rotor has two axes of symmetry
- For the purpose of describing synchronous machine characteristics, two axes are defined:
  - the direct (d) axis, centered magnetically in the centre of the north pole
  - The quadrature (q) axis, 90 electrical degrees ahead of the d-axis

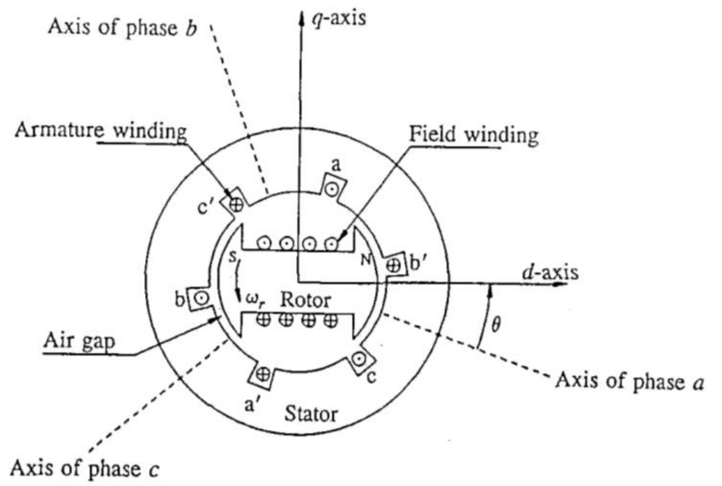


Figure 3.1: Schematic diagram of a 3-phase synchronous machine

Short Circuit Currents of a Synchronous Machine

- If a bolted three-phase fault is suddenly applied to a synchronous machine, the three phase currents are shown in Figure 3.25.

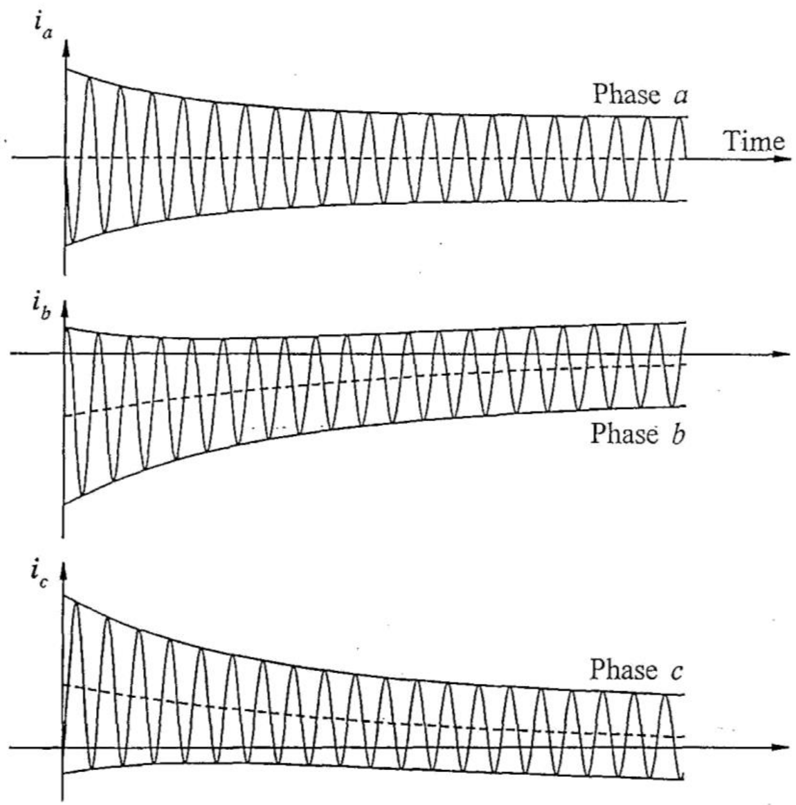


Figure 3.25: Three-phase short-circuit currents

Short Circuit Currents of a Synchronous Machine (cont'd)

- In general, fault current has two distinct components:
  a) a fundamental frequency component which decays initially very rapidly (a few cycles) and then relatively slowly (several seconds) to a steady state value

  b) a dc component which decays exponentially in several cycles
- This is similar to the short circuit current in the case of the simple RL circuit. However, the amplitude of the ac component is not constant
  - internal voltage, which is a function of rotor flux linkages, is not constant
  - the initial rapid decay is due to the decay of flux linking the subtransient circuits (high resistance)
  - the slowly decaying part of the ac component is due to the transient circuit (low resistance)
- The dc components have different magnitudes in the three phases

Elimination of dc Component by Neglecting Stator Transients

- For many classes of problems, considerable computational simplicity results if the effects of ac and dc components are treated separately
- Consider the stator voltage equations

$$\mathbf{e_d = p\psi_d - \bar{\omega}\psi_q - i_d R_a}$$
$$\mathbf{e_q = p\psi_q + \bar{\omega}\psi_d - i_q R_a}$$

transformer voltage terms: $p\psi_d$, $p\psi_q$

speed voltage terms: $\bar{\omega}\psi_q, \bar{\omega}\psi_d$

- The transformer voltage terms represent stator transients:
  - stator flux linkages (ψ_d, ψ_q) cannot change instantaneously
  - result in dc offset in stator phasor current
- If only fundamental frequency stator currents are of interest, stator transients ($p\psi_d$, $p\psi_q$) may be neglected.

Short Circuit Currents with Stator Transients Neglected

- The resulting stator phase currents following a disturbance has the wave shape shown in Figure 3.27
- The short circuit has only the ac component whose amplitude decays
- Regions of <u>subtransient</u>, <u>transient</u> and <u>steady state</u> periods can be readily identified from the wave shape of phase current

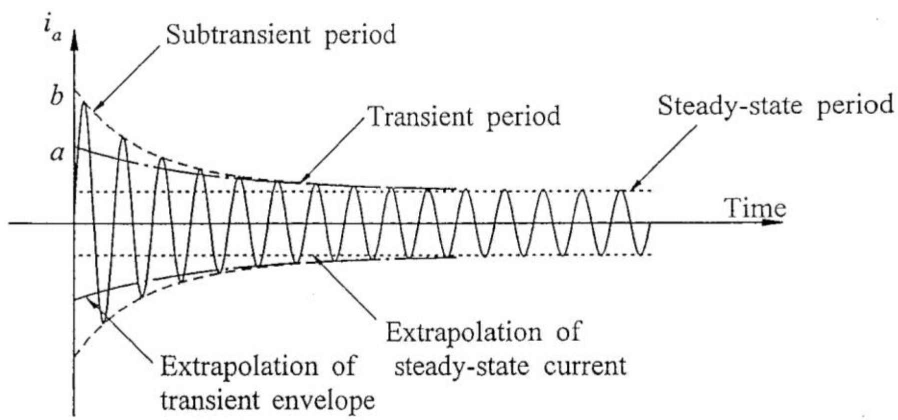


Figure 3.27: Fundamental frequency component of short circuit armature current

Synchronous Machine Representation in System Stability Studies

- Stator Transients ($p\psi_d$, $p\psi_q$) are usually neglected
  - accounts for only fundamental frequency components of stator quantities
  - dc offset either neglected or treated separately
  - allows the use of steady-state relationships for representing the transmission network

- Another simplifying assumption normally made is setting $\bar{\omega} = 1$ in the stator voltage equations
  - counter balances the effect of neglecting stator transients so far as the low-frequency rotor oscillations are concerned
  - with this assumption, in per unit air-gap power is equal to air-gap torque

Equation of Motion (Swing Equation)

- The combined inertia of the generator and prime-mover is accelerated by the accelerating torque:

$$J\frac{d\omega_m}{dt} = T_a = T_m - T_e$$

where

T_m = mechanical torque in N-M

T_e = electromagnetic torque in N-m

J = combined moment of inertia of generator and turbine, $kg \bullet m^2$

α_m = angular velocity of the rotor in mech. rad/s

t = time in seconds

Equation of Motion (cont'd)

- The above equation can be normalized in terms of per unit inertia constant H

$$H = \frac{1}{2}\frac{J\omega_{0m}^2}{VA_{base}}$$

where

α_{0m} = rated angular velocity of the rotor in mechanical radians per second

- Equation of motion in per unit form is

$$2H\frac{d\overline{\omega}_r}{dt} = \overline{T}_m - \overline{T}_e$$

where

$\overline{\omega}_r = \frac{\omega_m}{\omega_{0m}}$ = per unit rotor angular velocity

$\overline{T}_m = \frac{T_m\omega_{0m}}{VA_{base}}$ = per unit mechanical torque

$\overline{T}_e = \frac{T_e\omega_{0m}}{VA_{base}}$ = per unit electromechanical torque

- Often inertia constant M = 2H used

Magnetic Saturation

- Basic equations of synchronous machines developed so far ignored effects of saturation
  - analysis simple and manageable
  - rigorous treat a futile exercise
- Practical approach must be based on semi-heuristic reasoning and judiciously chosen approximations
  - consideration to simplicity, data availability, and accuracy of results
- Magnetic circuit data essential to treatment of saturation given by the <u>open-circuit characteristic</u> (OCC)

Assumptions Normally Made in the Representation of Saturation

- Leakage inductances are independent of saturation
- Saturation under loaded conditions is the same as under no-load conditions
- Leakage fluxes do not contribute to iron saturation
  - degree of saturation determined by the air-gap flux
- For salient pole machines, there is no saturation in the q-axis
  - flux is largely in air
- For round rotor machines, q-axis saturation assumed to be given by OCC
  - reluctance of magnetic path assumed homogeneous around rotor periphery

- **The effects of saturation is represented as**

$$L_{ad} = K_{sd} L_{adu} \qquad (3.182)$$

$$L_{aq} = K_{sq} L_{aqu} \qquad (3.183)$$

L_{adu} and L_{aqu} are unsaturated values. The saturation factors K_{sd} and K_{sq} identify the degrees of saturation.

- **As illustrated in Figure 3.29, the d-axis saturation is given by The OCC.**
- **Referring to Figure 3.29,**

$$\Psi_I = \Psi_{at0} - \Psi_{at} \qquad (3.186)$$

$$K_{sd} = \frac{\Psi_{at}}{\Psi_{at} + \Psi_I} \qquad (3.187)$$

- **For the nonlinear segment of OCC, Ψ_I can be expressed by a suitable mathematical function:**

$$\Psi_I = A_{sat} e^{B_{sat}\left(\Psi_{at} - \Psi_{TI}\right)} \qquad (3.189)$$

Open-Circuit Characteristic (OCC)

- **Under no load rated speed conditions**

$$\mathbf{i_d = i_q = \Psi_q = e_d = 0}$$

$$\mathbf{E_t = e_q = \Psi_d = L_{ad} i_{fd}}$$

- **Hence, OCC relating to terminal voltage and field current gives saturation characteristic of the d-axis**

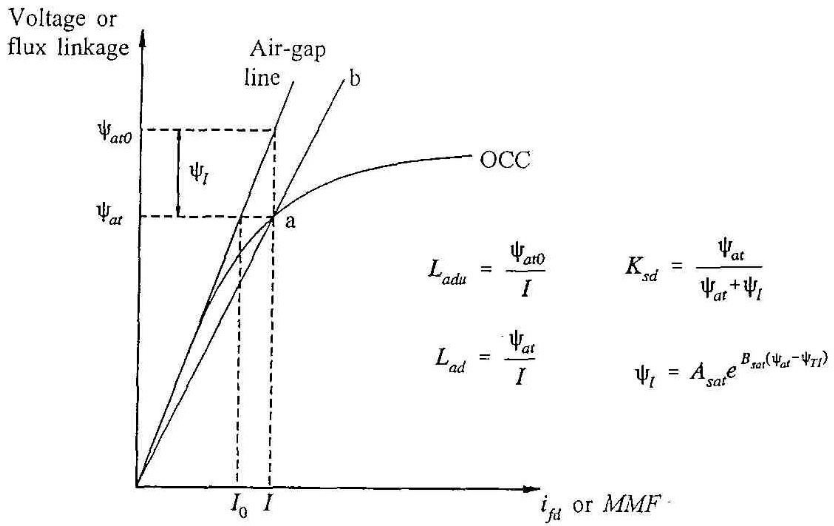


Figure 3.29: Open-circuit characteristic showing effects of saturation

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- For salient pole machines, since q-axis flux is largely in air, L_{aq} does not vary significantly with saturation
  - K_{sq}=1 for all loading conditions
- For round rotor machines, there is saturation in both axes
  - q-axis saturation characteristic not usually available
  - the general industry practice is to assume $K_{sq} = K_{sd}$
- For a more accurate representation, it may be desirable to better account for q-axis saturation of round rotor machines
  - q-axis saturates appreciably more than the d-axis, due to the presence of rotor teeth in the magnetic path
- Figure 3.32 shows the errors introduced by assuming q-axis saturation to be same as that of d-axis, based on actual measurements on a 500 MW unit at Lambton GS in Ontario
  - Figure shows differences between measured and computed values of rotor angle and field current
  - the error in rotor angle is as high as 10%, being higher in the underexcited region
  - the error in the field current is as high as 4%, being greater in the overexcited region

- **The q-axis saturation characteristic is not readily available**
  - **It can, however, be fairly easily determined from steady-state measurements of field current and rotor angle at different values of terminal voltage, active and reactive power output**
  - **Such measurements also provide d-axis saturation characteristics under load**
  - **Figure 3.33 shows the d- and q-axis saturation characteristics derived from steady-state measurements on the 500 MW Lambton unit**

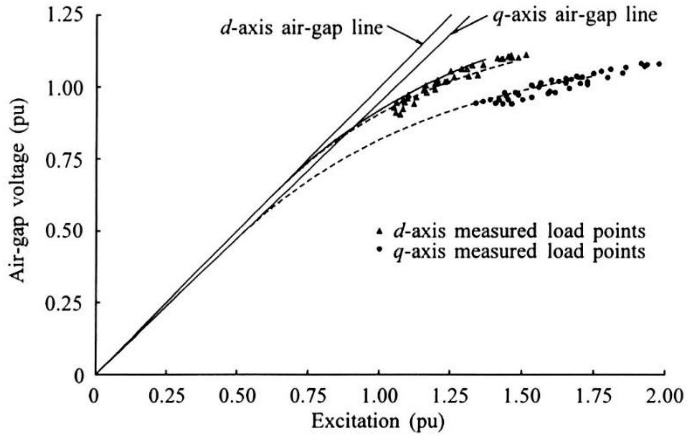


Figure 3.33: Lambton saturation curves derived from steady-state field current and rotor angle measurements

Example 3.3

- **Considers the 555 MVA unit at Lambton GS and examines**
  - **the effect of representing q-axis saturation characteristic distinct from that of d-axis**
  - **the effect of reactive power output on rotor angle**

- Table E3.1 shows results with q-axis saturation <u>assumed same</u> as d-axis saturation

Table E3.1

P_t	Q_t	E_a (pu)	K_{sd}	δ_i (deg)	i_{fd} (pu)
0	0	1.0	0.889	0	0.678
0.4	0.2	1.033	0.868	25.3	1.016
0.9	0.436	1.076	0.835	39.1	1.565
0.9	0	1.012	0.882	54.6	1.206
0.9	-0.2	0.982	0.899	64.6	1.089

- Table E3.2 shows results with <u>distinct</u> d- and q-axis saturation representation

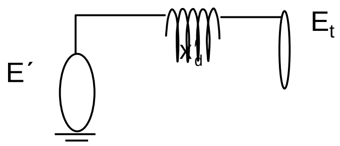
Table E3.2

P_t	Q_t	K_{sq}	K_{sd}	δ_i (deg)	i_{fd} (pu)
0	0	0.667	0.889	0	0.678
0.4	0.2	0.648	0.868	21.0	1.013
0.9	0.436	0.623	0.835	34.6	1.559
0.9	0	0.660	0.882	47.5	1.194
0.9	-0.2	0.676	0.899	55.9	1.074

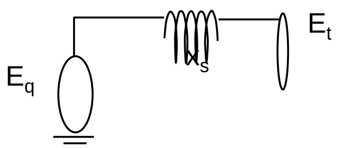
Simplified Models for Synchronous Machines

- **Neglect of Amortisseurs**
  - ☞ **first order of simplification**
  - ☞ **data often not readily available**

- **Classical Model (transient performance)**
  - ☞ **constant field flux linkage**
  - ☞ **neglect transient saliency ($x'_d = x'_q$)**



- **Steady-state Model**
  - ☞ **constant field current**
  - ☞ **neglect saliency ($x_d = x_q = x_s$)**



$E_q = X_{ad} i_{fd}$

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Reactive Capability Limits of Synchronous Machines

- In voltage stability and long-term stability studies, it is important to consider the reactive capability limits of synchronous machines
- Synchronous generators are rated in terms of maximum MVA output at a specified voltage and power factor which can be carried continuously without overheating
- The active power output is limited by the prime mover capability
- The continuous reactive power output capability is limited by three considerations
  - ☞ armature current limit
  - ☞ field current limit
  - ☞ end region heating limit

Armature Current Limit

- **Armature current results in power loss, and the resulting heat imposes a limit on the output**

  The per unit complex output power is

  $$S = P + jQ = \tilde{E}_t \tilde{I}_t^* = |E_t||I_t|(\cos\phi + j\sin\phi)$$

  where Φ is the power factor angle

- **In a P-Q plane the armature current limit, as shown in Fig. 5.12, appears as a circle with centre at the origin and radius equal to the MVA rating**

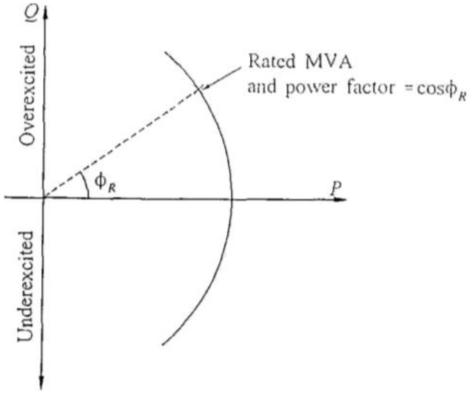

Fig 5.12: Armature current heating limit

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Field Current Limit

- Because of the heating resulting from $R_{fd}I^2_{fd}$ power loss, the field current imposes the second limit
- The phasor diagram relating E_t, I_t and E_q (with R_a neglected) is shown in Fig. 5.13

  Equating the components along and perpendicular to the phasor $\bar{E}_t$

$$X_{ad} i_{fd} \sin\delta_i = X_s I_t \cos\phi$$

$$X_{ad} i_{fd} \cos\delta_i = E_t + X_s I_t \sin\phi$$

  Therefore

$$P = E_t I_t \cos\phi = \frac{X_{ad}}{X_s} E_t i_{fd} \sin\delta_i$$

$$Q = E_t I_t \sin\phi = \frac{X_{ad}}{X_s} E_t i_{fd} \cos\delta_i - \frac{E_t^2}{X_s}$$

- The relationship between P and Q for a given field current is a circle centered at on the Q-axis and with as the radius. The effect of the maximum field current on the capability of the machine is shown in Fig. 5.14
- In any balanced design, the thermal limits for the field and armature intersect at a point (A) which represents the machine name-plate MVA and power factor rating

Field Current Limit

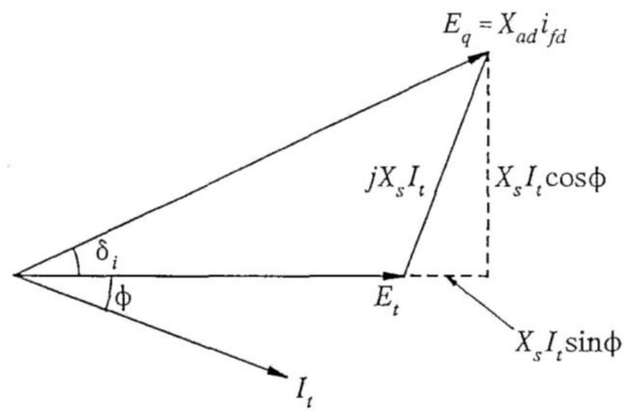

Fig. 5.13: Steady state phasor diagram

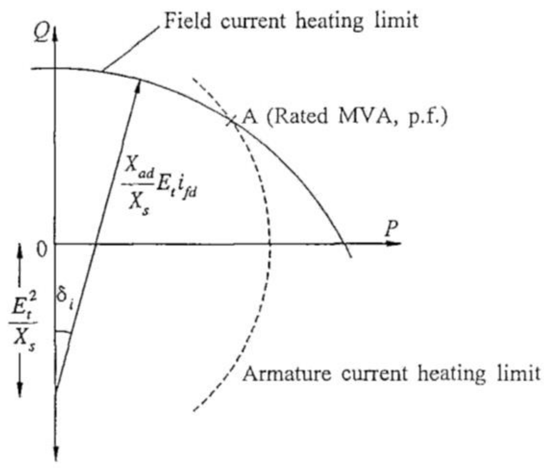

Fig. 5.14: Field current heating limit

End Region Heating Limit

- **The localized heating in the end region of the armature affects the capability of the machine in the <u>underexcited condition</u>**
- **The end-turn leakage flux, as shown in Fig. 5.15, enters and leaves in a direction perpendicular (axial) to the stator lamination. This causes eddy currents in the laminations resulting in localized heating in the end region**
- **The high field currents corresponding to the overexcited condition keep the retaining ring saturated, so that end leakage flux is small. However, in the underexcited region the field current is low and the retaining ring is not saturated; this permits an increase in armature end leakage flux**
- **Also, in the underexcited condition, the flux produced by the armature current adds to the flux produced by the field current. Therefore, the end-turn flux enhances the axial flux in the end region and the resulting heating effect may severely limit the generator output, particularly in the case of a round rotor machine**
- **Fig. 5.16 shows the locus of end region heating limit on a P-Q plane**

End Region Heating Limit

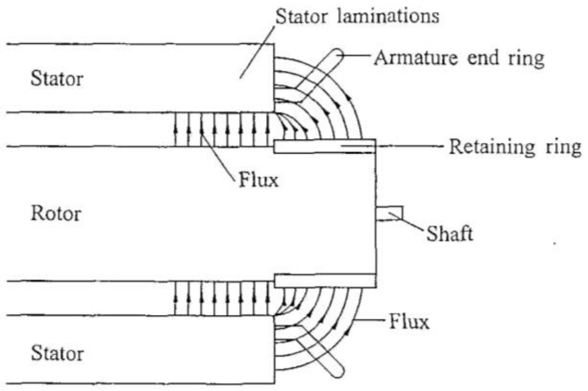

Fig. 5.15: Sectional view end region of a generator

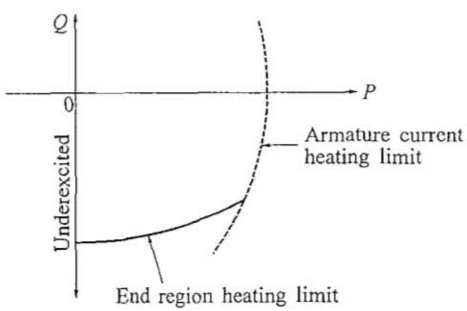

Fig. 5.16: End region heating limit

Reactive Capability Limit of a 400 MVA Hydrogen Cooled Steam Turbine Generator

- **Fig. 5.18 shows the reactive capability curves of a 400 MVA hydrogen cooled steam turbine driven generator at rated armature voltage**
  - **the effectiveness of cooling and hence the allowable machine loading depends on hydrogen pressure**
  - **for each pressure, the segment AB represents the field heating limit, the segment BC armature heating limit, and the segment CD the end region heating limit**

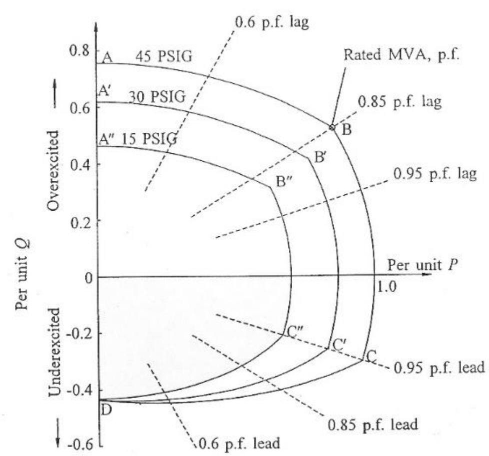

Fig. 5.18: Reactive capability curves of a hydrogen cooled generator at rated voltage

Effect of Changes in Terminal Voltage E_t

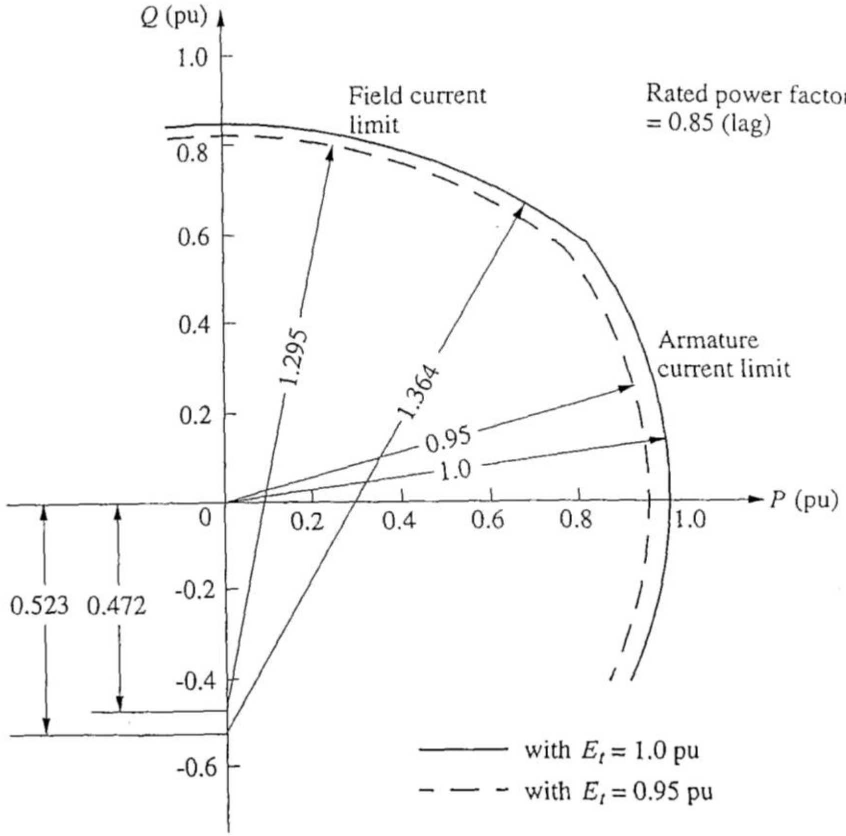

Fig. 5.17: Effect of reducing the armature voltage on the generator capability curve

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