RDDM

Recherche Design Developpement Management

Turbomachinary Lecture Series

Module 04 – Axial Flow Compressors [x]

Presented to - Présenté à

Ecole Polytechnique

AER4270 Propulsion Aeronautique

01 January 2011 – 01 Janvier 2011

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SUBJECT: *Literature research - axial flow compressors*

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INTRODUCTION

The axial flow compressor has the potential for both higher-pressure ratio and higher efficiency than the centrifugal (radial-flow) compressor. The centrifugal compressor often consists of a single stage and it is possible to obtain a higher-pressure rise over a single centrifugal compressor. Another major advantage that axial-flow compressors have, especially for turbo-jet and turbo-fan engines, is that a much larger flow rate is possible for a given frontal area. The axial-flow machine dominates the field for large powers, and centrifugal compressors are restricted to the lower end of the power requirements where the flow is too small to be handled efficiently by axial blade rows.

Through the decades, compressor aerodynamic research and development has resulted in a steady increase in axial-flow compressor pressure ratio, with the effect of reducing the number of stages required to produce those high pressure ratios. However, even though reducing the number of stages has the benefit of decreasing overall engine weight for a specific performance, higher pressure ratios result in high Mach numbers and large gas deflections in compressor stage blade rows.

The difficulties associated with the development of axial-flow compressors stems mainly from the fundamentally different nature of the flow process compared to that in axial-flow turbines. Whereas in the axial-flow turbine the flow relative to each blade row is accelerated, in the axial compressor the flow is decelerated. It is widely known that although a fluid can be rapidly accelerated through a passage and sustain a small or moderate loss in stagnation (total) pressure, the same is not true for a rapid deceleration. In the latter case large pressure losses would arise as a result of severe stall caused by a large adverse pressure gradient. So as to limit the total pressure losses during flow diffusion it is necessary for the rate of deceleration and turning in the compressor blade passage to be severely restricted. It is mainly because of these restrictions that axial-flow compressors need to have many stages for a given pressure ratio compared to axial-flow turbines which need only a few [3].

Careful designs of axial-flow compressor blade rows are based on both aerodynamic theory and experimental data. This is required not only to reduce the various losses that impact overall compressor stage efficiency, but also to ensure a minimum of stalling issues that are prevalent in axial-flow compressor designs.

Successful compressor designs are very much an art, and all the major engine manufacturers have developed a body of knowledge that is kept proprietary for competitive reasons [2]. However, once in a while, after a certain time period, one is able to gather information through publications, doctoral thesis, patents, even news clipping about the various design aspects for compressors. A simple Internet search can uncover valuable information that was once unavailable decades ago.

DISCUSSION – Axial-flow compressor theory & flow physics

A compressor stage is defined as a rotor blade row (rotating circumferentially about a central access at a specified RPM) followed by a stator blade row (which remains stationary). The working fluid is initially accelerated by the rotor blades thus gaining energy, and then decelerated in the stator blade passages wherein the kinetic energy is converted to static pressure. The flow is always subject to an adverse pressure gradient. This adverse pressure gradient, developed during compression, limits the amount of pressure rise that a single axial-flow compressor stage can provide. The higher the pressure ratio the more difficult it becomes to design the axial-flow compressor.

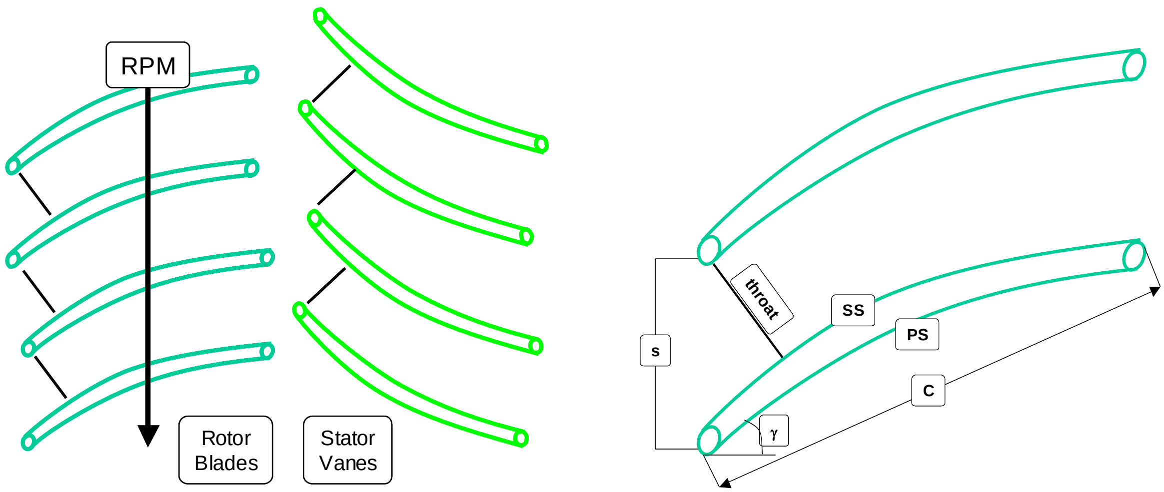

Figure 1 – Rotor-Stator configuration of a compressor stage

Compressor blade row nomenclature

SS	Suction Surface
PS	Pressure Surface
s	Pitch
c	Chord
γ	Stagger angle

All the power is absorbed in the compressor rotor. The stator merely transforms the kinetic energy to an increase in static pressure with the stagnation (total) temperature remaining constant if we assume the compression process to be adiabatic. The increase in stagnation (total) pressure is accomplished wholly in the rotor. The stator and rotor will create stagnation (total) pressure losses due to fluid friction and other loss creating flow mechanisms. There is a strong link and interdependence among the pressure rise, the temperature rise, and the compressor blade shapes. To understand this link one must first understand the impact of compressor velocity triangles. A typical axial-flow compressor velocity triangle is shown below.

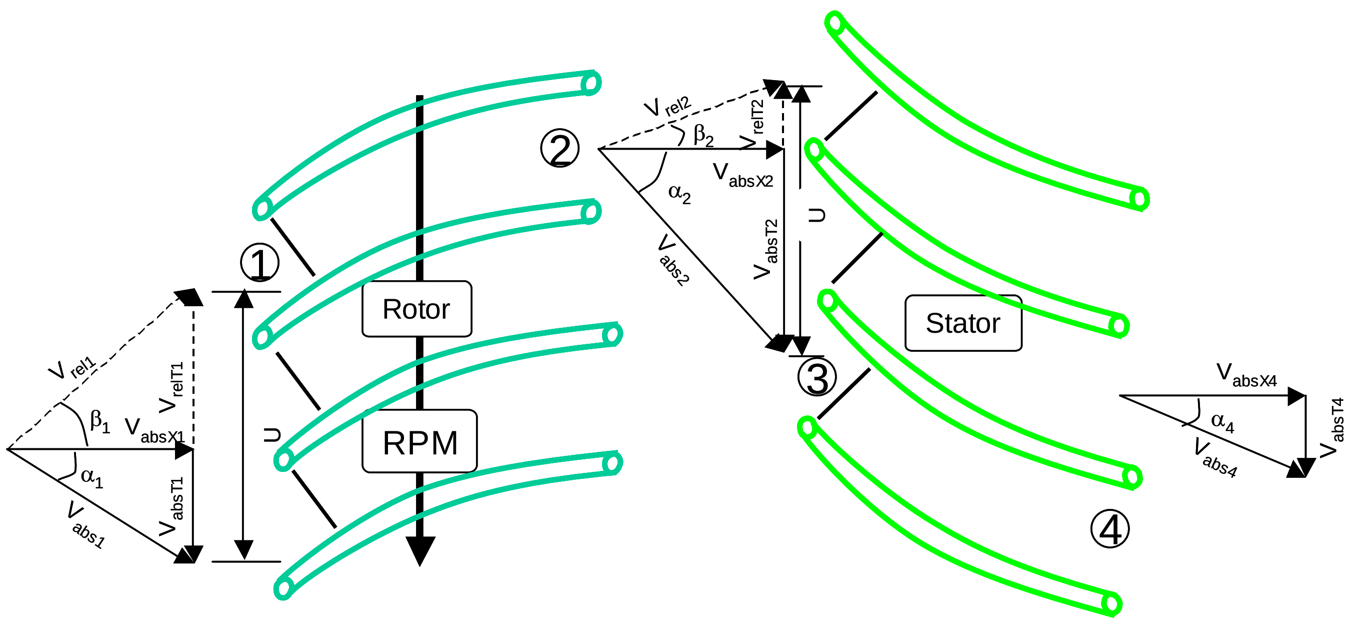

Figure 2 - Typical axial-flow compressor velocity triangles

The fluid enters the rotor with an absolute velocity $\mathbf{V_{abs1}}$ with a flow angle from the axial of α_1. Combining the absolute velocity with the blade speed **U** will give the relative velocity $\mathbf{V_{rel1}}$ and the relative inlet flow angle β_1 with respect to the rotor. The mechanical energy from the rotating rotors will be transferred to the working fluid, increasing the stagnation (total) temperature and stagnation (total) pressure. The energy absorption will also increase the absolute velocity of the fluid. At the exit of the rotor, the fluid will have a relative velocity of $\mathbf{V_{rel2}}$ and a relative exit flow angle of β_2. If there are no radial changes between rotor exit and stator inlet, then it may be assumed that what exits the rotor remains the same as it enters the stator. The fluid leaving the rotor is consequently the same fluid entering the stator rows (station 3 velocity vectors equals station 2 velocity vectors). Combining the relative exit velocity with the blade speed **U** will give the absolute velocity $\mathbf{V_{abs3}}$ and absolute flow angle α_3 that enters the stator. The fluid entering the stator will be further diffused and will leave the stator with an absolute velocity of $\mathbf{V_{abs4}}$ and an absolute flow angle of α_4.

NOTE 1: a so-called "normal" compressor stage is one where the absolute velocities and flow directions at compressor stage outlet (station 4) are the same as at the compressor stage inlet (station 1) [3].

NOTE 2: If there are radial differences between the rotor exit (station 2) and stator inlet (station 3) then the conservation of angular momentum is used to adjust the velocity vectors at station 3 from the velocity vectors at station 2, as follows:

$$\dot{m}_3 r_{m3} V_{absT3} = \dot{m}_2 r_{m2} V_{absT2} \qquad \therefore V_{absT3} = \frac{\dot{m}_2 r_{m2}}{\dot{m}_3 r_{m3}} V_{absT2}$$

The compression process consists of a series of diffusions, both in the rotor and the stator blade passages; although the absolute velocity of the fluid is increased in the rotor, the relative velocity in the rotor is decreased. Using the above velocity triangle nomenclature for a compressor stage, the power input of a compressor is given by (assuming constant mass flow and gas properties) [2]

$$Work = \dot{m} C_p \left(T_{O2} - T_{O1} \right)$$

$$\text{Specific Work} = \frac{W}{\dot{m}} = \Delta h_o = C_p \left(T_{O2} - T_{O1} \right)$$

$$\Delta h_O = h_{O2} - h_{O1} = C_p T_{02} - C_p T_{01}$$

$$\Delta h_O = \left(h_{S2} + \frac{V_{abs2}^2}{2} \right) - \left(h_{S1} + \frac{V_{abs1}^2}{2} \right)$$

The change of momentum between the flow entering and leaving the rotor can be used to calculate the force on the rotor, and the equal and opposite forces on the fluid. There are three principal force components:

- Axial
- Radial
- Tangential

The axial and radial force components cannot contribute to the work transfer between the working fluid and the rotor. Only the tangential force can produce a change in enthalpy through work transfer.

- The tangential forces on the rotor due to the entering fluid and exiting fluid are
  $$TangentialForce\,@\,Inlet = \dot{m} V_{absT1} \qquad TangentialForce\,@\,Exit = \dot{m} V_{absT2}$$
- The torque produced is calculated by the distance from the central axis to radial position of the rotor
  $$Torque\,@\,Inlet = r_1 \left(\dot{m} V_{absT1} \right) \quad Torque\,@\,Exit = r_2 \left(\dot{m} V_{absT2} \right)$$
- The net torque on the rotor will be the difference between the torque produced by the entering fluid and the torque produced by the exiting fluid

$$NetTorque = r_1(\dot{m}V_{absT1}) - r_2(\dot{m}V_{absT2})$$

- The energy transfer will be the net torque and the speed at which the rotor is spinning

$$EnergyTransfer = Work = \omega[r_1(\dot{m}V_{absT1}) - r_2(\dot{m}V_{absT2})]$$

- We know that ωr = the rotor peripheral speed at the radius of the fluid entering or exiting the rotor. If we let $\mathbf{U_m}$ be equal to rotor mean radius speed ωr_m then we'll get

$$EnergyTransfer = Work = U_{m1}(\dot{m}V_{absT1}) - U_{m2}(\dot{m}V_{absT2})$$

$$SpecificWork = \frac{Work}{\dot{m}} = \Delta h_O = U_{m1}V_{absT1} - U_{m2}V_{absT2}$$

The above equation is known as Euler's equation. Positive work means that work is delivered from the turbomachine shaft (the turbine delivers the work rather than the compressor). We also use this equation to calculate the energy transferred to the fluid by the by the compressor. A simple swap of the terms handles this case.

$$SpecificWork = \frac{Work}{\dot{m}} = \Delta h_O = U_2V_{absT2} - U_1V_{absT1}$$

Expanding $\mathbf{V_{absT}}$ using basic trigonometry we'll get

$$\Delta h_O = U_{m2}(V_{absX2}\tan\alpha_2) - U_{m1}(V_{absX1}\tan\alpha_1)$$

We would like to cast $\mathbf{\Delta h_o}$ in terms of the relative flow angles, because these angles will define the rotor inlet and exit metal angles (if we use the assumption that the design point solution equals the rotor metal angles). Using basic trigonometry and the velocity triangles, we will find that

$$U_{m1} = V_{absX1}(\tan\alpha_1 + \tan\beta_1) \qquad U_{m2} = V_{absX2}(\tan\alpha_2 + \tan\beta_2)$$

$$\therefore \tan\alpha_1 = \frac{U_{m1}}{V_{absx1}} - \tan\beta_1 \qquad \therefore \tan\alpha_2 = \frac{U_{m2}}{V_{absx2}} - \tan\beta_2$$

$$\therefore \Delta h_O = U_{m2}V_{absX2}\left(\frac{U_{m2}}{V_{absX2}} - \tan\beta_2\right) - U_{m1}V_{absX1}\left(\frac{U_{m1}}{V_{absX1}} - \tan\beta_1\right)$$

$$\therefore \Delta h_O = U_{m2}^2 - U_{m2}V_{absX2}\tan\beta_2 - U_{m1}^2 + U_{m1}V_{absX1}\tan\beta_1$$

$$\therefore \Delta h_O = (U_{m2}^2 - U_{m1}^2) - U_{m2}V_{absX2}\tan\beta_2 + U_{m1}V_{absX1}\tan\beta_1$$

For a "purely" axial compressor, $\mathbf{r_{m1}}=\mathbf{r_{m2}}$ then $\mathbf{U_{m2}}=\mathbf{U_{m1}}$, and if the inlet and exit area of the rotor are equivalent then $\mathbf{V_{absx1}} = \mathbf{V_{absx2}}$. Therefore $\mathbf{\Delta h_O}$ for a "purely" axial compressor becomes

$$\Delta h_O = U_mV_{absX}(\tan\beta_1 - \tan\beta_2)$$

Combining the following equations together we'll get

$$\Delta h_O = h_{O2} - h_{O1} = C_pT_{02} - C_pT_{01} \text{ and } \Delta h_O = U_mV_{absX}(\tan\beta_1 - \tan\beta_2)$$

$$C_pT_{02} - C_pT_{01} = U_mV_{absX}(\tan\beta_1 - \tan\beta_2)$$

$$\therefore T_{02} - T_{01} = \frac{U_m V_{absX}}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

It can be seen from the above equations that to obtain a high temperature rise in a stage, which is desirable to minimize the number of compressor stages for a given overall pressure ratio, the compressor must combine the following characteristics:

1. High blade speed, either by RPM or blade mean radial location, such that the mid-height compressor blade speed $\mathbf{U_m}$ is high.
2. High axial velocity, which may be related to high mass flow or high Mach numbers
3. High fluid deflection, or camber, in the rotor blades

However, blade stresses will limit the allowable maximum blade speed, plus aerodynamic considerations and the developed adverse pressure gradient will limit the mass flow rate and the camber (fluid deflection) of the compressor blades.

The velocity triangles can be joined together in several different ways to help visualize the changes in velocity entering and exiting the compressor rotor and stator. Two common methods are:

- Superimposed using the axial velocity as the datum. In doing so we're assuming that the axial velocity is constant throughout the compressor stage
- Or superimposed using the blade speed (U_m) as the common datum. In doing so we're assuming that the inlet and exit blade speeds are equal, i.e. they are at the same radial location.

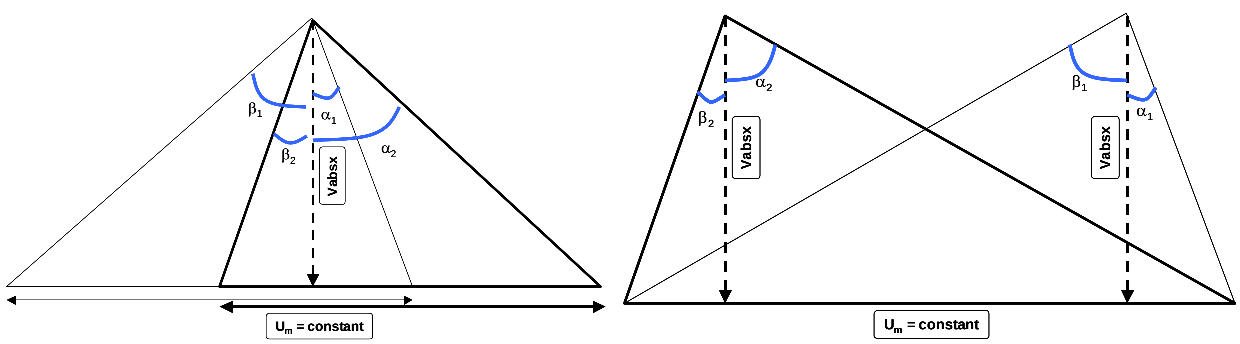

Figure 3 - Velocity Triangle Visualization

So far we've shown the impact of velocity triangles on temperature rise. But what about pressure rise? Do we just assume the compression leads to a pressure rise because we just use the word "compression"? Of course not, that's too simple. We do know however that temperatures and pressures are directly related to each other. Using the adiabatic isentropic relationship P_S/ρ_S together with $P_S=\rho_S RT_S$ we'll get the expression

$$\frac{T_{O2}}{T_{O1}} = \left(\frac{P_{O2}}{P_{O1}}\right)^{\gamma-1/\gamma}$$

So if we arrange the above equation as follows

$$T_{O1}\left(\frac{T_{02}}{T_{01}} - 1\right) = \frac{U_m V_{absX}}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

$$\therefore T_{O1}\left[\left(\frac{P_{O2}}{P_{O1}}\right)^{\gamma-1/\gamma} - 1\right] = \frac{U_m V_{absX}}{C_p}\left(\tan\beta_1 - \tan\beta_2\right)$$

Knowing the compressor temperature rise will directly lead itself to knowing the pressure rise.

o Rothalpy and Velocity Triangles

One of the more obscure assumptions employed, usually not well defined in gas turbine engine theory, is the conservation of rothalpy. Rothalpy, **I**, is defined as follows:

$$Rothalpy, I = h_{orel} - \frac{1}{2}U^2 = c_p T_{orel} - \frac{1}{2}U^2$$

Since the inlet Rothalpy must equal the exit rothalpy for a rotor

$$\left(h_{orel} - \frac{1}{2}U^2\right)_{IN} = \left(h_{orel} - \frac{1}{2}U^2\right)_{EX}$$

$$\left(c_p T_{orel} - \frac{1}{2}U^2\right)_{IN} = \left(c_p T_{orel} - \frac{1}{2}U^2\right)_{EX}$$

$$\therefore \left(\frac{1}{2}U^2\right)_{EX} - \left(\frac{1}{2}U^2\right)_{IN} = \left(c_p T_{orel}\right)_{EX} - \left(c_p T_{orel}\right)_{IN}$$

$$\therefore \frac{1}{2}\left(U_{EX}^2 - U_{IN}^2\right) = c_p\left(T_{orel-EX} - T_{orel-IN}\right)$$

This conservation of rothalpy is the same for both compressors and turbines

o Compressor Reaction

Following the above velocity triangle nomenclature, for the case of incompressible and reversible flow it is permissible to define the reaction of a compressor stage as the ratio of static pressure rise in the rotor to the static pressure rise across the stage, or the static enthalpy rise in the rotor to the stagnation (total) enthalpy rise through the compressor stage

$$\text{Reaction} = R = \frac{\Delta \text{Rotor P}_{static}}{\Delta \text{Stage P}_{static}} = \frac{P_{S2} - P_{S1}}{P_{S4} - P_{S1}}$$

$$\text{Reaction} = R = \frac{\Delta \text{Rotor h}_S}{\Delta \text{Stage h}_O} = \frac{h_{S2} - h_{S1}}{h_{O4} - h_{O1}}$$

If we assume that we have a "normal" compressor stage and that the axial velocity remains constant across the compressor stage, we can write the reaction in terms of velocity-triangles as follows (derivation not included):

$$\text{Reaction} = \frac{1}{2}\frac{V_{absX}}{U_m}\left(\tan\beta_1 + \tan\beta_2\right) = \frac{1}{2}\phi\left(\tan\beta_1 + \tan\beta_2\right)$$

The reaction ratio is a design parameter that has an important influence on stage efficiency. Stages having 50% reaction are widely used as the adverse pressure (retarding) gradient through the rotor and stator rows is equally shared. This choice of reaction minimizes the tendency of the blade boundary layers to separate from the solid surfaces, thus avoiding large stagnation (total) pressure losses [3].

- If reaction > 50% then $\beta_2 > \alpha_1$ and the velocity diagram is skewed towards the right. The static enthalpy rise in the rotor exceeds that in the stator.

- If reaction = 50% then $\beta_2 = \alpha_1$ (or the rotor inlet angle equals the rotor outlet angle) and the velocity triangle is symmetrical. The stage enthalpy rise is equally distributed between the rotor and stator rows.

- If reaction < 50% then $\beta_2<\alpha_1$ and the velocity diagram is skewed towards the left. The stator enthalpy rise exceeds that in the rotor.

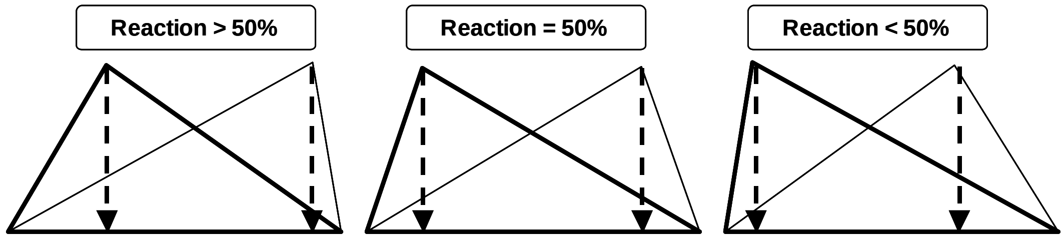

Figure 4 - Impact of Reaction on Velocity Triangles

In axial turbines the limitation on stage work output is imposed by the rotor blade stresses. In axial compressors, stage performance is limited by Mach number consideration. If Mach number effects could be ignored, the permissible temperature rise (based on incompressible flow cascade limits) increases with the amount of reaction. With a limit of 0.7 on the allowable Mach number, the temperature rise and efficiency for axial-flow compressors are at a maximum with a reaction of 50 % [3].

But in practice a higher stage reaction is preferred. Increasing the stage reaction results in a decrease in tangential velocity prior to the rotor. A smaller whirl will create a larger relative velocity to the rotor row making it easier for the rotor to increase the static pressure [4].

- **Stage loading coefficient [1, 3]**

The total enthalpy rise through a rotor blade is expressed by the Euler equation. It is often useful to introduce dimensionless stage performance parameters. One of those parameters is the “stage loading” coefficient (or factor), also known as the “work” or “temperature coefficient”. This value is positive for turbines and negative for compressors.

The “stage loading” coefficient is an important design parameter for a compressor stage rotor, and is one that strongly affects the off-design performance characteristics. Its value can be determined from the stage velocity diagram. In compressor stages a highly and lowly loaded stage would be as follows:

- Highly loaded $\psi > 0.5$
- Lowly loaded $\psi < 0.3$

$$\Delta h_O = U_{m2}V_{absT2} - U_{m1}V_{absT1}$$

$$\psi = \frac{\Delta h_O}{U_m^2} = \frac{U_{m2}V_{absT2} - U_{m1}V_{absT1}}{U_m^2}$$

$$\psi = \frac{\Delta h_O}{U_m^2} = \frac{c_p \Delta T_O}{U_m^2}$$

Assuming that $\mathbf{U_m}$ is constant at stations 1 and 2

$$\psi = \frac{\Delta h_O}{U_m^2} = \frac{V_{absT2} - V_{absT1}}{U_m}$$

Data from compressor cascade tests show that a design-point “stage loading” ψ should be in the range of 0.3 to 0.4 for the most efficient operation, however substantial variations of ψ can be expected at off-design conditions.

The "stage loading" coefficient alone is insufficient to specify the aerodynamic or boundary layer loading on the compressor blade rows and for the inner & outer walls. With the same value of ψ and a high or low value of axial velocity, the required flow deflection through any blade row could be low or high, respectively. We therefore need to define a second velocity-diagram dimensionless parameter known as the "stage flow" coefficient [1].

- **Stage flow coefficient**

The "stage flow" coefficient, also known as the "velocity" coefficient, expresses the ratio between the meridional (or axial velocity) and the blade rotational velocity. A high "stage flow" coefficient indicates a high flow through the stage relative to the blade rotational velocity. Assuming a "purely" axial compressor

$$\phi = \frac{V_{abs-m}}{U_m} = \frac{V_{absX}}{U_m}$$

NOTE: $\mathbf{V_{absX}}$ is directly linked the compressor stage mass flow rate and the stage area.

It can be shown that the "stage loading" is related to the "stage flow" coefficient by

$$\psi = 1 - \phi(\tan\alpha_3 + \tan\beta_2) \text{ or } \psi = \phi(\tan\beta_1 - \tan\beta_2)$$

For a "normal" compressor, where $\alpha_4 = \alpha_1$, we'll have

$$\psi = 1 - \phi(\tan\alpha_1 + \tan\beta_2)$$

Where

- β_2 is the rotor relative flow exit angle
- α_4 is the stator absolute flow exit angle

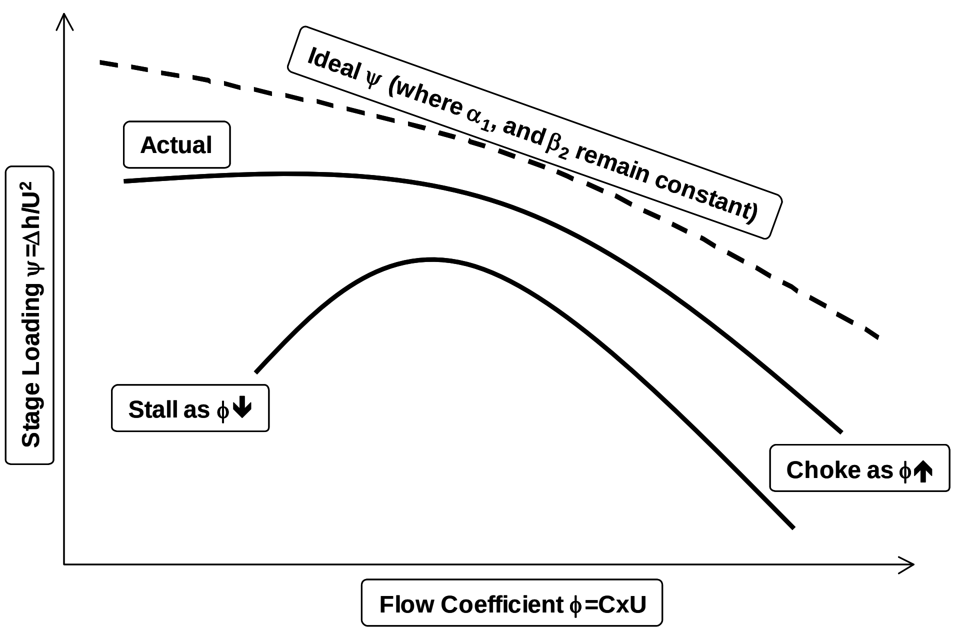

Figure 5 - Interaction of Flow Coefficient and Stage Loading

DISCUSSION – Compressor surge & stalling

An important part of compressor performance, as shown in a compressor map, is the limit to stable operation called the surge line. This limit can be reached by reducing the mass flow while the rotational speed is maintained. When a compressor goes into surge the effects are usually quite dramatic. Generally an increase in noise level is experienced, indicative of a pulsation of the airflow and mechanical vibration. Surge involves an axial oscillation of the total mass flow, a condition highly detrimental to efficient compressor design [3].

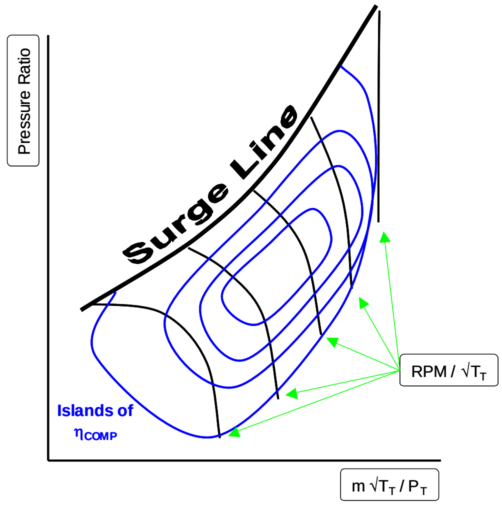

Figure 6 - Typical compressor map

In a typical compressor it is normal that if the mass flow is reduced the pressure ratio increases. At a certain point in an operating range the pressure rise is at its maximum. A further reduction in mass flow will lead to an abrupt and definite change in flow pattern in the compressor. This change in flow pattern is known as surge and can cause the flow to start oscillating backwards and forwards, and after a while the compressor will break down. A mild version of surge causes the operating point to orbit around the point of maximum pressure rise. An audible burble is a clear indicator when the compressor is on the limit of a more severe surge [4].

Stalling arises when the difference between the flow direction and the blade angle becomes too excessive. The fact that the pressure gradient is acting against the flow direction is always a danger to compressor flow stability. Flow reversals may occur at off-design conditions of mass flow rates and rotational speeds that are different from the design-point condition to which the compressor was developed for.

If the mass flow is reduced the axial velocity will, according to the continuity equation, also decrease. This will increase the air inlet angle and, due to the difference in air inlet angle and blade inlet angle, will create flow incidence. With an increasing incidence angle the flow will eventually separate from the surface at the trailing edge. The separation will grow with a further increase in incidence angle, and finally cover the whole blade. Stall changes the performance characteristics of an axial-flow compressor [4].

Two types of stall could be found in a compressor blade row:

- **Blade stall**
  It is a two-dimensional type of stall where a significant part of the blade has a large wake leaving the blade suction surface.

- **Wall stall**
  It is a type of stall related to the boundary layer growth of the flow over the gas path surfaces.

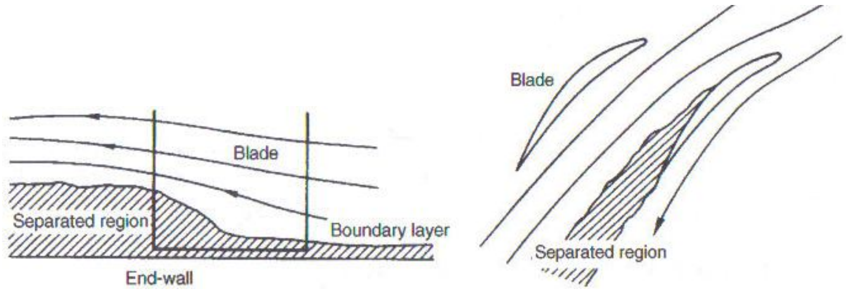

Figure 7 – Wall and Blade stall [3]

Rotating stall is a particular phenomenon of axial-flow compressors. Rotating stall is a mechanism that allows the compressor to adapt to a mass flow that is too small. Instead of trying to share the limited flow over the whole annulus, the flow is shared unequally such that some areas have a larger mass flow than other areas.

When a rotor blade row reaches the "stall point", the blades do not stall all together, but instead stall in patches and these patches travel around the compressor annulus. These stall patches propagate from blade to blade in the direction of rotation. The stall patch causes a partial obstruction to the flow that is deflected on both sides of it. The incidence of the flow on to the blades on the right of the "stall cell" is reduced. The incidence of the flow on the blades to the left of the "stall cell" is increased. As these blades (those to the left and to the right) are already close to stalling, the effect is for the "stall patch" to move to the left, and the motion is self-sustaining. "Stall patches" travelling around the blade rows load and unload each blade at some frequency related to the speed and number of the "stall patches". This frequency may be close to a natural frequency of blade vibration, which will induce undue vibratory stresses, and possible blade failure due to frequency resonance may occur.

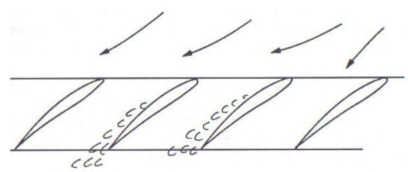

Figure 8 – stall cell propagation [3]

The cells always rotate in the direction of the rotor. Full-span cells extend axially through the whole compressor while part-span cells can exist in a single blade row [4].

- o Part-span cells very often rotate at close to 50% of the rotor speed
- o Full-span cells usually rotate more slowly in the range of 20 to 40% of the rotor speed.

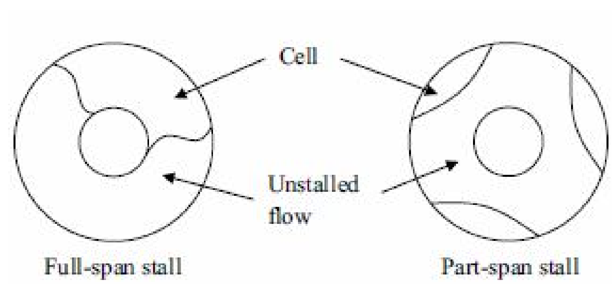

Figure 9 - Different types of rotating stall [4]

There is considerable freedom to choose the design value of "stage flow: coefficient, particularly in those cases where inlet Mach numbers is not a problem to be avoided. Compressor blades with the same "de Haller" ratio but different stagger (setting) angles will have different design point characteristics and different off-design characteristics.

- The high-work, high-flow, low stagger compressor blades tend to have a sharp stall, with a large fall is pressure rise and possibly a hysteresis effect that makes recovery from stall more difficult

- The low-work, low-flow, high-stagger compressor blades tend to have an almost imperceptible stall, and a stalled pressure rise higher than at design-point condition, with no drop off in pressure differential at the stall itself.

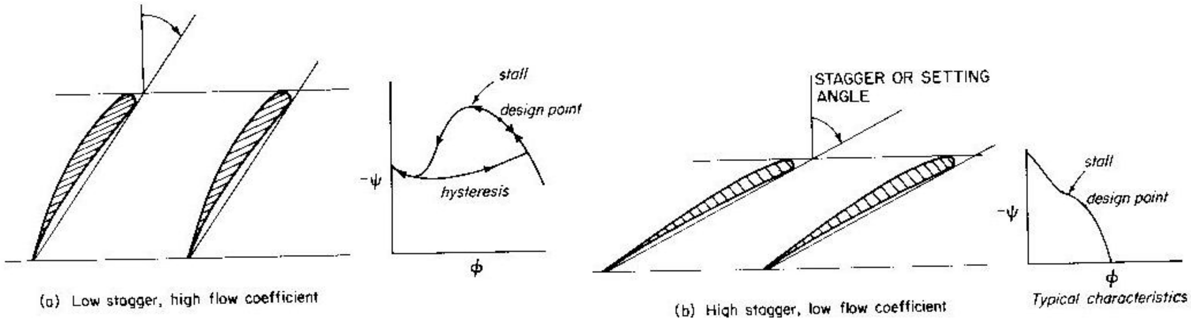

Figure 10 - low vs high stagger compressor blades and stall characteristics [1]

DISCUSSION – Compressor stage design

For assembly purposes, the rotor blades are fixed to the rotor drum (or shaft) and the stator blades are fixed to the outer casing. The stator rows are not fixed to an inner ring as turbine stator rows. Instead, they are only affixed from the outer rings creating stator hub clearances. In doing so, each compressor stage rotor and stator can be assembled by stacking one-by-one the individual blade rows.

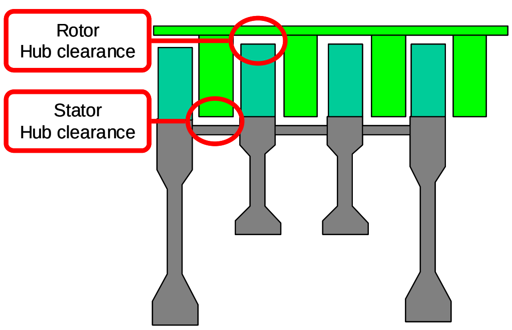

Figure 11- Assembly stacking of compressor blade rows

On some first stage axial-flow compressors, the first rotor row has an upstream row of inlet guide vanes (IGV). These are not considered to be a part of the compressor stage and are treated differently. Their function is quite different form the other compressor rotor or stator blade rows. The inlet guide vanes direct flow away from the axial direction, and they act to accelerate the flow rather than diffuse it. Functionally, inlet guide vanes are the same as turbine nozzles; they increase the kinetic energy of the flow at the expense of pressure energy [3].

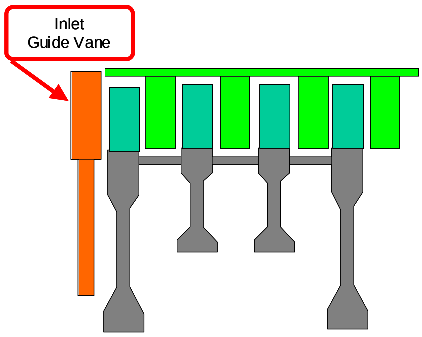

Figure 12 - Compressor stage with IGV

It is desirable to keep the axial velocity approximately constant throughout a multi-stage compressor. With the flow density increasing as it progresses through the stages, it will be necessary to reduce the flow area and hence the airfoil height. When the compressor is running at a lower speed than the design-point speed, the density in the rear stages will be far from the design value, resulting in incorrect axial velocities which will cause blade stalling and compressor surge. There are various design methodologies that can be used (individually or in combination) to overcome these problems, all of which result in mechanical complexity:

- Use a multi-spool configuration
- Use of variable stator blades
- Use of blow-off valves

In the field of aircraft gas turbines the engine designer is more concerned with maximizing the work done per stage while retaining an acceptable level of overall performance. Increased "stage loading" almost inevitably leads to some aerodynamic constraint. This constraint will be increased Mach number, possibly giving rise to shock-induced boundary layer separation or increased losses arising from poor diffusion of the flow [3].

o **Estimating the number of compressor stages for a given pressure-ratio**

It is possible to apply the various equations, described in this module, to determine a preliminary number of compressor stages for a given pressure-ratio. The procedure requires the calculation of pressure and temperature changes for a single stage, and the stage exit conditions are then used for the next compressor stage as the inlet conditions. This calculation is repeated for each stage in turn until the final conditions are satisfied.

However, for compressors having identical stages it is more convenient to resort to a simple compressible flow analysis [3]. The procedure is as follows:

1. Gather the list of requirements for
    a. Required overall pressure-ratio
    b. The compressor inlet temperature
    c. Stage RPM
    d. Stage reaction (assume as the same for all the stages)
    e. U_m, mean blade speed (assume as the same for all stages)
    f. ϕ, Flow coefficient (assume as the same for all the stages)
    g. ψ, Stage loading factor (assume as the same for all the stages)

2. Use the following equations to obtain the values for $\tan\beta_1$ and $\tan\beta_2$ as follows

$$\psi = \phi\left(\tan\beta_1 - \tan\beta_2\right)$$

$$\text{Reaction} = \frac{1}{2}\phi\left(\tan\beta_1 + \tan\beta_2\right)$$

$$\therefore \tan\beta_1 = \frac{\left(\text{Reaction} + \psi/2\right)}{\phi} \quad \text{and} \quad \tan\beta_2 = \frac{\left(\text{Reaction} - \psi/2\right)}{\phi}$$

3. Use the following equation to obtain the individual stage stagnation (total) temperature rise as follows

$$\psi = \frac{\Delta h_O}{U_m^2} = \frac{c_p \Delta T_O}{U_m^2}$$

4. Denoting the overall compressor inlet and outlet conditions as I and II use the following equation to assess the number of compressor stages

$$\frac{T_{OII}}{T_{OI}} = 1 + \frac{N_{STAGES}\Delta T_O}{T_{OI}} = \left(\frac{P_{OII}}{P_{OI}}\right)^{\gamma-1/\gamma\eta_c} \quad \therefore N_{STAGES} = \frac{T_{OI}}{\Delta T_O}\left[\left(\frac{P_{OII}}{P_{OI}}\right)^{\gamma-1/\gamma\eta_c} - 1\right]$$

It should be reminded that the above procedure is an overly simplified method to obtain a quick estimate for the number of compressor stages required to obtain a particular overall pressure-ratio. Assuming constant reaction, mean blade speed, stage loading, and flow coefficient across all the stages of a compressor is an idealized design assumption.

o **Work done factor [2, 3]**

Because of the adverse pressure gradient in compressors, the boundary layers along the annulus walls thicken as the flow progresses. The main effect is to reduce the area available for flow below the geometric area of the annulus. This will have a considerable effect on the axial velocity through the compressor and must be allowed for in the design process. The flow is extremely complex, with successive accelerations and decelerations combined with changes in tangential flow direction [2].

The radial distribution of axial velocity is not constant across the annulus, but becomes peaky as the flow proceeds, settling down to a fixed profile at about the fourth stage of a multi-stage compressor. Over the central region of the blade, the axial velocity is higher than the average value. The mean blade section (and most of the blade span) will, therefore, do less work than is estimated from the velocity triangles based on the mean axial velocity [3].

Due to the influence of both end wall boundary layers and clearance vortices, the total work capacity of a stage is decreased. This effect becomes more pronounced as the number of compressor stages is increased. The reduction in work capacity can be accounted for empirically, as shown in the figure below, with the "work done factor" λ, which is always a number less than 1.

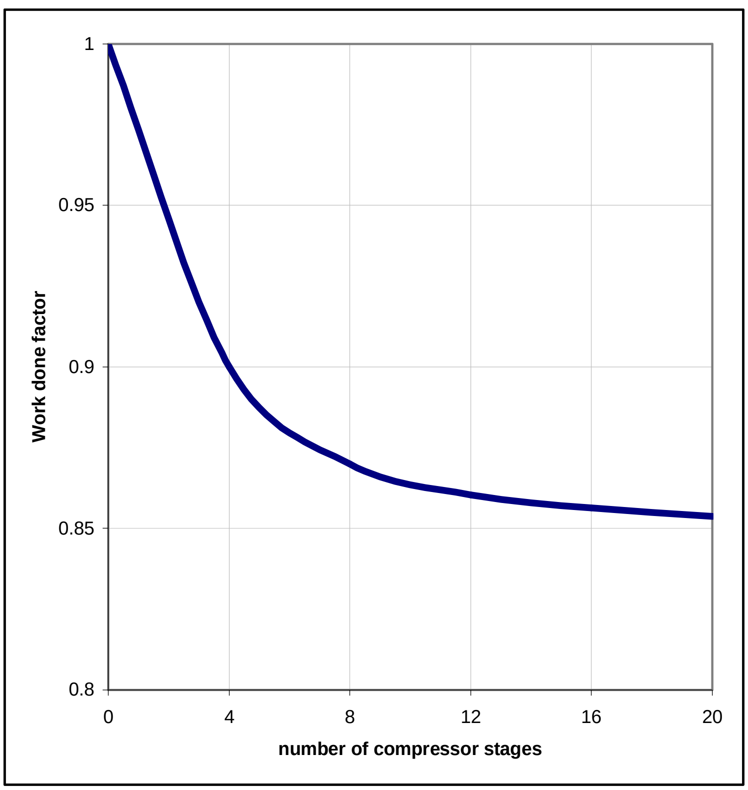

Figure 13 – approximated work done factor

The “work done” factor can be applied to the “stage loading” coefficient as shown below, and the estimation process mentioned above can be adjusted to reflect the change.

$$\psi = \lambda \cdot \phi(\tan \beta_1 - \tan \beta_2)$$

DISCUSSION – Compressor blade design

Three types of compressor airfoil design can be used for different Mach numbers:

- **Subsonic**
  For flows that are entirely subsonic, it has been found that the use of aerofoil section type blading could be used to obtain high stage efficiencies. The need to pass higher mass flow rates at high pressures increase the Mach numbers, which become especially critical at the tip of the first row of rotor blades.

- **Transonic**
  For transonic compressors, where the flow over part of the blade is supersonic, it’s been found that the most effective blade shape consists of sections of circular arcs, often referred to “biconvex” blading.

- **Supersonic**
  For much higher Mach numbers, it’s been found that blade profiles based on parabolas become more effective.

The choice of blade aspect ratio also has an impact on compressor design characteristics:

- Usage of high aspect ratio airfoil makes engines more compact in axial length.
- Usage of low aspect ratio airfoil in axial-flow compressors has high loading capabilities, high efficiency, and a good performance range.

There have been three types of compressor blade designs that have been investigated, and data recorded as performance maps, losses, or correlations. These blade types are the:

- National Advisory Committee for Aeronautics, NACA 65-series compressor blades
- National Gas-Turbine Establishment, British C-series, C4 and C7 compressor blades
- Double Circular Arcs (DCA) (**Source Unknown**)

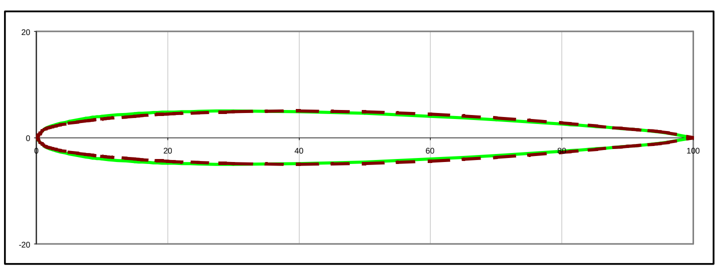

Figure 14 - British C4 (solid) and C7 (dashed) compressor-blade base profiles

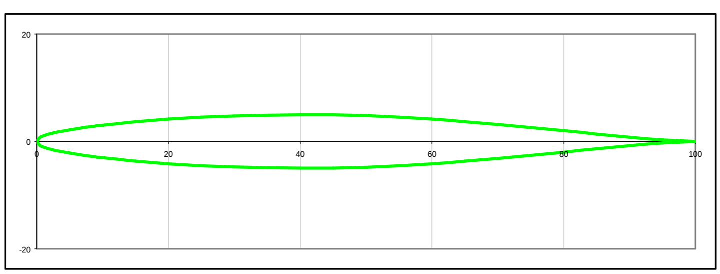

Figure 15 - NACA 65-series compressor-blade base profile

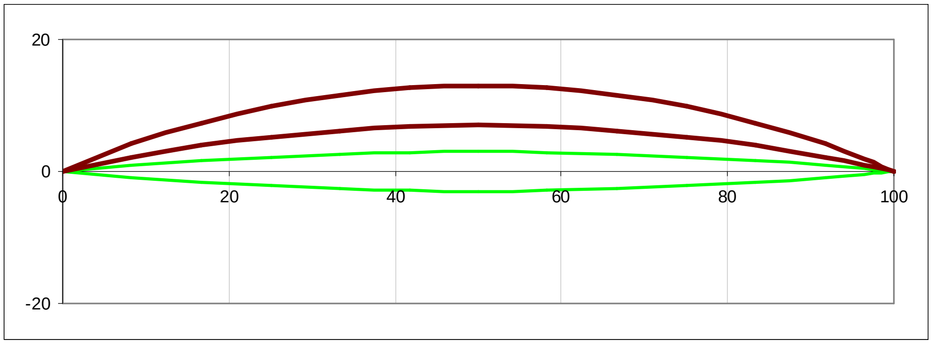

Figure 16 - Double Circular Arc

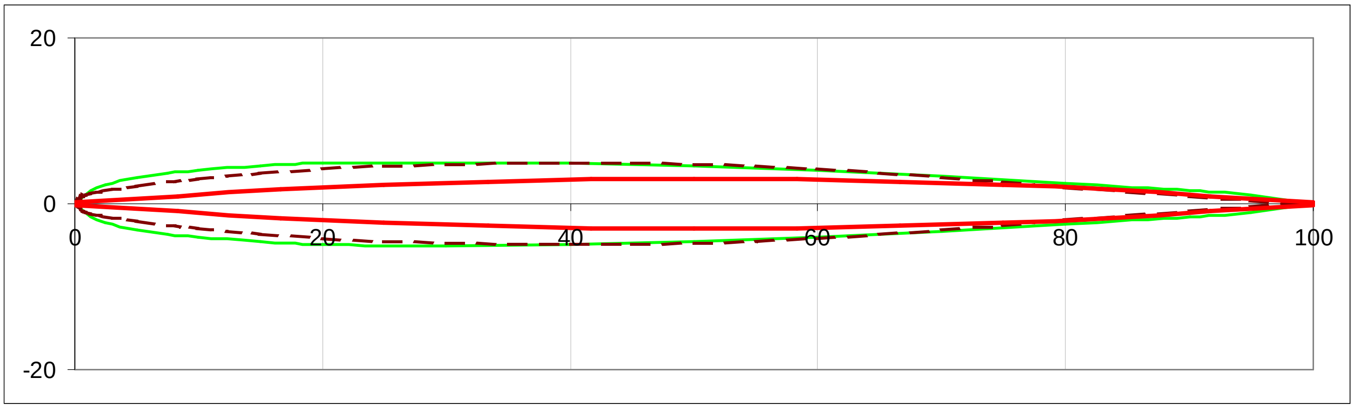

Figure 17 - comparison of compressor-blade base profiles of

British C4 (solid), NACA 65-series (dashed), and DCA (middle solid)

- **Incidence angles**

The "incidence" angle is the difference between the inlet blade angle and the inlet flow angle. As the fluid approaches the leading edge of a compressor airfoil it will experience an "induced" incidence.

Incidence is one of the pressure loss creating mechanisms found in axial-flow compressors.

- **Deviation angles**

The "deviation" angle is the difference between the exit blade angle and the exit flow angle. It arises from a combination of two effects. Firstly the flow is decelerating on the suction surface and accelerating on the pressure surface as it approaches the blade trailing edge. The result of this is that the streamlines are diverging from the suction surface and converging towards the pressure surface so that the mean (average) flow angle is less than the blade angle. This is an inviscid effect that increases in magnitude with the rate of diffusion and acceleration towards the trailing edge. Secondly, the rapid boundary layer growth on the suction surface towards the trailing edge "pushes" the streamline away from the surface, contributing to the deviation [4].

The deviation angle is dependent on blade camber and stagger angle, making it difficult to predict.

- **"de Haller" number**

As mentioned before, in an axial-flow compressor stage both the rotor and stator are designed to diffuse the flow. The flow kinetic energy is converted into an increase in the stage static enthalpy and the stage static pressure. The more the fluid is decelerated the bigger is the pressure rise, and with the rise in pressure is a rise in the adverse pressure gradient. Blade boundary layer growth and stall are the limiting flow mechanisms on a compressor blade design.

A simple dimensionless parameter is used (suggested by Mr. de Haller a Swiss engineer) by the compressor designer to choose an appropriate diffusion factor. This dimensionless parameter compares the rotor relative velocity ratio and the stator absolute velocity ratio as follows

$$\text{de Haller No.} = \left(\frac{V_{rel2}}{V_{rel1}}\right)_{rotor}, \left(\frac{V_{abs4}}{V_{abs3}}\right)_{stator}$$

Historically this value has been set to be more than 0.72 as an appropriate target. Lower values lead to excessive losses. Because of its extreme simplicity the "de Haller" number is still used in preliminary design work, but for final design calculations a criterion called the "diffusion factor" is preferred [2].

- **Diffusion Factor and Diffusion Ratio**

The air passing over a compressor blade will accelerate to a higher velocity on the suctions surface. In a stationary row this will give rise to a drop in static pressure. On the pressure surface, the fluid will be decelerated. The velocity distribution through the blade passage will be as shown below. The maximum velocity on the suction surface will occur at around 10 to 15% of the blade chord from the leading edge, and will then fall steadily until the outlet velocity is reached.

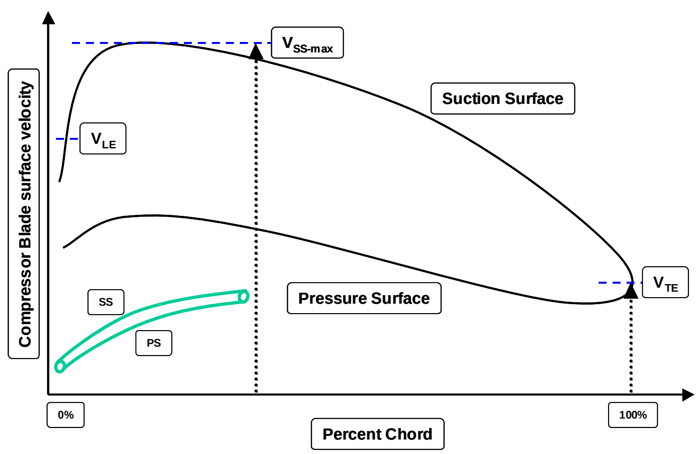

Figure 18 - compressor cascade blade surface velocity distribution

The losses in a compressor blade row arise primarily from the growth of boundary layers on the suction and pressure surfaces of the blade. These surface boundary layers come together at the blade trailing edge to form a wake, giving rise to a local drop in the stagnation (total) pressure. Relatively thick surface boundary layers, resulting in high losses, have been found to occur in regions where rapid changes of velocity are occurring [2].

The blade loading is usually assessed by the "diffusion factor", DF. This relates the peak velocity on the suction surface of the blade to the velocity at the trailing edge. Values of diffusion factors in excess of 0.6 are thought to indicate blade stall and values of 0.45 might be taken as a typical design choice.

$$DF = \frac{V_{SS-\max} - V_{TE}}{V_{LE}}$$

The "diffusion ratio" **DR** is that ratio between the maximum velocity and the outlet velocity

$$DR = \frac{V_{SS-\max}}{V_{TE}}$$

NACA Diffusion Factor [2,4]

The derivation of the NACA diffusion factor is based on the establishment of the velocity gradient on the suction surface in terms of V_{LE}, V_{TE}, and $V_{SS\text{-}max}$ in conjunction with results from cascade tests. From the cascade tests it was deduced that the maximum velocity on the suction surface is

$$V_{SS-\max} = V_{LE} + \frac{\Delta V_{relT}}{2}\frac{s}{c}$$

The above diffusion factor then becomes

$$DF = \frac{V_{SS-\max} - V_{TE}}{V_{LE}} = \frac{V_{LE} + \frac{\Delta V_{relT}}{2}\frac{s}{c} - V_{TE}}{V_{LE}} = 1 + \frac{\Delta V_{relT}}{2 \cdot V_{LE}}\frac{s}{c} - \frac{V_{TE}}{V_{LE}}$$

Lieblein's "equivalent" Diffusion Ratio [3]

Lieblein's correlation is based on the experimental observation that a large amount of velocity diffusion on blade surfaces tends to produce thick boundary layers and eventual flow separations. Lieblein states the general hypothesis that in the region of minimum loss, the wake thickness and consequently the magnitude of the loss in total pressure is proportional to the diffusion in velocity on the suction surface of the blade in that region. The hypothesis is based on the consideration that the boundary layer on the suction surface of conventional compressor blades contributes the largest share of the blade wake. Therefore the suction surface velocity distribution becomes the main factor in determining the pressure loss [3].

Knowledge of the suction surface velocities before an actual design is difficult to predict. As this data may not be available it is necessary to establish an "equivalent" diffusion ratio that approximates $V_{SS\text{-}max}/V_{TE}$.

$$\mathrm{DR}_{\mathrm{eq}} = \frac{V_{SS-\max}}{V_{TE}} = \frac{\cos\alpha_1}{\cos\alpha_2}\left[1.12 + 0.61\left(\frac{s}{c}\right)\cos^2\alpha_1\left(\tan\alpha_1 - \tan\alpha_2\right)\right] \text{[ref Wilson]}$$

$$\mathrm{DR}_{\mathrm{eq}} = \frac{V_{SS-\max}}{V_{TE}} = \frac{\cos\alpha_2}{\cos\alpha_1}\left[1.12 + 0.61\left(\frac{s}{c}\right)\cos^2\alpha_1\left(\tan\alpha_1 - \tan\alpha_2\right)\right] \text{[ref Dixon]}$$

$$\mathrm{DR}_{\mathrm{eq}} = \frac{V_{SS-\max}}{V_{TE}} = \frac{\cos\beta_2}{\cos\beta_1}\left[1.12 + 0.61\left(\frac{s}{c}\right)\cos^2\beta_1\left(\tan\beta_2 - \tan\beta_1\right)\right] \text{[ref Falck]}$$

WARNING: literature research has found 3 different representations for Lieblein's DR_{eq} equation. Currently there is no consensus as to which is the correct equation to use. The equations are to be used with caution.

Koch and Smith "equivalent" Diffusion Ratio

Koch & Smith further modified Lieblein's "equivalent" diffusion ratio by including the impact due to maximum thickness vs chord ratio (t_{max}/c) and a parameter called the Axial Velocity Density ratio (AVDR).

NOTE: Their derivation has not yet been included in this literature research

DISCUSSION - Compressor loss modeling

The design of an axial-flow compressor of high performance involves three-dimensional high-speed flow of compressible viscous gases through successive rows of closely spaced blades [5].

Various aspects of the problem have been treated theoretically, and the results of those studies are quite useful in design calculations. All such studies, however, have been on idealized flow, with the effects of one or more such physical realities as compressibility, finite blade spacing, and viscosity neglected. Consideration of viscosity effects has been particularly difficult [5].

Some of the information required can be obtained only by experiment in single-stage and multi-stage compressors. Much of the information, however, can be obtained more easily by isolating the effects of each parameter for detailed measurement. The effects of inlet angle, blade shape, angle of attack, and solidity on the turning angle and drag produced can be studied by tests of compressor blades in two-dimensional cascades tunnels. Cascade tests can provide many basic data concerning the performance of compressors under widely varying conditions of operation with relative ease, rapidly and at low cost. A more refined procedure, however, would use cascade data not as the final answer, but as a broad base from which to work out the three-dimensional relations [5].

The flow in a compressor is highly three-dimensional with many different loss (or entropy) creating flow mechanisms that impact the overall efficiency of a compressor stage. It is these loss mechanisms that the compressor designer will try to reduce to obtain both design-point and off-design performance characteristics that will be beneficial to the overall SFC of the gas turbine engine. The image below shows the various flow mechanisms that can be found in a compressor blade row, valid for both the rotor and stator.

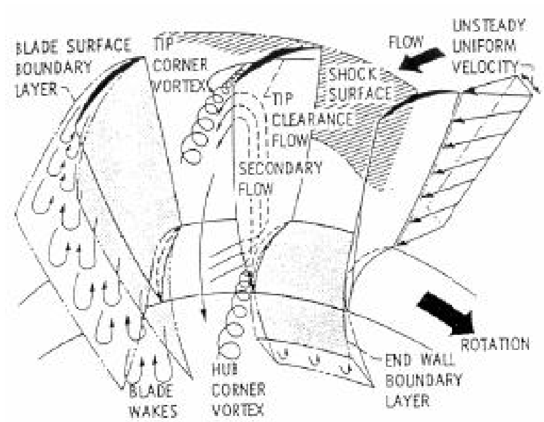

Figure 19 - Compressor flow mechanisms [4, original ref NA]

The various flow mechanisms in a compressor blade row are:
- o Profile losses, or blade surface boundary layer loss
- o Secondary losses; these could be considered to be general vortex flow losses or end wall losses
- o Clearance losses; tip clearance loss for rotors and hub clearance loss for stators

When testing a given compressor cascade, at different inlet flow angles with fixed inlet blade angles, the overall loss will vary with the flow incidence. There will be an increase in both positive and negative incidence angles with a range of low values of loss. The “reference incidence” angle is that angle defined to be within the half-range of incidence where the loss is twice the minimum loss value. Outside of this range blade stall is said to occur.

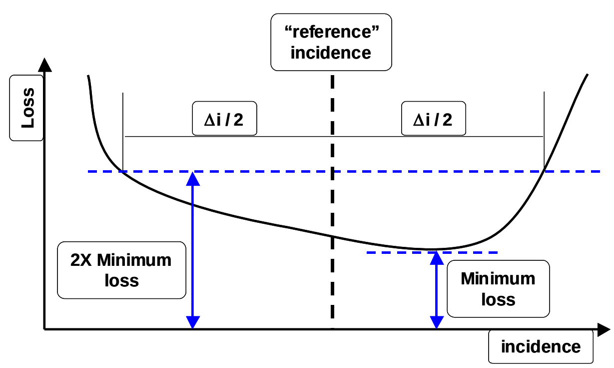

Figure 20 - definition of reference incidence angle

o Reference incidence angle correlation [4]

As mentioned above three types of compressor cascade airfoils have been analyzed: NACA 65-series, British C-series, and DCA. A majority of the test data was based on a compressor blade with a maximum thickness of 10% of the chord length. The "reference" incidences for these three types have been correlated as follows [4]

$$i_{ref} = K_{sh} \cdot K_{it} \cdot i_{010} + n \cdot \theta$$

Where

- K_{sh} is a compressor blade shape correction factor
- K_{it} is a compressor blade thickness correction factor
- i_{010} is the incidence angle based on a compressor blade with 10% thickness and zero camber
- n represents the incidence slope factor, in degrees
- θ is the compressor blade camber angle, in degrees
- α_1 is the relative inlet flow angle for the rotor and the absolute inlet flow angle for stators, in degrees

- K_{sh} is defined to be
  - 0.7 for DCA
  - 1.0 for NACA 65-series
  - 1.1 for British C-series

$$K_{it} = -0.0214 + 19.17\left(\frac{t_{max}}{c}\right) - 122.3\left(\frac{t_{max}}{c}\right)^2 + 312.5\left(\frac{t_{max}}{c}\right)^3$$

$$i_{010} = \left(0.0325 - 0.0674\frac{c}{s}\right) + \left(-0.002364 + 0.0913\frac{c}{s}\right)\alpha_1 + \left(1.64x10^{-5} - 2.38x10^{-4}\frac{c}{s}\right)\alpha_1^2$$

$$n = \left(-0.063 - 0.02274\frac{c}{s}\right) + \left(-0.0035 + 0.0029\frac{c}{s}\right)\alpha_1 - \left(3.79x10^{-5} + 1.11x10^{-5}\frac{c}{s}\right)\alpha_1^2$$

WARNING: the equations for i_{010} and n from reference [4, Falck's thesis] do not coincide with the graphs presented in the same reference. The equations are to be used with caution. The graphs have been reproduced based on the above equations.

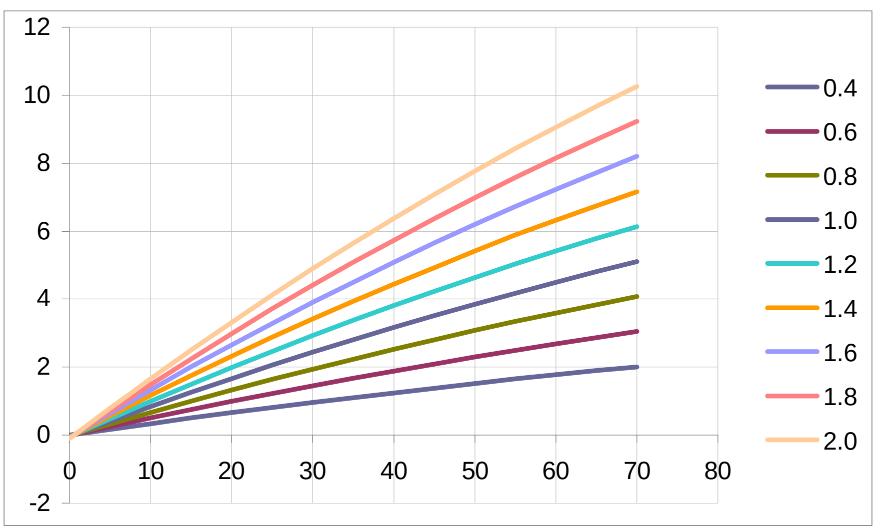

Figure 21 - i_{010} at zero camber and 10% thickness for various solidity

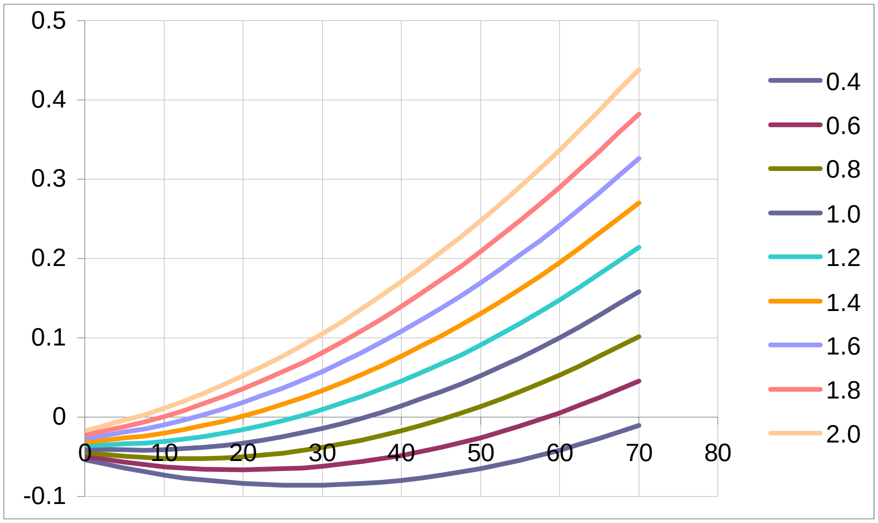

Figure 22 - slope factor "n" for various solidity

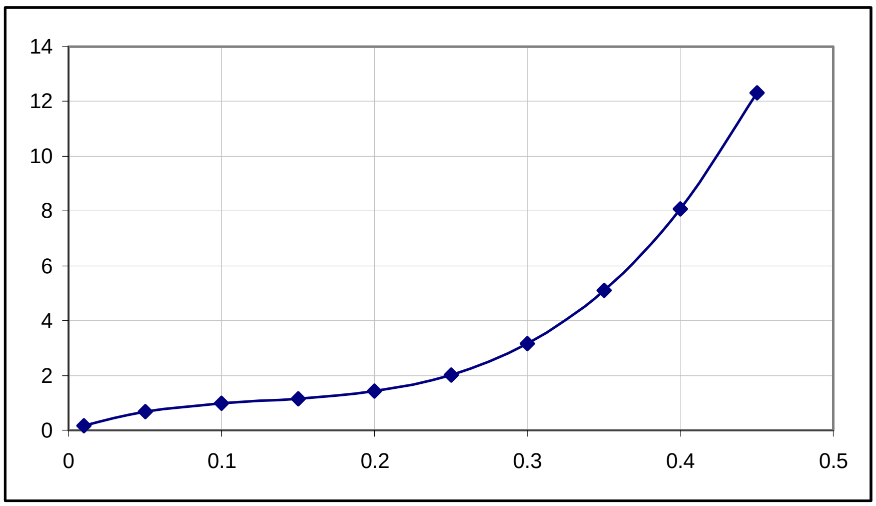

Figure 23 - Correction factor for maximum thickness, K_{it}

o **Deviation angle correlation**

The deviation angle is dependent on blade camber and stagger angle, making it difficult to predict.

Methodology 1 – Carter's Rule
One of the earliest deviation rules, suggested Mr. Carter, is defined as follows

$$\delta = \frac{m_c \theta}{\sqrt{\frac{c}{s}}} + x$$

Where

- m_c is an empirical function of stagger angle
- x is an experimental factor typically equal to 2

Methodology 2
The reference deviation angle is defined as follows; similar to the above mentioned reference incidence angle correlation:

$$\delta_{ref} = K_{sh} \cdot K_{\delta t} \cdot \delta_{010} + m \cdot \theta$$

Where

- K_{sh} is a compressor blade shape correction factor
- $K_{\delta t}$ is a compressor blade thickness correction factor
- δ_{010} is the deviation angle based on a compressor blade with 10% thickness and zero camber
- m represents the deviation slope factor, in degrees
- θ is the compressor blade camber angle, in degrees
- α_1 is the relative inlet flow angle for the rotor and the absolute inlet flow angle for stators, in degrees

- K_{sh} is defined to be (same as for the reference incidence angle correlation)
  - 0.7 for DCA
  - 1.0 for NACA 65-series
  - 1.1 for British C-series

$$K_{\delta t} = 0.0142 + 6.172\left(\frac{t_{\max}}{c}\right) + 36.61\left(\frac{t_{\max}}{c}\right)^2$$

$$\delta_{010} = \left(-0.0443 + 0.1057\frac{c}{s}\right) + \left(0.0209 - 0.0186\frac{c}{s}\right)\alpha_1 + \left(-0.0004 + 0.00076\frac{c}{s}\right)\alpha_1^2$$

$$m = \frac{m_{\text{Blade Type}}}{\left(\frac{c}{s}\right)^b}$$

$$b = 0.9655 + 2.538x10^{-3}\alpha_1 + 4.221x10^{-5}\alpha_1^2 - 1.3x10^{-6}\alpha_1^3$$

Where $m_{\text{Blade Type}}$ is for the different compressor blade types mentioned

$$m_{\text{NACA 65-series}} = 0.17 - 3.33x10^{-4}(1.0 - 0.1\alpha_1)\alpha_1$$

$$m_{\text{DCA}} = m_{\text{British C-series}} = 0.249 + 7.4x10^{-4}\alpha_1 - 1.32x10^{-5}\alpha_1^2 + 3.16x10^{-7}\alpha_1^3$$

WARNING: the equations for δ_{010} and m from reference [4, Falck's thesis] do not coincide with the graphs presented in the same reference. The equations are to be used with caution. The graphs have been reproduced based on the above equations.

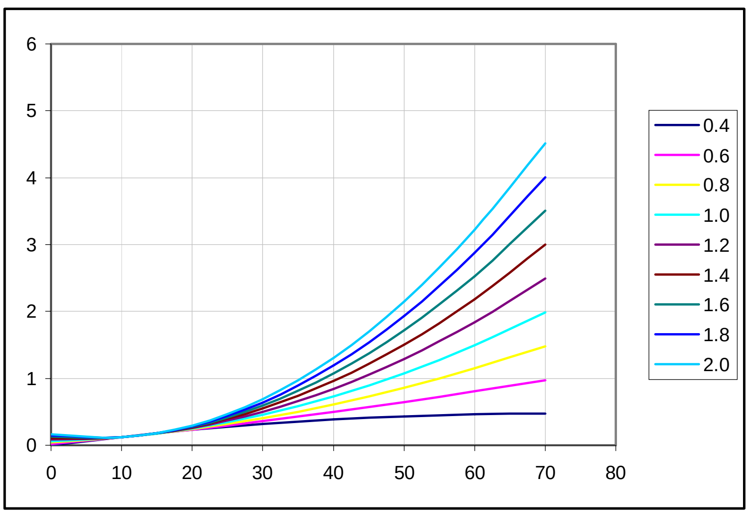

Figure 24 - δ_{010} at zero camber and 10% thickness for various solidity

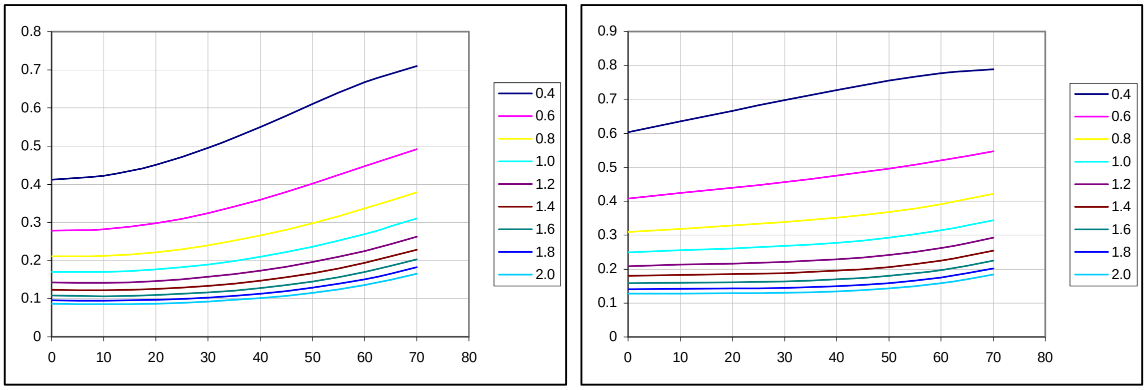

Figure 25- slope factor "m" for various solidity

NACA 65-seried (left), DCA & British C-series (right)

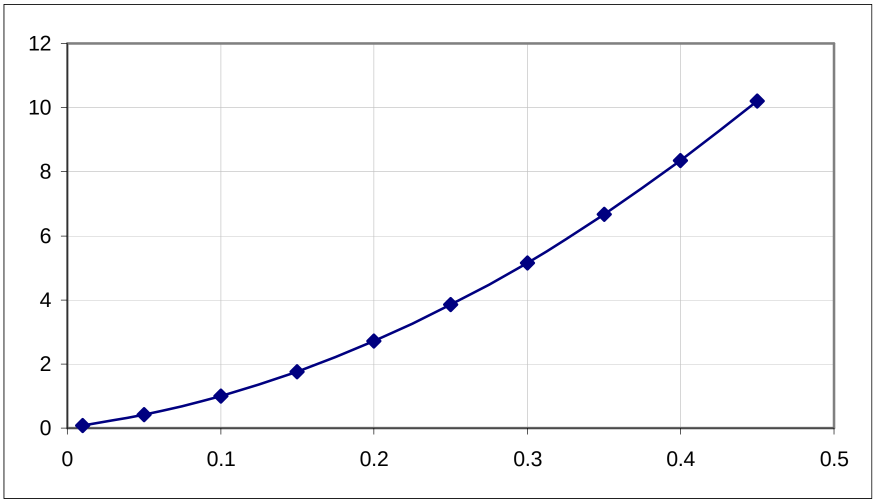

Figure 26 - Correction factor for maximum thickness, $K_{\delta t}$

- **Total loss**

The total loss for a compressor cascade is defined to be the combination of the blade profile loss and the end wall losses

$$\text{Total Loss} = \text{Profile} + \text{End Wall}$$

$$\omega_{TOTAL} = \omega_p + \omega_{ew}$$

$$\omega_{TOTAL} = \frac{\Delta P_O}{P_{O1} - P_{S1}} = \frac{P_{O1} - P_{O2}}{P_{O1} - P_{S1}}$$

- **Profile loss**

The compressor blade profile loss correlation is based on a modified version of the two-dimensional low-speed equation developed by Lieblein as follows.

$$\omega_p \frac{1}{2} \frac{V_{LE}^2}{V_{TE}^2} \cos\alpha_2 = \mathit{function}\left(M_{LE}, \mathrm{DR}_{eq}\right)$$

$$\omega_p \frac{1}{2}\left(\frac{V_{LE}}{V_{TE}}\right)^2 \cos\alpha_2 = function\left(M_{LE}, \mathrm{DR}_{\mathrm{eq}}\right)$$

$$\omega_p = 2\frac{function\left(M_{LE}, \mathrm{DR}_{\mathrm{eq}}\right)}{\left(\frac{V_{LE}}{V_{TE}}\right)^2 \cos\alpha_2}$$

NOTE: From the profile loss correlation for compressor cascade blades, we see that the “de Haller” number has a role in the derivation.

$$\text{de Haller No.} = \left(\frac{V_{rel2}}{V_{rel1}}\right)_{rotor}, \left(\frac{V_{abs4}}{V_{abs3}}\right)_{stator} \approx \left(\frac{V_{TE}}{V_{LE}}\right)$$

$$\omega_p = 2\frac{f\left(M_{LE}, \mathrm{DR}_{\mathrm{eq}}\right)(\text{de Haller No})^2}{\cos\alpha_2}$$

where

- ω_p is the Profile Loss Coefficient
- V_{LE} inlet velocity to the compressor blade
- V_{TE} exit velocity from the compressor blade
- α_2 exit flow angle from the compressor blade
- M_{LE} inlet Mach number
- DR_{eq} equivalent diffusion factor

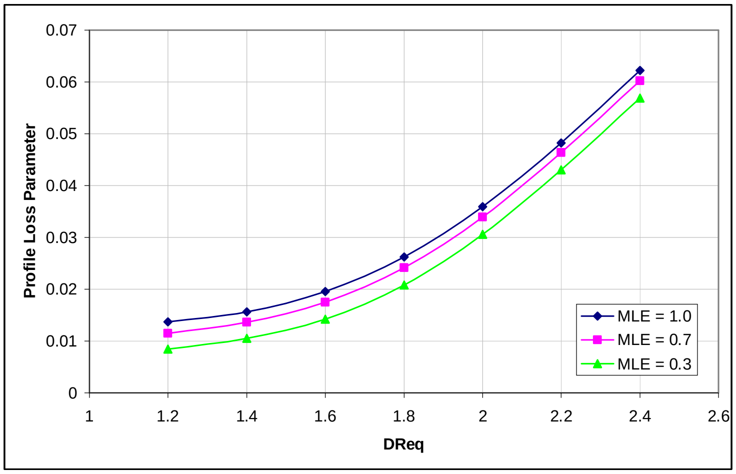

Figure 27 – Profile Loss Parameter wrt M_{LE}

o **End wall loss**

Based on numerous compressor data, the end wall loss has been correlated to be a function of the following parameters

- End wall clearance ratio ε/c = clearance/chord
- Blade aspect ratio h/c = span/chord
- Mean line loading

$$\omega_{ew} \frac{h}{c}\frac{V_{LE}^2}{V_{TE}^2} = function\left(\frac{\varepsilon}{c}, \mathrm{DF}\right)$$

$$\omega_{ew}\frac{h}{c}\left(\frac{V_{LE}}{V_{TE}}\right)^2 = function\left(\frac{\varepsilon}{c}, \mathrm{DF}\right)$$

$$\omega_{ew} = \frac{function\left(\frac{\varepsilon}{c}, \mathrm{DF}\right)}{\left(\frac{h}{c}\right)\left(\frac{V_{LE}}{V_{TE}}\right)^2}$$

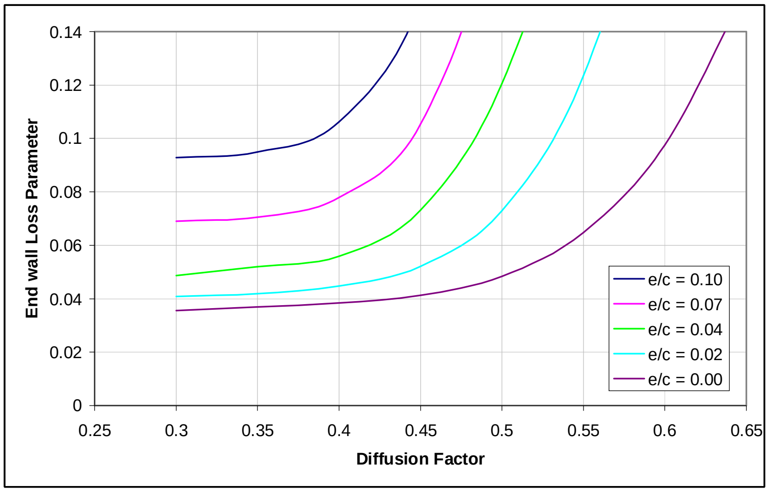

Figure 28 – End wall Loss Parameter with respect to end wall Clearances

DISCUSSION – Rules of thumb for compressor design

Throughout this module various axial-flow compressor stage and airfoil design criteria have been described. Below is a summary of the main rules-of-thumb that can be employed in the preliminary design of an axial-flow compressor.

Table 1- Some axial-flow compressor design ROT

Parameter	**Rule of Thumb**
Compressor Stage Reaction	50%
Compressor Stage Loading, ψ	$0.3 \le \psi \le 0.4$
Work done factor, λ	$\lambda \ge 0.86$, stage count dependent
Allowable Mach number	0.7
De Haller No.	≥ 0.72
Rotor tip Relative Mach number	o ≤ 0.9 if subsonic o ≤ 1.2 if transonic
Stator hub Absolute Mach number	≤ 0.9

DISCUSSION – Compressor blade stresses

Compressor blades will develop internal stresses due to the centrifugal forces caused by the rotational speeds. The blade stresses will be a function of:

- The rotational speed
- The blade material density
- And the length of the blade

The maximum centrifugal tensile stress, which occurs at the blade root, is given by

$$\left(\sigma_{ct}\right)_{\max} = \frac{\rho_b \omega^2}{a_r}\int_r^t a \cdot r \cdot dr$$

Where

- ρ_b blade material density
- ω Angular velocity
- a the cross sectional area of the blade at any radius
- r the radius of the blade section

And subscripts

- r the root radius
- t the tip radius

For simplicity, if we assume that the compressor blade cross sectional area is constant at a value “a” then the maximum tensile stress will become

$$\left(\sigma_{ct}\right)_{\max} = \frac{1}{2}\rho_b \left(2\pi N\right)^2 \left(r_t^2 - r_r^2\right) = 2\pi\rho_b \left(AN^2\right)$$

Where

- N the rotational speed (RPM)
- A the annulus area

Assuming that the blade material is selected from the beginning, we can see from the above equation that changing the rotational speed and/or the annulus area will have a direct impact on the blade stresses. It is for this reason that the compressor or turbine designer will quote and keep track of the quantity of **AN^2** as an indication of the stresses that the stress engineer will need to design for.

As shown in the figures below, there will be a compromise between the needs of the aerodynamicist versus the needs of the structural designer.

- The aerodynamicist would want to create stages at high RPM and large annulus areas to increase stage efficiency
- The structural designer would want to create stages at low RPM and small areas to reduce the overall stresses on the blades and the rotor disks that they are attached to.

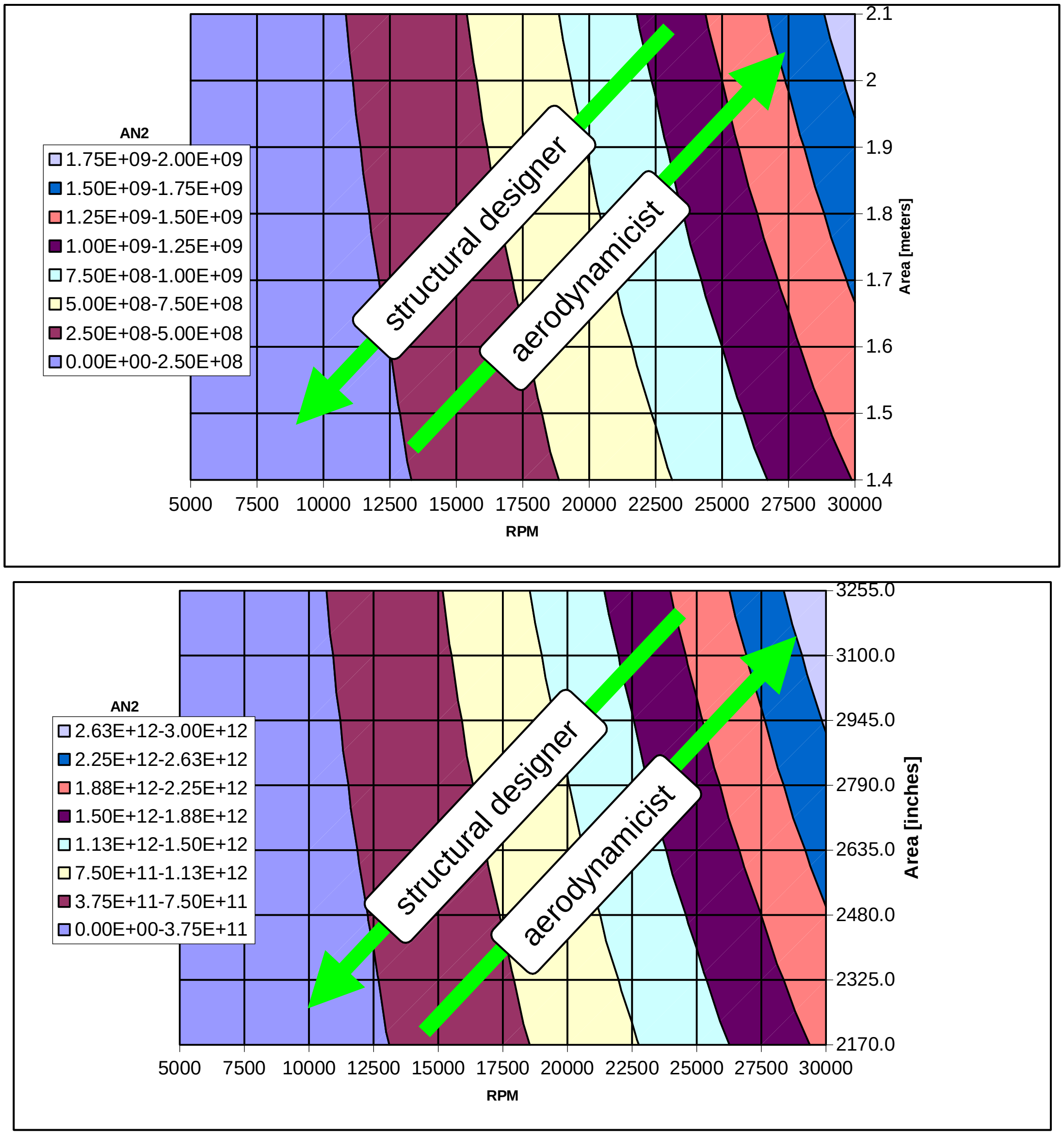

Figure 29 – carpet plot of AN^2 (SI on top, Imperial on bottom)

The tip speed can be written as $\mathbf{2\pi N r_t}$ so the above equation can be rewritten as a function of (r_r/r_t) normally referred to as the "hub-tip" ratio. It is immediately apparent that the centrifugal stress is proportional to the square of the tip speed, and that a reduction of the hub-tip ratio increases the blade stresses.

$$(\sigma_{ct})_{\max} = \frac{1}{2}\rho_b U_t^2 \left[1 - \left(\frac{r_r}{r_t}\right)^2\right]$$

In practice, the blade sectional area will be decreased with radius to relieve the blade root stresses and the loading on the disc carrying the blades. A simple analytical expression can be deduced for a linear variation of cross sectional area from root to tip. If the hub-tip ratio is "b" and the ratio of the cross-sectional area at the tip to that at the root is "d", the stress in a tapered blade is given by

$$(\sigma_{ct})_{\max} = \frac{1}{2}\rho_b U_t^2 K\left(1 - b^2\right)$$

where [2]

$$K = 1 - \left[\frac{(1-d)(2-b-b^2)}{3(1-b^2)} \right]$$

Typical values of **K** would range from 0.55 to 0.65 for tapered blades.

DISCUSSION – benchmarking compressor design

This section is to capture various compressor design geometry and parameter as to benchmark the various equations mentioned throughout this module.

No. Of Stages	Pressure Ratio	Efficiency	Stall Margin	Blade speed	Flow rate	Aspect ratio	Engine
5	12.1	81.9%	11%	457 m/s	192.5 kg/s/m2	1.2 1^{st} stage <1.0 last 3 stages	NA
1^{st} stage only	1.912	85.4%	11%			1.32 1^{st} stage	US air force

Company	Engine	Type	No. of shafts	Fan	LP Compressor	HP Compressor
Rolls-Royce [3, pg 139]	RB211 535E4	Turbo-fan	3	1 fan rotor no boost	6 stages with IGV	6 stages

END OF / FIN DE MODULE